# Risk Aversion Sensitive Real Business Cycles *

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**Abstract**

We study endogenous state-contingent technology choice in a production-based economy. Risk aversion, through technology choice, drives production substitution across states and exerts a first-order effect on macroeconomic quantities. The model simultaneously fits the volatility of the growth rate of consumption, investment, output, and total factor productivity and produces plausible autocorrelations, correlations, and cross-correlations. Further, our model provides a good fit to the interest rate and mimics macro-financial lead-lag linkages. For example, the model reproduces the negative relation between output and next period's interest rate and the positive relation between the interest rate and next period's output growth.

**Keywords:** State-contingent technology; Risk aversion; Volatility of the growth rate of total factor productivity and investment; Autocorrelations, correlations, and cross-correlations of macroeconomic quantities

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# 1 Introduction

The aim of the paper is to establish a link between asset prices and the macroeconomy through risk aversion, which is absent in the standard real business cycle (RBC) model. To achieve this goal, we investigate a production-based economy with flexible technology choice (similar to Cochrane (1988) and Belo (2010)) in which risk aversion drives production substitution across states and exerts a first-order effect on macroeconomic quantities. Our simple model produces one-period lead-lag linkages between the interest rate and consumption and the interest rate and output. We find empirical evidence in support of such lead-lag relations between the interest rate and macroeconomic quantities.

In a standard production-based economy, the representative firm, in lieu of the owner, manages investments to smooth consumption over time. However, the firm has no means to adjust production or technology across states. Since the intertemporal substitution plays the key role for how an agent smooths consumption in such an environment, the elasticity of intertemporal substitution (EIS) almost exclusively drives the time-variation in consumption, investment, and output in the standard production-based real business cycle model. Specifically, risk aversion cannot exert a first-order effect on the macroeconomy. This is an underappreciated cause for concern as it implies that risk aversion can be employed to match asset pricing moments without negatively affecting a models' fit to macroeconomic quantities, as shown in Tallarini (2000). Cochrane (2008) calls this defect of standard real business cycle models the divorce between asset pricing and macroeconomics.

To reunite the peculiar couple and show how technology choice makes for a happier marriage between asset prices and macroeconomics, we study an economy with production, where the firm employs technology choice to substitute productivity across states at the expense of average productivity.[1] Therefore, risk aversion and the EIS drive optimal investment and state-

[1] A practicable way to substitute productivity across states is through investing in various different production technologies. In the Online appendix A, we provide a theoretical connection between the reduced-form approach to technology choice that we adopt and investing in several technologies as in Jermann (2010). For example, it seems plausible, that the various different technologies of generating electricity, e.g., coal, natural gas, nuclear, oil, solar, wind, etc., are broadly consistent with Jermann (2010) and, therefore, also with our reduced-form approach. Specifically, since each technology has its own risk characteristics combining them allows to choose

contingent output. Through this channel, the model fits the point estimates of the volatility of the growth rate of investment or the autocorrelation of the growth rate of total factor productivity (TFP) and output, in addition to the volatilities of consumption and output growth. Also, macroeconomic quantities exhibit autocorrelations, correlations, and cross-correlations that are reminiscent of what we see in the data.

In a model without technology choice, the firm cannot modify the effect of negative shocks to productivity. In our model, technology choice shifts productivity to smooth consumption across states and over time. Depending on the cost to average output and risk aversion, productivity shifts from high productivity to low productivity states or vice versa, which can lead to endogenous productivity that is more or less volatile than the exogenous productivity. To reproduce the positive autocorrelation in TFP growth in the data requires that productivity shifts from high to low productivity states, which reduces volatility. When the incentive to smooth consumption over time is strong, the model with technology choice produces larger autocorrelation in the growth rate of investment and smaller autocorrelation in the growth rate of consumption relative to the autocorrelation in the growth rate of output, which is consistent with the data.

The difference in the autocorrelations of investment and consumption implies that the macroeconomic variables no longer comove perfectly, leading to more plausible correlations. For example, with technology choice the correlation between consumption and investment reduces from 1.0 to 0.62.

Through technology choice, consumption can be smoother, which might have dire consequences for the fit of the financial side of the model economy to the data. Since all our calibrations perfectly fit the point estimate of the consumption volatility, technology choice does not constrain the model's fit to financial markets. In fact the opposite is true, the model with technology choice produces a better fit to the volatility and autocorrelation of the interest rate, as well as, to the correlation between the log price-dividend ratio and the interest rate.

To analyze the model, we log-linearize the macroeconomy as in Jermann (1998) to use closed-

the risk profile of energy generation.

form log-normal pricing for assets, since then the stochastic discount factor, cash flows, and state variables are jointly conditionally log-normally distributed. Through the log-linearization, we see that risk aversion exerts a first-order effect over the macroeconomy with technology choice. We find that endogenous technology transforms the exogenous productivity, which evolves as an AR(1), into an ARMA(1,1). Precisely this feature of the model leads to autocorrelations in the growth rates of macroeconomic quantities and replicates the autocorrelations in the growth rates of consumption, investment, output, and TFP. As a plausibility test, we estimate an ARMA(1,1) process for the TFP series to compare the volatility of the exogenous productivity process implied by the data with the one in the model. This exercise shows that the volatility of the exogenous productivity in the model, which we set to match the output volatility, is very close to the one implied by the data.

Our model produces realistic macro-financial lead-lag linkages even though we do not calibrate it to match such linkages. In the data, the interest rate and changes in the interest rate predict macroeconomic growth rates, which the model reproduces as a result of the autocorrelations in the growth rates. For example, in the data and our model changes in the interest rate predict changes in output with a correlation of 0.4.

The paper speaks to the literature that explores the asset pricing implications of production transformation across states or technologies. To allow for production transformation across states, Cochrane (1993) proposes to allow firms to choose state-contingent productivity endogenously subject to a constraint set. In closely related works, Cochrane (1988) and Jermann (2010) back out the stochastic discount factor from producers' first-order conditions assuming complete technologies, i.e., that there are as many technologies as states of nature. The calibrated model in Jermann (2010) matches the mean and volatility of aggregate stock market returns and the interest rate and, in addition, produces a volatile Sharpe ratio. In similar spirit, Belo (2010) applies state-contingent productivity to derive a pure production-based pricing kernel in a partial equilibrium setting, which gives rise to a macro-factor asset pricing model that explains the cross-sectional variation in average stock returns.[2] The key takeaway from these

[2]Recent contributions to the literature on investment- or production-based asset pricing include Kaltenbrun-

papers is that state-contingent technology can explain asset prices both in the time-series and the cross-section. However, these studies do not look at the implications of state-contingent technology for the macroeconomy or for lead-lag linkages between macroeconomic and financial quantities. Our paper fills this gap in the literature.

Even though our focus differs, the predictions of our model are supportive of the broader production-based macroeconomic literature that emphasizes the importance of autocorrelations in macroeconomic quantities. We start with Burnside and Eichenbaum (1996); they argue that capital-utilization rates are an important source of propagation to the volatility of exogenous technology shocks. In their model, propagation is required to match the autocorrelations of output growth. In our model, technology choice helps to match the autocorrelations in the growth rates of consumption, investment, output, and TFP provided that the volatility of endogenous shocks is at least twenty percent smaller than the volatility of exogenous shocks. Boldrin, Christiano, and Fisher (2001) study an otherwise standard RBC model with habit preferences and a two-sector technology with limited intersectoral factor mobility. Their model also fits the autocorrelation of the growth rate of output, but the autocorrelation of consumption growth is negative in their model. Further, we differ from Boldrin, Christiano, and Fisher (2001) in that we match the volatility of investment while their model produces investment volatility that is too low compared to the data and in that they do not study the correlations of macroeconomic growth rates. Other than that, our paper joins Boldrin, Christiano, and Fisher (2001) in the task of integrating the analysis of asset returns and business cycles. We close by mentioning Cogley and Nason (1995). They discuss the autocorrelation in output and conclude, consistent with Burnside and Eichenbaum (1996), that models with weak propagation have to resort to exogenous sources of autocorrelation. Our model endogenously generates autocorrelations with additional flexibility rather than frictions.

In closing the introduction, we remark that our aim is to understand how a constant risk aversion coefficient drives production substitution across states and how it affects consumption

ner and Lochstoer (2010), Papanikolaou (2011), Gârleanu, Panageas, and Yu (2012), Ai, Croce, and Li (2013), and Belo, Lin, and Bazdresch (2014), among many others; none of these works, however, study state-contingent technology.

and investment. To help understand the model we contrast it with the standard RBC model. Yet, the standard RBC model in which the unobservable exogenous productivity follows an ARMA(1,1) process is observationally identical to our model, provided that risk aversion and the TFP process are specified consistently. For example, Croce (2014) uses an ARMA(1,1) process, instead of an AR(1), for exogenous productivity, which generates more realistic correlations and autocorrelations of macro quantities. However, in our model we cannot choose risk aversion independently from the fit to the macroeconomy. Our focus on the autocorrelations and correlations is not only because the model can explain them, but mainly because our theory predicts that risk aversion drives these quantities. We acknowledge that there are other sufficiently rich models in this respect in the literature. For example, another model with realistic correlations and autocorrelations of macro quantities is Papanikolaou (2011), who introduces an investment specific shock to break the perfect correlations of macroeconomic growth rates. Our contribution is, on the one hand, to provide a model that reunites the real and the financial sides of the economy and, on the other hand, to show how risk-aversion affects the macroeconomy. The predictions of the model pertaining to the correlations and autocorrelations of macro quantities and the macro-finance linkages indicate that our mechanism can matter.

# 2 A model with state-contingent technology

Consider a representative agent who owns an all-equity representative firm, which uses productive capital to generate one real good and operates in discrete time with infinite horizon.

## 2.1 Firms

Let $\Theta_t$ be the exogenous technological productivity level at time $t$. We assume that $\log \Theta_t$ follows an AR(1) process with trend,

$$\log \Theta_{t+1} = \log Z_{t+1} + \phi \left(\log \Theta_t - \log Z_t\right) + \varepsilon_{t+1}, \quad \text{and} \quad \log Z_t = \mu t, \tag{1}$$

where $|\phi| < 1$ and $\varepsilon_{t+1} \sim N(0, \sigma^2)$ denotes the exogenous shock.

Departing from the standard production economy, we assume that the representative firm modifies the underlying productivity shocks. Following Cochrane (1993) and Belo (2010), at time $t$ a state-contingent technology $\Omega_{t+1}$ is chosen through a CES aggregator

$$\mathbb{E}_t \left[ \frac{\Omega_{t+1}^{(1-\alpha)\nu}}{\Theta_{t+1}^{(1-\alpha)\nu}} \right] \leq 1, \tag{2}$$

where $\mathbb{E}_t$ is the conditional expectation operator at time $t$. In (2), the variable $\alpha \in (0, 1)$ stands for the capital share in output and the curvature $\nu$ captures the representative firm's technical ability to modify technology. When $\nu < 1$, increasing the volatility of technology choice also increases average productivity. For this reason, we assume that $\nu > 1$. With this assumption, as $\nu$ increases, distorting the underlying shocks reduces average productivity. When $\nu \to +\infty$, it is infinitely costly to modify the exogenous productivity. Therefore, we obtain $\Omega_{t+1} = \Theta_{t+1}$.

Appendix A provides intuition for the reduced-form approach in modeling technology choice. We interpret the technology modifications set in (2) as a simple abstract form of modeling state-contingent technologies implying flexibility for optimal future productivity. More specifically, constraint (2) determines the representative firm's ability to trade off higher realizations of shocks in some states at time $t + 1$ with lower realizations in other states. The optimal choice offsets the marginal benefit from smoothing consumption over time and states with the marginal cost of lower average productivity (or a tradeoff between static efficiency and flexibility similar to Mills and Schumann (1985)).

Output, $Y_t$, is given by

$$Y_t = K_t^{\alpha} \, \Omega_t^{1-\alpha}, \tag{3}$$

where $K_t$ denotes the capital stock at the beginning of period $t$.

Capital accumulates according to

$$K_{t+1} = (1 - \delta) K_t + g_t, \tag{4}$$

where $\delta$ is the depreciation rate and $g_t$ stands for the capital formation function. We specify $g$ as in Jermann (1998), i.e.,

$$g_t = \left[ \frac{a_1}{1 - 1/\chi} \left( \frac{I_t}{K_t} \right)^{1-1/\chi} + a_2 \right] K_t, \tag{5}$$

where $I_t$ denotes investment at time $t$, the curvature $\chi > 0$ governs capital adjustment costs, and $a_1$ and $a_2$ are constants.[3] These specifications imply that capital adjustment costs are high when $\chi$ is low and that capital adjustments are costless when $\chi \to \infty$. Following Boldrin, Christiano, and Fisher (2001), we set $a_1$ and $a_2$ such that there is no cost to capital adjustment in the deterministic steady-state

$$a_1 = (e^{\mu} - 1 + \delta)^{1/\chi} \quad \text{and} \quad a_2 = \frac{1}{1-\chi} (e^{\mu} - 1 + \delta) .$$

## 2.2 Households

To separate the elasticity of intertemporal substitution (EIS) from risk aversion, we assume that the representative agent exhibits recursive preferences (Kreps and Porteus (1978), Epstein and Zin (1989, 1991), and Weil (1989)), whose utility at time $t$ is represented by

$$U_t = \left\{ (1-\beta) C_t^{1-1/\psi} + \beta \mathbb{E}_t \left[ U_{t+1}^{1-\gamma} \right]^{\frac{1-1/\psi}{1-\gamma}} \right\}^{\frac{1}{1-1/\psi}}, \tag{6}$$

where $0 < \beta < 1$ denotes the subjective time discount factor, $C_t$ stands for aggregate consumption at time $t$, $\psi > 0$ represents the EIS, and the constant relative risk aversion (CRRA) is given by $\gamma > 0$.

Every period the representative agent maximizes her utility (6) by choosing consumption $C_t$ and investment $I_t$ given the aggregate output $Y_t = C_t + I_t$. In addition, the agent chooses the productivity $\Omega_{t+1}$ for every future state next period, given the conditional distribution of the exogenous productivity $\Theta_{t+1}$ and according to the constraint (2).

[3]The functional form for capital formation in (5) simplifies the log-linearized model.

The representative agent discounts consumption with her stochastic discount factor given by

$$M_{t,t+1} = \beta \left[ \frac{C_{t+1}}{C_t} \right]^{-\frac{1}{\psi}} \left[ \frac{U_{t+1}^{1-\gamma}}{\mathbb{E}_t \left( U_{t+1}^{1-\gamma} \right)} \right]^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}} . \tag{7}$$

Besides the macroeconomic quantities of the economy, we also study asset prices in the model with technology choice. Specifically, we compute the return $R_{f,t}$ on the risk-free asset, which pays one unit of consumption next period, and the return on the risky stock with next period dividends, $D_{t+1}$, as follows

$$R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t}, \tag{8}$$

where $P_t$ denotes the price of the dividend claim at time $t$. We price a claim to dividends of the aggregate stock market instead of a claim to aggregate consumption. Since we price dividends, these returns do not equate with investment returns. Specifically, we assume that the log growth in dividends, denoted by $\Delta d_{t+1}$, evolves according to

$$\Delta d_{t+1} = d_0 + d_1 \, \Delta c_{t+1} + d_2 \, u_{t+1}, \tag{9}$$

where $u_{t+1} \sim N(0,1)$, $\Delta c_{t+1}$ denotes log consumption growth, and $d_0, d_1, d_2$ are constant coefficients.

## 2.3 The equilibrium conditions

With recursive preferences, the current value Lagrangian function of the maximization problem with state-contingent technology can be written as

$$\begin{aligned} L_t &= \left\{ (1-\beta)\, C_t^{1-1/\psi} + \beta \mathbb{E}_t \left[ U_{t+1}^{1-\gamma} \right]^{\frac{1-1/\psi}{1-\gamma}} \right\}^{\frac{1}{1-1/\psi}} - \lambda_t^1 (C_t - K_t^{\alpha} \Omega_t^{1-\alpha} + I_t) \\ &\quad - \lambda_t^2 \left[ K_{t+1} - (1-\delta) K_t - g_t \right] - \lambda_t^3 \left\{ \mathbb{E}_t \left[ \frac{\Omega_{t+1}^{(1-\alpha)\nu}}{\Theta_{t+1}^{(1-\alpha)\nu}} \right] - 1 \right\}, \end{aligned} \tag{10}$$

where $\lambda_t^1$, $\lambda_t^2$, and $\lambda_t^3$ denote Lagrangian multipliers for the three constraints.

Six first-order conditions characterize equilibrium; the first three conditions are the constraints that appear in the Lagrangian in (10), that is, consumption and investment always equal aggregate production, the capital accumulation, and the productivity choice constraint. The remaining three conditions characterize the optimal amount of consumption and investment and the optimal productivity choice for next period.

The optimal amount of investment in period $t$ is characterized by the marginal $q$ condition,

$$\frac{1}{g_{I,t}} = \mathbb{E}_t \left[ M_{t,t+1} \left( \alpha \frac{Y_{t+1}}{K_{t+1}} + \frac{1-\delta+g_{K,t+1}}{g_{I,t+1}} \right) \right], \tag{11}$$

where $g_{I,t}$ and $g_{K,t}$ are the partial derivatives of the capital formation function with respect to investment and capital, respectively, in period $t$. The left hand side of (11) shows the marginal cost of investment which is the amount of investment required to generate a unit of productive capital. The right hand side of (11) describes the marginal benefit from an additional unit of capital, which stems from next period's production and the remaining marginal value of future capital stock. Thus, the firm optimally equates the marginal costs with the marginal benefits of investment.

