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Risk components and the market model: a pedagogical note

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Teaching modern finance involves familiarizing the student with terms like total risk, systematic risk, unique risk, beta, and R^2. Although each of these concepts may be relatively easy to communicate and digest one by one, it is harder to see their internal links. Using the logic of the market model, this note offers a simple framework for presenting the basic risk concepts in an integrated way.

I. INTRODUCTION

Students of modern finance are familiar with the market model, they know how to estimate its parameters, and they can distinguish between total risk, systematic risk, and unsystematic risk. However, although each of these concepts is easy to understand one by one, it is less straightforward to discover their internal relationship. As existing finance literature does not offer such a unified framework, the purpose of this note is to present an easily accessible structure using very familiar building blocks.

According to the market model, the return on asset j is:

$$r_j = \alpha_j + \beta_j r_m + \varepsilon_j$$

r_m is the market return, α_j is the expected return when r_m is zero, β_j is the market sensitivity of asset j, and ε_j is an error term with zero expectation. When ε_j and r_m are independent (homoskedastic), the return of asset j has a variance of:

$$\sigma_j^2 = \beta_j^2 \sigma_m^2 + \sigma_{\varepsilon j}^2 \tag{1}$$

A fundamental result from portfolio theory is that asset j contributes to the risk of an efficient portfolio primarily through the first term in Equation 1, as the second term tends to disappear through diversification (Fama, 1976, pp. 245–256). Thus, if risk is defined in terms of variance, σ_m^2 is called the market portfolio's risk (volatility), σ_j^2 is the total risk (volatility) of asset j, $\beta_j^2\sigma_m^2$ is the asset's systematic (non-diversifiable) risk, and $\sigma_{\varepsilon j}^2$ is its unique (diversifiable) risk.

An alternative to the variance-based approach in Equation 1 was introduced by Ben-Horim and Levy (1980), defining the risk components of the market model in terms of standard deviations:

$$\begin{aligned} &\text{market portfolio risk} = \sigma_m \qquad &&\text{Total risk} = \sigma_j \\ &\text{systematic risk} = \beta_j\sigma_m \qquad &&\text{unique risk} = \sigma_j - \beta_j\sigma_m \end{aligned} \tag{2}$$

This definition has two advantages. First, unlike the variance-based approach in Equation 1, Equation 2 can account for the fact that assets with a negative beta contribute negatively to (i.e. reduce) portfolio risk. Second, in terms of pricing-relevant risk, the asset's expected return in the CAPM is determined by $\beta_j\sigma_m$ from Equation 2 rather than $\beta_j^2\sigma_m^2$ from Equation 1.

When the market model is estimated empirically by OLS, the estimate of the asset's beta is:

$$\beta_j = \frac{\sigma_{jm}}{\sigma_m^2} \tag{3}$$

where σ_{jm} is the covariance between the returns of asset j and the market portfolio.[1] Finally, the regression's goodness of fit is given by the coefficient of determination:

$$R_j^2 = \frac{\beta_j^2\sigma_m^2}{\sigma_j^2} \tag{4}$$

Section 2 of this note uses one equation, scatterplots, and a simple graph to relate the risk components in Equations 1, 2 and 3 to each other and to the empirical fit of the market model in Equation 4. Section 3, explains why R_j^2 cannot be used in the traditional way to evaluate

[1] For expositional simplicity, we do not distinguish between sample estimates and true population parameters.

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the empirical quality of risk components from the market model. The fourth section summarizes the major points.

II. BETA, CORRELATION, AND VOLATILITY

The correlation coefficient between the return on asset j and the market portfolio is $\rho_j = (\sigma_{jm}/\sigma_j) - \sigma_m$. Using this definition and the standard deviation-based risk components in Equation 2, Expression 3 can be transformed into a simple relationship between beta, the correlation coefficient, total risk, and market portfolio risk:

$$\beta_j = \rho_j \frac{\sigma_j}{\sigma_m} \tag{5}$$

Empirically, negative correlations between market returns and asset returns are virtually non-existent. Thus, for expositional clarity, we only consider a non-negative ρ_j, which implies a non-negative β_j from Equation (5). This means the correlation coefficient ρ_j and the coefficient of determination R_j^2 have the same extreme values of 0 and 1, are monotone transformations of each other, and will be equivalent in terms of all our conclusions.[2]

Expression 5 offers several useful insights. To start with the most familiar cases, a *deterministic* asset return is defined by $\sigma_j = 0$ (no total risk in Equation 2), which implies $\beta_j = 0$ by Equation 5. This is plot 1 in Fig. 1, where the dots are the empirical observations and the broken line is the fitted regression (the two are inseparable in this case).

