# ONLINE APPENDIX <br> Optimists and Pessimists in (In)Complete Markets 

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June 6, 2019


#### Abstract

This Online Appendix serves as a companion to our paper "Optimists and Pessimists in (In)Complete Markets". It provides additional results and explanations not reported in the main text due to space constraints.


[^0]
## Model (1): Jumps in aggregate consumption

## I. Solving for the equilibrium

We consider two investors with identical recursive preferences. ${ }^{34}$ Investor $i$ 's $(i=1,2)$ value function at time $t$ is given as

$$
\begin{equation*}
J_{i, t}=E_{i, t}\left[\int_{t}^{\infty} f\left(C_{i, s}, J_{i, s}\right) d s\right] \tag{I.1}
\end{equation*}
$$

where $f\left(C_{i, t}, J_{i, t}\right)$ is her normalized aggregator function with

$$
\begin{equation*}
f\left(C_{i, t}, J_{i, t}\right)=\frac{\beta C_{i, t}^{1-\frac{1}{\psi}}}{\left(1-\frac{1}{\psi}\right)\left[(1-\gamma) J_{i, t}\right]^{\frac{1}{\theta}-1}}-\beta \theta J_{i, t}, \tag{I.2}
\end{equation*}
$$

where $\beta$ is the subjective time preference rate, $\gamma$ is the coefficient of relative risk aversion, $\psi$ denotes the elasticity of intertemporal substitution (EIS), and $\theta=\frac{1-\gamma}{1-\frac{1}{\psi}}$. In the following, we assume $\gamma>1$ and $\psi>1$, which implies $\gamma>\frac{1}{\psi}, \theta<0$, and that both investors exhibit a preference for early resolution of uncertainty.

All equilibrium quantities are functions of the pessimist's share of aggregate consumption, $w_{t}=\frac{C_{1, t}}{C_{t}}$, as the endogenous state variable. Its dynamics are:

$$
\begin{equation*}
d w_{t}=\mu_{w}\left(w_{t}\right) d t+\sigma_{w}\left(w_{t}\right) d W_{t}+L_{w}\left(w_{t}\right) d N_{t}(\lambda), \tag{I.3}
\end{equation*}
$$

where the coefficient functions $\mu_{w} \equiv \mu_{w}\left(w_{t}\right), \sigma_{w} \equiv \sigma_{w}\left(w_{t}\right)$, and $L_{w} \equiv L_{w}\left(w_{t}\right)$ are determined in equilibrium. The dynamics of investor 1's and investor 2's level of consumption then follow

[^1]from Ito's lemma:
\[

$$
\begin{aligned}
\frac{d C_{1}}{C_{1}}= & \left\{\mu_{C}+\frac{1}{w_{t}} \mu_{w}+\frac{1}{w_{t}} \sigma_{w} \sigma_{C}\right\} d t+\left\{\sigma_{C}+\frac{1}{w_{t}} \sigma_{w}\right\} d W_{t} \\
& +\left\{\frac{1}{w_{t}} L_{w}+L_{C}\left(1+\frac{1}{w_{t}} L_{w}\right)\right\} d N_{t}\left(\lambda_{1}\right) \\
\equiv & \mu_{C_{1}} d t+\sigma_{C_{1}} d W_{t}+L_{C_{1}} d N_{t}\left(\lambda_{1}\right) \\
\frac{d C_{2}}{C_{2}}= & \left\{\mu_{C}-\frac{1}{1-w_{t}} \mu_{w}-\frac{1}{1-w_{t}} \sigma_{w} \sigma_{C}\right\} d t+\left\{\sigma_{C}-\frac{1}{1-w_{t}} \sigma_{w}\right\} d W_{t} \\
& +\left\{-\frac{1}{1-w_{t}} L_{w}+L_{C}\left(1-\frac{1}{1-w_{t}} L_{w}\right)\right\} d N_{t}\left(\lambda_{2}\right) \\
\equiv & \mu_{C_{2}} d t+\sigma_{C_{2}} d W_{t}+L_{C_{2}} d N_{t}\left(\lambda_{2}\right)
\end{aligned}
$$
\]

From Equation (I.1), we get

$$
\begin{equation*}
\mathbb{E}_{i, t}\left[d J_{i, t}+f_{i}\left(C_{i, t}, J_{i, t}\right) d t\right]=0 \tag{I.4}
\end{equation*}
$$

Following Campbell, Chacko, Rodriguez, and Viceira (2004) and Benzoni, Collin-Dufresne, and Goldstein (2011), we employ the following guess for the individual value function $J_{i, t}$ :

$$
\begin{equation*}
J_{i, t}=\frac{C_{i, t}^{1-\gamma}}{1-\gamma} \beta^{\theta} e^{\theta v_{i, t}} \tag{I.5}
\end{equation*}
$$

where $v_{i, t} \equiv v_{i}\left(w_{t}\right)$ denotes investor $i$ 's $\log$ wealth-consumption ratio. An application of Ito's lemma to $v_{i, t}$ yields
$d v_{i, t}=\left\{\frac{\partial v_{i, t}}{\partial w_{t}} \mu_{w}+\frac{1}{2} \frac{\partial^{2} v_{i, t}}{\partial w_{t}^{2}} \sigma_{w}^{2}\right\} d t+\left\{\frac{\partial v_{i, t}}{\partial w_{t}} \sigma_{w}\right\} d W_{t}+\left\{v_{i}\left(w_{t}+L_{w}\right)-v_{i}\left(w_{t}\right)\right\} d N_{t}\left(\lambda_{i}\right)$
$(\mathrm{I} .6) \equiv \mu_{v_{i}} d t+\sigma_{v_{i}} d W_{t}+L_{v_{i}} d N_{t}\left(\lambda_{i}\right)$.

Plugging the guess from (I.5) into Equation (I.4) results in the following PDE for $v_{i, t}$ :

$$
\begin{align*}
0= & e^{-v_{i, t}}-\beta+\left(1-\frac{1}{\psi}\right)\left[\mu_{C_{i}}-\frac{1}{2} \gamma \sigma_{C_{i}}^{2}\right]+\mu_{v_{i}}+\frac{1}{2} \theta \sigma_{v_{i}}^{2} \\
& +(1-\gamma) \sigma_{C_{i}} \sigma_{v_{i}}+\frac{1}{\theta}\left[\left(1+L_{C_{i}}\right)^{1-\gamma} e^{\theta L_{v_{i}}}-1\right] \lambda_{i} . \tag{I.7}
\end{align*}
$$

As pointed out in Section A of the main text, the two PDEs for $v_{1, t}$ and $v_{2, t}$ are connected through $\mu_{w}, \sigma_{w}$, and $L_{w}$. Hence, they have to be solved simultaneously.

Following Duffie and Skiadas (1994), investor $i$ 's pricing kernel $\xi_{i, t}$ at time $t$ is given as

$$
\xi_{i, t}=e^{-\beta \theta t-(1-\theta) \int_{0}^{t} e^{-v_{i, s}} d s} e^{(\theta-1) v_{i, t}} C_{i, t}^{-\gamma} \beta^{\theta}
$$

with dynamics

$$
\begin{aligned}
\frac{d \xi_{i, t}}{\xi_{i, t}}= & -\left\{\beta+\frac{1}{\psi} \mu_{C_{i}}-\frac{1}{2}\left(1+\frac{1}{\psi}\right) \gamma \sigma_{C_{i}}^{2}-\frac{1}{2}(1-\theta) \sigma_{v_{i}}^{2}-(1-\theta) \sigma_{C_{i}} \sigma_{v_{i}}\right. \\
& \left.+\left(1-\frac{1}{\theta}\right)\left[\left(1+L_{C_{i}}\right)^{1-\gamma} e^{\theta L_{v_{i}}}-1\right] \lambda_{i}\right\} d t \\
& -\left\{\gamma \sigma_{C_{i}}+(1-\theta) \sigma_{v_{i}}\right\} d W_{t}+\left\{\left(1+L_{C_{i}}\right)^{-\gamma} e^{(\theta-1) L_{v_{i}}}-1\right\} d N_{t}\left(\lambda_{i}\right) .
\end{aligned}
$$

From this, we obtain the investor-specific market prices of risk $\eta_{i, t}$ as the exposures of the pricing kernel to the different risk factors. For diffusion risks, this yields

$$
\begin{equation*}
\eta_{i, t}^{W}=\gamma \sigma_{C_{i}}+(1-\theta) \sigma_{v_{i}} . \tag{I.8}
\end{equation*}
$$

The first term is the standard market price for individual consumption risk, which also results in a CRRA economy, while the second term gives the additional market prices of risk for the diffusive volatility of the log wealth-consumption ratio caused by the state variable $w_{t}$. Analogously, the individual market prices of jump risk $\eta_{i, t}^{N}$ are given as

$$
\begin{equation*}
\eta_{i, t}^{N}=\left(1+L_{C_{i}}\right)^{-\gamma} e^{(\theta-1) L_{v_{i}}}-1, \tag{I.9}
\end{equation*}
$$

where the first term on the right-hand side is the product of the market price of consumption jump risk with CRRA utility, $\left(1+L_{C_{i}}\right)^{-\gamma}$, and an adjustment for jump risk in the individual wealth-consumption ratios resulting from the state variable $w_{t}$.

Finally, the subjective risk-free rate $r_{i, t}^{f}$ equals the negative of the expected relative change of the pricing kernel, i.e.,

$$
\begin{align*}
r_{i, t}^{f}= & \beta+\frac{1}{\psi} \mu_{C_{i}}-\frac{1}{2}\left(1+\frac{1}{\psi}\right) \gamma \sigma_{C_{i}}^{2}-\frac{1}{2}(1-\theta) \sigma_{v_{i}}^{2}-(1-\theta) \sigma_{C_{i}}^{\prime} \sigma_{v_{i}} \\
& -\left[\eta_{i}^{N}-\left(1-\frac{1}{\theta}\right)\left[\left(1+L_{C_{i}}\right)^{1-\gamma} e^{\theta L_{v_{i}}}-1\right]\right] \lambda_{i} \tag{I.10}
\end{align*}
$$

where the terms have the usual interpretation reflecting the impact of impatience, the individual consumption growth rate, and precautionary savings due to uncertainty about individual consumption, about the evolution of the log wealth-consumption ratio, about its covariance with individual consumption and, finally, the precautionary savings due to jump risk.