In our model, the representative firm in a period $t$ optimally chooses the productivity $\Omega_{t+1}$ state-by-state for next period, which is given by

$$\left( \frac{\Omega_{t+1}}{\Theta_{t+1}} \right)^{(1-\alpha)\nu} = \frac{\left( M_{t,t+1} \Theta_{t+1}^{1-\alpha} \right)^{\frac{\nu}{\nu-1}}}{\mathbb{E}_t \left[ \left( M_{t,t+1} \Theta_{t+1}^{1-\alpha} \right)^{\frac{\nu}{\nu-1}} \right]}, \tag{12}$$

where the ratio on the left hand side is the transformation of the exogenous productivity. Equation (12) describes the tradeoff embedded in the distribution of $\Omega$. On the one hand, it can be beneficial to increase productivity in states where the productivity is exogenously high and decrease it where the productivity is exogenously low. In this way, next period's average productivity is maximized since the cost of deviating from the exogenous productivity is a function of the ratio of transformation. We see this from the case of risk neutrality, $\gamma = 0$, and non-recursive utility, $\gamma = 1/\psi$, where the stochastic discount factor is constant and, thus, cancels out from (12). As a result, the log optimal endogenous technology is proportional to the log exogenous productivity,

$$\log \Omega_{t+1} \;\propto\; \frac{\nu}{\nu - 1} \log \Theta_{t+1}. \tag{13}$$

On the other hand, when the representative agent is risk averse it can be optimal to shift productivity to high "value" states, that is, states of high marginal utility $M$. For non-recursive utility, $\gamma = 1/\psi$, these are the states of low consumption. With recursive utility, the value of a state also depends on the continuation utility and whether the agent prefers early, $\gamma > 1/\psi$, or late, $\gamma < 1/\psi$, resolution of uncertainty. For example, when agents have preference for early resolution of uncertainty, the value of a state decreases with the continuation utility. The continuation utility is mainly driven by the exogenous state $\theta$, which means that the representative agent also wants to shift productivity to exogenously low productivity states. Given the above tradeoff in the model with endogenous technology choice, it can be optimal to amplify or reduce exogenous volatility and, also, it can be optimal to chose a positive or negative correlation between endogenous and exogenous productivity.

# 3 The log-linearized real economy

This section presents and discusses the economic mechanism behind technology choice. We, first, summarize the log-linearized model economy in a proposition. Second, we discuss the nested standard RBC model without technology choice. Third, we discuss optimal technology choice. Hereby, we focus on how technology choice affects the autocorrelations of macroeconomic variables and the correlations between those macroeconomic variables. Appendix B contains proofs and additional details of the log-linearization.

The log-linear RBC model depends on the three state variables $\theta_t$, $k_t$, and $\omega_t$, which measure the percentage deviation from the steady-state value of the detrended variables $\Theta_t$, $K_t$, and $\Omega_t$, respectively.

**Proposition 1.** *The percentage deviations of utility, consumption, and investment from their steady-state values are given by*

$$\begin{aligned} u_t &= u_k k_t + \tilde{u}_\theta \theta_{t-1} + \sigma_u \epsilon_t, \\ c_t &= c_k k_t + \tilde{c}_\theta \theta_{t-1} + \sigma_c \epsilon_t, \\ i_t &= i_k k_t + \tilde{i}_\theta \theta_{t-1} + \sigma_i \epsilon_t, \end{aligned} \tag{14}$$

*where* $\tilde{x}_\theta = \phi\,(x_\omega + x_\theta)$ *and* $\sigma_x = \sigma_\omega x_\omega + x_\theta$ *for* $x \in \{u, c, i\}$*. Expressions for the coefficients* $x_k$*,* $x_\omega$*, and* $x_\theta$ *for* $x \in \{u, c, i\}$ *are found at Appendix B. The law of motion of the percentage deviations from the steady-state values of exogenous productivity, capital, and endogenous productivity are*

$$\begin{aligned} \theta_{t+1} &= \phi\,\theta_t + \epsilon_{t+1}, \\ k_{t+1} &= \frac{1-\delta}{e^\mu} k_t + \left(1 - \frac{1-\delta}{e^\mu}\right) i_t, \\ \omega_{t+1} &= \phi\,\theta_t + \sigma_\omega \epsilon_{t+1}, \end{aligned} \tag{15}$$

*where* $\epsilon \sim \mathbb{N}(0, \sigma^2)$*. Finally, the sensitivity of the endogenous productivity to exogenous shocks is given by*

$$\sigma_\omega = \frac{(1-\alpha)\nu + m_\theta}{(1-\alpha)(\nu - 1) - m_\omega}, \tag{16}$$

*where the coefficients* $m_\theta = -\frac{1}{\psi} c_\theta - (\gamma - \frac{1}{\psi}) u_\theta$ *and* $m_\omega = -\frac{1}{\psi} c_\omega - (\gamma - \frac{1}{\psi}) u_\omega$ *represent derivatives of the log stochastic discount factor with respect to* $\theta$ *and* $\omega$*, respectively.*

In Proposition 1, the sensitivities with respect to $\omega$, i.e., $x_\omega$, represent the sensitivities with respect to the current level of productivity, whether this is endogenous or exogenous. Whereas, the sensitivities with respect to $\theta$, i.e., $x_\theta$, represent the sensitivities to the expected endogenous productivity and, thus, depend on the persistence of the exogenous shocks. If

$\phi = 0$, then all sensitivities with respect to $\theta$ are zero. What differentiates the dynamics of the technology choice model from the dynamics of the standard RBC model is $\sigma_\omega$, which represents the optimal technology choice in the log-linearized model and depends on risk aversion, $\gamma$. None of the coefficients $x_k$, $x_\theta$, and $x_\omega$ depend on $\gamma$.

## 3.1 The standard RBC economy

In the standard RBC economy, the expressions in (14) simplify since $\omega = \theta$.[4] Since $\sigma_\omega = 1$ and, thus, $\sigma_\omega$ does not depend on $\gamma$, macroeconomic quantities are only risk sensitive, $\epsilon_t$, but not risk aversion sensitive, $\gamma$, as shown by Tallarini (2000).[5] Nevertheless, the following corollary shows that the economy without technology choice can be observationally identical to the economy with technology choice provided that the exogenous productivity follows the same process as the endogenous productivity and risk aversion is identical in both economies.

**Corollary 1.** *The economy without technology choice ($\nu = \infty$), where the exogenous productivity process is given by*

$$\omega_{t+1} = \phi\, \theta_t + \tau_\omega \epsilon_{t+1}$$

*is isomorphic in its pricing and macroeconomic implications with the technology choice economy provided that risk aversions are identical and $\tau_\omega = \sigma_\omega$.*

Even so, the Tallarini (2000) result still holds since $\tau_\omega$ is exogenous and independent of risk aversion.

## 3.2 Technology choice

We now show that with technology choice, risk aversion exerts a first-order effect on macroeconomic quantities. From Proposition 1, it follows that the endogenous productivity in period

[4] The standard RBC economy is a special case of the economy with technology choice since $\sigma_\omega = 1$ when $\nu = \infty$.

[5] Risk aversion can exert second-order effects on macroeconomic quantities. Therefore, we verify that second-order effects are negligible in all our calibrated economies.

$t+1$ can be viewed as a "weighted" average of exogenous productivity levels $\theta_t$ and $\theta_{t+1}$, i.e.,

$$\omega_{t+1} = \phi\,\theta_t + \sigma_\omega \epsilon_{t+1} = \phi(1-\sigma_\omega)\theta_t + \sigma_\omega \theta_{t+1}. \tag{17}$$

For example, when $\phi$ is close to 1, then $1-\sigma_\omega$ and $\sigma_\omega$ are simple weights on the current and next period's exogenous productivity. Since $\sigma_\omega$ critically depends on the firm's ability to chose technology, $\nu$, and on risk aversion, $\gamma$, we see that risk aversion drives technology modifications through $\sigma_\omega$ and, hence, affects macroeconomic quantities.

The representative firm shifts productivity across states depending on the tradeoff between maximizing average productivity and transferring productivity from low value states to high value states. The value of the state is determined by the value of the stochastic discount factor $M$. When agents are risk-neutral and have non-recursive utility, then the log-linear solution for $\sigma_\omega$, with general solution shown in (16), takes the same value, $\nu/(\nu-1)$, as the exact solution derived in (13). In this case, to maximize average productivity requires shifting productivity to high productivity states. More specifically, the lower is $\nu$, the more productivity is shifted to high productivity states.

With recursive preferences, $\sigma_\omega$ can be positive or negative and smaller or greater than one. In the limiting case, where $\phi = 0$ and $\gamma \to \infty$, we have that $\sigma_\omega \to 0$, that is, it is optimal to eliminate all one-period risk.[6] Generally, $\sigma_\omega$ depends on all structural parameters. This dependency generates a rich structure for technology choice. For example, with a certain parametrization, (i) when $\sigma_\omega > 1$, it is optimal to choose amplified shocks that comove with the underlying shocks; (ii) when $0 \leq \sigma_\omega \leq 1$, it is optimal to choose less volatile shocks that comove with the underlying shocks; (iii) when $-1 \leq \sigma_\omega \leq 0$, it is optimal to choose less volatile shocks that partly offset the underlying shocks; and, (iv) when $\sigma_\omega \leq -1$, it is optimal to choose amplified shocks that more than offset the underlying shocks. This can also be seen from comparing the unconditional variance of the exogenous productivity with that of the

[6]This corresponds to the case of utility smoothing discussed in Backus, Routledge, and Zin (2013).

endogenously determined productivity:

$$\sigma(\omega_t)^2 = \left[\phi^2 + \sigma_\omega^2 \left(1 - \phi^2\right)\right] \sigma(\theta_t)^2, \tag{18}$$

where $\sigma(\theta_t)^2 = \frac{\sigma^2}{1-\phi^2}$. If $|\sigma_\omega| > 1$, then the endogenous productivity is more volatile than the exogenous productivity, i.e., $\sigma(\omega_t) > \sigma(\theta_t)$. The productivity reaches the minimum unconditional volatility $\frac{\phi^2\sigma^2}{1-\phi^2}$ when $\sigma_\omega = 0$.

The optimal value of $\sigma_\omega$ depends on the values of $m_\omega$ and $m_\theta$, that is, it depends on how the marginal utility is affected by the current endogenous productivity and the expected endogenous productivity in the future. Typically, the sensitivity of the continuation utility $u_\theta$ is the most important element of $m_\theta$. With preference for early resolution of uncertainty, $\gamma > 1/\psi$, the representative firm shifts productivity to low exogenous productivity states. It is optimal to do so, because it allows the firm to boost investment to reduce the impact of negative and persistent exogenous shocks to productivity. This is driven by the negative $i_\theta$, that is, investment increases as a response to negative shocks to the expected endogenous productivity in the future.

However, as the firm shifts resources to states with low exogenous productivity, consumption shifts from high to low exogenous productivity states. This changes the relative value of the states through $c_\omega$, which typically is the main driver of $m_\omega$. This means that the representative firm stops shifting production to low exogenous productivity states when the relative value of consumption in the high exogenous productivity states is sufficiently high. Essentially, the optimal technology choice depends also on the tradeoff between increasing investment in low exogenous states and decreasing consumption in high exogenous states. Because of this mechanism and depending on the structural parameters of the model, investment can be more volatile than consumption and output.

When $\sigma_\omega$ differs from one, then the endogenous productivity follows an ARMA(1,1) process instead of an AR(1) process, which generates significant autocorrelations in the growth rates of macroeconomic variables. The following proposition summarizes the transformation from AR(1) to ARMA(1,1) and its effect on the growth rate of $\omega$.

**Proposition 2.** *Endogenous technology follows an ARMA(1,1) process, where the AR(1) term originates from the exogenous productivity process,*

$$\omega_{t+1} = \phi\,\omega_t + \sigma_\omega \epsilon_{t+1} + \phi\,(1-\sigma_\omega)\epsilon_t. \tag{19}$$

*The unconditional volatility and the first-order autocorrelation of the growth rate of the endogenous technology, denoted by $\Delta\omega_{t+1}$, are given by:*

$$\sigma(\Delta\omega)^2 = 2\left[\frac{\phi^2}{1+\phi} + \sigma_\omega^2 - \phi\sigma_\omega\right]\sigma^2, \tag{20}$$

*and*

$$ac_1(\Delta\omega) = -\frac{1}{2}\frac{(1-\phi)\sigma_\omega^2 + \phi(1-\sigma_\omega)(\phi - 2\sigma_\omega - \phi^2)}{\sigma_\omega^2 + \phi(1-\sigma_\omega)(\phi-\sigma_\omega)}. \tag{21}$$

*For the case without technology choice, $\sigma_\omega = 1$, we obtain $\sigma(\Delta\theta)^2 = \frac{2\sigma^2}{1+\phi}$ and $ac_1(\Delta\theta) = \frac{\phi-1}{2}$.*

Equation (20) implies that if $\sigma_\omega < \phi - 1$ or $\sigma_\omega > 1$ then $\sigma(\Delta\omega)^2 > \sigma(\Delta\theta)^2$. That is, the endogenous TFP growth rate can be more volatile than the underlying TFP growth rate, as shown in Panel A of Figure 1. Otherwise, the technology choice attenuates the exogenous shocks. For example, when $\sigma_\omega = \frac{\phi}{2}$, then $\sigma(\Delta\omega)^2$ reaches its minimum.

In the standard model, when $\phi$ is close to 1 the autocorrelation of the TFP growth is close to 0. With technology choice, the autocorrelation can be either significantly negative or significantly positive depending on the optimal technology choice, $\sigma_\omega$, as shown in Panel B of Figure 1. When $\sigma_\omega = \frac{\phi}{1+\phi}$, $ac_1(\Delta\omega)$ reaches its maximum value of $\frac{\phi}{2}$. Also, when $|\sigma_\omega| \to \infty$ we have that $ac_1(\Delta\omega) \to -0.5$. Moreover, Panels A and B show that when the TFP growth is positively (negatively) autocorrelated, it is less (more) volatile than the exogenous productivity growth.

# 4 Solution method and asset prices

We solve the model numerically by log-linearizing the economy and by using log-normal pricing for the financial quantities similar to Jermann (1998).[7] Modulo the log-linearized approximation, prices are closed-form since the stochastic discount factor, the cash flows, and the state variables are jointly conditionally log-normally distributed.

Starting with cash-flows, the following proposition presents the log consumption growth for the log-linearized approximation of the model's equilibrium.

**Proposition 3.** *Given the log-linear approximation of the equilibrium in Proposition 1, the log consumption growth is conditionally normal, $\Delta c_{t+1} = \mu_t + \sigma_c \, \epsilon_{t+1}$. Its conditional mean is given by*

$$\mu_t = \mu + \mu_k \, k_t + \mu_\theta \, \theta_{t-1} + \sigma_\mu \, \epsilon_t, \tag{22}$$

*where $\mu_k = \delta c_k(i_k - 1)$, $\mu_\theta = \delta c_k i_\theta - c_\theta(1 - \phi)$, and $\sigma_\mu = \delta c_k \sigma_i + c_\theta - \sigma_c$. The coefficients $\sigma_c$ and $\sigma_i$ are defined in (14).*

The stochastic discount factor in (7) is also log-normally distributed in the log-linear approximation.

**Proposition 4.** *Given the log-linear approximation of equilibrium in Proposition 1, the log stochastic discount factor is conditionally normal:*

$$\log M_{t,t+1} = \log \hat{\beta} - \frac{1}{\psi} \mu_t - \sigma_m \, \epsilon_{t+1}, \tag{23}$$

[7] We verify that for the calibrated models the second-order perturbation method produces almost identical results (see also Tallarini (2000) and Kaltenbrunner and Lochstoer (2010)).

*where*

$$\log \hat{\beta} = \log \beta + \frac{1}{2}(1-\gamma)\left(\gamma - \frac{1}{\psi}\right)\sigma_u^2\sigma^2, \tag{23a}$$

$$\sigma_m = \gamma\sigma_c + \left(\gamma - \frac{1}{\psi}\right)(\sigma_u - \sigma_c), \tag{23b}$$

*where $\mu_t$ is given in Proposition 3, $\sigma_u$ is defined in (14), and $\sigma$ denotes the standard deviation of the exogenous shock $\varepsilon$ defined in (1).*

Since technology choice changes the sensitivities to the exogenous shock, the $\sigma_x$'s, the price of risk changes when technology choice is introduced. One concern is that the extra flexibility of the economy could significantly decrease the one-period consumption risk. If so, then the model could not generate a high price of risk. Fortunately, this is not the case because the endogenous productivity is not only driven by the motive to smooth consumption across states but also by the motive to smooth consumption over time.