Second, $\beta_j = 0$ may also hold for stochastic asset returns ($\sigma_j > 0$, i.e. positive total risk), provided $\rho_j = 0$. In such

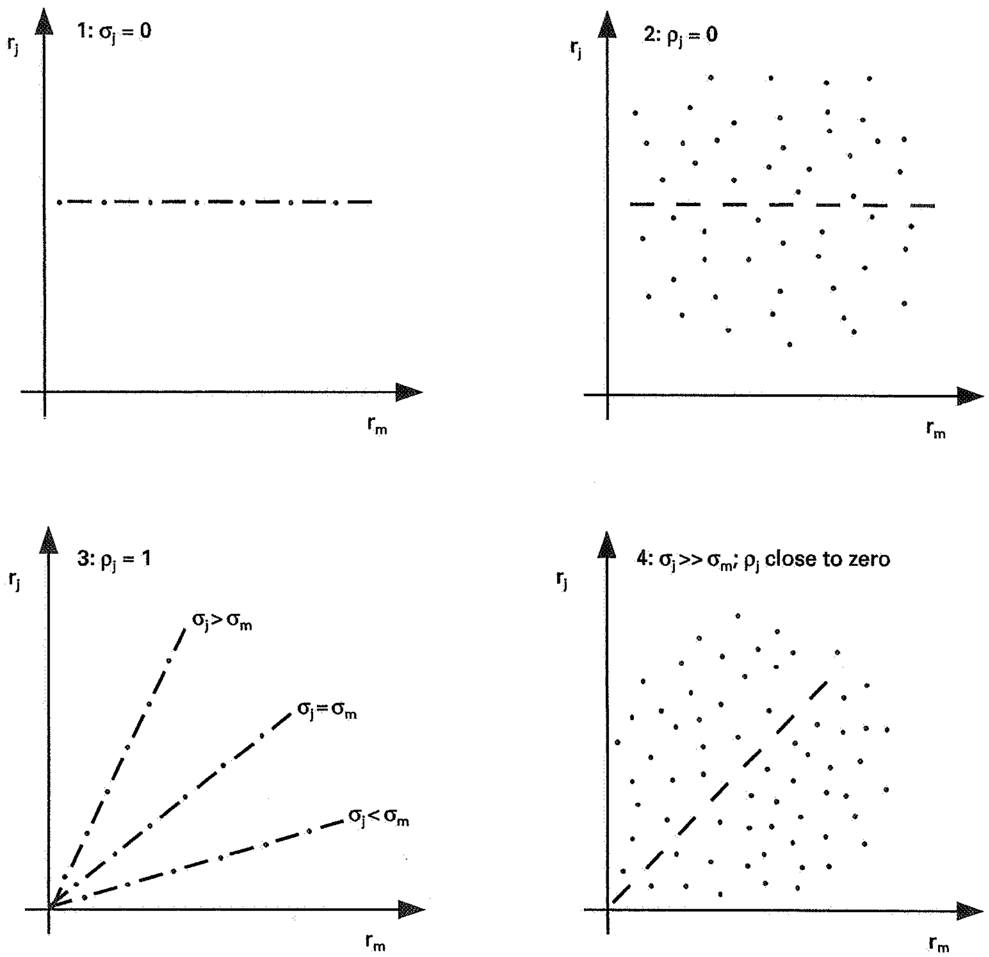

Fig. 1. *Scatterplots of market return* (r_m) *versus asset returns* (r_j). *The dots represent empirical observations, and the broken lines are OLS regression lines (characteristic lines). The total risk of asset j is* σ_j, *market portfolio risk is* σ_m, *and* ρ_j *is the coefficient of correlation between the return on asset j and the market portfolio return*

[2] In fact, when β_j is non-negative, $R_j = \rho_j$, and Equation 5 is just the square root of Equation 4 solved for β_j.

a case, the asset has *no systematic risk*, i.e. $\beta_j\sigma_m = 0$ in Equation 2. The diffuse scatter in the second plot reflects this case, where asset returns fluctuate independently of market movements. In Equation 1 or 2, the asset's total risk equals its unique risk, which disappears in a well-diversified portfolio. Under CAPM pricing, $\rho_j = 0$ implies a riskless expected return regardless of whether total risk is high or low.

Third, the case of *no unique risk* is defined by $\sigma_j = \beta_j\sigma_m$ in Equation 2, corresponding to $\sigma_j^2 = \beta_j^2\sigma_m^2$ and hence $\sigma_{\varepsilon j}^2 = 0$ in Equation 1. This means $\rho_j = 1$ from Equation 5. Such an asset contributes to an efficient portfolio's risk by its full volatility, as no part of total risk is washed out by diversification. Plot 3 in Fig. 1 illustrates that when all the asset's risk is systematic, every combination of asset return and market return falls exactly on the regression line.