## A. Equilibrium on the complete market

As explained in Section A of the main text, $\sigma_{w}$ is found using the condition that the investor's subjective markets price of diffusion risk are equal, i.e., $\eta_{1}^{W}=\eta_{2}^{W}$ yielding $\frac{1}{w_{t}} \sigma_{w}=-\frac{1}{1-w_{t}} \sigma_{w}$. This holds true, if $\sigma_{w}$ is equal to zero which implies that the investors share no diffusive risks. The drift $\mu_{w}$ follows from the restriction that the investors have to agree on the risk-free rate, i.e., that $r_{1, t}^{f}=r_{2, t}^{f}$ :

$$
\mu_{w}=-\sigma_{w} \sigma_{C}+\psi w_{t}\left(1-w_{t}\right)
$$

$$
\times\left\{\frac{1}{2}\left(1+\frac{1}{\psi}\right) \gamma\left[\sigma_{C_{1}}^{2}-\sigma_{C_{2}}^{2}\right]+\frac{1}{2}(1-\theta)\left[\sigma_{v_{1}}^{2}-\sigma_{v_{2}}^{2}\right]+(1-\theta)\left[\sigma_{C_{1}} \sigma_{v_{1}}-\sigma_{C_{2}} \sigma_{v_{2}}\right]\right.
$$

$$
+\left[\eta_{1, t}^{N}-\left(1-\frac{1}{\theta}\right)\left[\left(1+L_{C_{1}}\right)^{1-\gamma} e^{\theta L_{v_{1}}}-1\right]\right] \lambda_{1}
$$

$$
\begin{equation*}
\left.-\left[\eta_{2, t}^{N}-\left(1-\frac{1}{\theta}\right)\left[\left(1+L_{C_{2}}\right)^{1-\gamma} e^{\theta L_{v_{2}}}-1\right]\right] \lambda_{2}\right\} . \tag{I.11}
\end{equation*}
$$

The jump size $L_{w}$ is found using the condition that the investor-specific risk-neutral jump intensities are equal, i.e., $\lambda_{1, t}^{\mathbb{Q}}=\lambda_{2, t}^{\mathbb{Q}}$ :

$$
\begin{equation*}
L_{w}=\frac{e^{\frac{1}{\gamma}\left[(\theta-1)\left(L_{v_{1}}-L_{v_{2}}\right)+\ln \frac{\lambda_{1}}{\lambda_{2}}\right]}-1}{\frac{1}{w_{t}}+\frac{1}{1-w_{t}} e^{\frac{1}{\gamma}\left[(\theta-1)\left(L_{v_{1}}-L_{v_{2}}\right)+\ln \frac{\lambda_{1}}{\lambda_{2}}\right]}} . \tag{I.12}
\end{equation*}
$$

The equilibrium solution is then found by simultaneously solving the two PDEs in (I.7) for $v_{1, t}$ and $v_{2, t}$, using the expressions for $\mu_{w}, \sigma_{w}$, and $L_{w}$.

## B. Equilibrium on the incomplete market

When the market is incomplete, we still have to solve the two PDEs in (I.7) simultaneously for the two investors' log wealth-consumption ratios, $v_{1, t}$ and $v_{2, t}$, which are coupled through $\mu_{w}, \sigma_{w}$, and $L_{w}$. In contrast to the complete market, these coefficients of the consumption share process cannot be obtained from the different components of the pricing kernel since, when the market is incomplete, the individual pricing kernels will not coincide anymore in general. But still the investors have to agree on the prices of the remaining traded assets, i.e., on the log price-cash flow ratio of the consumption claim and the risk-free rate paid by the money market account.

First, let $\nu_{i, t} \equiv \nu_{i}\left(w_{t}\right)$ denote investor $i$ 's log price-cash flow ratio with dynamics:

$$
\begin{aligned}
d \nu_{i, t} & =\left\{\frac{\partial \nu_{i, t}}{\partial w_{t}} \mu_{w}+\frac{1}{2} \frac{\partial^{2} \nu_{i, t}}{\partial w_{t}^{2}} \sigma_{w}^{2}\right\} d t+\left\{\frac{\partial \nu_{i, t}}{\partial w_{t}} \sigma_{w}\right\} d W_{t}+\left\{\nu_{i}\left(w_{t}+L_{w}\right)-\nu_{i}\left(w_{t}\right)\right\} d N_{t}\left(\lambda_{i}\right) \\
& \equiv \mu_{\nu_{i}} d t+\sigma_{\nu_{i}} d W_{t}+L_{\nu_{i}} d N_{t}\left(\lambda_{i}\right)
\end{aligned}
$$

The sum of expected price change and cash flows have to be equal to zero, i.e.,

$$
\frac{1}{d t} \mathbb{E}_{i, t}\left[\frac{d\left(\xi_{i, t} C_{t} e^{\nu_{i, t}}\right)}{\xi_{i, t} C_{t} e^{\nu_{i, t}}}\right]+e^{-\nu_{i, t}}=0
$$

which yields the following PDEs for $\nu_{i, t}(i=1,2)$ :

$$
\begin{align*}
0= & e^{-\nu_{i, t}}+\mu_{\xi_{i}}+\mu_{C}+\mu_{\nu_{i}}+\frac{1}{2} \sigma_{\nu_{i}}^{2}+\sigma_{\xi_{i}} \sigma_{C}+\sigma_{\xi_{i}} \sigma_{\nu_{i}}+\sigma_{C} \sigma_{\nu_{i}} \\
& +\left[\left(1+L_{C}\right)\left(1+L_{C_{i}}\right)^{-\gamma} e^{(\theta-1) L_{v_{i}}} e^{L_{\nu_{i}}}-1\right] \lambda_{i} . \tag{I.13}
\end{align*}
$$

The investors have to agree on the price of the consumption claim, $\nu_{1, t}=\nu_{2, t}=\nu_{t}$, so that $\nu_{t}$ has to solve the two PDEs in Equation (I.13) for $i=1,2$. Second, the investors also have to agree on the risk-free rate, i.e., $r_{1, t}^{f}=r_{2, t}^{f}=r_{t}^{f}$, where $r_{i, t}^{f}$ is given in Equation (I.10).

While all exposures are obtainable for each investor on the complete market, the investors can only take positions which are obtainable by positions in the remaining traded assets, i.e., in the consumption claim and in the money market account, when the market is incomplete. Therefore, investor $i$ constructs her financing portfolio out of the corresponding portfolio weights $\pi_{i, C, t}$ and $\pi_{i, M, t}$ in such a way that its return $R_{i, t}^{\Pi}$ with dynamics

$$
\begin{aligned}
d R_{i, t}^{\Pi}= & \pi_{i, C, t}\left(\frac{d P_{i, t}^{C}}{P_{i, t}^{C}}+e^{-\nu_{i, t}} d t\right)+\pi_{i, M, t} r_{t}^{f} d t \\
= & \left\{\pi_{i, C, t}\left[\bar{\mu}_{C}+X_{t}+\mu_{\nu_{i}}+\frac{1}{2} \sigma_{\nu_{i}}^{2}+\sigma_{C}^{\prime} \sigma_{\nu_{i}}+e^{-\nu_{i, t}}\right]+\pi_{i, M, t} r_{t}^{f}\right\} d t \\
& +\left\{\pi_{i, C, t}\left(\sigma_{C}+\sigma_{\nu_{i}}\right)\right\} d W_{t}+\left\{\pi_{i, C, t}\left(e^{L_{\nu_{i}}}-1\right)\right\} d N_{t}\left(\lambda_{i}\right)
\end{aligned}
$$

equals the total return $R_{i, t}^{V}$ (including consumption) on her individual wealth, $V_{i, t} \equiv C_{i, t} e^{v_{i, t}}$, which has dynamics

$$
\begin{align*}
d R_{i, t}^{V}= & \frac{d V_{i, t}}{V_{i, t}}+e^{-v_{i, t}} d t \\
= & \left\{\mu_{C_{i}}+\mu_{v_{i}}+\frac{1}{2} \sigma_{v_{i}}^{2}+\sigma_{C_{i}} \sigma_{v_{i}}+e^{-v_{i, t}}\right\} d t+\left\{\sigma_{C_{i}}+\sigma_{v_{i}}\right\} d W_{t} \\
& +\left\{\left(1+L_{C_{i}}\right) e^{L_{v_{i}}}-1\right\} d N_{t}\left(\lambda_{i}\right) \\
\equiv & \left(\mu_{V_{i}}+e^{-v_{i}}\right) d t+\sigma_{V_{i}} d W_{t}+L_{V_{i}} d N_{t}\left(\lambda_{i}\right) \tag{I.14}
\end{align*}
$$

Since individual wealth and financing portfolio have to have the same exposures to diffusive shocks and the jump component, respectively, the following conditions have to hold:

$$
\begin{align*}
\sigma_{C_{i}}+\sigma_{v_{i}} & =\pi_{i, C, t}\left(\sigma_{C}+\sigma_{\nu_{i}}\right)  \tag{I.15}\\
\left(1+L_{C_{i}}\right) e^{L_{v_{i}}}-1 & =\pi_{i, C, t}\left[\left(1+L_{C}\right) e^{L_{\nu_{i}}}-1\right] . \tag{I.16}
\end{align*}
$$

The equilibrium solution on the incomplete market is then found by simultaneously solving the following seven equations: the two PDEs for the individual log wealth-consumption ratios represented by Equation (I.7) for $i=1,2$, the two PDEs for the individual log pricedividend ratios of the claim on aggregate consumption given in (I.13) for $i=1,2$, the equation obtained through the restriction that the individual risk-free rates given in (I.10) are the same, and the two equations for the portfolio weights (I.15) and (I.16).

We solve those seven equations for the following seven variables of interest: the two individual $\log$ wealth-consumption ratios $v_{1, t}$ and $v_{2, t}$, the log price-dividend ratio of the traded consumption claim $\nu_{t}$, the drift $\mu_{w}$, the volatility $\sigma_{w}$, and the jump size $L_{w}$ of the consumption share process, as well as the portfolio weight for the claim on aggregate consumption $\pi_{1, C, t}$. The portfolio weight for investor 2 is determined via the market clearing condition $\pi_{1, C, t} C_{1, t} e^{v_{1, t}}+\pi_{2, C, t} C_{2, t} e^{v_{2, t}}=C_{t} e^{v_{t}}$, and the weight of the money market account is given by $\pi_{i, M, t} \equiv 1-\pi_{i, C, t}$.

## II. Further results from quantitative analysis

## A. Investor survival

Figure II. 1 shows the kernel density estimates of the pessimist's consumption share on the complete and incomplete market following from a Monte Carlo simulation of the pessimist's consumption share over a period of $100,500,1,000$, and 10,000 years.

Next, we present the results for each of the channels resulting from the Borovicka (2018) decomposition in Equation (3). The savings channel, the speculative volatility channel, the speculative jump channel, and the "risk premium" channel are shown in Figure II. 2 to II. 5. As explained in the main text, we see barely any differences between the complete and the incomplete market.

## B. Varying the true jump intensity

Table II. 1 provides the aggregate asset pricing moments from the Monte Carlo simulation for $T=1,000$ years when we vary the true jump intensity $\lambda$. Since we have already seen that the evolution of the consumption share is barely affected by market incompleteness, it comes as no surprise that the differences between the two market structures with respect to the risk-free rate, the risk premium, and the return volatility are fairly small, too.

## C. Reducing the jump size

Figure II. 6 illustrates the impact of reducing the jump size (in absolute terms) in a ceteris paribus analysis from $L_{C}=-0.40$ to $L_{C}=-0.20$. This has three effects. First, there is less speculation on the complete market, so that $L_{w}$ drops compared to the case of $L_{C}=-0.40$. Second, choosing a less negative $L_{C}$ makes jump risk less and diffusion risk more important in relative terms. Hence, when the market is incomplete, the consumption claim resembles less an insurance product against jump risk. So on the incomplete market, $L_{w}$ is lower for $L_{C}=-0.20$ than for $L_{C}=-0.40$. Third, since market incompleteness matters more, the difference between the complete and the incomplete market becomes larger for $L_{C}=-0.20$.