Since log cash-flows, the conditional mean of consumption growth, and the log stochastic discount factor are jointly normally distributed, the natural logarithm of the prices of zero-coupon bonds, $q_t^{(n)}$, that pay a unit of consumption $n$ periods ahead and the natural logarithm of the price-dividend ratios of dividend strips, $p_t^{(n)}$, that pay a dividend $n$ periods ahead are affine in the state variables and their coefficients can be computed recursively, as shown in Appendix C. We are interested in the one-period interest rate or risk-free rate and the price of the stock, which is a claim to the dividend stream defined in (9). The prices are given by

$$R_{f,t} = \exp\left(q_t^{(1)}\right) - 1 \qquad \text{and} \qquad P_t = D_t \sum_{n=1}^{\infty} \exp\left(p_t^{(n)}\right).$$

To guarantee convergence, we compute the stock price by the sum of the first 5000 terms.

Since risk aversion is constant, the price of risk, which is given by $\sigma_m$, is also constant. As a result, stock prices and bond prices vary only due to changes in cash-flow expectations as shown in the following proposition.

**Proposition 5.** *Given the stochastic discount factor in Proposition 4, the continuously compounded one-period risk-free rate is*

$$r_{f,t} = -\log \hat{\beta} + \frac{1}{\psi}\mu_t - \frac{1}{2}\sigma_m^2\sigma^2. \tag{24}$$

*The log-linear approximation of the stock price-dividend ratio is given by*

$$p_t - d_t \approx \overline{p-d} + \left(d_1 - \frac{1}{\psi}\right)\mathbb{E}_t \sum_{\tau=0}^{\infty} \hat{J}^{\tau}\left(\mu_{t+\tau} - \mu\right), \tag{25}$$

*where $d_1$ is a constant coefficient in (9) and where $\hat{J}$ and $\overline{p-d}$ are defined in Appendix C.*

We see that the dynamics of stock and bond prices depend on the dynamics of consumption growth expectations, $\mu_t$, and, in general, the dynamics of the term-structure of consumption growth expectations, $\mu_{t+\tau}, \tau \geq 0$. These can be inferred from Proposition 3 and are determined by the dependence of $\mu_t$ on the state and the dynamics of the state variables.

# 5 Calibration

We calibrate the model to analyze its implications on macroeconomic quantities, asset prices, and macro-finance linkages. We discuss three calibrations with technology choice: Models 1 and 3 have low EIS while Model 2 has high EIS.

## 5.1 Data

We collect data for the period 1947$Q$1 to 2012$Q$4. We use quarterly CRSP value-weighted returns as the market return and the Fama 3-month T-bill rate as the risk-free rate from WRDS. Real returns equal nominal returns deflated with inflation computed from the CPI index of the Bureau of Labor Statistics. The price-dividend ratio is inferred from the CRSP value-weighted returns with and without dividends. Macroeconomic variables are from the NIPA tables. Output series are taken to be the total output reported, the consumption series is the consumption of non-durables and services, and the investment series is the non-residential fixed investments. All macroeconomic variables are deflated by CPI and normalized by the civilian noninstitutional population with age over 16, from the Current Population Survey (Serial ID LNU00000000Q). The total factor productivity (TFP) is inferred from the output series and the capital series constructed by Fernald (2014).

## 5.2 Parameter selection

Certain parameters are fixed across model calibrations, which we discuss first. The long-term quarterly growth rate, $\mu$, is set to 0.4%; this value is close to the average growth rates of output, consumption, and investment in the data. The capital share, $\alpha$, is set to 0.36, which is similar to the capital share in Boldrin, Christiano, and Fisher (2001), while the quarterly depreciation rate, $\delta$, is computed from the average investment to capital ratio over the data sample period from the NIPA tables as 0.026. Finally, the dividend process parameters, $d = (0.0025, 1.65, 0.04)$, are chosen to fit the mean and volatility of the annual growth in aggregate dividends as well as its correlation with the annual growth in consumption.

The technology choice parameter, $\nu$, is set to fit the TFP autocorrelation with low EIS in Model 1 and with high EIS in Model 2. Model 3 instead is calibrated to fit the volatility of investment growth for which the model requires a low EIS.[8] Each model with technology choice is compared to a model without technology choice, $\nu = \infty$, referred to as the standard model throughout. To highlight the role of technology choice, the standard model has identical parameters, except for the exogenous volatility which is adjusted to fit the TFP growth volatility. Alternatively, by virtue of Corollary 1, we can use the same ARMA(1,1) process for exogenous productivity to obtain identical behavior for the macroeconomy, with the additional freedom to improve the fit of asset prices by choosing risk aversion. A comparison of the best fit of the standard RBC model with an AR(1) productivity process and the technology choice model can be found in Tables 8 - 11 in the Online appendix D.

[8] Untabulated results show that the model with technology choice and high EIS cannot simultaneously match the TFP growth volatility and investment growth volatility.

We focus on a set of models where we calibrate $\gamma$ to fit the Sharpe ratio of the stock market.[9] For the high EIS case, we set $\psi$ equal to 1.5 and for the models with low EIS, we set $\psi$ as low as possible requiring that the subjective discount factor, $\beta$, stays below one. The remaining parameters are determined as follows: the volatility of the exogenous shock, $\sigma$, is set to fit the volatility of TFP growth, the capital adjustment cost parameter, $\chi$, is set to match the consumption growth volatility, the subjective discount factor is adjusted to fit the average price-to-dividend ratio of the stock market, and, finally, the autocorrelation of the exogenous shock, $\phi$, is chosen to fit the autocorrelation of the price-dividend ratio.

## 5.3 The macroeconomy

Panel A in Table 2 shows the following macroeconomic quantities: volatility of the log growth rate of consumption, investment, and output.[10] Each volatility is standardized by the volatility of the log growth rate of TFP, which we also report. In Panel A, all models with technology choice perfectly match the point estimate of the volatility of consumption, output, and TFP growth. Model 3 also matches perfectly the point estimate of the volatility of investment growth. The first two models produce lower investment growth volatility, nevertheless we cannot reject the hypothesis that the data, including the volatility of the growth rate of investment, are generated by the calibrated models with technology choice. Panel E in Table 2 further substantiates the fit of the calibrated models by showing that the log growth rate and volatility of aggregate consumption and aggregate dividends and their correlations are reproduced when we annualize the data through time aggregation. Again, the hypothesis that the data are generated by the models cannot be rejected for all four quantities in each model.

Comparing the standard model with the technology choice model, we see that for Model 2

[9] In the Online appendix D, we consider two models (Model 4 and 5) with a fixed CRRA, namely $\gamma = 5$. The implications of the model are the same with the exception that with low $\gamma$ it cannot fit the stock market Sharpe ratio.

[10] To facilitate comparison between model calibrations and the data, each table shows the $t - statistic$ of the corresponding quantities with respect to the data estimate; i.e., a $t - statistic$ is computed as the difference between the data estimate and the model average scaled by the square root of the sum of the squared standard errors of the data estimate and the model average. Standard errors of the data estimates are Newey and West (1987) corrected, using 16 lags.

with high EIS the macroeconomic volatilities are almost identical. Whereas for Models 1 and 3 with low EIS, the economies with technology choice show relatively smoother consumption and more volatile investment. Turning to the other results, we find that the standard model produces zero first-order autocorrelations and zero cross-correlations for the macroeconomic variables shown in Panels B and D, respectively. Further, Panel C shows that log growth rates of consumption, investment, and output correlate perfectly with each other and with TFP shocks.

From Panel B, we see that Model 1 with technology choice matches almost perfectly the point estimate of the macroeconomic autocorrelations, despite being calibrated to match only the autocorrelation of the TFP growth. In the data, the autocorrelations of TFP and output are around 0.25, the autocorrelation of consumption growth is close to zero, 0.04, while the autocorrelation of investment equals 0.37. The hypothesis that the macroeconomic autocorrelations are generated by Model 1 with technology choice cannot be rejected for the four autocorrelations. Model 2, which exhibits high EIS is less successful in this regard as the autocorrelations of the macroeconomic growth rates are all close to 0.25. Hence, we reject the hypothesis that the autocorrelation of consumption growth is generated by Model 2. Model 3, which matches the investment growth volatility but cannot simultaneously match the autocorrelation of TFP growth, generates overall higher autocorrelations than in the data. Nevertheless, this economy produces a low autocorrelation of consumption growth and a high autocorrelation of investment growth. Hence, we cannot reject the hypothesis that the autocorrelation of consumption and investment growth is generated by Model 3.

Panel C shows that Model 1 and 2 with technology choice also produce almost perfect correlations between the growth rates of macroeconomic variables; except that the correlation between consumption growth and investment growth in Model 1 is 0.94. Yet, from Model 3 we learn that technology choice can lead to low correlations between the macroeconomic growth rates. For instance, the correlation between consumption and investment growth, which at 0.43 is particularly low in the data, in Model 3 is 0.67. While Model 3 fails to fit the point estimate in the data and while all but one of the hypotheses that the data are generated by the model are rejected, we see a significant improvement relative to the standard model or the technology choice Models 1 and 2. This improvement, however, comes at the cost of somewhat higher autocorrelations and cross-correlations, as is evident from Panels B and D.

When judging the performance of the model concerning correlations, autocorrelations, and cross-correlations it is useful to recall that in the data output is not equal to consumption plus investment, which certainly contributes, at least, to the low correlation between output and investment and output and consumption. In the model, we have $Y = C + I$ at all times, therefore it is difficult for the model to fit those point estimates.[11] Lastly, we point to Model 6 in Appendix D, which produces a consumption-investment growth correlation as low as 0.46.

## 5.4 Asset prices

Table 3 shows that each model with technology choice fits the stock market Sharpe ratio, albeit with a high risk aversion, as in Tallarini (2000). Meanwhile, the high risk aversion also requires high technology modification costs, that is a high $\nu$, to generate a high price of risk. Compared to the standard model, the technology choice model exhibits roughly a 50% higher Sharpe ratio since for the latter consumption is smoother. The excess returns on the stock market are, therefore, also higher in each of the technology choice models.[12]

All models replicate the average log price-dividend ratio and its first-order autocorrelation. However, none of the models generates significant volatility for the log price-dividend ratio, which implies that, consistent with the results of Kaltenbrunner and Lochstoer (2010),[13] they also cannot fit the volatility of the risk-free rate and the equity premium.

Given that we calibrate the models to macroeconomic data perhaps it is not surprising that

[11] If we impose on the data that $Y = C + I$ holds, then the point estimates for the correlation of output growth with investment growth increases from 0.62 to 0.77 and between output and consumption growth the correlation increases from 0.68 to 0.90.

[12] Recall that for the standard model we intentionally use identical parameters, except for the exogenous volatility. When the standard model is calibrated to consumption volatility, then it can produce a higher Sharpe ratio.

[13] Roughly, all models generate stock returns only half as volatile as those observed in the data, which comes mostly from the volatility of dividend growth. Only a significantly lower EIS than what we use can reproduce the stock market volatility. Using an EIS of 0.05, Kaltenbrunner and Lochstoer (2010) generate sufficient stock return volatility but require a subjective discount factor higher than 1. Here, we focus on cases where $\beta \leq 1$; thus, the level of EIS in the model cannot be as low as 0.05.

they do not generate sufficient volatility of the risk-free rate and the equity premium. What might seem surprising is that technology choice, which allows for consumption smoothing across states, does not restrict the performance of the model pertaining to asset pricing moments. To the contrary, we see that the moments of the risk-free rate improve: The technology choice model produces a lower risk-free rate, a higher volatility of the risk-free rate, a lower first-order autocorrelation of the risk-free rate, and lower absolute correlation between the log price-dividend ratio and the risk-free rate than the standard model.[14]

## 5.5 Macro-finance linkages

Since technology choice modifies the exogenous shock over one period which makes the endogenous TFP follow an ARMA(1,1), we focus on the macro-finance linkages one period ahead or one period lagged. For the same reason, technology choice affects only short horizon consumption or output expectations, as shown in Proposition 3. As a result, its effect on macro-finance linkages is ideally measured through short lived assets. Consequently, we focus on the relation between the risk-free rate and macroeconomic quantities.

Table 4 reports correlations between output and changes in the risk-free rate one period ahead, changes in output and changes in lagged risk-free rate, and changes in output and lagged risk-free rate. We also study the corresponding correlations between consumption and the risk-free rate and investment and the risk-free rate.

According to our technology choice model, output predicts the risk-free rate. For example, when output is high, then the risk-free rate next period is expected to be low. From Table 4, we see that in the data output also predicts the risk-free rate with a negative coefficient of $-0.18$. The standard model produces a positive sign for this relation, while each model of the economy with technology choice produces a negative coefficient: $-0.27$ in Model 1 and $-0.21$ in Models 2 and 3.

In the technology choice model, the risk-free rate and changes in the risk-free rate predict

[14] Many economists argue that the expected volatility of the real risk-free rate is only half of the realized volatility. If so, our model reproduces all point estimates involving the risk-free rate.

next period's growth rate in output. In the data, the risk-free rate predicts changes in output with a positive sign and a large coefficient of 0.34. In the models with technology choice, the risk-free rate predicts changes in output with a positive sign and large coefficients of 0.28, 0.27 and 0.53, for Model 1, 2 and 3, respectively. In the data, changes in the risk-free rate predict changes in output with a positive sign and a large coefficient of 0.40. Model 3 with technology choice produces changes in the risk-free rate that predict changes in output with a positive sign and an equally large coefficient of 0.42, while for Model 1 and 2 this correlation is 0.21 and 0.20, respectively. In the standard model, the risk-free rate and changes in the risk-free rate have virtually no predictive power.

Briefly, correlations between consumption and the risk-free rate have the same pattern as the correlations between output and the risk-free rate. Consumption predicts the risk-free rate with a negative sign while the two other correlation coefficients are large and positive. Again, the standard model cannot produce such a correlation structure. The technology choice economies reproduce the negative and the two positive correlation coefficients. We cannot reject the null that the negative correlation between consumption and the risk-free rate in the data are generated by the three technology choice based economies. However, the two positive correlation coefficients are too small compared to the data; thus, the nulls are rejected.

Regarding the linkages between investments and the risk-free rate, we see that the risk-free rate and changes in the risk-free rate predict positively next period's investment growth while investment does not predict the risk-free rate. For Models 1 and 2 we cannot reject the null that the positive correlation between changes in investments and lagged changes in the risk-free rate in the data are generated by the technology choice based economies. The same holds true for the correlation between investment growth and lagged risk-free rate for Model 2. All other $t - statistic$ are too large and, thus, we reject the nulls.

To sum up, these results showcase that technology choice allows to produce realistic macro-finance linkages. We emphasize that none of the economies shown in Table 4 are calibrated to match the reported macro-finance linkages.

# 6 Model analysis

Table 5 presents the log-linear solutions for all the models. Panel A shows the sensitivities to capital, $k_t$, the exogenous productivity, $\theta_t$, and the endogenous productivity, $\omega_t$, in the log-linear solution

$$x_t = x_k\, k_t + x_\theta\, \theta_t + x_\omega\, \omega_t.$$

We see that $\omega_t$ matters because it represents current productivity; $\theta_t$ also matters to the extent that it determines expected productivity. From the law of motion of $\omega$ in Proposition 1, we have that $\mathbb{E}_t(\omega_{t+\tau}) = \phi^\tau \theta_t$, $\tau > 0$.[15] Henceforth, we refer to a coefficient $x_\theta$ as the sensitivity to expected productivity. Panel B shows the log-linear solution from Proposition 1, where we substitute the solution for $\omega$ into the log-linear solution above. We see that the coefficients $\sigma_x = \sigma_\omega\, x_\omega + x_\theta$ depend on technology choice whereas the coefficients $\tilde{x}_\theta = \phi(x_\theta + x_\omega)$ do not. This is because technology choice changes only $\sigma_\omega$, which is the only coefficient that depends on risk aversion, $\gamma$.

## 6.1 The macroeconomy

From Panel B of Figure 1, we gauge that to match the 0.25 autocorrelation in TFP growth in the data requires $\sigma_\omega < 1$. Indeed, Panel B of Table 5 shows that $\sigma_\omega$ is close to 0.77 in Models 1 and 2. Using maximum likelihood, we estimate an ARMA(1,1) process for the detrended TFP series yielding an estimate for $\sigma_\omega$ of 0.80. Panel A of Figure 1 shows that these estimates of $\sigma_\omega$ imply that the endogenous productivity is less volatile than the exogenous. Therefore, to fit the 1.81% volatility of TFP growth requires $\sigma$ to be around 2.25% for Model 1 and Model 2. This value is close to the 2.20% implied by the estimated ARMA(1,1) process. Model 3, which is calibrated to fit the volatility of investment growth, instead, has a lower $\sigma_\omega$ of 0.58 and, thus, a higher $\sigma$ of 2.55%. In all models and the data, output growth has almost the same autocorrelation as TFP growth.

The technology choice model generates imperfect correlations and differing autocorrelations

[15] If we set $\phi$ to zero, then all coefficients $x_\theta$ are zero.

in the growth rates of macroeconomic quantities. This is because $\omega$ and $\theta$ differ and $c_\theta$ and $i_\theta$ are non-zero. Without technology choice, since there is no difference between $\omega$ and $\theta$, macroeconomic autocorrelations are identical and macroeconomic correlations are perfect, even though the coefficients $c_\theta$ and $i_\theta$ are non-zero. Panel A of Table 5 shows that in all the three models consumption increases and investment decreases as a response to a positive shock in expected productivity, albeit with different coefficients.[16] These coefficients are driven by the desire to smooth consumption over time and, for this reason, the EIS determines their magnitude. In Model 2, where the EIS is high and, thus, fluctuations in expected consumption growth over time are less costly, $c_\theta$ and $i_\theta$ are close to zero. Consequently, autocorrelations are flat and correlations are perfect. Whereas, in Models 1 and 3, because of the low EIS, the coefficients are large. In Model 3, we see the largest coefficients. This is because of the lowest capital adjustment costs, i.e., the highest $\chi$.

