Risk Aversion Sensitive Real Business Cycles *

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Abstract

We study technology choice in equilibrium by allowing for production substitution across states of nature. In this model, risk aversion is an important determinant of the optimal productivity risk and of volatilities, correlations, and autocorrelations of macroeconomic growth rates. Through a propagation effect, the model reproduces the autocorrelation of total factor productivity and it matches autocorrelations of consumption, investment and output. The endogenous autocorrelation implies that the risk-free rate predicts output growth, which the data corroborate, and the model explains the low correlation between consumption and investment and the high volatility of investment.

Keywords: State-contingent technology; Risk aversion; Volatility of the growth rate of total factor productivity and investment; Autocorrelations and correlations of macroeconomic quantities; Testable linkages between financial markets and the macroeconomy

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1 Introduction

The paper studies how risk aversion affects the macroeconomy, which is a mechanism that is absent from the standard real business cycle (RBC) model, even though risk aversion is a prominent determinant of asset pricing moments. In this regard, we investigate an economy with flexible technology choice in which risk aversion affects production substitution across states of nature and, through that, exerts first-order effects on financial markets and macroeconomic quantities. In the calibrated model, we see that endogenous technology choice generates autocorrelation in the growth rates of macroeconomic quantities, exhibiting a propagation effect. Specifically, in our preferred calibration the economy with technology choice matches almost perfectly the point estimates of the macroeconomic autocorrelations, despite being calibrated to match only the autocorrelation of the total factor productivity (TFP) growth. Further, technology choice can lead to low correlations between the macroeconomic growth rates. For instance, the correlation between consumption and investment growth, which is particularly low in the data, is similarly low in the model. In addition, we show that the autocorrelation in growth rates implies that output growth is predictable, which establishes testable linkages between financial markets and the macroeconomy. For example, we see in the model that the risk-free rate predicts output growth, which the data corroborate.

In a standard production-based economy, the representative firm, in lieu of the owner, manages investments to smooth consumption over time. However, the firm has no means to adjust production or technology across states, since increasing investment results in an increase of production in all states next period. For this reason, the intertemporal substitution plays the key role for how an agent smooths consumption in such an environment and the elasticity of intertemporal substitution (EIS) almost exclusively drives the time-variation in consumption, investment, and output in the standard production-based real business cycle model. Specifically, risk aversion cannot exert a first-order effect on the macroeconomy. This is an underappreciated cause for concern as it implies that risk aversion can be employed to match asset pricing moments without negatively affecting a model's fit to macroeconomic quantities,

as shown in Tallarini (2000). Cochrane (2008) calls this defect of standard real business cycle models the divorce between asset pricing and macroeconomics.

We analyze a model economy with production, where the firm employs technology choice to substitute productivity across states.[1] As in the standard model, there is an exogenous productivity that follows an AR(1) process; however, every period the endogenous productivity choice can deviate from the exogenous at some cost, that depends on the amount of productivity transformation. In equilibrium, the optimal choice depends on the cost of transformation and the degree of risk aversion, although all structural parameters play a role. This creates a rich structure where the optimal endogenous productivity may be more or less volatile than the exogenous productivity and may move counter to the exogenous shocks. For the calibrated model, this is pinned down by the connection between the optimal endogenous productivity and its autocorrelation. Specifically, a positive autocorrelation requires a decrease in productivity risk, that is, the endogenous productivity needs to be less volatile than the exogenous.

Through a log-linearization, we find that the endogenous technology transforms the exogenous productivity into an ARMA(1,1) process.[2] Precisely this feature of the model leads to autocorrelations in the growth rates of macroeconomic quantities and allows us to calibrate the model to match the autocorrelation of TFP. The MA(1) term arises from the fact that technology choice differentiates the current (endogenous) productivity from the expected (endogenous) productivity. The calibrated model, generates a propagation mechanism, resulting from the fact that risk aversion decreases the productivity risk: a positive shock in the endogenous productivity implies an even greater positive shock in expected productivity, since the latter is driven

[1] A practicable way to substitute productivity across states is through investing in various different production technologies. In the Online appendix A, we provide a theoretical connection between the reduced-form approach to technology choice that we adopt and investing in several technologies as in Jermann (2010). For example, it seems plausible, that the various different technologies of generating electricity, e.g., coal, natural gas, nuclear, oil, solar, wind, etc., are broadly consistent with Jermann (2010) and, therefore, also with our reduced-form approach. Specifically, since each technology has its own risk characteristics combining them allows choosing the risk profile of energy generation.