## D. Varying the individual beliefs

In Figures II. 7 to II. 12 we provide the consumption share dynamics, the portfolio weights, and the Borovicka (2018) decomposition for two cases in which we vary the beliefs relative to our benchmark parametrization, i.e., $\lambda_{1}=0.017, \lambda_{2}=0.001$, and $\lambda=0.017$. As a measure of disagreement, we use $\Delta=\frac{\lambda_{2}-\lambda_{1}}{\lambda}$.

First, we reduce $\lambda_{1}$ to 0.012 , while keeping $\lambda_{2}$ and $\lambda$ unchanged. The results are shown in Figures II. 7 to II.9. Less disagreement ( $\Delta$ drops from 0.94 to 0.65 ) leads to less speculation, and less extreme portfolio positions for both investors. For instance, on the incomplete market, the pessimist no longer takes a short position in the consumption claim. Based on the Borovicka (2018) decomposition, the implications for investor survival are qualitative similar as for our benchmark case with $\lambda_{1}=0.017$.

Next, we proceed in a similar fashion by increasing $\lambda_{2}$ to 0.06 , while keeping $\lambda_{1}$ and $\lambda$ unchanged, as shown in Figures II. 10 to II.12. This case leads to the same disagreement ( $\Delta=0.65$ ), but there is significantly less speculation on the complete and on the incomplete market which is also supported by the portfolio weights. In terms of investor survival, both market structures now lead to virtually identical results.

Overall, adjusting the jump intensity of the optimist whose beliefs are further away from the true model seems to have a larger impact on speculation and the investors' portfolio strategies, while changing the beliefs of the pessimist whose beliefs are close to the true jump intensity appears to have a smaller effect.
Figure II.1. Investor survival in model (1)
The figure shows for the model with jumps in aggregate consumption presented in Section II, the kernel density estimates for the pessimist's consumption share $w_{T}$ for $T$ years into the future under the true measure $\lambda=0.017$. The results after 100 years are given by the gray dashed (solid) line, those after 1,000 years ( 10,000 years) by the black dashed (solid) line. The graph on the left shows the results on the complete market, the one on the right those on the incomplete market. All quantities are determined by a Monte Carlo simulation of the dynamics of the consumption share shown in Equation (I.3) over 10,000 paths with a starting value of $w_{0}=0.5$. The coefficients $\mu_{w}, \sigma_{w}$, and $L_{w}$ are obtained by interpolating the grids for these quantities obtained as part of the equilibrium solution. The parameters are given in Table 1.



## Figure II.2. Savings channel and consumption-wealth ratios in model (1)

The figure shows for the model with jumps in aggregate consumption presented in Section II, the individual wealth-consumption ratios, $e^{-v_{i, t}}$ for $i=1,2$, and the resulting savings channel, ( $e^{-v_{2, t}}-e^{-v_{1, t}}$ ), as defined in Equation (3). The dotted (dashed) line shows the pessimist's (optimist's) individual consumption-wealth ratio, whereas the solid line indicates the savings channel. The graphs on the left (right) show the results for the complete (incomplete) market. All quantities are determined under the true measure $\lambda=0.017$ and shown as functions of the pessimist's consumption share $w_{t}$. The parameters are given in Table 1.



## Figure II.3. Speculative volatility channel and exposures of individual wealth to

 diffusive risks in model (1)The figure depicts for the model with jumps in aggregate consumption presented in Section II, (from left to right) each investor's exposure of individual wealth to diffusive consumption shocks ( $\sigma_{V_{i}, C}$ for $i=1,2$ ) and the resulting speculative volatility channel, $\frac{1}{2} \sigma_{V_{2}}^{\prime} \sigma_{V_{2}}-\frac{1}{2} \sigma_{V_{1}}^{\prime} \sigma_{V_{1}}$ as defined in Equation (3). The dotted (dashed) line represents the results for the pessimist (optimist), whereas the solid line indicates the speculative volatility channel. The top (bottom) row refers to the complete (incomplete) market. All quantities are determined under the true measure $\lambda=0.017$ and shown as functions of the pessimist's consumption share $w_{t}$. The parameters are given in Table 1.


## Figure II.4. Speculative jump channel and exposures of individual wealth to jump risk in model (1)

The figure depicts for the model with jumps in aggregate consumption presented in Section II, (from left to right) each investor's exposure of individual wealth to jumps in the long-run growth rate $\left(L_{V_{i}}\right.$ for $\left.i=1,2\right)$, as given in Equation (I.14), and the resulting speculative jump channel, $\left(\log \left(1+L_{V_{1}}\right) \lambda-\log \left(1+L_{V_{2}}\right) \lambda\right.$, as defined in Equation (3). The dotted (dashed) line represents results for the pessimist (optimist), whereas the solid line gives those for the speculative jump channel. The top (bottom) row refers to the complete (incomplete) market. All quantities are determined under the true measure $\lambda=0.017$ and shown as functions of the pessimist's consumption share $w_{t}$. The parameters are given in Table 1.




Figure II.5. "Risk premium" channel in model (1)
The figure depicts for the model with jumps in aggregate consumption presented in Section II, (from left to right) the part of the "risk premium" channel that is due to diffusive shocks, $\left(\sigma_{V_{1}} \eta_{1}^{W}-\sigma_{V_{2}} \eta_{2}^{W}\right)$, due to jump risk, $\left(-L_{V_{1}} \lambda_{1}^{\mathbb{Q}}\right)-\left(-L_{V_{2}} \lambda_{2}^{\mathbb{Q}}\right)$, and the sum over both parts, $\left(\sigma_{V_{1}} \eta_{1}^{W}-L_{V_{1}} \lambda_{1}^{\mathbb{Q}}\right)-\left(\sigma_{V_{2}} \eta_{2}^{W}-L_{V_{2}} \lambda_{2}^{\mathbb{Q}}\right)$, as defined in Equation (3). The dotted (dashed) line represents the results for the pessimist (optimist), whereas the solid line gives those for the difference between the pessimist and the optimist. The top (bottom) row refers to the complete (incomplete) market. All quantities are determined under the true measure $\lambda=0.017$ and shown as functions of the pessimist's consumption share $w_{t}$. The parameters are given in Table 1.






Table II.1. Impact of varying the true measure on aggregate asset pricing quantities in model (1)
The table shows for the model with jumps in aggregate consumption presented in Section II, the risk-free rate, the risk premium, and the return volatility obtained from a Monte Carlo simulation of the model $T=1,000$ years into the future under the true measure $\lambda$ which we vary in each column. The top (bottom) panel gives the results on the complete (incomplete) market. The risk-free rate is given in Equation (I.10) in the Online Appendix, the risk premium in (5), and the return volatility in (6). The evolution of the pessimist's consumption share is given in Table 6. The parameters are given in Table 4.

| $\begin{aligned} & T=1,000 \\ & \text { (years) } \end{aligned}$ |  |  |  |  | $\lambda$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 | 0.10 |
|  | Complete market |  |  |  |  |  |  |  |  |  |
| Risk-free rate | 0.0403 | 0.0361 | 0.0302 | 0.0233 | 0.0167 | 0.0109 | 0.0065 | 0.0037 | 0.0022 | 0.0015 |
| Risk premium | 0.0017 | 0.0021 | 0.0036 | 0.0060 | 0.0084 | 0.0100 | 0.0100 | 0.0082 | 0.0049 | 0.0007 |
| Return volatility | 0.0068 | 0.0081 | 0.0095 | 0.0108 | 0.0122 | 0.0135 | 0.0148 | 0.0161 | 0.0174 | 0.0187 |
| Incomplete market |  |  |  |  |  |  |  |  |  |  |
| Risk-free rate | 0.0410 | 0.0377 | 0.0319 | 0.0250 | 0.0181 | 0.0120 | 0.0072 | 0.0042 | 0.0025 | 0.0016 |
| Risk premium | 0.0013 | 0.0019 | 0.0017 | 0.0042 | 0.0069 | 0.0088 | 0.0092 | 0.0077 | 0.0046 | 0.0005 |
| Return volatility | 0.0068 | 0.0081 | 0.0095 | 0.0108 | 0.0122 | 0.0135 | 0.0148 | 0.0166 | 0.0174 | 0.0187 |

Figure II.6. Consumption share dynamics in model (1) for $L_{C}=-0.20$
The figure depicts for the model with jumps in aggregate consumption, the coefficients in the dynamics of the pessimist's consumption share. From left to right, the graphs show the drift $\left(\mu_{w}\right)$, and the coefficients for diffusive consumption shocks $\left(\sigma_{w}\right)$ and jumps in aggregate consumption $\left(L_{w}\right)$, respectively. The gray (black) line represents the results on the complete (incomplete) market. All quantities are determined under the true measure $\lambda=0.017$ and shown as functions of the pessimist's consumption share $w_{t}$. The parameters are given in Table 1 of the main text.

Figure II.7. Consumption share dynamics in model (1) for $\lambda_{1}=0.012, \lambda_{2}=0.001$, and $\lambda=0.017$
The figure depicts for the model with jumps in aggregate consumption, the coefficients in the dynamics of the pessimist's consumption share. From left to right, the graphs show the drift $\left(\mu_{w}\right)$, and the coefficients for diffusive consumption shocks $\left(\sigma_{w}\right)$ and jumps in aggregate consumption $\left(L_{w}\right)$, respectively. The gray (black) line represents the results on the complete (incomplete) market. All quantities are determined under the true measure $\lambda=0.017$ and shown as functions of the pessimist's consumption share $w_{t}$. The parameters are given in Table 1 of the main text.



Figure II.8. Portfolio weights in model (1) for $\lambda_{1}=0.012, \lambda_{2}=0.001$, and $\lambda=0.017$
The figure shows for the model with jumps in aggregate consumption, the investors' asset holdings on the complete (top row) and on the incomplete market (bottom row), respectively. From left to right, the graphs show the fraction of wealth invested in the consumption claim, $\pi_{i, C}$, the jump insurance product $I, \pi_{i, I}$, and the money market account, $\pi_{i, M}$. The pessimist's (optimist's) portfolio weights are indicated by the dotted (dashed) line. All quantities are determined under the true measure $\lambda=0.017$ and shown as functions of the pessimist's consumption share $w_{t}$. The parameters are given in Table 1 of the main text.