Given that technology choice generates autocorrelation in the growth rate of output, it must be that either consumption growth, investment growth or both are autocorrelated. In Models 1 and 3, consumption growth is less autocorrelated than investment growth. To understand why, consider that consumption growth is autocorrelated when the expected consumption growth $\mu_t$ is autocorrelated and fluctuates significantly. Technology choice increases the volatility of its unpredictable component $\sigma_\mu \sigma$ in equation (22). With low EIS, fluctuations in $\mu_t$ are more costly and, in equilibrium, $\mu_t$ shows low volatility. In Panel B of Table 5, $\sigma_\mu \sigma$ is 0.089 in Model 1 and 0.114 in Model 3, whereas in Model 2 it is about twice as large and equal to 0.224. Another way to look at this is that technology choice and low EIS reduce the sensitivity of investment to shocks and its autocorrelation increases as the volatility of the unpredictable component decreases. This happens because $\sigma_i = \sigma_\omega i_\omega + i_\theta$ decreases as $i_\theta$ is negative and increases in absolute value.

To understand the correlations, consider the following decomposition of the covariance be-

[16] The solution in the Online appendix B shows that $y_\theta = 0$ and $i_\theta = -c_\theta C/I$. Hence, output is independent of expected productivity implying that the changes in consumption and investment have the same magnitude but with opposite sign.

tween consumption and investment growth:

$$Cov(\Delta i_t, \Delta c_t) = Cov(i_k \Delta k_t + \tilde{i}_\theta \Delta\theta_{t-1}, c_k \Delta k_t + \tilde{c}_\theta \Delta\theta_{t-1}) + \sigma_c \sigma_i \sigma^2.$$

Technology choice and low EIS lead to a low value for $\sigma_c\sigma_i\sigma^2$. In Table 5, we see that in Model 1 $\sigma_c\,\sigma$ increases with technology choice from 0.699 to 0.783 and $\sigma_i\,\sigma$ decreases from 2.364 to 1.972. The net decrease is 0.108, which reduces the correlation by 0.062 given that $\sigma(\Delta i)\,\sigma(\Delta c)$ is 1.744. In Model 3, we see that the reduction in the correlation from the standard model to the model with technology choice is larger, which is mainly due to the reduction in $\sigma_i$. Since in Model 3 $\nu$ is 80 but in Model 1 it is 155, $\sigma_\omega$ is significantly smaller, while the lower capital adjustment costs decreases $i_\theta$. Of course, the lower $\sigma_\omega$ requires a higher $\sigma$. Nevertheless, the net effect is negative for this correlation. Hence, there is a tension in the model since a lower $\sigma_\omega$, on the one hand, decreases correlations but, on the other hand, increases autocorrelations.

The reduction in the correlation between consumption and investment growth allows to also fit investment volatility in Model 3. To understand why, consider first the standard model and technology choice model with EIS=1.5. Since in these economies there is almost perfect correlation between the macroeconomic variables, the volatility of output growth is given by a weighted average of the volatilities of consumption and investment growth:

$$\sigma(\Delta y) = \frac{C}{Y}\sigma(\Delta c) + \frac{I}{Y}\sigma(\Delta i), \qquad \text{where} \qquad \frac{I}{Y} = \frac{e^{\mu} - (1-\delta)}{e^{\mu/\psi} - \beta(1-\delta)}\alpha\beta,$$

and where $Y$, $C$, and $I$ denote steady state values. For these models to fit all three volatilities, $I/Y$ has to decrease significantly. Since the parameters $\alpha$, $\delta$, and $\mu$ are fixed across all models, the remaining free parameters are $\beta$ and $\psi$. To match the required weight, either $\psi$ or $\beta$ need to be significantly lower. Unfortunately, in either case the risk-free rate increases, which makes it impossible to fit the average price-to-dividend ratio and the average risk-free rate. In Model 3, however, the relatively low correlation between consumption and investment, implies that the output volatility is lower than the weighted average of $\sigma(\Delta c)$ and $\sigma(\Delta i)$.[17]

[17]If consumption and investment were perfectly correlated in Model 3, the volatility of output growth nor-

## 6.2 Asset prices

Our model improves the fit to the moments of the risk-free rate compared to the standard model. The improvement is due to the fact that $\sigma_\mu$, the response of expected consumption growth $\mu_t$ to exogenous shocks, increases. From Proposition 5, we know that the risk-free rate in the model is entirely driven by $\mu_t$. Panel B of Table 5 shows that in Model 1, $\sigma_\mu$ with technology choice is 0.089% compared to 0.003% in the standard model, 0.224% compared to 0.018% for Model 2 and 0.114% compared to 0.008% in Model 3. The increase in $\sigma_\mu$ increases the volatility of the risk-free rate and decreases its autocorrelation and its correlation with the price-to-dividend ratio. The last feature of the model is explained by the fact that the price-to-dividend ratio is not affected by short-lived shocks to $\mu_t$, but is entirely driven by long-run expectations. Thus, the dynamics of $p - d$ are virtually unaffected by the increase in $\sigma_\mu$ and by Proposition 3 depend on the coefficients $\mu_k$ and $\tilde{\mu}_\theta$ that are independent of technology choice.

## 6.3 Macro-finance linkages

Technology choice augments the power of the risk-free rate, and its changes, to predict output, consumption, and investment growth, enabling the model to reproduce the data. To understand why, it suffices to consider that technology choice not only augments the variability of $\mu_t$, that is, of the predictable component of consumption growth, but also of the predictable components of investment and output. Then, a decrease in the EIS reduces the volatility of $\mu_t = \mathbb{E}_t(\Delta c_{t+1})$ and increases the volatility of $\mathbb{E}_t(\Delta i_{t+1})$. This explains why in Model 2 consumption is as predictable as investment and output, whereas in Models 1 and 2 consumption is less predictable and investment is more predictable than output, as shown in Table 4.

Finally, we discuss why technology choice generates negative correlations between lagged (HP filtered) macroeconomic quantities and the risk-free rate, while the standard model generates positive correlations. This is an in-sample property generated by the HP-filter, which estimates a trend in the data. As a result, the HP-filtered series mean revert and, therefore, the

malized by the volatility of TFP growth would be 0.70 instead of 0.64.

detrended series predict the shocks with a negative sign. Now, since with technology choice the risk-free rate is more sensitive to shocks, the HP-filtered macro quantities predict the risk-free rate with a negative sign. This is not the case in the standard model, where $\sigma_\mu$ is small. In the standard model, the risk-free rate is highly autocorrelated and driven by the level of productivity as well as the level of capital, which explains why the detrended series predict the risk-free rate with a positive sign.

# 7 Conclusions

In this paper, we embark on an abstract theoretical exploration of technology choice or state-contingent technology in a production-based economy. Our point of departure is that from a theoretical point of view, we have no reason to believe that risk aversion does not affect real life investment and production decisions. We are also not aware of empirical evidence that would unambiguously support such a hypothesis. Unfortunately, the feature that risk aversion can have, at most, a second-order effect is hardwired into the standard real business cycle model. In this respect, we find it important to analyze a general equilibrium model where risk-aversion, not just the elasticity of intertemporal substitution, matters. The empirics in the model are promising. Despite the fact that we adopt a reduced-form approach to characterize technology choice, the model reproduces several moments that the standard model is unable to account for, e.g., the volatility of investment growth, the low correlation between investment and consumption growth, the autocorrelations of the macroeconomic growth series, and several lead-lag relations between the interest rate and the macroeconomy. Therefore, we feel that this line of research is promising for our understanding of how the macroeconomy operates as well as for its link with the financial side. We close by acknowledging that we should develop a micro-founded model of technology choice and that we should incorporate time-varying risk aversion into the model since it has been successful in explaining a number of asset pricing facts that our current model cannot account for. We leave these more ambitious goals for future research.

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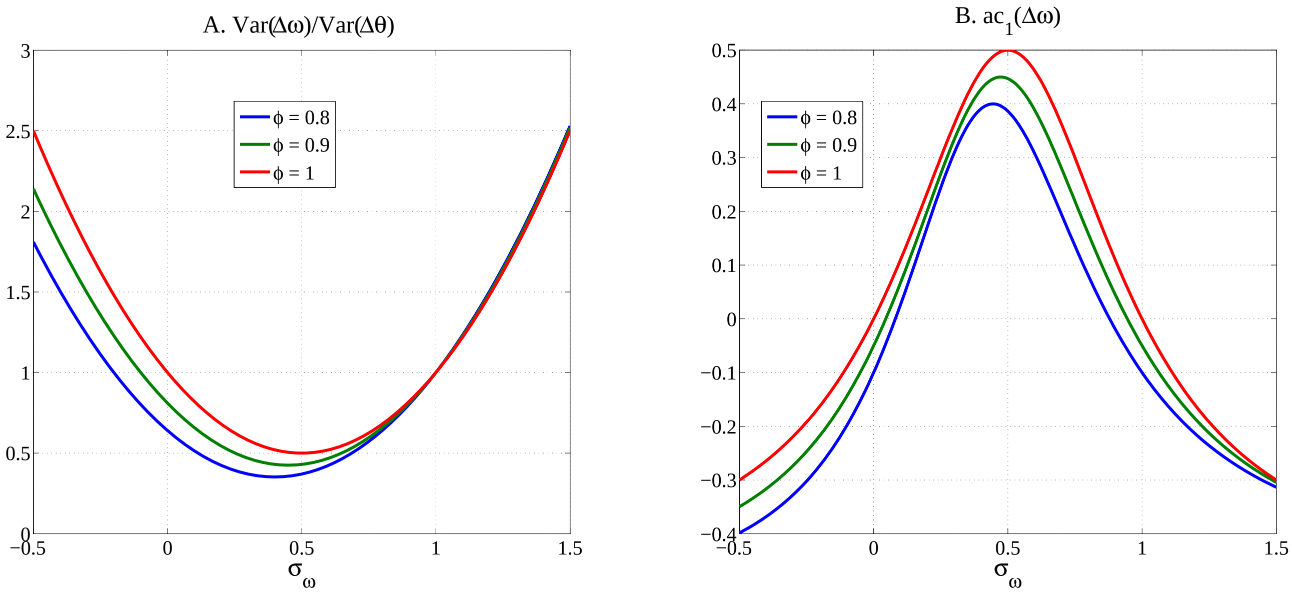


The figure plots the unconditional volatility and first-order autocorrelation of endogenous shocks with respect to the optimal technology choice ($\sigma_\omega$), assuming different persistence ($\phi$) of exogenous shocks.

Figure 1: Unconditional volatility and autocorrelation of endogenous shocks

## Table 1: Macroeconomic moments

The data statistics are computed from quarterly macroeconomic data obtained from the NIPA tables over 1947Q1-2012Q4. The model statistics are averages of 1000 simulated paths of 300 quarters with a burn-in of 100 quarters. Parentheses show the $t-statistics$ of the hypotheses that the data estimates are different from zero, $H_1$, or that they are different from the model averages, $H_2$. The standard errors of the data estimates are Newey and West (1987) corrected with 16 lags. $\Delta x$ denotes the first-difference of the natural logarithm of a variable $X$. $y$ denotes (the natural logarithm of) total output; $c$ denotes total consumption; $i$ denotes total investment and $\omega$ denotes total factor productivity. For a variable $x$, $\sigma(x)$ denotes its volatility; $ac_1(x)$ is its first-order autocorrelation and $\rho(x,z)$ is its correlation with variable $z$. The models with technology choice are calibrated to fit the TFP growth volatility, the consumption growth volatility, the first-order autocorrelation of the stock market log price-to-dividend ratio, the first-order autocorrelation of the TFP growth and the CRRA was chosen to fit the Sharpe ratio of the stock market portfolio. The exogenous volatility ($\sigma$) of the models without technology choice ($\nu=\infty$) is adjusted to fit the TFP growth volatility. For all models the average growth rate in the economy is $\mu=0.4\%$, the capital share is $\alpha=0.36$, the capital depreciation is $\delta=0.026$ and the dividend process parameters are $d=(-0.0026, 1.65, 0.04)$.

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | A. Volatilities | | | | | | | | | | | | | |
| | | | Model 1 | | | | Model 2 | | | | Model 3 | | | |
| | | | $\psi=0.3,\gamma=110,\chi=3.6$ $\beta=0.9997,\phi=0.970$ | | | | $\psi=1.5,\gamma=200,\chi=4.5$ $\beta=0.9935,\phi=0.980$ | | | | $\psi=0.3,\gamma=100,\chi=6.5$ $\beta=0.9995,\phi=0.970$ | | | |
| | Data | | $\nu=\infty$ $\sigma=1.80$ | | $\nu=155$ $\sigma=2.26$ | | $\nu=\infty$ $\sigma=1.80$ | | $\nu=260$ $\sigma=2.25$ | | $\nu=\infty$ $\sigma=1.80$ | | $\nu=80$ $\sigma=2.55$ | |
| | *est.* | $H_1$ | *avg.* | $H_2$ | *avg.* | $H_2$ | *avg.* | $H_2$ | *avg.* | $H_2$ | *avg.* | $H_2$ | *avg.* | $H_2$ |
| $\sigma(\Delta y)/\sigma(\Delta\omega)$ | 0.64 | | 0.64 | (0.01) | 0.64 | (0.02) | 0.64 | (0.02) | 0.64 | (0.05) | 0.64 | (0.00) | 0.64 | (0.03) |
| $\sigma(\Delta c)/\sigma(\Delta\omega)$ | 0.44 | | 0.39 | (0.79) | 0.44 | (0.04) | 0.43 | (0.17) | 0.44 | (0.06) | 0.35 | (1.29) | 0.44 | (0.04) |
| $\sigma(\Delta i)/\sigma(\Delta\omega)$ | 1.40 | | 1.31 | (0.42) | 1.21 | (0.85) | 1.12 | (1.27) | 1.11 | (1.30) | 1.40 | (0.01) | 1.40 | (0.00) |
| $\sigma(\Delta\omega)$ | 1.81 | | 1.81 | (0.01) | 1.81 | (0.05) | 1.81 | (0.04) | 1.81 | (0.00) | 1.81 | (0.01) | 1.81 | (0.05) |
| | B. Autocorrelations | | | | | | | | | | | | | |
| $ac_1(\Delta y)$ | 0.24 | (3.65) | -0.00 | (3.73) | 0.26 | (0.33) | 0.00 | (3.63) | 0.26 | (0.32) | -0.00 | (3.70) | 0.46 | (3.29) |
| $ac_1(\Delta c)$ | 0.04 | (0.58) | 0.01 | (0.49) | 0.11 | (1.05) | 0.03 | (0.15) | 0.28 | (3.50) | 0.01 | (0.37) | 0.15 | (1.52) |
| $ac_1(\Delta i)$ | 0.37 | (4.52) | -0.01 | (4.66) | 0.38 | (0.07) | -0.01 | (4.69) | 0.25 | (1.46) | -0.01 | (4.67) | 0.45 | (0.90) |
| $ac_1(\Delta\omega)$ | 0.25 | (3.67) | -0.02 | (3.92) | 0.25 | (0.03) | -0.01 | (3.86) | 0.25 | (0.04) | -0.02 | (3.92) | 0.45 | (2.89) |