However, note that although the asset correlates perfectly with market returns, β_j may still differ from 1. Clearly, with $\rho_j = 1$ in Equation 5, β_j equals 1 if and only if the asset and the market has the same volatility, i.e. $\sigma_j = \sigma_m$. When the asset is more (less) volatile than the market, β_j will be larger (smaller) than one, even if the asset return responds strictly proportionally to market movements. For instance, given perfect correlation, $\sigma_j = 20\%$ and $\sigma_m = 10\%$ implies a β_j of 2. Conversely, when the market is twice as volatile as the asset, $\beta_j = 0.5$.

Notice also that because $\rho_j = 1$ means $\beta_j = \sigma_j/\sigma_m$, the CAPM predicts that expected return is proportional to the asset's total risk. That is, the asset is priced like an efficient portfolio along the capital market line. Whenever $\rho_j < 1$, some part of σ_j is not reflected in β_j, and the asset will be priced below the capital market line.

Fourth, it also follows from Equation 5 that *the strength of the correlation* between asset returns and market portfolio returns (i.e. the size of R_j^2 or ρ_j, which reflects the concentration of the scatterplot) implies nothing about the size of beta. In particular, Equation 5 shows that a high beta is fully consistent with a low correlation. This happens when the asset's total risk (a large part of which would be unique) is considerably higher than the market portfolio's risk. Plot 4 of Fig. 1 illustrates the case of an asset with high beta (steep regression line), low correlation with the market (small ρ_j), and high total risk relative to market portfolio risk ($\sigma_j \gg \sigma_m$). The latter point is reflected in the fact that asset returns are scattered over a considerably larger range than market returns.

This case can be directly related to the typical New York Stock Exchange (NYSE) stock. Based on monthly CRSP data over the period 1926–1990, the average σ_j/σ_m is found to be 2.6, ranging from a minimum of 1.5 (in 1929 and 1962) to a maximum of 6.8 (1964). The average σ_j/σ_m for 1980–1990 is 2.5. Moreover, as the average beta of all stocks is 1, Expression 5 suggests that with an average σ_j/σ_m of 2.5, the average ρ_j is 0.4 and hence the average R_j^2 is 0.16. Thus, in terms of the standard deviation based definition of risk components in Equation 2, 40% of a typical NYSE stock's risk is systematic. With the variance-based definition in Equation 1, the fraction becomes 16%.

Figure 2 summarizes the insights from the four scatterplots. This figure, which is the graphical representation of Expression 5, shows the relationship between ρ_j and β_j for three different levels of total risk ($\sigma_A < \sigma_B < \sigma_C$). The ratio of total asset risk (σ_j) to market portfolio risk (σ_m) is thus constant along a given line, but increases with the steepness of a line. For each line, the minimum beta of zero occurs when $\rho_j = 0$, and the maximum value of $\beta_j = \sigma_j/\sigma_m$ occurs when $\rho_j = 1$, corresponding to $R_j^2 = 1$. For a given total asset risk and market portfolio risk, beta increases linearly with the correlation coefficient. If R_j^2 is used instead of ρ_j on the horizontal axis, the three increasing curves would be concave between the same endpoints.

Case 1 ($\sigma_j = 0$) and case 2 ($\sigma_j > 0$ and $\rho_j = 0$) from Fig. 1 are both represented by the origins in Fig. 1. The third case ($\rho_j = 1$) is illustrated by the assets at the end points A, B, and C. These three assets all have perfect correlation with the market (no unique risk), but different volatility (total risk). Clearly, $\beta_C > \beta_B > \beta_A$, but whether these betas are high or low relative to the market beta of one is completely determined by the ratio of the asset's total risk to the market portfolio's risk (σ_j/σ_m).

Case 4 clarifies that high betas are fully compatible with low correlations. This is seen by comparing asset D to asset E, which both have the same, moderate correlation with the