Figure II.9. Borovicka (2018) decomposition in model (1) for $\lambda_{1}=0.012, \lambda_{2}=0.001$, and $\lambda=0.017$
The figure depicts for the model with jumps in aggregate consumption, (from left to right) the following channels: (1) the savings channel, ( $\left.e^{-v_{2, t}}-e^{-v_{1, t}}\right)$; (2) the speculative volatility channel, ( $\frac{1}{2} \sigma_{V_{2}}^{\prime} \sigma_{V_{2}}-\frac{1}{2} \sigma_{V_{1}}^{\prime} \sigma_{V_{1}}$ ); (3) the speculative jump channel, $\left(\log \left(1+L_{V_{1}}\right) \lambda-\log \left(1+L_{V_{2}}\right) \lambda\right)$; (4) the "risk premium" channel, $\left(\sigma_{V_{1}} \eta_{1}^{W}-L_{V_{1}} \lambda_{1}^{\mathbb{Q}}\right)-\left(\sigma_{V_{2}} \eta_{2}^{W}-L_{V_{2}} \lambda_{2}^{\mathbb{Q}}\right)$; (5) the sum over those four channels. The gray (black) line represents the results on the complete (incomplete) market. All quantities are determined under the true measure $\lambda=0.017$ and shown as functions of the pessimist's consumption share $w_{t}$. The parameters are given in Table 1 of the main text.




Figure II.10. Consumption share dynamics in model (1) for $\lambda_{1}=0.017, \lambda_{2}=0.006$, and $\lambda=0.017$
The figure depicts for the model with jumps in aggregate consumption, the coefficients in the dynamics of the pessimist's consumption share. From left to right, the graphs show the drift $\left(\mu_{w}\right)$, and the coefficients for diffusive consumption shocks $\left(\sigma_{w}\right)$ and jumps in aggregate consumption $\left(L_{w}\right)$, respectively. The gray (black) line represents the results on the complete (incomplete) market. All quantities are determined under the true measure $\lambda=0.017$ and shown as functions of the pessimist's consumption share $w_{t}$. The parameters are given in Table 1 of the main text.



Figure II.11. Portfolio weights in model (1) for $\lambda_{1}=0.017, \lambda_{2}=0.006$, and $\lambda=0.017$
The figure shows for the model with jumps in aggregate consumption, the investors' asset holdings on the complete (top row) and on the incomplete market (bottom row), respectively. From left to right, the graphs show the fraction of wealth invested in the consumption claim, $\pi_{i, C}$, the jump insurance product $I, \pi_{i, I}$, and the money market account, $\pi_{i, M}$. The pessimist's (optimist's) portfolio weights are indicated by the dotted (dashed) line. All quantities are determined under the true measure $\lambda=0.017$ and shown as functions of the pessimist's consumption share $w_{t}$. The parameters are given in Table 1 of the main text.





Figure II.12. Borovicka (2018) decomposition in model (1) for $\lambda_{1}=0.017, \lambda_{2}=0.006$, and $\lambda=0.017$
The figure depicts for the model with jumps in aggregate consumption, (from left to right) the following channels: (1) the savings channel, ( $\left.e^{-v_{2, t}}-e^{-v_{1, t}}\right)$; (2) the speculative volatility channel, ( $\left.\frac{1}{2} \sigma_{V_{2}}^{\prime} \sigma_{V_{2}}-\frac{1}{2} \sigma_{V_{1}}^{\prime} \sigma_{V_{1}}\right)$; (3) the speculative jump channel, $\left(\log \left(1+L_{V_{1}}\right) \lambda-\log \left(1+L_{V_{2}}\right) \lambda\right)$; (4) the "risk premium" channel, $\left(\sigma_{V_{1}} \eta_{1}^{W}-L_{V_{1}} \lambda_{1}^{\mathbb{Q}}\right)-\left(\sigma_{V_{2}} \eta_{2}^{W}-L_{V_{2}} \lambda_{2}^{\mathbb{Q}}\right)$; (5) the sum over those four channels. The gray (black) line represents the results on the complete (incomplete) market. All quantities are determined under the true measure $\lambda=0.017$ and shown as functions of the pessimist's consumption share $w_{t}$. The parameters are given in Table 1 of the main text.





## Model (2): Long-run risk model with jumps in the long-run growth rate

## III. Solving for the equilibrium

As in model (1), we consider two investors with identical recursive preferences. Hence, the investor's individual value function and her normalized aggregator function are those given in Equations (I.1) and (I.2).

The dynamics of investor 1's and investor 2's level of consumption follow from Ito's lemma:

$$
\begin{align*}
\frac{d C_{1, t}}{C_{1, t}}= & \left\{\bar{\mu}_{C}+X_{t}+\frac{1}{w_{t}} \mu_{w}+\frac{1}{w_{t}} \sigma_{w}^{\prime} \sigma_{C}\right\} d t \\
& +\left\{\sigma_{C}+\frac{1}{w_{t}} \sigma_{w}\right\}^{\prime} d W_{t}+\left\{\frac{1}{w_{t}} L_{w}\right\} d N_{t}\left(\lambda_{1}\right) \\
\equiv & \mu_{C_{1}} d t+\sigma_{C_{1}}^{\prime} d W_{t}+L_{C_{1}} d N_{t}\left(\lambda_{1}\right)  \tag{III.1}\\
\frac{d C_{2, t}}{C_{2, t}}= & \left\{\bar{\mu}_{C}+X_{t}-\frac{1}{1-w_{t}} \mu_{w}-\frac{1}{1-w_{t}} \sigma_{w}^{\prime} \sigma_{C}\right\} d t \\
& +\left\{\sigma_{C}-\frac{1}{1-w_{t}} \sigma_{w}\right\}^{\prime} d W_{t}+\left\{-\frac{1}{1-w_{t}} L_{w}\right\} d N_{t}\left(\lambda_{2}\right) \\
\equiv & \mu_{C_{2}} d t+\sigma_{C_{2}}^{\prime} d W_{t}+L_{C_{2}} d N_{t}\left(\lambda_{2}\right) . \tag{III.2}
\end{align*}
$$

The dynamics of investor $i$ 's $\log$ wealth-consumption ratio $v_{i, t}=v_{i}\left(w_{t}, X_{t}\right)$ are given as

$$
\begin{aligned}
d v_{i, t}= & \left\{\frac{\partial v_{i, t}}{\partial w_{t}} \mu_{w}+\frac{1}{2} \frac{\partial^{2} v_{i, t}}{\partial w_{t}^{2}} \sigma_{w}^{\prime} \sigma_{w}-\frac{\partial v_{i, t}}{\partial X_{t}} \kappa_{X} X_{t}+\frac{1}{2} \frac{\partial^{2} v_{i, t}}{\partial X_{t}^{2}} \sigma_{X}^{\prime} \sigma_{X}+\frac{\partial^{2} v_{i, t}}{\partial w_{t} \partial X_{t}} \sigma_{w}^{\prime} \sigma_{X}\right\} d t \\
& +\left\{\frac{\partial v_{i, t}}{\partial w_{t}} \sigma_{w}+\frac{\partial v_{i, t}}{\partial X_{t}} \sigma_{X}\right\}^{\prime} d W_{t}+\left\{v_{i}\left(w_{t}+L_{w}, X_{t}+L_{X}\right)-v_{i}\left(w_{t}, X_{t}\right)\right\} d N_{t}\left(\lambda_{i}\right) \\
\equiv & \mu_{v_{i}} d t+\sigma_{v_{i}}^{\prime} d W_{t}+L_{v_{i}} d N_{t}\left(\lambda_{i}\right) .
\end{aligned}
$$

Following the steps outlined in Online Appendix II.A leads to the same PDE for $v_{i, t}$ as in Equation (I.7). The dynamics of investor $i$ 's pricing kernel $\xi_{i, t}$ are also the same as those
derived in Section II.A, so that the investor-specific market prices of diffusion risk $\eta_{i, t}^{W}$, the market prices of jump risk $\eta_{i, t}^{N}$, and the subjective risk-free rate $r_{i, t}^{f}$ have the same form as shown in Equations (I.8) to (I.10).

## A. Equilibrium on the complete market

As explained in Section A in the main text, we have to determine the coefficients $\mu_{w}$, $\sigma_{w}$, and $L_{w}$ of the consumption share process (7). The volatility $\sigma_{w}$ is obtained by equating the investors' subjective market prices of diffusion risk, i.e., through the condition $\eta_{1, t}^{W}=\eta_{2, t}^{W}$ :

$$
\begin{equation*}
\sigma_{w}=\frac{w_{t}\left(1-w_{t}\right)(1-\theta)\left[\frac{\partial v_{2, t}}{\partial X_{t}}-\frac{\partial v_{1, t}}{\partial X_{t}}\right]}{\gamma+w_{t}\left(1-w_{t}\right)(1-\theta)\left[\frac{\partial v_{1, t}}{\partial w_{t}}-\frac{\partial v_{2, t}}{\partial w_{t}}\right]} \sigma_{X} . \tag{III.3}
\end{equation*}
$$

$\mu_{w}$ follows from equating both investors' risk-free rates and coincides with Equation (I.11). $L_{w}$ is determined by equating the risk-neutral jump intensities $\lambda_{i, t}^{\mathbb{Q}}=\lambda_{i}\left(1+\eta_{i, t}^{N}\right)$ for $i=1,2$ so that Equation (I.12) remains unchanged. Note that $\mu_{w}, \sigma_{w}$, and $L_{w}$ are essentially functions of $v_{1, t}$ and $v_{2, t}$ and, thus, capture the connection between the two individual $\log$ wealthconsumption ratios. Finally, the equilibrium is determined numerically by simultaneously solving the two PDEs in (I.7) for $v_{1, t}$ and $v_{2, t}$, using the above equations for $\mu_{w}, \sigma_{w}$, and $L_{w} .{ }^{35}$

## B. Equilibrium on the incomplete market

On the incomplete market, the investors have to agree on the prices of the traded assets, i.e., on the log price-cash flow ratio of the consumption claim and the risk-free rate paid by the money market account. First, let $\nu_{i, t} \equiv \nu_{i}\left(w_{t}, X_{t}\right)$ denote investor $i$ 's $\log$ price-cash flow

[^2]ratio of the consumption claim. It follows:
\[

$$
\begin{aligned}
d \nu_{i, t} \equiv & \left\{\frac{\partial \nu_{i, t}}{\partial w_{t}} \mu_{w}+\frac{1}{2} \frac{\partial^{2} \nu_{i, t}}{\partial w_{t}^{2}} \sigma_{w}^{\prime} \sigma_{w}-\frac{\partial \nu_{i, t}}{\partial X_{t}} \kappa_{X} X_{t}+\frac{1}{2} \frac{\partial^{2} \nu_{i, t}}{\partial X_{t}^{2}} \sigma_{X}^{\prime} \sigma_{X}+\frac{\partial^{2} \nu_{i, t}}{\partial w_{t} \partial X_{t}} \sigma_{w}^{\prime} \sigma_{X}\right\} d t \\
& +\left\{\frac{\partial \nu_{i, t}}{\partial w_{t}} \sigma_{w}+\frac{\partial \nu_{i, t}}{\partial X_{t}} \sigma_{X}\right\}^{\prime} d W_{t}+\left\{\nu_{i}\left(w_{t}+L_{w}, X_{t}+L_{X}\right)-\nu_{i}\left(w_{t}, X_{t}\right)\right\} d N_{t}\left(\lambda_{i}\right) \\
\equiv & \mu_{\nu_{i}} d t+\sigma_{\nu_{i}} d W_{t}+L_{\nu_{i}} d N_{t}\left(\lambda_{i}\right)
\end{aligned}
$$
\]

Furthermore, $\nu_{i, t}$ solves the PDE

$$
\begin{align*}
0= & e^{-\nu_{i, t}}+\mu_{\xi_{i}}+\mu_{C}+\mu_{\nu_{i}}+\frac{1}{2} \sigma_{\nu_{i}}^{\prime} \sigma_{\nu_{i}}+\sigma_{\xi_{i}}^{\prime} \sigma_{C}+\sigma_{\xi_{i}}^{\prime} \sigma_{\nu_{i}}+\sigma_{C}^{\prime} \sigma_{\nu_{i}} \\
& +\left[\left(1+L_{C_{i}}\right)^{-\gamma} e^{(\theta-1) L_{v_{i}}} e^{L_{\nu_{i}}}-1\right] \lambda_{i} . \tag{III.4}
\end{align*}
$$

Since the investors have to agree on the price of the consumption claim, $\nu_{1, t}=\nu_{2, t}=\nu_{t}$ has to solve the two PDEs represented by Equation (III.4) for $i=1,2$. Second, the investors have to agree on the price of the money market account and, thus, on the risk-free rate, i.e., $r_{1, t}^{f}=r_{2, t}^{f}=r_{t}^{f}$ where $r_{i, t}^{f}$ is given in Equation (I.10).