Continued on next page

Table 2 – continued from previous page

| | Data | | Model 1 | | | | Model 2 | | | | Model 3 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | $\psi = 0.3, \gamma = 110, \chi = 3.6$ $\beta = 0.9997, \phi = 0.970$ | | | | $\psi = 1.5, \gamma = 200, \chi = 4.5$ $\beta = 0.9935, \phi = 0.980$ | | | | $\psi = 0.3, \gamma = 100, \chi = 6.5$ $\beta = 0.9995, \phi = 0.970$ | | | |
| | | | $\nu = \infty$ $\sigma = 1.80$ | | $\nu = 155$ $\sigma = 2.26$ | | $\nu = \infty$ $\sigma = 1.80$ | | $\nu = 260$ $\sigma = 2.25$ | | $\nu = \infty$ $\sigma = 1.80$ | | $\nu = 80$ $\sigma = 2.55$ | |
| | *est.* | $H_1$ | *avg.* | $H_2$ | *avg.* | $H_2$ | *avg.* | $H_2$ | *avg.* | $H_2$ | *avg.* | $H_2$ | *avg.* | $H_2$ |
| **C. Correlations** | | | | | | | | | | | | | | |
| $\rho(\Delta y, \Delta i)$ | 0.68 | | 1.00 | (6.36) | 0.99 | (6.08) | 1.00 | (6.32) | 1.00 | (6.30) | 1.00 | (6.35) | 0.90 | (4.36) |
| $\rho(\Delta y, \Delta c)$ | 0.62 | | 1.00 | (9.99) | 0.99 | (9.63) | 1.00 | (9.94) | 1.00 | (9.92) | 1.00 | (9.98) | 0.93 | (8.11) |
| $\rho(\Delta c, \Delta i)$ | 0.43 | | 1.00 | (8.79) | 0.94 | (7.94) | 0.99 | (8.67) | 0.99 | (8.60) | 1.00 | (8.75) | 0.67 | (3.74) |
| $\rho(\Delta \omega, \Delta y)$ | 0.99 | | 1.00 | (2.33) | 1.00 | (1.24) | 1.00 | (3.03) | 1.00 | (2.33) | 1.00 | (2.05) | 0.99 | (0.08) |
| $\rho(\Delta \omega, \Delta c)$ | 0.67 | | 0.99 | (6.62) | 0.98 | (6.34) | 0.99 | (6.56) | 0.99 | (6.48) | 0.99 | (6.56) | 0.89 | (4.49) |
| $\rho(\Delta \omega, \Delta i)$ | 0.61 | | 1.00 | (10.45) | 0.98 | (10.03) | 1.00 | (10.47) | 1.00 | (10.47) | 1.00 | (10.46) | 0.93 | (8.50) |
| **D. Cross-correlations** | | | | | | | | | | | | | | |
| $\rho(\Delta y, \Delta i_{-1})$ | 0.16 | (1.76) | -0.01 | (1.83) | 0.27 | (1.24) | 0.00 | (1.75) | 0.26 | (1.09) | -0.00 | (1.82) | 0.55 | (4.30) |
| $\rho(\Delta y, \Delta c_{-1})$ | 0.11 | (2.03) | -0.00 | (2.10) | 0.25 | (2.53) | 0.00 | (1.99) | 0.26 | (2.83) | -0.00 | (2.07) | 0.31 | (3.70) |
| $\rho(\Delta c, \Delta i_{-1})$ | 0.08 | (1.19) | 0.01 | (1.10) | 0.10 | (0.36) | 0.02 | (0.87) | 0.27 | (2.94) | 0.01 | (1.01) | 0.09 | (0.16) |
| $\rho(\Delta \omega, \Delta y_{-1})$ | 0.23 | (3.37) | -0.02 | (3.74) | 0.24 | (0.23) | -0.02 | (3.62) | 0.24 | (0.21) | -0.02 | (3.74) | 0.43 | (3.08) |
| $\rho(\Delta \omega, \Delta c_{-1})$ | 0.15 | (1.59) | -0.03 | (1.89) | 0.25 | (1.08) | -0.02 | (1.82) | 0.23 | (0.88) | -0.03 | (1.92) | 0.53 | (4.05) |
| $\rho(\Delta \omega, \Delta i_{-1})$ | 0.07 | (1.48) | -0.02 | (1.90) | 0.23 | (3.01) | -0.01 | (1.72) | 0.25 | (3.43) | -0.02 | (1.89) | 0.29 | (4.19) |
| **E. Annual (time-aggregated) statistics** | | | | | | | | | | | | | | |
| $\mu(\Delta c)$ | 1.79 | | 1.60 | (0.80) | 1.60 | (0.80) | 1.60 | (0.80) | 1.60 | (0.80) | 1.60 | (0.80) | 1.60 | (0.80) |
| $\sigma(\Delta c)$ | 1.49 | | 1.18 | (2.06) | 1.45 | (0.24) | 1.37 | (0.79) | 1.68 | (1.27) | 1.10 | (2.61) | 1.51 | (0.11) |
| $\mu(\Delta d)$ | 2.09 | | 1.61 | (0.53) | 1.61 | (0.53) | 1.61 | (0.53) | 1.61 | (0.53) | 1.61 | (0.53) | 1.61 | (0.53) |
| $\sigma(\Delta d)$ | 7.09 | | 6.88 | (0.24) | 7.02 | (0.08) | 6.97 | (0.13) | 7.15 | (0.06) | 6.84 | (0.27) | 7.05 | (0.05) |
| $\rho(\Delta c, \Delta d)$ | 0.32 | | 0.28 | (0.42) | 0.34 | (0.15) | 0.32 | (0.02) | 0.38 | (0.60) | 0.26 | (0.59) | 0.35 | (0.25) |

Table 2: Asset pricing moments

The empirical data statistics are computed from quarterly return data obtained from CRSP and WRDS Federal Reserve data (3-month T-bill yields) over 1947Q1-2012Q4. The model statistics are computed as averages from 1000 simulated paths, where each path has 300 quarters with a burn-in of 100 quarters. Parentheses show the $t-statistics$ of the hypothesis that the data estimates are different from the model averages ($H_2$). The standard errors of the data estimates are Newey and West (1987) corrected with 16 lags. $p-d$ denotes the log price-to-dividend ratio of the stock market portfolio; $R_f$ is the quarterly risk-free rate and $R$ is the quarterly stock market return. For a variable $x$ $\mu(x)$ is the mean; $\sigma(x)$ is the standard deviation and $ac_1(x)$ is the first-order autocorrelation. The Sharpe ratio, $SR$, equals $\mu(R-R_f)/\sigma(R-R_f)$ and $\rho(p-d, r_f)$ denotes the correlation between $p-d$ and the log risk-free rate. For the model calibrations and other parameter values see caption of Table 2.

| | | Model 1 | | | | Model 2 | | | | Model 3 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $\psi = 0.3, \gamma = 110, \chi = 3.6$<br>$\beta = 0.9997, \phi = 0.970$ | | | | $\psi = 1.5, \gamma = 200, \chi = 4.5$<br>$\beta = 0.9935, \phi = 0.980$ | | | | $\psi = 0.3, \gamma = 100, \chi = 6.5$<br>$\beta = 0.9995, \phi = 0.970$ | | | |
| | Data | $\nu = \infty$<br>$\sigma = 1.80$ | | $\nu = 155$<br>$\sigma = 2.26$ | | $\nu = \infty$<br>$\sigma = 1.80$ | | $\nu = 260$<br>$\sigma = 2.25$ | | $\nu = \infty$<br>$\sigma = 1.80$ | | $\nu = 80$<br>$\sigma = 2.55$ | |
| | *est.* | *avg.* | $H_2$ | *avg.* | $H_2$ | *avg.* | $H_2$ | *avg.* | $H_2$ | *avg.* | $H_2$ | *avg.* | $H_2$ |
| $\mu(p-d)$ | 4.87 | 4.81 | (0.57) | 4.87 | (0.03) | 5.06 | (1.93) | 4.88 | (0.06) | 4.78 | (0.87) | 4.87 | (0.02) |
| $\sigma(p-d)$ | 0.42 | 0.04 | (6.60) | 0.05 | (6.43) | 0.04 | (6.62) | 0.05 | (6.49) | 0.04 | (6.61) | 0.05 | (6.34) |
| $ac_1(p-d)$ | 0.98 | 0.98 | (0.31) | 0.99 | (0.68) | 0.99 | (0.71) | 0.99 | (1.04) | 0.99 | (0.65) | 0.99 | (0.97) |
| $\mu(R_f)$ | 0.23 | 0.57 | (2.77) | 0.28 | (0.43) | 0.53 | (2.46) | 0.43 | (1.66) | 0.73 | (4.15) | 0.32 | (0.74) |
| $\sigma(R_f)$ | 0.94 | 0.12 | (7.05) | 0.34 | (5.16) | 0.05 | (7.72) | 0.16 | (6.73) | 0.14 | (6.92) | 0.43 | (4.38) |
| $ac_1(R_f)$ | 0.34 | 0.99 | (6.69) | 0.26 | (0.84) | 0.95 | (6.35) | 0.21 | (1.36) | 0.97 | (6.52) | 0.28 | (0.63) |
| $\mu(R_m)$ | 2.03 | 1.32 | (1.62) | 1.28 | (1.71) | 1.12 | (2.06) | 1.25 | (1.76) | 1.34 | (1.57) | 1.27 | (1.72) |
| $\sigma(R_m)$ | 8.26 | 4.41 | (7.04) | 4.47 | (6.93) | 4.13 | (7.54) | 4.16 | (7.48) | 4.33 | (7.18) | 4.42 | (7.01) |
| $ac_1(R_m)$ | 0.08 | -0.00 | (1.37) | 0.03 | (0.92) | -0.00 | (1.39) | 0.01 | (1.26) | 0.00 | (1.35) | 0.03 | (0.80) |
| $\mu(R_m - R_f)$ | 1.79 | 0.75 | (2.51) | 0.99 | (1.92) | 0.59 | (2.88) | 0.82 | (2.33) | 0.60 | (2.86) | 0.95 | (2.02) |
| $\sigma(Rm - R_f)$ | 8.19 | 4.40 | (6.90) | 4.45 | (6.81) | 4.13 | (7.39) | 4.16 | (7.34) | 4.32 | (7.05) | 4.40 | (6.90) |
| $ac_1(R_m - R_f)$ | 0.08 | -0.00 | (1.49) | -0.00 | (1.48) | -0.00 | (1.48) | -0.00 | (1.48) | -0.00 | (1.48) | -0.00 | (1.48) |
| $SR$ | 0.22 | 0.17 | (0.80) | 0.22 | (0.07) | 0.14 | (1.24) | 0.20 | (0.34) | 0.14 | (1.30) | 0.22 | (0.04) |
| $\rho(p-d, r_f)$ | 0.04 | -0.69 | (5.75) | -0.27 | (2.45) | 0.27 | (1.82) | 0.11 | (0.52) | -0.49 | (4.17) | -0.21 | (1.99) |

Table 3: Risk-free rate correlations with macroeconomic quantities

The data risk-free rate is taken to be the quarterly 3-month T-bill yield obtained from WRDS Federal Reserve data, while the quarterly macroeconomic data are obtained from NIPA. The data period is from 1947Q1 to 2012Q4. The model statistics are computed as averages from 1000 simulated paths, where each path has 300 quarters with a burn-in of 100 quarters. Parentheses show the $t-statistics$ of the hypotheses that the data estimates are different from zero, $H_1$, or that they are different from the model averages, $H_2$. The standard errors of the data estimates are Newey and West (1987) corrected with 16 lags. $\Delta x$ denotes the first-difference of the natural logarithm of a variable $X$. $y$ denotes (the HP filtered natural logarithm of) total output and similarly, $c$ corresponds to consumption, $i$ corresponds to investment and $\omega$ corresponds to total factor productivity. $r_f$ denotes the log risk-free rate. For the model calibrations and other parameter values see caption of Table 2.

| | | | Model 1 | | | | Model 2 | | | | Model 3 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | $\psi=0.3,\gamma=110,\chi=3.6$ $\beta=0.9997,\phi=0.970$ | | | | $\psi=1.5,\gamma=200,\chi=4.5$ $\beta=0.9935,\phi=0.980$ | | | | $\psi=0.3,\gamma=100,\chi=6.5$ $\beta=0.9995,\phi=0.970$ | | | |
| | Data | | $\nu=\infty$ $\sigma=1.80$ | | $\nu=155$ $\sigma=2.26$ | | $\nu=\infty$ $\sigma=1.80$ | | $\nu=260$ $\sigma=2.25$ | | $\nu=\infty$ $\sigma=1.80$ | | $\nu=80$ $\sigma=2.55$ | |
| | *est.* | $H_1$ | *avg.* | $H_2$ | *avg.* | $H_2$ | *avg.* | $H_2$ | *avg.* | $H_2$ | *avg.* | $H_2$ | *avg.* | $H_2$ |
| $\rho(y_t, r_{f,t+1})$ | -0.18 | (1.95) | 0.09 | (2.91) | -0.27 | (1.03) | 0.29 | (5.17) | -0.21 | (0.35) | 0.21 | (4.30) | -0.21 | (0.32) |
| $\rho(\Delta y_t, \Delta r_{f,t-1})$ | 0.40 | (5.47) | 0.01 | (5.33) | 0.21 | (2.64) | 0.00 | (5.41) | 0.20 | (2.67) | 0.00 | (5.41) | 0.42 | (0.27) |
| $\rho(\Delta y_t, r_{f,t-1})$ | 0.34 | (3.59) | 0.09 | (2.68) | 0.28 | (0.61) | 0.04 | (3.12) | 0.27 | (0.74) | 0.06 | (2.98) | 0.53 | (1.92) |
| $\rho(c_t, r_{f,t+1})$ | -0.23 | (2.17) | 0.08 | (2.88) | -0.28 | (0.44) | 0.27 | (4.64) | -0.24 | (0.08) | 0.20 | (3.98) | -0.22 | (0.11) |
| $\rho(\Delta c_t, \Delta r_{f,t-1})$ | 0.47 | (5.31) | 0.00 | (5.27) | 0.08 | (4.42) | 0.01 | (5.19) | 0.19 | (3.14) | 0.01 | (5.25) | 0.10 | (4.20) |
| $\rho(\Delta c_t, r_{f,t-1})$ | 0.56 | (7.33) | 0.10 | (6.04) | 0.14 | (5.47) | 0.10 | (5.96) | 0.30 | (3.31) | 0.09 | (6.13) | 0.17 | (5.09) |
| $\rho(i_t, r_{f,t+1})$ | 0.00 | (0.03) | 0.09 | (1.38) | -0.26 | (3.86) | 0.31 | (4.63) | -0.18 | (2.68) | 0.22 | (3.31) | -0.19 | (2.84) |
| $\rho(\Delta i_t, \Delta r_{f,t-1})$ | 0.27 | (4.02) | 0.02 | (3.79) | 0.32 | (0.71) | -0.00 | (4.04) | 0.21 | (0.89) | 0.00 | (3.97) | 0.63 | (4.93) |
| $\rho(\Delta i_t, r_{f,t-1})$ | 0.15 | (2.50) | 0.08 | (1.21) | 0.41 | (4.27) | -0.01 | (2.61) | 0.24 | (1.48) | 0.03 | (1.92) | 0.74 | (9.21) |

Table 4: Log-linear solution

The table provides the parameters of the log-linear solution of the three calibrated models:

$$x_t = x_k\,k_t + x_\theta\,\theta_t + x_\omega\,\omega_t \qquad \Rightarrow \qquad x_t = x_k\,k_t + \tilde{x}_\theta\,\theta_{t-1} + \sigma_x\,\epsilon_t,$$

where $x_t$ is the log-deviation of variable $X$ from the steady-state, and where $\tilde{x}_\theta = \phi(x_\omega + x_\theta)$ and $\sigma_x = \sigma_\omega x_\omega + x_\theta$. When $x = k$, $x_t$ refers to $k_{t+1}$, which is determined at the end of period $H_2$. See Proposition 1 for further information. For each model configuration, the standard model differs from the technology choice model only in the parameters $\nu$ and $\sigma$, which affect only $\sigma_\omega$ and, hence, the $\sigma_x$'s. The parameters in Panel A and the parameters $\tilde{x}_\theta$ in Panel B are the same for the standard and the technology choice model. Panel B also provides the coefficients of the exogenous shocks, $\sigma_x$'s, in percentages, and after being multiplied by the volatility of the exogenous shocks, $\sigma$. The coefficients are provided for TFP ($\omega$), capital ($k$), output ($y$), consumption ($c$), investment ($i$), expected consumption growth ($\mu$), and utility ($u$). The last row of Panel B shows the coefficient of the stochastic discount factor ($m$) to the exogenous shock, whose absolute value is the price of risk, i.e. the maximum conditional Sharpe ratio.

| | Model 1 | | | Model 2 | | | Model 3 | | |
|---|---|---|---|---|---|---|---|---|---|
| | $\psi = 0.3, \gamma = 110, \chi = 3.6$ | | | $\psi = 1.5, \gamma = 200, \chi = 4.5$ | | | $\psi = 0.3, \gamma = 100, \chi = 6.5$ | | |
| | $\beta = 0.9997, \phi = 0.970$ | | | $\beta = 0.9935, \phi = 0.980$ | | | $\beta = 0.9995, \phi = 0.970$ | | |
| | A. Coefficients to state vector $(k_t, \theta_t, \omega_t)$ | | | | | | | | |
| $x$ | $x_k$ | $x_\theta$ | $x_\omega$ | $x_k$ | $x_\theta$ | $x_\omega$ | $x_k$ | $x_\theta$ | $x_\omega$ |
| $k$ | 0.982 | -0.017 | 0.056 | 0.967 | -0.001 | 0.034 | 0.981 | -0.019 | 0.061 |
| $y$ | 0.360 | 0.000 | 0.640 | 0.360 | 0.000 | 0.640 | 0.360 | 0.000 | 0.640 |
| $c$ | 0.342 | 0.210 | 0.179 | 0.563 | 0.013 | 0.416 | 0.354 | 0.236 | 0.118 |
| $i$ | 0.408 | -0.562 | 1.875 | -0.100 | -0.029 | 1.148 | 0.376 | -0.637 | 2.047 |
| $\mu$ | -0.006 | 0.161 | -0.159 | -0.018 | 0.406 | -0.396 | -0.007 | 0.101 | -0.097 |
| $u$ | 0.121 | 0.206 | 0.008 | 0.077 | 0.186 | 0.005 | 0.123 | 0.209 | 0.009 |
| | B. Coefficients of $\theta_{t-1}$ and $\epsilon_t$ | | | | | | | | |
| | | $\sigma_x\ (\times 100\sigma)$ | | | $\sigma_x\ (\times 100\sigma)$ | | | $\sigma_x\ (\times 100\sigma)$ | |
| | | $\nu = \infty$ | $\nu = 155$ | | $\nu = \infty$ | $\nu = 260$ | | $\nu = \infty$ | $\nu = 80$ |
| $x$ | $\tilde{x}_\theta$ | $\sigma = 1.80$ | $\sigma = 2.26$ | $\tilde{x}_\theta$ | $\sigma = 1.80$ | $\sigma = 2.25$ | $\tilde{x}_\theta$ | $\sigma = 1.80$ | $\sigma = 2.55$ |
| $\omega$ | 0.970 | 1.800 | 1.729 | 0.980 | 1.800 | 1.743 | 0.970 | 1.800 | 1.488 |
| $k$ | 0.038 | 0.071 | 0.059 | 0.033 | 0.060 | 0.058 | 0.041 | 0.076 | 0.042 |
| $y$ | 0.621 | 1.152 | 1.106 | 0.627 | 1.152 | 1.116 | 0.621 | 1.152 | 0.953 |
| $c$ | 0.377 | 0.699 | 0.783 | 0.420 | 0.771 | 0.753 | 0.344 | 0.638 | 0.779 |
| $i$ | 1.274 | 2.364 | 1.972 | 1.096 | 2.013 | 1.935 | 1.367 | 2.537 | 1.421 |
| $\mu$ | 0.002 | 0.003 | 0.089 | 0.010 | 0.018 | 0.224 | 0.004 | 0.008 | 0.114 |
| $u$ | 0.208 | 0.386 | 0.480 | 0.187 | 0.343 | 0.426 | 0.211 | 0.391 | 0.545 |
| $m$ | | -43.503 | -53.827 | | -68.868 | -85.462 | | -39.955 | -55.303 |

# A Online appendix (Not for publication)

## A Technology choice

The following section provides some economic intuition for the reduced-form formulation of technology choice, which we borrow from Cochrane (1993), in our economy. Suppose that the central planner can choose to invest in a complete set of different technologies as in Jermann (2010). With a complete set we mean that there are as many independent technologies, indexed by $i = [1, \ldots, I]$, as there are states of nature denoted by $s = [1, \ldots, S]$. The productivity of a technology $i$ is denoted by $\Theta_i(s)$ for state $s$. Without loss of generality, let also $\Theta_1(s)$ be the productivity next period for the exogenous benchmark technology which is log-normally distributed,

$$\log \Theta_1 = \mu + \epsilon, \tag{A1}$$

where $\epsilon \sim N(0, \sigma^2)$. Define

$$\vartheta_i(s) = \frac{\Theta_i(s)}{\Theta_1(s)}, \forall i = 1, ..., I,$$

where by definition $\vartheta_1(s) = 1$.