[2] To analyze the model, we log-linearize the macroeconomy as in Jermann (1998) to use closed-form log-normal pricing for assets, since then the stochastic discount factor, cash flows, and state variables are jointly conditionally log-normally distributed. The log-linear solution highlights the first-order effects of risk aversion. We have also solved the model with a second-order perturbation method and verified that the macroeconomic dynamics are close.

by the exogenous process. As a plausibility test, we estimate an ARMA(1,1) process for the TFP series to compare the volatility of the exogenous productivity process implied by the data with the one in the model. This exercise shows that the volatility of the exogenous productivity in the model, which we set to match the output volatility, is very close to the one implied by the data.

The solution of the model clarifies how macroeconomic quantities react to shocks to current and expected productivity.[3] This allows us to uncover an essential mechanism that explains several features of the model. Specifically, for a large parameter set and in all our calibrations, investment moves counter to an expected productivity shock. Essentially, the firm finds it optimal to counter negative and persistent shocks to expected productivity through an increase in investments. When the value of doing so is high, for example, when the EIS is sufficiently low and risk aversion is sufficiently high, and when the capital adjustment costs are also sufficiently low, then this mechanism leads to endogenous productivity choice that is large and negative, relative to the exogenous productivity. This mechanism also explains why with low EIS the model reproduces the high autocorrelation of investment growth and the low autocorrelation of consumption growth in the data. In this case, technology choice makes investment less sensitive and consumption more sensitive to exogenous shocks. As a result, investment growth becomes less volatile and highly autocorrelated and consumption growth becomes more volatile with low autocorrelation. As a final point, since consumption and investment differ in how they react to shocks to expected productivity, we see a decrease in the correlation of their growth rates.

Next, we look at how macroeconomic quantities vary when we vary the level of risk aversion. Specifically, we show that the level of risk aversion significantly affects the volatilities, autocorrelations, and correlations of the growth rates. For example, as risk aversion increases the elasticities of consumption, investment, and output with respect to exogenous shocks decrease and can become negative. As a result, the volatilities of their growth rates initially decrease but then increase again. Correlations show a similar U-shape pattern as they initially decrease

[3] In the standard model, we cannot separate between the expected and the current productivity, because both are driven by the same variable.

with risk aversion but beyond certain values they increase again. Autocorrelations exhibit the opposite behavior, where they initially increase with risk aversion, reach a maximum around 0.5, and then decrease until they turn negative.

The ability to manage productivity risk through technology choice might have dire consequences for the fit of the financial side of the model economy to the data. This turns out not to be true in relation to the price of risk, because all our calibrations perfectly fit the point estimate of the consumption volatility. Also, since technology choice mainly affects short-run expectations, its impact on the valuation of long-lived assets is limited. For instance, the properties of the price-to-dividend ratio are essentially identical with or without technology choice. Yet, the model with technology choice produces a better fit to the volatility and autocorrelation of the interest rate, as well as, to the correlation between the log price-dividend ratio and the interest rate. In addition, we find that the real interest rate in the data predicts output growth, which the model reproduces as a result of the autocorrelations in the growth rates.

The paper speaks to the literature that explores the asset pricing implications of production transformation across states or technologies. To allow for production transformation across states, Cochrane (1993) proposes to allow firms to choose state-contingent productivity endogenously subject to a constraint set. In closely related works, Cochrane (1988) and Jermann (2010) back out the stochastic discount factor from producers' first-order conditions assuming complete technologies, i.e., that there are as many technologies as states of nature. The calibrated model in Jermann (2010) matches the mean and volatility of aggregate stock market returns and the interest rate and, in addition, produces a volatile Sharpe ratio. In similar spirit, Belo (2010) applies state-contingent productivity to derive a pure production-based pricing kernel in a partial equilibrium setting, which gives rise to a macro-factor asset pricing model that explains the cross-sectional variation in average stock returns.[4] The key takeaway from these papers is that state-contingent technology can explain asset prices both in the time-series and

[4]Recent contributions to the literature on investment- or production-based asset pricing include Kaltenbrunner and Lochstoer (2010), Papanikolaou (2011), Gârleanu, Panageas, and Yu (2012), Ai, Croce, and Li (2013), Belo, Lin, and Bazdresch (2014), and Kung (2015), among many others; none of these works, however, study state-contingent technology.

the cross-section and suggest that risk aversion matters for the macroeconomy. However, these studies do not look at the implications of state-contingent technology for the macroeconomy. Our paper fills this gap in the literature.