As in Section I.B, investor $i$ constructs her financing portfolio in such a way that its return coincides with the return on her individual wealth. The return on the financing portfolio $R_{i, t}^{\Pi}$ and the return on wealth $R_{i, t}^{V}$ follow the dynamics

$$
\begin{aligned}
d R_{i, t}^{\Pi}= & \left\{\pi_{i, C, t}\left[\mu_{C}+\mu_{\nu_{i}}+\frac{1}{2} \sigma_{\nu_{i}}^{\prime} \sigma_{\nu_{i}}+\sigma_{C}^{\prime} \sigma_{\nu_{i}}+e^{-\nu_{i, t}}\right]+\pi_{i, M, t} r_{t}^{f}\right\} d t \\
& +\left\{\pi_{i, C, t}\left(\sigma_{C}+\sigma_{\nu_{i}}\right)\right\}^{\prime} d W_{t}+\left\{\pi_{i, C, t}\left[\left(1+L_{C}\right) e^{L_{\nu_{i}}}-1\right]\right\} d N_{t}\left(\lambda_{i}\right) \\
d R_{i, t}^{V}= & \left\{\mu_{C_{i}}+\mu_{v_{i}}+\frac{1}{2} \sigma_{v_{i}}^{\prime} \sigma_{v_{i}}+\sigma_{C_{i}}^{\prime} \sigma_{v_{i}}+e^{-v_{i, t}}\right\} d t \\
& +\left\{\sigma_{C_{i}}+\sigma_{v_{i}}\right\}^{\prime} d W_{t}+\left\{\left(1+L_{C_{i}}\right) e^{L_{v_{i}}}-1\right\} d N_{t}\left(\lambda_{i}\right) .
\end{aligned}
$$

Since $R_{i, t}^{\Pi}$ and $R_{i, t}^{V}$ have to have equal exposures to the diffusion and the jump component,
respectively, the following conditions hold for each investor:

$$
\begin{align*}
\sigma_{C_{i}, C}+\sigma_{v_{i}, C} & =\pi_{i, C, t}\left(\sigma_{c}+\sigma_{\nu_{i}, C}\right),  \tag{III.5}\\
\sigma_{C_{i}, X}+\sigma_{v_{i}, X} & =\pi_{i, C, t} \sigma_{\nu_{i}, X}  \tag{III.6}\\
\left(1+L_{C_{i}}\right) e^{L_{v_{i}}}-1 & =\pi_{i, C, t}\left(e^{L_{\nu_{i}}}-1\right), \tag{III.7}
\end{align*}
$$

where $\sigma_{,, C}$ and $\sigma_{\cdot, X}$ refer to the first and second component of the respective volatility vector.
The equilibrium solution on the incomplete market is found by simultaneously solving the following eight equations: the two PDEs for the individual log wealth-consumption ratios represented by Equation (I.7) for $i=1,2$, the two PDEs for the individual log price-cash flow ratios of the claim on aggregate consumption given in (III.4) for $i=1,2$, the equation obtained through the restriction that the individual risk-free rates given in (I.10) are equal, and the three equations (III.5) to (III.7) for the portfolio weights. ${ }^{36}$

We solve those eight equations for the following eight variables: the two individual log wealth-consumption ratios $v_{1, t}$ and $v_{2, t}$, the log price-dividend ratio of the traded consumption claim $\nu_{t}$, the drift $\mu_{w}$, the two elements of the volatility vector $\sigma_{w}$, and the jump size $L_{w}$ of the consumption share process, and the portfolio weight for the claim on aggregate consumption $\pi_{1, C, t}$. The portfolio weight for investor 2 is determined via the market clearing condition $\pi_{1, C, t} C_{1, t} e^{v_{1, t}}+\pi_{2, C, t} C_{2, t} e^{v_{2, t}}=C_{t} e^{v_{t}}$, and the weight of the money market account is given by $\pi_{i, M, t} \equiv 1-\pi_{i, C, t}$.

[^3]
## IV. Insurance products and portfolio weights on the complete market

To find the agents' portfolio weights one has to set the dynamics of the individual wealth component by component equal to the dynamics of a portfolio containing the set of tradable assets, i.e., wealth changes and changes in the value of the portfolio have to have identical exposures to each risk factor. In the complete market case, the tradable assets are the claim on aggregate consumption, the money market account, and the insurance products linked to jump and diffusive risks in $X_{t}$, respectively.

Next, we specify the payoffs of the insurance products. To trade the diffusive risk $W_{t}^{X}$, the agents use a claim labeled $Z_{t}$ with cash flow dynamics

$$
\frac{d Z_{t}}{Z_{t}}=\mu_{Z} d t+\sigma_{Z}^{\prime} d W_{t}
$$

where the drift $\mu_{Z}$ and the volatility $\sigma_{Z}^{\prime}=\left(0, \sigma_{z}\right)$ are exogenous constants. Let $\zeta_{i, t}=$ $\zeta_{i}\left(w_{t}, X_{t}\right)$ denote the $\log$ price-to-cash-flow ratio of this asset from investor $i$ 's perspective. Analogously to Equations (I.6) and (I.7), the dynamics of the log price-to-cash-flow ratio $\zeta_{i, t}$ of the insurance asset $Z_{t}$ are

$$
\begin{aligned}
d \zeta_{i, t}= & \left\{\frac{\partial \zeta_{i, t}}{\partial w_{t}} \mu_{w}+\frac{1}{2} \frac{\partial^{2} \zeta_{i, t}}{\partial w_{t}^{2}} \sigma_{w}^{\prime} \sigma_{w}-\frac{\partial \zeta_{i, t}}{\partial X_{t}} \kappa_{X} X_{t}+\frac{1}{2} \frac{\partial^{2} \zeta_{i, t}}{\partial X_{t}^{2}} \sigma_{X}^{\prime} \sigma_{X}+\frac{\partial^{2} \zeta_{i, t}}{\partial w_{t} \partial X_{t}} \sigma_{w}^{\prime} \sigma_{X}\right\} d t \\
& +\left\{\frac{\partial \zeta_{i, t}}{\partial w_{t}} \sigma_{w}+\frac{\partial \zeta_{i, t}}{\partial X_{t}} \sigma_{X}\right\}^{\prime} d W_{t}+\left\{\zeta_{i}\left(w_{t}+L_{w}, X_{t}+L_{X}\right)-\zeta_{i}\left(w_{t}, X_{t}\right)\right\} d N_{t}\left(\lambda_{i}\right) \\
(\text { IV.1 }) \equiv & \mu_{\zeta_{i}} d t+\sigma_{\zeta_{i}}^{\prime} d W_{t}+L_{\zeta_{i}} d N_{t}\left(\lambda_{i}\right),
\end{aligned}
$$

and $\zeta_{i, t}$ solves the PDE

$$
\begin{aligned}
0= & e^{-\zeta_{i, t}}+\mu_{\xi_{i}}+\mu_{Z}+\mu_{\zeta_{i}}+\frac{1}{2} \sigma_{\zeta_{i}}^{\prime} \sigma_{\zeta_{i}}+\sigma_{\xi_{i}}^{\prime} \sigma_{Z}+\sigma_{\xi_{i}}^{\prime} \sigma_{\zeta_{i}}+\sigma_{Z}^{\prime} \sigma_{\zeta_{i}} \\
& +\left[\left(1+L_{C_{i}}\right)^{-\gamma} e^{(\theta-1) L_{v_{i}}} e^{L_{\zeta_{i}}}-1\right] \lambda_{i} .
\end{aligned}
$$

Since the investors agree on the price of this instrument, $\zeta_{1, t}=\zeta_{2, t} \equiv \zeta_{t}$.
Analogously, the payoff from the jump-linked instrument is denoted by $I_{t}$ and evolves as

$$
\frac{d I_{t}}{I_{t}}=\mu_{I} d t+L_{I} d N_{t}\left(\lambda_{i}\right)
$$

where the coefficients $\mu_{I}$ and $L_{I}$ are given exogenously. The insurance product $I_{t}$ has a price-to-cash flow ratio denoted by $\varpi_{i, t}=\varpi_{i}\left(w_{t}, X_{t}\right)$ with dynamics

$$
\begin{aligned}
d \varpi_{i, t}= & \left\{\frac{\partial \varpi_{i, t}}{\partial w_{t}} \mu_{w}+\frac{1}{2} \frac{\partial^{2} \varpi_{i, t}}{\partial w_{t}^{2}} \sigma_{w}^{\prime} \sigma_{w}-\frac{\partial \varpi_{i, t}}{\partial X_{t}} \kappa_{X} X_{t}+\frac{1}{2} \frac{\partial^{2} \varpi_{i, t}}{\partial X_{t}^{2}} \sigma_{X}^{\prime} \sigma_{X}+\frac{\partial^{2} \varpi_{i, t}}{\partial w_{t} \partial X_{t}} \sigma_{w}^{\prime} \sigma_{X}\right\} d t \\
& +\left\{\frac{\partial \varpi_{i, t}}{\partial w_{t}} \sigma_{w}+\frac{\partial \varpi_{i, t}}{\partial X_{t}} \sigma_{X}\right\}^{\prime} d W_{t}+\left\{\varpi_{i}\left(w_{t}+L_{w}, X_{t}+L_{X}\right)-\varpi_{i}\left(w_{t}, X_{t}\right)\right\} d N_{t}\left(\lambda_{i}\right) \\
(\text { IV.2 } \equiv & \mu_{\varpi_{i}} d t+\sigma_{\varpi_{i}}^{\prime} d W_{t}+L_{\varpi_{i}} d N_{t}\left(\lambda_{i}\right) .
\end{aligned}
$$