Each technology produces the same good and the production of a technology $i$ is given by

$$Y_i(s) = K_i^{\alpha} \Theta_i(s)^{1-\alpha},$$

where $K_i$ is the capital invested in technology $i$ at the beginning of the current period. The central planner has a total of $K$ capital to allocate over the set of technologies. Let $w_i$ be the fraction invested in technology $i$, i.e.,

$$w_i = \frac{K_i}{K}.$$

Then, total production can be expressed as follows:

$$Y = K^{\alpha} \Theta_1^{1-\alpha} \sum_{i=1}^{I} w_i^{\alpha} \vartheta_i(s)^{1-\alpha}.$$

Let us now define

$$T(\mathbf{w}, s) = \sum_{i=1}^{I} w_i^{\alpha} \vartheta_i(s)^{1-\alpha} \qquad \text{and} \qquad \Omega(s) = \Theta_1(s) T(\mathbf{w}, s)^{1/1-\alpha}.$$

Then, aggregate output can be rewritten as

$$Y = K^{\alpha}\Omega^{1-\alpha},$$

where $\Omega$ becomes the endogenously chosen productivity or technology next period through the choice of the portfolio of technologies $\mathbf{w} = [w_1, \ldots, w_I]$. Since, the production technology market is complete, instead of choosing $\mathbf{w}$ the social planner can directly choose $\Omega$ (or $T$) in all future states given, of course, the joint productivity distribution of the technologies. Instead of specifying, however, the joint productivity distribution of the available technologies, we adopt the reduced-form assumption by which we can choose $T$ given the constraint

$$\mathbb{E}\left[T^{\nu}\right] \leq 1, \tag{A2}$$

for some constant $\nu$. This implies that the endogenously chosen productivity $\Omega$ can have any conditional distribution as long as (A2) holds. Since we log-linearize the economy, the endogenous productivity next period $\Omega$ can be expressed as

$$\log \Omega = \log X + \sigma_{\omega}\epsilon + \sigma_u u, \tag{A3}$$

where $u \sim N(0,1)$ is an innovation to productivity orthogonal to $\epsilon$. The central planner can therefore choose, $\sigma_{\omega}$, $\sigma_u$ and $X$ according to a certain objective and subject to the constraint (A3). Choosing $\sigma_{\omega} = 1$, $\sigma_u = 0$ and $\log X = \mu$ ensures that $\Omega = \Theta$.

To understand the role of the parameter $\nu$, we can derive the optimal choice for $\sigma_{\omega}$, $\sigma_u$, and $\log X$ from maximizing average production next period, which is given by

$$\mathbb{E}\left[\Omega^{1-\alpha}\right] = X^{1-\alpha} \exp\left[\frac{1}{2}(1-\alpha)^2(\sigma^2\sigma_{\omega}^2 + \sigma_u^2)\right].$$

Then, we can investigate the cost to average production from deviating from such a choice. Note, first, that the productivity choice constraint (A2) implies that

$$X^{1-\alpha} \leq \exp\left\{(1-\alpha)\mu + \frac{1}{\nu} - \frac{1}{2}\nu(1-\alpha)^2\left[(\sigma_{\omega}-1)^2 + \sigma_u^2\right]\right\}.$$

Assuming, therefore, that the above constraint is binding at the optimum, we have that the average productivity next period is given by

$$\mathbb{E}\left[\Omega^{1-\alpha}\right] = \exp\left\{(1-\alpha)\mu + \frac{1}{\nu} + \frac{1}{2}(1-\alpha)^2\left[\sigma_{\omega}^2\sigma^2 - \nu(\sigma_{\omega}-1)^2\sigma^2 + (1-\nu)\sigma_u^2\right]\right\}.$$

Maximizing next period's average production would then mean that

$$\max_{\sigma_\omega,\sigma_u} \qquad \sigma_\omega^2\sigma^2 - \nu(\sigma_\omega - 1)^2\sigma^2 + (1-\nu)\sigma_u^2.$$

Given this maximization problem, if $\nu$ was less than one then increasing $\sigma_u$ as much as possible would be the optimal decision. To avoid such examples, we restrict to cases where $\nu > 1$ and, therefore, the optimal solution is $\sigma_u^* = 0$. The optimal exposure to the exogenous productivity of the benchmark technology becomes then

$$\sigma_\omega^* = \frac{\nu}{\nu - 1},$$

which ensures the maximum average production next period. If any other exposure $\sigma_\omega = \sigma_\omega^* - \Delta$ is chosen, then the cost to the average production is proportional to $(\nu-1)\Delta^2$. Therefore, the larger the parameter $\nu$ is, the larger is the cost to average production from a deviation $\Delta$ from the growth optimal choice. When $\nu \to \infty$, then it becomes infinitely costly to deviate from the exogenous benchmark productivity and $\sigma_\omega^* \to 1$.

# B Loglinearization

## B.1 Equilibrium conditions

With a slight abuse of notation, all variables below are normalized by the time trend. The equilibrium conditions for recursive preferences with technology choice are summarized as follows:

$$\begin{aligned}
\lambda_t^1 &= (1-\beta)U_t^{\frac{1}{\psi}}C_t^{-\frac{1}{\psi}}, && \text{(B4)}\\
\lambda_t^1 &= \lambda_t^2 g'_{I_t}, && \text{(B5)}\\
\lambda_t^2 e^{\mu} &= \mathbb{E}_t\left[\frac{\partial U_t}{\partial U_{t+1}}\frac{\partial U_{t+1}}{\partial K_{t+1}}\right], && \text{(B6)}\\
0 &= \frac{\partial U_t}{\partial U_{t+1}}\frac{\partial U_{t+1}}{\partial \Omega_{t+1}} - \lambda_t^3\nu\left(\frac{\Omega_{t+1}^{1-\alpha}}{\Theta_{t+1}^{1-\alpha}}\right)^{\nu-1}\frac{1-\alpha}{\Theta_{t+1}^{1-\alpha}}\Omega_{t+1}^{-\alpha}, && \text{(B7)}\\
\Omega_t^{1-\alpha}K_t^{\alpha} &= C_t + I_t, && \text{(B8)}\\
K_{t+1}e^{\mu} &= (1-\delta)K_t + g_t, && \text{(B9)}\\
1 &= \mathbb{E}_t\left[\frac{\Omega_{t+1}^{1-\alpha}}{\Theta_{t+1}^{1-\alpha}}\right]^{\nu}, && \text{(B10)}\\
U_t &= \left\{(1-\beta)\,C_t^{1-1/\psi} + \beta e^{\mu(1-\gamma)/\theta}\mathbb{E}_t\left[U_{t+1}^{1-\gamma}\right]^{\frac{1-1/\psi}{1-\gamma}}\right\}^{\frac{1}{1-1/\psi}}. && \text{(B11)}
\end{aligned}$$

The key variables in the deterministic steady-state of the economy are described by

$$\begin{aligned}
K &= \left[\frac{e^{\mu/\psi} - \beta(1-\delta)}{\alpha\beta}\right]^{\frac{1}{\alpha-1}}, \\
C &= K^{\alpha} - (e^{\mu} - 1 + \delta)K, \\
I &= (e^{\mu} - 1 + \delta)K, \\
U &= C\left[\frac{1-\beta}{1-\beta e^{\mu(1-1/\psi)}}\right]^{\frac{1}{1-1/\psi}}, \\
\lambda^1 &= (1-\beta)\left[\frac{1-\beta}{1-\beta e^{\mu(1-1/\psi)}}\right]^{\frac{1}{\psi-1}}, \\
\lambda^2 &= (1-\beta)\left[\frac{1-\beta}{1-\beta e^{\mu(1-1/\psi)}}\right]^{\frac{1}{\psi-1}}, \\
\lambda^3 &= \frac{\beta K^{\alpha} e^{\mu(1-1/\psi)}\lambda^1}{\nu},
\end{aligned}$$

where variables without subscripts indicate steady-state values. Clearly, the deterministic state is independent of risk aversion $\gamma$, and only $\lambda^3$ depends on the technology choice curvature $\nu$.

## B.2 Loglinearization: Recursive preferences with technology choice

By convention, the percentage deviation of variable $X_t$ from its detrended steady-state value $(X)$ is defined as $x_t = logX_t - logX$. For example, the exogenous technology shock process can be rewritten as $\theta_t = \phi\,\theta_{t-1} + \epsilon_t$ where $\epsilon \sim \mathbb{N}(0, \sigma^2)$. The loglinearized model depends on the three state variables $\theta_t$, $k_t$, and $\omega_t$, which measure the percentage deviation from the steady-state values of the detrended variables $\Theta_t$, $K_t$, and $\Omega_t$.

The percentage deviations of consumption, investment, and utility can be summarized as follows

$$c_t = c_k k_t + c_\omega \omega_t + c_\theta \theta_t, \tag{B12}$$

$$i_t = i_k k_t + i_\omega \omega_t + i_\theta \theta_t, \tag{B13}$$

$$u_t = u_k k_t + u_\omega \omega_t + u_\theta \theta_t, \tag{B14}$$

where $c_k$, $c_\omega$, $c_\theta$, $i_k$, $i_\omega$, $i_\theta$,$u_k$, $u_\omega$, and $u_\theta$ are coefficients to be determined.

Log-linearizing the above equilibrium conditions (B7) and (B10) gives the optimal technology choice

$$\omega_{t+1} = \phi(1-\sigma_\omega)\theta_t + \sigma_\omega\theta_{t+1} = \phi\theta_t + \sigma_\omega\epsilon_{t+1}, \tag{B15}$$

where

$$\sigma_\omega = \frac{(1-\alpha)\nu - \frac{1}{\psi}c_\theta + (\frac{1}{\psi}-\gamma)u_\theta}{(1-\alpha)(\nu-1) + \frac{1}{\psi}c_\omega + (\gamma - \frac{1}{\psi})u_\omega}. \tag{B16}$$

Loglinearizing the equilibrium conditions gives the coefficients. For example, $c_k$ is the positive root from the following quadratic equation

$$0 = B\Big[\frac{\alpha(C+I)k_2}{I} + k_1\Big] - \frac{\alpha(C+I)-I}{\chi I} - \Big(\frac{Bk_2C}{I} - \frac{1}{\psi} - \frac{C}{\chi I}\Big)c_k, \tag{B17}$$

where

$$B = \frac{\alpha K^{\alpha-1}(\alpha-1)}{\alpha K^{\alpha-1}+1-\delta} - \frac{c_k}{\psi} + \frac{1}{\chi(\alpha K^{\alpha-1}+1-\delta)}\left[\frac{\alpha(C+I)}{I} - \frac{C}{I}c_k - 1\right], \tag{B18}$$

$$k_1 = \frac{1-\delta}{e^\mu}, \tag{B19}$$

$$k_2 = \frac{e^\mu - 1 + \delta}{e^\mu}. \tag{B20}$$

The other coefficients are given by:

$$c_\omega = \frac{\frac{(\alpha-1)(C+I)}{\chi I} + B\frac{k_2}{I}(1-\alpha)(C+I)}{Bk_2\frac{C}{I} - \frac{1}{\psi} - \frac{C}{\chi I}}, \tag{B21}$$

$$c_\theta = \frac{\phi\left\{\frac{\alpha K^{\alpha-1}(1-\alpha)}{\alpha K^{\alpha-1}+1-\delta} - \frac{c_\omega}{\psi} + \frac{1}{\chi(\alpha K^{\alpha-1}+1-\delta)}\left[\frac{(1-\alpha)(C+I)}{I} - \frac{C}{I}c_\omega\right]\right\}}{\phi\left(\frac{1}{\psi} + \frac{C}{(\alpha K^{\alpha-1}+1-\delta)\chi I}\right) + \frac{Bk_2C}{I} - \frac{1}{\psi} - \frac{C}{\chi I}}, \tag{B22}$$

$$i_k = \frac{\alpha(C+I)}{I} - \frac{C}{I}c_k, \tag{B23}$$

$$i_\omega = \frac{(1-\alpha)(C+I)}{I} - \frac{C}{I}c_\omega, \tag{B24}$$

$$i_\theta = -\frac{C}{I}c_\theta, \tag{B25}$$

$$u_k = \frac{u_1 c_k}{1 - u_2 k_1 - u_2 k_2 i_k}, \tag{B26}$$

$$u_\omega = u_1 c_\omega + u_2 k_2 u_k i_\omega, \tag{B27}$$

$$u_\theta = \frac{u_1 c_\theta + u_2 k_2 u_k i_\theta + \phi u_2 u_\omega}{1 - \phi u_2}, \tag{B28}$$

where

$$u_1 = \frac{1-\beta}{1-\beta+\beta\frac{1-\beta}{e^{\mu(\frac{1}{\psi}-1)}-\beta}}, \tag{B29}$$

$$u_2 = \frac{\beta\frac{1-\beta}{e^{\mu(\frac{1}{\psi}-1)}-\beta}}{1-\beta+\beta\frac{1-\beta}{e^{\mu(\frac{1}{\psi}-1)}-\beta}}. \tag{B30}$$

From the above equations, we see that coefficients $u_k, u_\omega, u_\theta, c_k, c_\omega, c_\theta, i_k, i_\omega, i_\theta$ are dependent on EIS ($\psi$) but independent of the risk aversion ($\gamma$) and technology choice curvature ($\nu$). Moreover, from equation (B16), $\sigma_\omega$ depends on the risk aversion ($\gamma$) and technology choice curvature ($\nu$). Thus, in a standard RBC economy without technology choice, macroeconomic quantities are not risk aversion sensitive. Introducing technology choice makes macroeconomic quantities sensitive to the risk aversion. Proposition 1 concludes the above subsection.

# C Stock and bond prices

The stochastic discount factor, $M$, is log-normally distributed, as shown in Proposition 4, and can be expressed in the following form:

$$\log M_{t,t+1} = \log\hat{\beta} - \frac{1}{\psi}\mu_t - \sigma_m\epsilon_{t+1} = m_0 + m_1\, z_t - \sigma_m\, \epsilon_{t+1},$$

where $z_t = [k_{t+1}, \theta_t, c_t]'$ is a sufficient state vector, which is also normally distributed, and whose law of motion can be expressed as,

$$z_{t+1} = Z\, z_t + \Sigma_z\, \epsilon_{t+1}.$$

The coefficients $Z$ and $\Sigma_z$ are inferred from equations (14) and (15), while the coefficients of the stochastic discount factor are $m_0 = \log\hat{\beta} - \mu/\psi$ and $m_1 = -[c_k, c_\theta, -1]/\psi$.