Even though our focus differs, the predictions of our model are supportive of the broader production-based macroeconomic literature that emphasizes the importance of autocorrelations in macroeconomic quantities. We start with Burnside and Eichenbaum (1996); they argue that capital-utilization rates are an important source of propagation of technological shocks, which is required to match the autocorrelation of output growth. In our model, technology choice helps to match the autocorrelations in the growth rates of consumption, investment, output, and TFP. Boldrin, Christiano, and Fisher (2001) study an otherwise standard RBC model with habit preferences and a two-sector technology with limited intersectoral factor mobility. Their model also fits the autocorrelation of the growth rate of output, but the autocorrelation of consumption growth is negative in their model. Further, we differ from Boldrin, Christiano, and Fisher (2001) in that we match the volatility of investment while their model produces investment volatility that is too low compared to the data and in that they do not study the correlations of macroeconomic growth rates. Other than that, our paper joins Boldrin, Christiano, and Fisher (2001) in the task of integrating the analysis of asset returns and business cycles. We also mention Cogley and Nason (1995), who discuss the autocorrelation in output and conclude, consistent with Burnside and Eichenbaum (1996), that models with weak propagation have to resort to exogenous sources of autocorrelation, which our model endogenously generates with additional flexibility rather than frictions.

Finally, we point to the fact that the standard RBC model in which the exogenous productivity follows an ARMA(1,1) process is observationally identical to our model, provided that risk aversion and the TFP process are specified consistently. For example, Croce (2014) uses an ARMA(1,1) process, instead of an AR(1), for exogenous productivity, which generates more realistic correlations and autocorrelations of macroeconomic quantities. The focus of his study is on the asset pricing implications of persistent shocks to expected productivity. Kung and Schmid (2015) generate such propagation and realistic autocorrelations in growth rates

endogenously through innovation and also focus on asset prices. Another model with realistic correlations and autocorrelations of macro quantities is, for example, Papanikolaou (2011), who introduces an investment specific shock to break the perfect correlations of macroeconomic growth rates. Our model differs in that it produces realistic correlations and autocorrelations of macro quantities through production substitution across states of nature. Further, none of the above mentioned papers studies the role of risk aversion for the macroeconomy and produces predictions pertaining to macro-finance linkages.

2 A model with state-contingent technology

Consider a representative agent who owns an all-equity representative firm, which uses productive capital to generate one real good and operates in discrete time with infinite horizon.

2.1 Firms

Let Θ_t be the exogenous technological productivity level at time t. We assume that $\log \Theta_t$ follows an AR(1) process with trend,

$$\log \Theta_{t+1} \quad = \log Z_{t+1} + \phi \left(\log \Theta_t - \log Z_t \right) + \varepsilon_{t+1}, \quad \text{and} \quad \log Z_t = \mu t, \tag{1}$$

where $|\phi| < 1$ and $\varepsilon_{t+1} \sim N(0, \sigma^2)$ denotes the exogenous shock.

Departing from the standard production economy, we assume that the representative firm modifies the underlying productivity shocks. Following Cochrane (1993) and Belo (2010), at time t, a state-contingent technology Ω_{t+1} is chosen through a CES aggregator

$$\mathbb{E}_t \left[\frac{\Omega_{t+1}^{(1-\alpha)\nu}}{\Theta_{t+1}^{(1-\alpha)\nu}} \right] \leq 1, \tag{2}$$

where $\mathbb{E}_t$ is the conditional expectation operator. In (2), the variable $\alpha \in (0, 1)$ stands for the capital share in output and the curvature ν captures the representative firm's technical ability

to modify technology. When $\nu < 1$, increasing the volatility of technology choice also increases average productivity. For this reason, we assume that $\nu > 1$. With this assumption, as ν increases, distorting the underlying shocks reduces average productivity. When $\nu \to +\infty$, it is infinitely costly to modify the exogenous productivity. Therefore, we obtain $\Omega_{t+1} = \Theta_{t+1}$.

Appendix A provides intuition for the reduced-form approach in modeling technology choice. We interpret the technology modifications set in (2) as a simple abstract form of modeling state-contingent technologies implying flexibility for optimal future productivity. More specifically, constraint (2) determines the representative firm's ability to trade off higher realizations of shocks in some states at time $t+1$ with lower realizations in other states. The optimal choice offsets the marginal benefit from smoothing consumption over time and states with the marginal cost of lower average productivity (or a tradeoff between static efficiency and flexibility similar to Mills and Schumann (1985)).

Output, Y_t, is given by

$$Y_t = K_t^{\alpha}\,\Omega_t^{1-\alpha}, \tag{3}$$

where K_t denotes the capital stock at the beginning of period t.