Finally, $\varpi_{i}$ solves the PDE

$$
\begin{aligned}
0= & e^{-\varpi_{i, t}}+\mu_{\xi_{i}}+\mu_{I}+\mu_{\varpi_{i}}+\frac{1}{2} \sigma_{\varpi_{i}}^{\prime} \sigma_{\varpi_{i}}+\sigma_{\xi_{i}}^{\prime} \sigma_{\varpi_{i}} \\
& +\left[\left(1+L_{I}\right)\left(1+L_{C_{i}}\right)^{-\gamma} e^{(\theta-1) L_{v_{i}}} e^{L_{\varpi_{i}}}-1\right] \lambda_{i} .
\end{aligned}
$$

As for $Z_{t}$, the investors must agree on the price of $I_{t}$, i.e., $\varpi_{1, t}=\varpi_{2, t} \equiv \varpi_{t}$.
The aggregate $\log$ wealth-consumption ratio $v_{t}=\log \left(w_{t} e^{v_{1, t}}+\left(1-w_{t}\right) e^{v_{2, t}}\right)$ has dynamics

$$
\begin{aligned}
d v_{t}= & \left\{\frac{\partial v_{t}}{\partial w_{t}} \mu_{w}+\frac{1}{2} \frac{\partial^{2} v_{t}}{\partial w_{t}^{2}} \sigma_{w}^{\prime} \sigma_{w}-\frac{\partial v_{t}}{\partial X_{t}} \kappa_{X} X_{t}+\frac{1}{2} \frac{\partial^{2} v_{t}}{\partial X_{t}^{2}} \sigma_{X}^{\prime} \sigma_{X}+\frac{\partial^{2} v_{t}}{\partial w_{t} \partial X_{t}} \sigma_{w}^{\prime} \sigma_{X}\right\} d t \\
& +\left\{\frac{\partial v_{t}}{\partial w_{t}} \sigma_{w}+\frac{\partial v_{t}}{\partial X_{t}} \sigma_{X}\right\}^{\prime} d W_{t}+\left\{v\left(w_{t}+L_{w}, X_{t}+L_{X}\right)-v\left(w_{t}, X_{t}\right)\right\} d N_{t}\left(\lambda_{i}\right) \\
(\text { IV.3 } \equiv & \mu_{v} d t+\sigma_{v}^{\prime} d W_{t}+L_{v} d N_{t}\left(\lambda_{i}\right) .
\end{aligned}
$$

Investor $i$ 's total wealth $V_{i, t}$ is equal to the value of her holdings (in units) $Q_{i, C, t}, Q_{i, M, t}$,
$Q_{i, Z, t}$, and $Q_{i, I, t}$ in the consumption claim, the money market account, and the two insurance products with prices $P_{t}^{C}, P_{t}^{M}, P_{t}^{Z}$, and $P_{t}^{I}$, respectively. Let $\Pi_{i, t}$ denote the value of this portfolio. With $\pi_{i, C, t}, \pi_{i, M, t} \pi_{i, Z, t}$ and $\pi_{i, I, t}$ denoting the relative share of investor $i$ 's wealth invested in the four assets, the total return $d R_{i, t}^{\Pi}$ on her portfolio can be represented as

$$
\begin{aligned}
d R_{i, t}^{\Pi}= & \pi_{i, C, t}\left(\frac{d P_{t}^{C}}{P_{t}^{C}}+e^{-v_{t}} d t\right)+\pi_{i, M, t} r_{i, t}^{f} d t+\pi_{i, Z, t}\left(\frac{d P_{t}^{Z}}{P_{t}^{Z}}+e^{-\zeta_{t}} d t\right)+\pi_{i, I, t}\left(\frac{d P_{t}^{I}}{P_{t}^{I}}+e^{-\varpi_{t}} d t\right) \\
= & \left\{\pi_{i, C, t}\left(\bar{\mu}_{C}+X_{t}+\mu_{v}+\frac{1}{2} \sigma_{v}^{\prime} \sigma_{v}+\sigma_{C}^{\prime} \sigma_{v}+e^{-v_{t}}\right)+\pi_{i, M, t} r_{i, t}^{f}\right. \\
& +\pi_{i, Z, t}\left(\mu_{Z}+\mu_{\zeta}+\frac{1}{2} \sigma_{\zeta}^{\prime} \sigma_{\zeta}+\sigma_{Z}^{\prime} \sigma_{\zeta}+e^{-\zeta_{t}}\right) \\
& \left.+\pi_{i, I, t}\left(\mu_{I}+\mu_{\varpi}+\frac{1}{2} \sigma_{\varpi}^{\prime} \sigma_{\varpi}+e^{-\varpi_{t}}\right)\right\} d t \\
& +\left\{\pi_{i, C, t}\left(\sigma_{C}+\sigma_{v}\right)+\pi_{i, Z, t}\left(\sigma_{Z}+\sigma_{\zeta}\right)+\pi_{i, I, t} \sigma_{\varpi}\right\}^{\prime} d W_{t} \\
& +\left\{\pi_{i, C, t}\left(e^{L_{v}}-1\right)+\pi_{i, Z, t}\left(e^{L_{\zeta}}-1\right)+\pi_{i, I, t}\left[\left(1+L_{I}\right) e^{L_{\varpi}}-1\right]\right\} d N_{t}\left(\lambda_{i}\right) .
\end{aligned}
$$

The portfolio shares are determined by the condition that investor $i$ 's wealth and her financing portfolio react in the same way to the shocks in the model. With respect to diffusive shocks the condition is thus

$$
\begin{equation*}
\sigma_{C_{i}}+\sigma_{v_{i}}=\pi_{i, C, t}\left(\sigma_{C}+\sigma_{v}\right)+\pi_{i, Z, t}\left(\sigma_{Z}+\sigma_{\zeta}\right)+\pi_{i, I, t} \sigma_{\varpi} \tag{IV.4}
\end{equation*}
$$

where the diffusion coefficients for investor $i$ 's wealth on the left-hand side were derived in (I.14).

Look at $\pi_{i, C, t}$. From Equations (III.1), (III.2), and (III.3) we see that the first component of the vector $\sigma_{C_{i}}$ is equal to $\sigma_{c}$, since the first component of $\sigma_{w}$ (a multiple of $\sigma_{X}$ ) equals zero. Equation (I.6), furthermore, shows that $\sigma_{v_{i}}$ is a multiple of $\sigma_{X}$, so that its first component is also zero. Overall, the first component of the vector $\sigma_{C_{i}}+\sigma_{v_{i}}$ is, thus, equal to $\sigma_{c}$. The same is true for the volatility vectors of investor $i$ 's portfolio, as can be seen from the definitions of $\sigma_{C}$ and $\sigma_{Z}$ and from Equations (IV.1), (IV.2), and (IV.3). Taken together,
this implies $\pi_{i, C, t}=1$ for $i=1,2$. So both agents invest $100 \%$ of their respective wealth into the claim on aggregate consumption, implying that the positions in the other three assets add up to zero in value for each agent individually.
$\pi_{i, Z, t}$ and $\pi_{i, I, t}$ follow from equating the reactions of wealth and the financing portfolio to diffusive shocks $W_{t}^{X}$ and jumps $N_{t}$. This gives two conditions, where the first one refers to the second components of the vectors $\sigma_{C_{i}}+\sigma_{v_{i}}$ and $\left(\sigma_{C}+\sigma_{v}\right)+\pi_{i, Z, t}\left(\sigma_{Z}+\sigma_{\zeta}\right)+\pi_{i, I, t} \sigma_{\varpi}$, respectively. The second one is obtained by matching the terms in front of $d N_{t}$ in the total return on wealth and on the financing portfolio, using $\pi_{i, C, t}=1$. This implies
(IV.5) $\left(1+L_{C_{i}}\right) e^{L_{v_{i}}}-1 \stackrel{!}{=}\left[e^{L_{v}}-1\right]+\pi_{i, Z, t}\left[e^{L_{\zeta}}-1\right]+\pi_{i, I, t}\left[\left(1+L_{I}\right) e^{L_{\sigma}}-1\right]$.

The resulting two equations can then be solved numerically for $\pi_{i, Z, t}$ and $\pi_{i, I, t}$. The portfolio weights for the other investor are found via the aggregate supply condition for the insurance products, which says that their total value in the economy equals zero, i.e., $\pi_{1, Z, t} V_{1, t}+$ $\pi_{2, Z, t} V_{2, t}=0$ and $\pi_{1, I, t} V_{1, t}+\pi_{2, I, t} V_{2, t}=0$. Finally, investor $i$ 's position in the money market account is

$$
\begin{equation*}
\pi_{i, M, t}=-\left(\pi_{i, Z, t}+\pi_{i, I, t}\right) \tag{IV.6}
\end{equation*}
$$

## V. Numerical Implementation

We now describe how we implemented the model in MATLAB, using the corresponding toolbox provided by the Numerical Algorithms Group (NAG). We solve the model numerically on a two-dimensional grid for the pessimist's consumption share $w_{t}$ and the long-run growth rate $X_{t}$. For $w_{t}$, we use 41 points over the interval $(0,1)$, while there are 39 points over the
interval $[-0.1560,0.1440]$ for $X_{t}$. The vectors of grid points are given as follows:

$$
\begin{aligned}
{\left[w_{1}, \ldots, w_{41}\right] } & =\left[\cos \left(\frac{(2 s-1) \pi}{2 \cdot 41}\right)+1\right] \frac{1}{2} \\
{\left[X_{1}, \ldots, X_{39}\right] } & =\left[\cos \left(\frac{(2 t-1) \pi}{2 \cdot 39}\right)+1\right] \frac{0.1440+0.1560}{2}-0.1560
\end{aligned}
$$

with $s=[41,40,39, \ldots, 2,1]$ and $t=[39,38,37, \ldots, 2,1]$.
Table V. 1 shows that the interval we will use for the long-run growth rate $X_{t}$ in our numerical implementation, $[-0.1560,0.1440]$, is actually larger than an interval ranging from the $0.0001 \%$ to the $99.9999 \%$ quantile of the long-run distribution of $X_{t}$. Widening this interval has no impact on the results.

## Table V.1. Quantiles for the long-run growth rate $X_{t}$ in model (2)

The table shows for the long-run risk model with jumps in the long-run growth rate presented in Section III, the quantiles of the long-run growth rate $X_{t}$ determined by a Monte Carlo simulation. We simulate the $X_{t}$ process daily and take the end of the month values over 10,000 paths of length 1,000 years each under the true measure $\lambda=0.02$. The parameters are given in Table 4.

| Quantiles | $X_{t}$ |
| ---: | ---: |
| $0.0001 \%$ | -0.1526 |
| $0.0010 \%$ | -0.1342 |
| $0.0100 \%$ | -0.1151 |
| $0.1000 \%$ | -0.0948 |
| $1.0000 \%$ | -0.0714 |
| $10.0000 \%$ | -0.0409 |
| $50.0000 \%$ | -0.0056 |
| $90.0000 \%$ | 0.0285 |
| $99.0000 \%$ | 0.0559 |
| $99.9000 \%$ | 0.0760 |
| $99.9900 \%$ | 0.0922 |
| $99.9990 \%$ | 0.1061 |
| $99.9999 \%$ | 0.1185 |

## A. Complete Market

To obtain boundary conditions for the PDE in Equation (I.7), we study the limiting cases $w_{t} \rightarrow 0$ and $w_{t} \rightarrow 1$. In either case, we have one very large and one very small investor. Motivated by Borovicka (2018), we assume that the large investor sets prices and risk premia, while the small one takes these quantities as exogenous.