## C.1 Zero coupon bonds

The natural logarithm of the prices of the zero-coupon bonds $q_t^{(n)}$, that pay a unit of consumption $n$ periods ahead, satisfy recursively the standard Euler equation

$$q_t^{(n)} = \log\mathbb{E}_t\left[M_{t,t+1}\exp\left(q_{t+1}^{(n-1)}\right)\right]. \tag{C31}$$

As a result $q_t^{(n)}$ are affine in the state $z_t$, that is,

$$q_t^{(n)} = q_0^{(n)} + q_1^{(n)}\, z_t, \tag{C32}$$

and the above coefficients are defined recursively as follows,

$$q_0^{(n)} = m_0 + q_0^{(n-1)} + \frac{1}{2}\sigma^2 \left( q_1^{(n-1)}\, \Sigma_z - \sigma_m \right)^2, \tag{C33}$$

$$q_1^{(n)} = m_1 + q_1^{(n-1)} Z, \tag{C34}$$

for $n \geq 1$ where $q_0^{(0)} = 0,\ q_1^{(0)} = [0, 0, 0]$. The one-period risk-free rate is given by

$$r_{f,t} = -m_0 - \frac{1}{2}(\sigma_n \sigma)^2 - m_1 z_t = -\log \hat{\beta} + \frac{1}{\psi}\mu_t - \frac{1}{2}\sigma_m^2 \sigma^2. \tag{C35}$$

### C.2 Zero coupon stocks

Similarly, the natural logarithm of the price-dividend ratios of the zero-coupon stocks $p_t^{(n)}$, that pay the dividend $D_{t+n}$, $n$ periods ahead, satisfy recursively the Euler equation,

$$p_t^{(n)} = \log \mathbb{E}_t \left[ M_{t,t+1} \exp\left( \Delta d_{t+1} + p_{t+1}^{(n-1)} \right) \right], \tag{C36}$$

and as such are also affine in the state $z_t$,

$$p_t^{(n)} = p_0^{(n)} + p_1^{(n)}\, z_t. \tag{C37}$$

Given the definition of the dividend process specified in (9), the coefficients of the coefficients of the zero-coupon stock price-dividend ratios are defined recursively according to,

$$p_0^{(n)} = m_0 + d_0 + d_1\, \mu + \frac{1}{2}\, d_2^2 + p_0^{(n-1)} + \frac{1}{2}\sigma^2 \left( d_1\, \sigma_c - \sigma_m + p_1^{(n-1)}\, \Sigma_z \right)^2, \tag{C38}$$

$$p_1^{(n)} = (1 - d_1\, \psi)\, m_1 + p_1^{(n-1)} Z, \tag{C39}$$

for $n \geq 1$ where $p_0^{(0)} = p_1^{(0)} = 0$.

### C.3 Log-linear approximation of the stock price-dividend ratio

The Euler equation of the stock is given as follows

$$e^{p_t - d_t} = \mathbb{E}_t \left[ J_{t,t+1} \left( e^{p_{t+1} - d_{t+1}} + 1 \right) \right], \tag{C40}$$

where $J_{t,t+1} = M_{t,t+1} D_{t+1}/D_t$. The log of the price-dividend ratio is approximated to be linear in the state $z_t$,

$$p_t - d_t \approx \overline{p-d} + b\, z_t, \tag{C41}$$

where $\overline{p-d}$ is the average log price-dividend ratio, since the mean of the vector $z_t$ is zero, and we need to solve for the coefficients $b$. If $z_t = 0$, then $p_{t+1} - d_{t+1} = \overline{p-d} + b\Sigma_z \epsilon_{t+1}$. Solving the Euler equation when the state is $z_t = 0$, we obtain the following:

$$e^{\overline{p-d}} = \hat{J} e^{\overline{p-d}} + J, \tag{C42}$$

where

$$\log\, J = \log \hat{\beta} + d_0 + \left(d_1 - \frac{1}{\psi}\right)\mu + \frac{1}{2}\left[d_2^2 + \sigma^2 (d_1\sigma_c - \sigma_m)^2\right], \tag{C43}$$

$$\log\, \hat{J} = \log\, J + (d_1\sigma_c - \sigma_m)\, b\Sigma_z + \frac{1}{2}(b\Sigma_z)^2, \tag{C44}$$

and, therefore, $\overline{p-d} = \log\left(J/(1-\hat{J})\right)$. Solving the Euler equation for a general state and applying a first-order approximation, we obtain the following:

$$(p_t - d_t) - \overline{p-d} \approx \hat{J}\, \mathbb{E}_t\left[(p_{t+1} - d_{t+1}) - \overline{p-d}\right] + \left(d_1 - \frac{1}{\psi}\right)(\mu_t - \mu)\,. \tag{C45}$$

Solving forward the above equation, we obtain the following expression:

$$p_t - d_t \approx \overline{p-d} + \left(d_1 - \frac{1}{\psi}\right)\sum_{\tau=0}^{\infty} \hat{J}^{\tau}\mathbb{E}_t\left(\mu_{t+\tau} - \mu\right). \tag{C46}$$

Finally, using the proposed approximation (C41) in equation (C45), we obtain that

$$b = (1 - d_1\psi)\, m_1 \left(\mathbf{I} - \hat{J}Z\right)^{-1}, \tag{C47}$$

where $\mathbf{I}$ indicates the unit matrix, which in this case is three dimensional. In the above equation, $b$ is solved numerically since $\hat{J}$ also includes $b$.

# D Alternative calibrations

Table 5: Macroeconomic moments ($\gamma = 5$)

The data statistics are computed from quarterly macroeconomic data obtained from the NIPA tables over 1947Q1-2012Q4. The model statistics are averages of 1000 simulated paths of 300 quarters with a burn-in of 100 quarters. Parentheses show the $t-statistics$ of the model averages in relation to the data estimates where the standard errors of the data estimates are Newey and West (1987) corrected with 16 lags. $\Delta x$ denotes the first-difference of the natural logarithm of a variable $X$. $y$ denotes (the natural logarithm of) total output; $c$ denotes total consumption; $i$ denotes total investment and $\omega$ denotes total factor productivity. For a variable $x$, $\sigma(x)$ denotes its volatility; $ac_1(x)$ is its first-order autocorrelation and $\rho(x,z)$ is its correlation with variable $z$. The models with technology choice are calibrated to fit the TFP growth volatility, the consumption growth volatility, the first-order autocorrelation of the stock market log price-to-dividend ratio, the first-order autocorrelation of the TFP growth and the CRRA was chosen to fit the Sharpe ratio of the stock market portfolio. The exogenous volatility ($\sigma$) of the models without technology choice ($\nu = \infty$) is adjusted to fit the TFP growth volatility. For all models the average growth rate in the economy is $\mu = 0.4\%$, the capital share is $\alpha = 0.36$, the capital depreciation is $\delta = 0.026$ and the dividend process parameters are $d = (-0.0026, 1.65, 0.04)$.

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | A. Volatilities | | | | | | | | | | | | |
| | | Model 4 | | | | Model 5 | | | | Model 6 | | | |
| | | $\psi = 0.35, \gamma = 5, \chi = 3.6$ $\beta = 0.9990, \phi = 0.970$ | | | | $\psi = 1.5, \gamma = 5, \chi = 4.5$ $\beta = 0.9903, \phi = 0.960$ | | | | $\psi = 0.25, \gamma = 90, \chi = 8.5$ $\beta = 0.9999, \phi = 0.970$ | | | |
| | | $\nu = \infty$ $\sigma = 1.80$ | | $\nu = 6.2$ $\sigma = 2.25$ | | $\nu = \infty$ $\sigma = 1.79$ | | $\nu = 2.6$ $\sigma = 2.27$ | | $\nu = \infty$ $\sigma = 1.80$ | | $\nu = 70$ $\sigma = 2.60$ | |
| | Data *est.* | *avg.* | $t-stat$ | *avg.* | $t-stat$ | *avg.* | $t-stat$ | *avg.* | $t-stat$ | *avg.* | $t-stat$ | *avg.* | $t-stat$ |
| $\sigma(\Delta y)/\sigma(\Delta\omega)$ | 0.64 | 0.64 | (0.01) | 0.64 | (0.02) | 0.64 | (0.01) | 0.64 | (0.03) | 0.64 | (0.00) | 0.64 | (0.01) |
| $\sigma(\Delta c)/\sigma(\Delta\omega)$ | 0.44 | 0.39 | (0.74) | 0.44 | (0.06) | 0.43 | (0.10) | 0.44 | (0.02) | 0.35 | (1.36) | 0.44 | (0.08) |
| $\sigma(\Delta i)/\sigma(\Delta\omega)$ | 1.40 | 1.27 | (0.58) | 1.19 | (0.97) | 1.17 | (1.06) | 1.16 | (1.08) | 1.48 | (0.34) | 1.62 | (0.98) |
| $\sigma(\Delta\omega)$ | 1.81 | 1.81 | (0.01) | 1.81 | (0.05) | 1.81 | (0.05) | 1.81 | (0.04) | 1.81 | (0.01) | 1.81 | (0.04) |
| | B. Autocorrelations | | | | | | | | | | | | |
| $ac_1(\Delta y)$ | 0.24 | -0.00 | (3.73) | 0.26 | (0.24) | -0.01 | (3.82) | 0.26 | (0.33) | -0.00 | (3.69) | 0.49 | (3.78) |
| $ac_1(\Delta c)$ | 0.04 | 0.01 | (0.48) | 0.12 | (1.18) | 0.01 | (0.43) | 0.28 | (3.46) | 0.02 | (0.35) | 0.12 | (1.20) |
| $ac_1(\Delta i)$ | 0.37 | -0.01 | (4.67) | 0.36 | (0.13) | -0.02 | (4.81) | 0.25 | (1.45) | -0.01 | (4.66) | 0.31 | (0.75) |
| $ac_1(\Delta\omega)$ | 0.25 | -0.02 | (3.92) | 0.25 | (0.06) | -0.02 | (4.01) | 0.25 | (0.01) | -0.02 | (3.92) | 0.48 | (3.35) |

Continued on next page

Table 6 – continued from previous page

| | | Model 4 | | | | Model 5 | | | | Model 6 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $\psi = 0.35, \gamma = 5, \chi = 3.6$ $\beta = 0.9990, \phi = 0.970$ | | | | $\psi = 1.5, \gamma = 5, \chi = 4.5$ $\beta = 0.9903, \phi = 0.960$ | | | | $\psi = 0.25, \gamma = 90, \chi = 8.5$ $\beta = 0.9999, \phi = 0.970$ | | | |
| | Data | $\nu = \infty$ $\sigma = 1.80$ | | $\nu = 6.2$ $\sigma = 2.25$ | | $\nu = \infty$ $\sigma = 1.79$ | | $\nu = 2.6$ $\sigma = 2.27$ | | $\nu = \infty$ $\sigma = 1.80$ | | $\nu = 70$ $\sigma = 2.60$ | |
| | *est.* | *avg.* | $t-stat$ | *avg.* | $t-stat$ | *avg.* | $t-stat$ | *avg.* | $t-stat$ | *avg.* | $t-stat$ | *avg.* | $t-stat$ |
| **C. Correlations** | | | | | | | | | | | | | |
| $\rho(\Delta y, \Delta i)$ | 0.68 | 1.00 | (6.36) | 0.99 | (6.13) | 1.00 | (6.34) | 1.00 | (6.32) | 1.00 | (6.35) | 0.82 | (2.74) |
| $\rho(\Delta y, \Delta c)$ | 0.62 | 1.00 | (9.99) | 0.99 | (9.71) | 1.00 | (9.95) | 1.00 | (9.92) | 1.00 | (9.98) | 0.89 | (7.01) |
| $\rho(\Delta c, \Delta i)$ | 0.43 | 1.00 | (8.78) | 0.95 | (8.11) | 1.01 | (8.70) | 0.99 | (8.64) | 1.00 | (8.75) | 0.46 | (0.42) |
| $\rho(\Delta \omega, \Delta y)$ | 1.01 | 1.00 | (2.48) | 1.00 | (1.49) | 1.00 | (3.32) | 1.00 | (2.73) | 1.00 | (1.74) | 1.01 | (0.84) |
| $\rho(\Delta \omega, \Delta c)$ | 0.67 | 1.01 | (6.62) | 0.98 | (6.39) | 1.01 | (6.61) | 0.99 | (6.54) | 1.01 | (6.55) | 0.81 | (2.76) |
| $\rho(\Delta \omega, \Delta i)$ | 0.61 | 1.00 | (10.46) | 0.99 | (10.12) | 1.00 | (10.47) | 1.00 | (10.47) | 1.00 | (10.45) | 0.88 | (7.39) |
| **D. Cross-correlations** | | | | | | | | | | | | | |
| $\rho(\Delta y_t, \Delta i_{t-1})$ | 0.16 | -0.01 | (1.83) | 0.26 | (1.16) | -0.01 | (1.92) | 0.26 | (1.07) | -0.00 | (1.81) | 0.66 | (5.51) |
| $\rho(\Delta y_t, \Delta c_{t-1})$ | 0.11 | -0.00 | (2.10) | 0.24 | (2.47) | -0.01 | (2.17) | 0.27 | (2.91) | -0.00 | (2.06) | 0.23 | (2.21) |
| $\rho(\Delta c_t, \Delta i_{t-1})$ | 0.08 | 0.01 | (1.10) | 0.11 | (0.51) | 0.01 | (1.04) | 0.28 | (3.04) | 0.01 | (1.00) | 0.05 | (0.37) |
| $\rho(\Delta \omega_t, \Delta y_{t-1})$ | 0.23 | -0.02 | (3.73) | 0.24 | (0.14) | -0.03 | (3.78) | 0.24 | (0.24) | -0.02 | (3.75) | 0.46 | (3.53) |
| $\rho(\Delta \omega_t, \Delta c_{t-1})$ | 0.15 | -0.03 | (1.89) | 0.24 | (1.00) | -0.03 | (1.96) | 0.23 | (0.88) | -0.03 | (1.92) | 0.64 | (5.25) |
| $\rho(\Delta \omega_t, \Delta i_{t-1})$ | 0.07 | -0.02 | (1.89) | 0.22 | (2.97) | -0.02 | (1.89) | 0.25 | (3.52) | -0.02 | (1.90) | 0.20 | (2.54) |
| **E. Annual (time-aggregated) statistics** | | | | | | | | | | | | | |
| $\mu(\Delta c)$ | 1.79 | 1.60 | (0.80) | 1.60 | (0.80) | 1.60 | (0.80) | 1.60 | (0.80) | 1.60 | (0.81) | 1.60 | (0.80) |
| $\sigma(\Delta c)$ | 1.49 | 1.19 | (1.99) | 1.46 | (0.19) | 1.33 | (1.08) | 1.64 | (1.01) | 1.09 | (2.68) | 1.51 | (0.15) |
| $\mu(\Delta d)$ | 2.09 | 1.61 | (0.53) | 1.61 | (0.53) | 1.62 | (0.52) | 1.62 | (0.52) | 1.61 | (0.53) | 1.61 | (0.53) |
| $\sigma(\Delta d)$ | 7.09 | 6.88 | (0.23) | 7.02 | (0.08) | 6.95 | (0.16) | 7.12 | (0.03) | 6.83 | (0.28) | 7.05 | (0.05) |
| $\rho(\Delta c, \Delta d)$ | 0.32 | 0.28 | (0.40) | 0.34 | (0.16) | 0.31 | (0.12) | 0.37 | (0.51) | 0.26 | (0.62) | 0.35 | (0.27) |

Table 6: Asset pricing moments ($\gamma = 5$)

The empirical data statistics are computed from quarterly return data obtained from CRSP and WRDS Federal Reserve data (3-month T-bill yields) over 1947Q1-2012Q4. The model statistics are computed as averages from 1000 simulated paths, where each path has 300 quarters with a burn-in of 100 quarters. Parentheses show the $t-statistics$ of the model averages in relation to the data estimates where the standard errors of the data estimates are Newey and West (1987) corrected with 16 lags. $p-d$ denotes the log price-to-dividend ratio of the stock market portfolio; $R_f$ is the quarterly risk-free rate and $R$ is the quarterly stock market return. For a variable $x$ $\mu(x)$ is the mean; $\sigma(x)$ is the standard deviation and $ac_1(x)$ is the first-order autocorrelation. The Sharpe ratio, $SR$, equals $\mu(R-R_f)/\sigma(R-R_f)$ and $\rho(p-d,r_f)$ denotes the correlation between $p-d$ and the log risk-free rate. For the model calibrations and other parameter values see caption of Table 2.