Capital accumulates according to

$$K_{t+1} = (1-\delta)K_t + g_t, \tag{4}$$

where δ is the depreciation rate and g_t stands for the capital formation function. We specify g as in Jermann (1998), i.e.,

$$g_t = \left[\frac{a_1}{1-1/\chi}\left(\frac{I_t}{K_t}\right)^{1-1/\chi} + a_2\right] K_t, \tag{5}$$

where I_t denotes investment at time t, the curvature $\chi > 0$ governs capital adjustment costs, and a_1 and a_2 are constants.[5] These specifications imply that capital adjustment costs are high when χ is low and that capital adjustments are costless when $\chi \to \infty$. Following Boldrin,

[5]The functional form for capital formation in (5) simplifies the log-linearized model.

Christiano, and Fisher (2001), we set a_1 and a_2 such that there is no cost to capital adjustment in the deterministic steady-state

$$a_1 = \left(e^{\mu} - 1 + \delta\right)^{1/\chi} \quad \text{and} \quad a_2 = \frac{1}{1-\chi}\left(e^{\mu} - 1 + \delta\right).$$

2.2 Households

To separate the elasticity of intertemporal substitution (EIS) from risk aversion, we assume that the representative agent exhibits recursive preferences (Kreps and Porteus (1978), Epstein and Zin (1989, 1991), and Weil (1989)), whose utility at time t is represented by

$$U_t = \left\{(1-\beta)\, C_t^{1-1/\psi} + \beta \mathbb{E}_t \left[U_{t+1}^{1-\gamma}\right]^{\frac{1-1/\psi}{1-\gamma}}\right\}^{\frac{1}{1-1/\psi}}, \tag{6}$$

where $0 < \beta < 1$ denotes the subjective time discount factor, C_t stands for aggregate consumption at time t, $\psi > 0$ represents the EIS, and the constant relative risk aversion (CRRA) is given by $\gamma > 0$.

Every period the representative agent maximizes her utility (6) by choosing consumption C_t and investment I_t given the aggregate output $Y_t = C_t + I_t$. In addition, the agent chooses the productivity Ω_{t+1} for every future state next period, given the conditional distribution of the exogenous productivity Θ_{t+1} and according to the constraint (2).

The representative agent discounts consumption with her stochastic discount factor given by

$$M_{t,t+1} = \beta \left[\frac{C_{t+1}}{C_t}\right]^{-\frac{1}{\psi}} \left[\frac{U_{t+1}^{1-\gamma}}{\mathbb{E}_t\left(U_{t+1}^{1-\gamma}\right)}\right]^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}}. \tag{7}$$

Besides the macroeconomic quantities, we also study asset prices in the model with technology choice. Specifically, we compute the returns $R_{f,t}$ on the risk-free asset, which pays one unit of consumption next period, and the returns $R_{i,t}$ on real investment, which are equal to the returns on the aggregate consumption claim (Restoy and Rockinger (1994)). In addition,

we study the returns on a risky stock with next period dividends, D_{t+1}, as follows

$$R_{s,t+1} = \frac{P_{t+1} + D_{t+1}}{P_t}, \tag{8}$$

where P_t denotes the price of the dividend claim at time t. Since, the properties of the representative firm dividends generated by the model differ from those of the dividends of the aggregate stock market in the data, we also price a claim to a dividend process, whose growth, denoted by Δd_{t+1}, evolves according to

$$\Delta d_{t+1} = d_0 + d_1 \, \Delta c_{t+1} + d_2 \, u_{t+1}, \tag{9}$$

where $u_{t+1} \sim N(0,1)$, Δc_{t+1} denotes log consumption growth, and d_0, d_1, d_2 are constant coefficients.

2.3 The equilibrium conditions

With recursive preferences, the current value Lagrangian function of the maximization problem with state-contingent technology can be written as

$$\begin{aligned} L_t \;=\; & \left\{ (1-\beta)\, C_t^{1-1/\psi} + \beta \mathbb{E}_t \left[U_{t+1}^{1-\gamma} \right]^{\frac{1-1/\psi}{1-\gamma}} \right\}^{\frac{1}{1-1/\psi}} - \lambda_t^1 (C_t - K_t^{\alpha} \Omega_t^{1-\alpha} + I_t) \\ & - \lambda_t^2 \left[K_{t+1} - (1-\delta) K_t - g_t \right] - \lambda_t^3 \left\{ \mathbb{E}_t \left[\frac{\Omega_{t+1}^{(1-\alpha)\nu}}{\Theta_{t+1}^{(1-\alpha)\nu}} \right] - 1 \right\}, \end{aligned} \tag{10}$$

where λ_t^1, λ_t^2, and λ_t^3 denote Lagrangian multipliers for the three constraints.