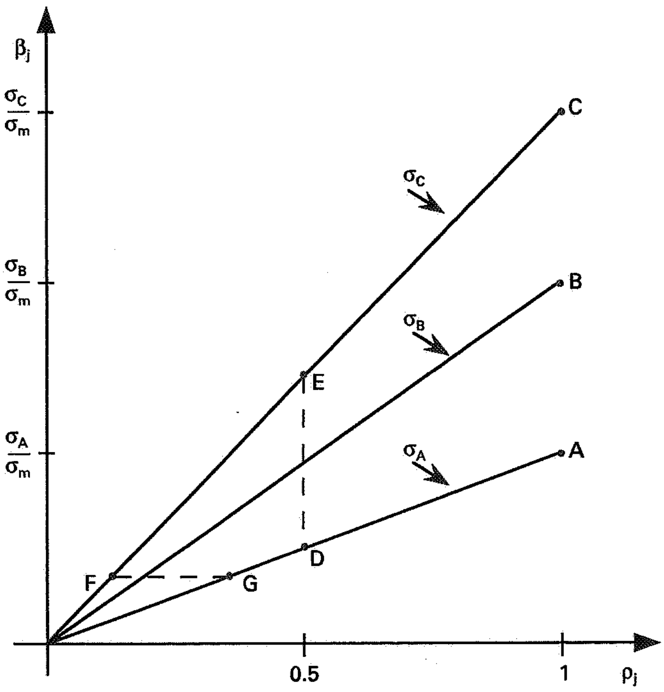

Fig. 2. *Market correlation (ρ_j) and beta (β_j) for three different levels of total risk ($\sigma_A < \sigma_B < \sigma_C$). The risk of the market portfolio is σ_m*

market portfolio (low ρ_j). Because asset E has a higher total risk than asset D, the beta of E is almost three times higher.

Notice also that when ρ_j is large a given difference in total risk between two assets transforms into a higher difference in betas: The total risk difference between E and D equals the difference between C and A. Still, the difference in beta is larger between C and A ($\rho_j = 1$) than between E and D ($\rho_j = 0.5$).

Finally, for two assets F and G to have *the same beta*, Equation 5 requires that:

$$\frac{\rho_F}{\rho_G} = \frac{\sigma_G}{\sigma_F}$$

For instance, if asset F has half the market correlation of asset G (i.e. four times the R_j^2), it must have twice the total risk (as defined by Equation 2) for β_F to equal β_G. Suppose $\sigma_F = 0.3$ and $\rho_F = 0.2$, and that asset G has $\rho_G = 0.4$, i.e. twice as strong a market correlation. The betas of F and G will still be equal if the volatility of asset G is just 0.15, i.e. half of σ_F.

This general relationship can be seen by comparing assets F and G in Fig. 2. The higher market correlation of G is neutralized by a correspondingly lower total risk, producing identical betas, i.e. $(\sigma_F \rho_F)/\sigma_m = (\sigma_G \rho_G)/\sigma_m$.

III. R^2 AND THE QUALITY OF RISK ESTIMATES

When linear regression is used to test a theory's predictions, a high R_j^2 is normally desirable, as it reflects a case where the independent variables predict the dependent variables quite well. Correspondingly, a low R_j^2 is considered unattractive, as it signals low predictive power. However, this general guideline for theory evaluation is not valid in our context, where the purpose is not to test a theory, but rather to estimate an asset's risk components as specified by the market model. As seen from Equations 1 and 4, R_j^2 is the fraction of the asset's (variance-based) total risk which is systematic, whereas $(1 - R_j^2)$ represents the unique part. Therefore, a low R_j^2 simply means that most of the asset's volatility is unique, because the asset's return cannot be well predicted by market movements. Such risk information is as valuable as finding a high R_j^2 for a different asset, which just tells that unique (non-market) risk accounts for only a small part of that asset's volatility. In terms of the right-hand side of Expression 1, the second term dominates total risk in the former case, and the first term dominates in the latter.

A quite different question is how well the market model *per se* explains cross-sectional differences in asset returns. A low cross-sectional R^2 in a market-wide regression does indeed suggest that systematic risk (market movements) is an unimportant determinant of asset returns. However, this is a question of relative model performance (e.g. single-factor versus multi-factor models) rather than estimation of risk components within a given model. In terms of Equations 1 or 2, what matters is the statistical significance of the separate risk components of the asset return, not the significance of the market return as a predictor of asset return.

IV. SUMMARY

This pedagogical note supplements existing literature by integrating well-known risk concepts into a simple framework. We used Equation 5, scatterplots of market returns versus asset returns (Fig. 1), and a graph of correlation versus beta (Fig. 2) to illustrate the essential properties of an asset's risk components as implied by the market model.

Beta is zero whenever the asset's return has either no total risk (volatility) or no correlation with the market returns. Perfect correlation with the market means unsystematic risk is zero, but implies nothing about the asset's beta. In particular, the asset's beta equals the market portfolio's beta of one only if the market portfolio's volatility equals the volatility of the perfectly correlated asset.

An asset's beta is multiplicatively determined by the asset's total risk and its correlation with market returns. Thus, even a low correlation (reflecting that most of the asset's total risk is unique) produces a high beta when the asset is considerably more volatile than the market. Moreover, two assets may have the same beta even though their market correlations differ widely. A high correlation may be neutralized by a correspondingly smaller total risk.

When the market model is used to estimate the risk components of asset returns, the coefficient of determination does not reflect the usefulness of the regression. What matters is the statistical significance of the separate risk components, not how well market returns predict individual asset returns.

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