When $w_{t}$ is very close to zero, we are basically in a one-investor economy, so that $\frac{1}{1-w_{t}} \mu_{w, t}, \frac{1}{1-w_{t}} \sigma_{w, t}$, and $L_{w, t}$ are zero as well. In this case, the PDE (I.7) for the log wealthconsumption ratio of the large investor 2, using the Equations (III.1), (III.2), and (I.6), simplifies to

$$
\begin{aligned}
0= & e^{-v_{2, t}}-\beta+\left(1-\frac{1}{\psi}\right)\left[\bar{\mu}_{C}+X_{t}-\frac{1}{2} \gamma \sigma_{C}^{\prime} \sigma_{C}\right]-\frac{\partial v_{2, t}}{\partial X_{t}} \kappa_{X} X_{t}+\frac{1}{2} \frac{\partial^{2} v_{2, t}}{\partial X_{t}^{2}} \sigma_{X}^{\prime} \sigma_{X} \\
& +\frac{1}{2} \theta\left(\frac{\partial v_{2, t}}{\partial X_{t}}\right)^{2} \sigma_{X}^{\prime} \sigma_{X}+(1-\gamma) \frac{\partial v_{2, t}}{\partial X_{t}} \sigma_{C}^{\prime} \sigma_{X}+\frac{1}{\theta}\left[e^{\theta\left[v_{2}\left(w_{t}+L_{w, t}, X_{t}\right)-v_{2}\left(w_{t}, X_{t}\right)\right]}-1\right] \lambda_{2} .
\end{aligned}
$$

To solve the above PDE numerically, we use the wealth-consumption ratio in a one-investor economy without a state variable as our starting value, i.e., $v_{2, t}=-\log \left[\beta-\left(1-\frac{1}{\psi}\right)\left(\bar{\mu}_{C}-\frac{1}{2} \gamma \sigma_{C}^{\prime} \sigma_{C}\right)\right]$.

For the small investor, the fact that $1-w_{t}$ is very close to one implies (based on Equations (III.3), (I.11), and (I.12)) in the limit as $w_{t}$ tends to 0 that

$$
\begin{aligned}
\frac{1}{w_{t}} \mu_{w, t}=\psi\{ & \frac{1}{2}\left(1+\frac{1}{\psi}\right) \gamma\left[\sigma_{C_{1}}^{\prime} \sigma_{C_{1}}-\sigma_{C_{2}}^{\prime} \sigma_{C_{2}}\right]+\frac{1}{2}(1-\theta)\left[\sigma_{v_{1}}^{\prime} \sigma_{v_{1}}-\sigma_{v_{2}}^{\prime} \sigma_{v_{2}}\right] \\
& +(1-\theta)\left[\sigma_{C_{1}}^{\prime} \sigma_{v_{1}}-\sigma_{C_{2}} \sigma_{v_{2}}\right] \\
& +\left[e^{(\theta-1)\left[v_{1}\left(w_{t}, X_{t}+L_{X}\right)-v_{1}\left(w_{t}, X_{t}\right)\right]}-1-\left(1-\frac{1}{\theta}\right)\left[e^{\theta\left[v_{1}\left(w_{t}, X_{t}+L_{X}\right)-v_{1}\left(w_{t}, X_{t}\right)\right]}-1\right]\right] \lambda_{1} \\
& \left.-\left[e^{(\theta-1)\left[v_{2}\left(w_{t}, X_{t}+L_{X}\right)-v_{2}\left(w_{t}, X_{t}\right)\right]}-1-\left(1-\frac{1}{\theta}\right)\left[e^{\theta\left[v_{2}\left(w_{t}, X_{t}+L_{X}\right)-v_{2}\left(w_{t}, X_{t}\right)\right]}-1\right]\right] \lambda_{2}\right\}, \\
\frac{1}{w_{t}} \sigma_{w, t}= & \frac{1}{\gamma}(1-\theta)\left(\frac{\partial v_{2, t}}{\partial X_{t}}-\frac{\partial v_{1, t}}{\partial X_{t}}\right) \sigma_{X},
\end{aligned}
$$

and that $v_{1}\left(w_{t}, X_{t}+L_{X}\right)-v_{1}\left(w_{t}, X_{t}\right)=v_{2}\left(w_{t}, X_{t}+L_{X}\right)-v_{2}\left(w_{t}, X_{t}\right)-\frac{1}{\theta-1} \ln \left(\frac{\lambda_{1}}{\lambda_{2}}\right)$. Thus,
the PDE becomes

$$
\begin{aligned}
0= & e^{-v_{1, t}}-\beta+\left(1-\frac{1}{\psi}\right)\left[\bar{\mu}_{C}+X_{t}+\frac{1}{w_{t}} \mu_{w, t}-\frac{1}{2} \gamma\left(\sigma_{C}^{\prime} \sigma_{C}-\frac{1}{w_{t}^{2}} \sigma_{w, t}^{\prime} \sigma_{w, t}\right)\right]-\frac{\partial v_{1, t}}{\partial X_{t}} \kappa_{X} X_{t} \\
& +\frac{1}{2} \frac{\partial^{2} v_{1, t}}{\partial X_{t}^{2}} \sigma_{X}^{\prime} \sigma_{X}+\frac{1}{2} \theta\left(\frac{\partial v_{1, t}}{\partial X_{t}}\right)^{2} \sigma_{X}^{\prime} \sigma_{X}+(1-\gamma) \frac{\partial v_{2, t}}{\partial X_{t}}\left(\sigma_{C}+\frac{1}{w_{t}} \sigma_{w, t}\right)^{\prime} \sigma_{X} \\
& +\frac{1}{\theta}\left[e^{\theta\left[v_{2}\left(w_{t}+L_{w, t}, X_{t}\right)-v_{2}\left(w_{t}, X_{t}\right)-\frac{1}{\theta-1} \ln \left(\frac{\lambda_{1}}{\lambda_{2}}\right)\right]}-1\right] \lambda_{2} .
\end{aligned}
$$

Again, we rely on $v_{1, t}=-\log \left[\beta-\left(1-\frac{1}{\psi}\right)\left[\bar{\mu}_{C}-\frac{1}{2} \gamma \sigma_{C}^{\prime} \sigma_{C}\right]\right]$ as our starting value for the numerical solution. The maximum errors in the solutions of the investor-specific partial differential equations (I.7) are always less than $10^{-6}$.

## B. Incomplete Market

On an incomplete market, we also need starting values for the optimization problem described in Online Appendix III.B. In addition to the complete market solution, we use $\nu=\frac{1}{2} v_{1}+\frac{1}{2} v_{2}$ and $\pi_{1}^{C}=1$ for $w<0.5$ or $\pi_{2}^{C}=1$ for $w \leq 0.5$, respectively. The maximum errors in the solutions for the investor-specific PDEs (I.7) and (III.4), the conditions for $r_{1, t}^{f}=r_{2, t}^{f}$ and for the portfolio weights in Equations (III.5) to (III.7) are always smaller than $10^{-13}$.

## VI. Further results from quantitative analysis

## A. Wealth-consumption ratios

Figure VI. 1 shows the wealth-consumption ratios, which are part of the equilibrium solution on the complete and on the incomplete market. The left column plots these as function of the pessimist's consumption share $w_{t}$ and on the right as function the long-run growth rate $X_{t}$. While it might be okay to assume that the wealth-consumption ratio is an
exponentially affine function the exogenous state variable $X_{t}$, adopting this assumption for the endogenous state variable $w_{t}$ would be invalid. Therefore, we have to solve the model numerically, as highlighted in Section A in the main text.

## B. Investor survival

Figure VI. 2 depicts the kernel density estimates of the pessimist's consumption share on the complete and the incomplete market following a Monte Carlo simulation of the pessimist's consumption share over a period of $100,500,1,000$, and 10,000 years.

## C. Varying the true jump intensity

Table VI. 1 provides the aggregate asset pricing moments for the Monte Carlo simulation for $T=1,000$ years when we vary the true jump intensity $\lambda$. The risk-free rate is consistently higher on the incomplete market, while we find the opposite for the risk premium. For the return volatility, there are mostly no substantial differences between the two economies, however, in three cases it is slightly higher on the complete market.

To understand these results, take a look at Figure 14 in the main text which shows all three quantities under the true measure with $\lambda=0.02$. The plots for the other values of the true jump intensities would look qualitatively similar. According to Table 6 in the main text, the pessimist dies out quite fast on the incomplete market, so that for all three asset pricing moments the incomplete market result is around $w_{t} \approx 0$, while the corresponding point for the complete market varies between 0.08 and 0.87 . The risk-free rate is a decreasing function of $w_{t}$, so it has to be higher on the incomplete market. Since the equity premium is an increasing function of $w_{t}$, it is higher when the market is complete. For the return volatility, the results for the two markets are quite similar, so that they coincide in most of the cases we look at. As emphasized in Section III.B. 4 in the main text, the differences between the
results for the two market structures are fairly small. Hence, it seems difficult to conclude that market incompleteness matters with respect to aggregate asset pricing moments.

## D. Varying the individual beliefs

In Figures VI. 3 to VI.8, we provide the consumption share dynamics, the portfolio weights, and the Borovicka (2018) decomposition for two cases in which we vary the beliefs relative to our benchmark parametrization, i.e., $\lambda_{1}=0.020, \lambda_{2}=0.001$, and $\lambda=0.020$. As a measure of disagreement, we use $\Delta=\frac{\lambda_{2}-\lambda_{1}}{\lambda}$.

First, we decrease $\lambda_{1}$ to 0.015 , while keeping $\lambda_{2}$ and $\lambda$ unchanged. The results are shown in Figures VI. 3 to VI.5. As for model (1), we observe less disagreement ( $\Delta$ drops from 0.95 to 0.70 ) which leads to less speculation, and less extreme portfolio positions for both investors on both markets. The implications for survival seem qualitatively similar as for our benchmark case.

In Figures VI. 6 to VI.8, we increase $\lambda_{2}$ to 0.06 and find a substantial drop in speculation as well as more conservative portfolio strategies for both investors on the complete and on the incomplete market, although the disagreement is the same as in the first case $(\Delta=0.70)$. When it comes to investor survival, the difference between both market structures persists, similar to the benchmark case.

Overall, adjusting the jump intensity of the optimist whose beliefs are further away from the true model seems to have a larger impact on speculation and the investors' portfolio strategies, while changing the beliefs of the pessimist whose beliefs are close to the true jump intensity appears to have a smaller effect.