| | | Model 4 | | | | Model 5 | | | | Model 6 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $\psi = 0.35, \gamma = 5, \chi = 3.6$ $\beta = 0.9990, \phi = 0.970$ | | | | $\psi = 1.5, \gamma = 5, \chi = 4.5$ $\beta = 0.9903, \phi = 0.960$ | | | | $\psi = 0.25, \gamma = 90, \chi = 8.5$ $\beta = 0.9999, \phi = 0.970$ | | | |
| | Data | $\nu = \infty$ $\sigma = 1.80$ | | $\nu = 6.2$ $\sigma = 2.25$ | | $\nu = \infty$ $\sigma = 1.79$ | | $\nu = 2.6$ $\sigma = 2.27$ | | $\nu = \infty$ $\sigma = 1.80$ | | $\nu = 70$ $\sigma = 2.60$ | |
| | *est.* | *avg.* | $t-stat$ | *avg.* | $t-stat$ | *avg.* | $t-stat$ | *avg.* | $t-stat$ | *avg.* | $t-stat$ | *avg.* | $t-stat$ |
| $\mu(d-p)$ | 4.87 | 4.87 | (0.03) | 4.88 | (0.04) | 4.87 | (0.01) | 4.87 | (0.00) | 4.61 | (2.62) | 4.75 | (1.19) |
| $\sigma(d-p)$ | 0.42 | 0.03 | (6.74) | 0.04 | (6.62) | 0.03 | (6.79) | 0.04 | (6.66) | 0.05 | (6.44) | 0.07 | (6.07) |
| $ac_1(d-p)$ | 0.98 | 0.98 | (0.23) | 0.98 | (0.61) | 0.98 | (0.16) | 0.98 | (0.55) | 0.99 | (0.82) | 0.99 | (1.07) |
| $\mu(R_f)$ | 0.23 | 1.22 | (8.18) | 1.21 | (8.11) | 1.24 | (8.36) | 1.24 | (8.35) | 0.87 | (5.31) | 0.35 | (0.96) |
| $\sigma(R_f)$ | 0.94 | 0.11 | (7.16) | 0.32 | (5.38) | 0.04 | (7.78) | 0.16 | (6.71) | 0.17 | (6.66) | 0.47 | (4.09) |
| $ac_1(R_f)$ | 0.34 | 0.99 | (6.69) | 0.25 | (0.97) | 1.01 | (6.72) | 0.13 | (2.22) | 0.97 | (6.49) | 0.36 | (0.11) |
| $\mu(R_m)$ | 2.03 | 1.27 | (1.73) | 1.27 | (1.73) | 1.26 | (1.75) | 1.26 | (1.75) | 1.50 | (1.21) | 1.37 | (1.49) |
| $\sigma(R_m)$ | 8.26 | 4.37 | (7.11) | 4.42 | (7.01) | 4.12 | (7.56) | 4.13 | (7.54) | 4.34 | (7.16) | 4.45 | (6.95) |
| $ac_1(R_m)$ | 0.08 | -0.00 | (1.38) | 0.02 | (0.97) | -0.00 | (1.40) | 0.00 | (1.28) | 0.00 | (1.33) | 0.04 | (0.74) |
| $\mu(R_m-R_f)$ | 1.79 | 0.05 | (4.20) | 0.06 | (4.17) | 0.01 | (4.27) | 0.02 | (4.26) | 0.63 | (2.80) | 1.02 | (1.85) |
| $\sigma(Rm-R_f)$ | 8.19 | 4.36 | (6.97) | 4.41 | (6.89) | 4.12 | (7.41) | 4.13 | (7.40) | 4.33 | (7.02) | 4.43 | (6.85) |
| $ac_1(R_m-R_f)$ | 0.08 | -0.00 | (1.49) | -0.00 | (1.49) | -0.00 | (1.48) | -0.00 | (1.48) | -0.00 | (1.48) | -0.00 | (1.48) |
| $SharpeRatio$ | 0.22 | 0.01 | (3.43) | 0.01 | (3.39) | 0.00 | (3.54) | 0.00 | (3.53) | 0.14 | (1.23) | 0.23 | (0.21) |
| $\rho(d-p,r_f)$ | 0.04 | -0.68 | (5.68) | -0.25 | (2.29) | 0.77 | (5.76) | 0.18 | (1.08) | -0.48 | (4.09) | -0.25 | (2.28) |

Table 7: Alternative standard RBC models

The benchmark models refer to the models without technology choice and are calibrated to fit the TFP growth volatility, the consumption growth volatility, the first order autocorrelation of the stock market log price-to-dividend ratio, and the Sharpe ratio of the stock market index. The models with technology choice are calibrated to additionally fit the first order autocorrelation of the TFP growth. For all models the average growth rate in the economy is $\mu = 0.4\%$, the capital share is $\alpha = 0.36$, the capital depreciation is $\delta = 0.026$ and the dividend process parameters are $d = (-0.0026, 1.65, 0.04)$.

| | Low EIS | | High EIS | |
|---|---|---|---|---|
| | Benchmark 1 | Model 1 | Benchmark 2 | Model 2 |
| $\beta$ | 0.9984 | 0.9997 | 0.9954 | 0.9935 |
| $\gamma$ | 80 | 110 | 65 | 200 |
| $\psi$ | 0.30 | 0.30 | 1.50 | 1.50 |
| $\chi$ | 100 | 3.6 | 4.2 | 4.5 |
| $\phi$ | 0.990 | 0.970 | 0.997 | 0.980 |
| $\nu$ | $\infty$ | 155 | $\infty$ | 260 |
| $\sigma$ | 1.81 | 2.26 | 1.81 | 2.25 |

## Table 8: Macroeconomic moments (alternative standard RBC models)

The data statistics are computed from quarterly macroeconomic data obtained from the NIPA tables over 1947Q1-2012Q4. The model statistics are averages of 1000 simulated paths of 300 quarters with a burn-in of 100 quarters. Parentheses show the $t-statistics$ of the model averages in relation to the data estimates where the standard errors of the data estimates are Newey and West (1987) corrected with 16 lags. $\Delta x$ denotes the first-difference of the natural logarithm of a variable $X$. $y$ denotes (the natural logarithm of) total output; $c$ denotes total consumption; $i$ denotes total investment and $\omega$ denotes total factor productivity. For a variable $x$, $\sigma(x)$ denotes its volatility; $ac_1(x)$ is its first-order autocorrelation and $\rho(x,z)$ is its correlation with variable $z$.

| | | Low EIS ($\psi=0.3$) | | | | High EIS ($\psi=1.5$) | | | |
|---|---|---|---|---|---|---|---|---|---|
| | Data | Benchmark 1 | | Model 1 | | Benchmark 2 | | Model 2 | |
| **A. Volatilities** | | | | | | | | | |
| | | Avg. | t-stat | Avg. | t-stat | Avg. | t-stat | Avg. | t-stat |
| $\sigma(\Delta y)/\sigma(\Delta\omega)$ | 0.64 | 0.64 | (0.03) | 0.64 | (0.02) | 0.64 | (0.04) | 0.64 | (0.05) |
| $\sigma(\Delta c)/\sigma(\Delta\omega)$ | 0.44 | 0.44 | (0.04) | 0.44 | (0.04) | 0.44 | (0.01) | 0.44 | (0.06) |
| $\sigma(\Delta i)/\sigma(\Delta\omega)$ | 1.40 | 1.19 | (0.94) | 1.21 | (0.85) | 1.06 | (1.53) | 1.11 | (1.30) |
| $\sigma(\Delta\omega)$ | 1.81 | 1.81 | (0.01) | 1.81 | (0.05) | 1.81 | (0.04) | 1.81 | (0.00) |
| **B. Autocorrelations** | | | | | | | | | |
| $ac_1(\Delta y)$ | 0.24 | 0.01 | (3.51) | 0.26 | (0.33) | 0.01 | (3.48) | 0.26 | (0.32) |
| $ac_1(\Delta c)$ | 0.04 | 0.02 | (0.26) | 0.11 | (1.05) | 0.04 | (0.06) | 0.28 | (3.50) |
| $ac_1(\Delta i)$ | 0.37 | -0.00 | (4.54) | 0.38 | (0.07) | -0.01 | (4.61) | 0.25 | (1.46) |
| $ac_1(\Delta\omega)$ | 0.25 | -0.01 | (3.79) | 0.25 | (0.03) | -0.01 | (3.76) | 0.25 | (0.04) |
| **C. Correlations** | | | | | | | | | |
| $\rho(\Delta y,\Delta i)$ | 0.68 | 1.00 | (6.36) | 0.99 | (6.08) | 1.00 | (6.32) | 1.00 | (6.30) |
| $\rho(\Delta y,\Delta c)$ | 0.62 | 1.00 | (9.98) | 0.99 | (9.63) | 1.00 | (9.94) | 1.00 | (9.92) |
| $\rho(\Delta c,\Delta i)$ | 0.43 | 1.00 | (8.78) | 0.94 | (7.94) | 0.99 | (8.65) | 0.99 | (8.60) |
| $\rho(\Delta\omega,\Delta y)$ | 0.99 | 1.00 | (1.87) | 1.00 | (1.24) | 1.00 | (2.70) | 1.00 | (2.33) |
| $\rho(\Delta\omega,\Delta c)$ | 0.67 | 0.99 | (6.61) | 0.98 | (6.34) | 0.99 | (6.53) | 0.99 | (6.48) |
| $\rho(\Delta\omega,\Delta i)$ | 0.61 | 1.00 | (10.45) | 0.98 | (10.03) | 1.00 | (10.47) | 1.00 | (10.47) |
| **D. Cross-correlations** | | | | | | | | | |
| $\rho(\Delta y,\Delta i_{t-1})$ | 0.16 | 0.01 | (1.65) | 0.27 | (1.24) | 0.01 | (1.61) | 0.26 | (1.09) |
| $\rho(\Delta y,\Delta c_{t-1})$ | 0.11 | 0.01 | (1.88) | 0.25 | (2.53) | 0.01 | (1.86) | 0.26 | (2.83) |
| $\rho(\Delta c,\Delta i_{t-1})$ | 0.08 | 0.02 | (0.93) | 0.10 | (0.36) | 0.03 | (0.76) | 0.27 | (2.94) |
| $\rho(\Delta\omega,\Delta y_{-1})$ | 0.23 | -0.01 | (3.56) | 0.24 | (0.23) | -0.01 | (3.49) | 0.24 | (0.21) |
| $\rho(\Delta\omega,\Delta c_{-1})$ | 0.15 | -0.01 | (1.75) | 0.25 | (1.08) | -0.01 | (1.70) | 0.23 | (0.88) |
| $\rho(\Delta\omega,\Delta i_{-1})$ | 0.07 | -0.01 | (1.68) | 0.23 | (3.01) | -0.01 | (1.59) | 0.25 | (3.43) |
| **E. Annual (time-aggregated) statistics** | | | | | | | | | |
| $\mu(\Delta c)$ | 1.79 | 1.60 | (0.81) | 1.60 | (0.80) | 1.60 | (0.83) | 1.60 | (0.80) |
| $\sigma(\Delta c)$ | 1.49 | 1.40 | (0.61) | 1.45 | (0.24) | 1.45 | (0.26) | 1.68 | (1.27) |
| $\mu(\Delta d)$ | 2.09 | 1.61 | (0.53) | 1.61 | (0.53) | 1.60 | (0.54) | 1.61 | (0.53) |
| $\sigma(\Delta d)$ | 7.09 | 6.99 | (0.11) | 7.02 | (0.08) | 7.02 | (0.08) | 7.15 | (0.06) |
| $\rho(\Delta c,\Delta d)$ | 0.32 | 0.33 | (0.05) | 0.34 | (0.15) | 0.34 | (0.15) | 0.38 | (0.60) |

Table 9: Asset pricing moments (alternative standard RBC models)

The empirical data statistics are computed from quarterly return data obtained from CRSP and WRDS Federal Reserve data (3-month T-bill yields) over 1947Q1-2012Q4. The model statistics are computed as averages from 1000 simulated paths, where each path has 300 quarters with a burn-in of 100 quarters. Parentheses show the $t-statistics$ of the model averages in relation to the data estimates where the standard errors of the data estimates are Newey and West (1987) corrected with 16 lags. $p-d$ denotes the log price-to-dividend ratio of the stock market portfolio; $R_f$ is the quarterly risk-free rate and $R$ is the quarterly stock market return. For a variable $x$ $\mu(x)$ is the mean; $\sigma(x)$ is the standard deviation and $ac_1(x)$ is the first-order autocorrelation. The Sharpe ratio, $SR$, equals $\mu(R-R_f)/\sigma(R-R_f)$ and $\rho(p-d, r_f)$ denotes the correlation between $p-d$ and the log risk-free rate.

| | | Low EIS ($\psi = 0.3$) | | | | High EIS ($\psi = 1.5$) | | | |
|---|---|---|---|---|---|---|---|---|---|
| | Data | Benchmark 1 | | Model 1 | | Benchmark 2 | | Model 2 | |
| | | Avg. | t-stat | Avg. | t-stat | Avg. | t-stat | Avg. | t-stat |
| $\mu(p-d)$ | 4.87 | 4.87 | (0.01) | 4.87 | (0.03) | 4.87 | (0.01) | 4.88 | (0.06) |
| $\sigma(p-d)$ | 0.42 | 0.04 | (6.53) | 0.05 | (6.43) | 0.03 | (6.84) | 0.05 | (6.49) |
| $ac_1(p-d)$ | 0.98 | 0.99 | (1.28) | 0.99 | (0.68) | 0.98 | (0.44) | 0.99 | (1.04) |
| $\mu(R_f)$ | 0.23 | 0.38 | (1.25) | 0.28 | (0.43) | 0.34 | (0.87) | 0.43 | (1.66) |
| $\sigma(R_f)$ | 0.94 | 0.22 | (6.24) | 0.34 | (5.16) | 0.07 | (7.53) | 0.16 | (6.73) |
| $ac_1(R_f)$ | 0.34 | 0.96 | (6.39) | 0.26 | (0.84) | 0.95 | (6.29) | 0.21 | (1.36) |
| $\mu(R)$ | 2.03 | 1.26 | (1.74) | 1.28 | (1.71) | 1.27 | (1.73) | 1.25 | (1.76) |
| $\sigma(R)$ | 8.26 | 4.27 | (7.28) | 4.47 | (6.93) | 4.35 | (7.14) | 4.16 | (7.48) |
| $ac_1(R)$ | 0.08 | 0.00 | (1.30) | 0.03 | (0.92) | -0.00 | (1.39) | 0.01 | (1.26) |
| $\mu(R-R_f)$ | 1.79 | 0.88 | (2.19) | 0.99 | (1.92) | 0.93 | (2.07) | 0.82 | (2.33) |
| $\sigma(R-R_f)$ | 8.19 | 4.26 | (7.15) | 4.45 | (6.81) | 4.35 | (6.99) | 4.16 | (7.34) |
| $ac_1(R-R_f)$ | 0.08 | -0.00 | (1.48) | -0.00 | (1.48) | -0.00 | (1.49) | -0.00 | (1.48) |
| *Sharpe Ratio* | 0.22 | 0.21 | (0.19) | 0.22 | (0.07) | 0.21 | (0.09) | 0.20 | (0.34) |
| $\rho(p-d, r_f)$ | 0.04 | -0.33 | (2.91) | -0.27 | (2.45) | 0.49 | (3.53) | 0.11 | (0.52) |

Table 10: Risk-free rate correlations with macroeconomic quantities (alternative standard RBC models)

The data risk-free rate is taken to be the quarterly 3-month T-bill yield obtained from WRDS Federal Reserve data, while the quarterly macroeconomic data are obtained from NIPA. The data period is from 1947Q1 to 2012Q4. The model statistics are computed as averages from 1000 simulated paths, where each path has 300 quarters with a burn-in of 100 quarters. Parentheses show the $t-statistics$ of the model averages in relation to the data estimates where the standard errors of the data estimates are Newey and West (1987) corrected with 16 lags. $\Delta x$ denotes the first-difference of the natural logarithm of a variable $X$. $y$ denotes (the HP filtered natural logarithm of) total output and similarly, $c$ corresponds to consumption, $i$ corresponds to investment and $\omega$ corresponds to total factor productivity. $r_f$ denotes the log risk-free rate.

| | | Low EIS ($\psi = 0.3$) | | | | High EIS ($\psi = 1.5$) | | | |
|---|---|---|---|---|---|---|---|---|---|
| | Data | Benchmark 1 | | Model 1 | | Benchmark 2 | | Model 2 | |
| $\rho(y_t, r_{f,t+1})$ | -0.18 | 0.31 | (5.37) | -0.27 | (1.03) | 0.35 | (5.80) | -0.21 | (0.36) |
| $\rho(\Delta y_t, \Delta r_{f,t-1})$ | 0.40 | 0.00 | (5.40) | 0.21 | (2.65) | 0.01 | (5.39) | 0.20 | (2.68) |
| $\rho(\Delta y_t, r_{f,t-1})$ | 0.34 | 0.04 | (3.16) | 0.28 | (0.61) | 0.03 | (3.30) | 0.27 | (0.74) |
| $\rho(c_t, r_{f,t+1})$ | -0.23 | 0.30 | (4.96) | -0.28 | (0.44) | 0.33 | (5.23) | -0.24 | (0.08) |
| $\rho(\Delta c_t, \Delta r_{f,t-1})$ | 0.47 | 0.01 | (5.22) | 0.08 | (4.42) | 0.02 | (5.12) | 0.19 | (3.15) |
| $\rho(\Delta c_t, r_{f,t-1})$ | 0.56 | 0.08 | (6.34) | 0.14 | (5.48) | 0.11 | (5.95) | 0.30 | (3.34) |
| $\rho(i_t, r_{f,t+1})$ | 0.00 | 0.32 | (4.72) | -0.26 | (3.89) | 0.36 | (5.39) | -0.18 | (2.69) |
| $\rho(\Delta i_t, \Delta r_{f,t-1})$ | 0.27 | 0.00 | (4.01) | 0.32 | (0.71) | -0.00 | (4.07) | 0.21 | (0.89) |
| $\rho(\Delta i_t, r_{f,t-1})$ | 0.15 | 0.01 | (2.41) | 0.41 | (4.37) | -0.04 | (3.15) | 0.24 | (1.49) |