## Figure VI.1. Wealth-consumption ratios in model (2)

The figure shows for the long-run risk model with jumps in the long-run growth rate presented in Section III, the individual wealth-consumption ratios, $e^{v_{i, t}}$ for $i=1,2$, and the aggregate wealthconsumption ratio, $e^{v_{t}}=w_{t} e^{v_{1, t}}+\left(1-w_{t}\right) e^{v_{2, t}}$. The solid line represents the aggregate, the dotted (dashed) line the pessimist's (optimist's) wealth-consumption ratio. In the top (bottom) row, the graphs depict the results for the complete (incomplete) market. All quantities are shown in the left column as functions of the pessimist's consumption share $w_{t}$ with the stochastic part of the expected growth rate of consumption fixed at $X_{t}=-0.0060$ and in the right column as functions of the long-run growth rate $X_{t}$ for $w_{t}=0.5$. The parameters are given in Table 4.




Figure VI.2. Investor survival in model (2)
The figure shows for the long-run risk model with jumps in the long-run growth rate presented in Section III, the kernel density estimates for the pessimist's consumption share $w_{T}$ for $T$ years into the future under the true measure $\lambda=0.02$. The results after 100 years ( 500 years) are given by the gray dashed (solid) line, those after 1,000 years ( 10,000 years) by the black dashed (solid) line. The graph on the left shows the results on a complete market, the one on the right those on an incomplete market. All quantities are determined by a Monte Carlo simulation of the dynamics of the consumption share shown in Equation (7) over 10,000 paths with a starting value of $w_{0}=0.5$. The coefficients $\mu_{w}, \sigma_{w}$, and $L_{w}$ are obtained by interpolating the grids for these quantities obtained as part of the equilibrium solution. The parameters are given in Table 4.


Table VI.1. Impact of varying the true measure on aggregate asset pricing quantities in model (2)
The table shows for the long-run risk model with jumps in the long-run growth rate presented in Section III, the risk-free rate, the risk premium, and the return volatility obtained from a Monte Carlo simulation of the model $T=1,000$ years into the future under the true measure $\lambda$ which we vary in each column. The top (bottom) panel gives the results on the complete (incomplete) market. The risk-free rate is given in Equation (I.10) in the Online Appendix, the risk premium in (5), and the return volatility in (6). The evolution of the pessimist's consumption share is given in Table 6. The parameters are given in Table 4.

| $\begin{aligned} & \hline=1,000 \\ & \text { (years) } \\ & \hline \end{aligned}$ |  |  |  |  | $\lambda$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 | 0.10 |
|  | Complete market |  |  |  |  |  |  |  |  |  |
| Risk-free rate | 0.0131 | 0.0108 | 0.0086 | 0.0063 | 0.0040 | 0.0017 | -0.0006 | -0.0029 | -0.0052 | -0.0075 |
| Risk premium | 0.0401 | 0.0394 | 0.0389 | 0.0386 | 0.0383 | 0.0380 | 0.0378 | 0.0375 | 0.0373 | 0.0370 |
| Return volatility | 0.0139 | 0.0141 | 0.0143 | 0.0145 | 0.0146 | 0.0148 | 0.0150 | 0.0151 | 0.0153 | 0.0154 |
| Incomplete market |  |  |  |  |  |  |  |  |  |  |
| Risk-free rate | 0.0133 | 0.0113 | 0.0094 | 0.0074 | 0.0055 | 0.0035 | 0.0016 | -0.0003 | -0.0023 | -0.0043 |
| Risk premium | 0.0397 | 0.0386 | 0.0375 | 0.0364 | 0.0354 | 0.0343 | 0.0332 | 0.0322 | 0.0312 | 0.0301 |
| Return volatility | 0.0139 | 0.0141 | 0.0143 | 0.0144 | 0.0146 | 0.0148 | 0.0149 | 0.0151 | 0.0152 | 0.0154 |

Figure VI.3. Consumption share dynamics in model (2) for $\lambda_{1}=0.015, \lambda_{2}=0.001$, and $\lambda=0.020$
The figure depicts for the long-run risk model with jumps in the long-run growth, the coefficients in the dynamics of the pessimist's consumption share. The gray (black) line represents the results on the complete (incomplete) market. From left to right the graphs show the drift ( $\mu_{w}$ ), and the coefficients for diffusive consumption shocks ( $\sigma_{w, C}$ ), diffusive expected growth rate shocks ( $\sigma_{w, X}$ ), and jumps in the expected growth rate $\left(L_{w}\right)$, respectively. All quantities are determined under the true measure $\lambda=0.02$ and shown as functions of the pessimist's consumption share $w_{t}$ with the long-run growth rate being fixed at $X_{t}=-0.0060$. The parameters are given in Table 4 of the main text.




Figure VI.4. Portfolio weights in model (2) for $\lambda_{1}=0.015, \lambda_{2}=0.001$, and $\lambda=0.020$
The figure shows for the long-run risk model with jumps in the long-run growth rate, the investors' asset holdings on the complete (top row) and on the incomplete market (bottom row), respectively. In the top row, the graphs show from left to right show the fraction of wealth invested in the consumption claim, $\pi_{i, C}$, the diffusion insurance product $Z, \pi_{i, Z}$, the jump insurance product $I, \pi_{i, I}$, and the money market account, $\pi_{i, M}$. In the bottom row, the left (right) graph is for the consumption claim, $\pi_{i, C}$ (money market account, $\pi_{i, M}$ ). The pessimist's (optimist's) portfolio holdings are indicated by the dotted (dashed) line. All quantities are determined under the true measure $\lambda=0.02$ and shown as functions of the pessimist's consumption share $w_{t}$ with the stochastic part of the expected growth rate of consumption fixed at $X_{t}=-0.0060$. The parameters are given in Table 4 of the main text.






Figure VI.5. Borovicka (2018) decomposition in model (2) for $\lambda_{1}=0.015, \lambda_{2}=0.001$, and $\lambda=0.020$
The figure depicts for the long-run risk model with jumps in the long-run growth rate, (from left to right) the following channels: (1) the savings channel, $\left(e^{-v_{2, t}}-e^{-v_{1, t}}\right) ;(2)$ the speculative volatility channel, $\left(\frac{1}{2} \sigma_{V_{2}}^{\prime} \sigma_{V_{2}}-\frac{1}{2} \sigma_{V_{1}}^{\prime} \sigma_{V_{1}}\right)$; (3) the speculative jump channel, $\left(\log \left(1+L_{V_{1}}\right) \lambda-\log \left(1+L_{V_{2}}\right) \lambda\right)$; (4) the "risk premium" channel, $\left(\sigma_{V_{1}} \eta_{1}^{W}-L_{V_{1}} \lambda_{1}^{\mathbb{Q}}\right)-\left(\sigma_{V_{2}} \eta_{2}^{W}-L_{V_{2}} \lambda_{2}^{\mathbb{Q}}\right)$. The plot on the furthest to the right labeled with "Total" shows the sum over these four channels, i.e., the difference in the investors' expected log wealth growth rates. The gray (black) line represents the results on the complete (incomplete) market. All quantities are determined under the true measure $\lambda=0.02$ and shown as functions of the pessimist's consumption share $w_{t}$ with the long-run growth rate being fixed at $X_{t}=-0.0060$. The parameters are given in Table 4 of the main text.





Figure VI.6. Consumption share dynamics in model (2) for $\lambda_{1}=0.020, \lambda_{2}=0.006$, and $\lambda=0.020$
The figure depicts for the long-run risk model with jumps in the long-run growth rate, the coefficients in the dynamics of the pessimist's consumption share. The gray (black) line represents the results on the complete (incomplete) market. From left to right the graphs show the drift ( $\mu_{w}$ ), and the coefficients for diffusive consumption shocks ( $\sigma_{w, C}$ ), diffusive expected growth rate shocks ( $\sigma_{w, X}$ ), and jumps in the expected growth rate $\left(L_{w}\right)$, respectively. All quantities are determined under the true measure $\lambda=0.02$ and shown as functions of the pessimist's consumption share $w_{t}$ with the long-run growth rate being fixed at $X_{t}=-0.0060$. The parameters are given in Table 4 of the main text.





## Figure VI.7. Portfolio weights in model (2) for $\lambda_{1}=0.020, \lambda_{2}=0.006$, and $\lambda=0.020$

The figure shows for the long-run risk model with jumps in the long-run growth rate, the investors' asset holdings on the complete (top row) and on the incomplete market (bottom row), respectively. In the top row, the graphs show from left to right show the fraction of wealth invested in the consumption claim, $\pi_{i, C}$, the diffusion insurance product $Z, \pi_{i, Z}$, the jump insurance product $I, \pi_{i, I}$, and the money market account, $\pi_{i, M}$. In the bottom row, the left (right) graph is for the consumption claim, $\pi_{i, C}$ (money market account, $\pi_{i, M}$ ). The pessimist's (optimist's) portfolio holdings are indicated by the dotted (dashed) line. All quantities are determined under the true measure $\lambda=0.02$ and shown as functions of the pessimist's consumption share $w_{t}$ with the stochastic part of the expected growth rate of consumption fixed at $X_{t}=-0.0060$. The parameters are given in Table 4 of the main text.






Figure VI.8. Borovicka (2018) decomposition in model (2) for $\lambda_{1}=0.020, \lambda_{2}=0.006$, and $\lambda=0.020$
The figure depicts for the long-run risk model with jumps in the long-run growth rate, (from left to right) the following channels: (1) the savings channel, $\left(e^{-v_{2, t}}-e^{-v_{1, t}}\right) ;(2)$ the speculative volatility channel, ( $\left.\frac{1}{2} \sigma_{V_{2}}^{\prime} \sigma_{V_{2}}-\frac{1}{2} \sigma_{V_{1}}^{\prime} \sigma_{V_{1}}\right)$; (3) the speculative jump channel, $\left(\log \left(1+L_{V_{1}}\right) \lambda-\log \left(1+L_{V_{2}}\right) \lambda\right)$; (4) the "risk premium" channel, $\left(\sigma_{V_{1}} \eta_{1}^{W}-L_{V_{1}} \lambda_{1}^{\mathbb{Q}}\right)-\left(\sigma_{V_{2}} \eta_{2}^{W}-L_{V_{2}} \lambda_{2}^{\mathbb{Q}}\right)$. The plot on the furthest to the right labeled with "Total" shows the sum over these four channels, i.e., the difference in the investors' expected log wealth growth rates. The gray (black) line represents the results on the complete (incomplete) market. All quantities are determined under the true measure $\lambda=0.02$ and shown as functions of the pessimist's consumption share $w_{t}$ with the long-run growth rate being fixed at $X_{t}=-0.0060$. The parameters are given in Table 4 of the main text.








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[^1]:    ${ }^{34}$ See Epstein and Zin (1989) for the discrete-time setup and Duffie and Epstein (1992) for the extension to continuous-time stochastic differential utility.

[^2]:    ${ }^{35}$ In Online Appendix IV, we explain the pricing of the insurance products and how to determine the portfolio weights. The numerical implementation of the complete market equilibrium is discussed in Online Appendix V.A.

[^3]:    ${ }^{36}$ The numerical implementation of the incomplete market equilibrium is discussed in detail in Online Appendix V.B.