Four sets of first-order conditions characterize equilibrium; the first two conditions are the first two constraints that appear in the Lagrangian in (10), that is, consumption equals aggregate output minus investment and that capital accumulates according to (4).[6] The third condition characterizes the optimal amount of investment and the fourth is a set of conditions that determine the optimal productivity choice for every state next period.

[6] The third constraint is automatically satisfied by the optimal productivity choice.

The optimal amount of investment in period t is characterized by the marginal q condition,

$$\frac{1}{g_{I,t}} = \mathbb{E}_t \left[M_{t,t+1} \left(\alpha \frac{Y_{t+1}}{K_{t+1}} + \frac{1 - \delta + g_{K,t+1}}{g_{I,t+1}} \right) \right], \tag{11}$$

where $g_{I,t}$ and $g_{K,t}$ are the partial derivatives of the capital formation function with respect to investment and capital, respectively, in period t. The left hand side of (11) shows the marginal cost of investment which is the amount of investment required to generate a unit of productive capital. The right hand side of (11) describes the marginal benefit from an additional unit of capital, which stems from next period's production and the remaining marginal value of future capital stock. Thus, the firm optimally equates the marginal costs with the marginal benefits of investment. From this first order conditions, returns on an additional unit of investment are:

$$R_{i,t+1} = g_{I,t} \left(\alpha \frac{Y_{t+1}}{K_{t+1}} + \frac{1 - \delta + g_{K,t+1}}{g_{I,t+1}} \right). \tag{12}$$

In our model, the representative firm in a period t optimally chooses the productivity Ω_{t+1} state-by-state for next period, which is given by

$$\left(\frac{\Omega_{t+1}}{\Theta_{t+1}} \right)^{(1-\alpha)\nu} = \frac{\left(M_{t,t+1} \Theta_{t+1}^{1-\alpha} \right)^{\frac{\nu}{\nu-1}}}{\mathbb{E}_t \left[\left(M_{t,t+1} \Theta_{t+1}^{1-\alpha} \right)^{\frac{\nu}{\nu-1}} \right]}, \tag{13}$$

where the ratio on the left hand side is the transformation of the exogenous productivity.[7] Equation (13) describes the tradeoff embedded in the distribution of Ω. On the one hand, it can be beneficial to increase productivity in states where the productivity is exogenously high and decrease it where the productivity is exogenously low. In this way, next period's average productivity is maximized since the cost of deviating from the exogenous productivity is a function of the ratio of transformation.[8] We see this from the case of CRRA preferences, $\gamma = 1/\psi$, and risk neutrality, $\gamma = 0$, where the stochastic discount factor is constant and, thus,

[7] Ω_{t+1} is a function of the state in period t and Θ_{t+1}.

[8] A ten percent increase in productivity when θ is high has the same cost as a ten percent increase in productivity when θ is low. Therefore, increasing productivity when θ is high and decreasing it when it is low maximizes average productivity.

cancels out from (13). As a result, the log optimal endogenous technology is proportional to the log exogenous productivity,

$$\log \Omega_{t+1} \;\propto\; \frac{\nu}{\nu - 1} \log \Theta_{t+1}. \tag{14}$$

On the other hand, when the representative agent is risk averse it can be optimal to shift productivity to high "value" states, that is, states of high marginal utility M. Given the above tradeoff in the model with endogenous technology choice, it can be optimal to amplify or reduce exogenous volatility and, also, it can be optimal to chose a positive or negative correlation between endogenous and exogenous productivity.

3 The log-linearized real economy

This section presents and discusses the economic mechanism behind technology choice. We, first, summarize the log-linearized model economy in a proposition. Second, we discuss the nested standard RBC model without technology choice. Third, we discuss the model economy with optimal technology choice. Appendix B contains proofs and additional details of the log-linearization.

The log-linear model depends on the three state variables θ_t, k_t, and ω_t, which measure the percentage deviations from the steady-state values of the detrended variables Θ_t, K_t, and Ω_t, respectively. Using the law of motion of θ and the equilibrium law of motion of ω, we express the state variables in terms of k_t, θ_{t-1}, and the exogenous shock $\epsilon(t)$.

Proposition 1. *The percentage deviations of utility, consumption, and investment from their steady-state values are given by*

$$\begin{aligned} u_t &= u_k k_t + \tilde{u}_\theta \theta_{t-1} + \sigma_u \epsilon_t, \\ c_t &= c_k k_t + \tilde{c}_\theta \theta_{t-1} + \sigma_c \epsilon_t, \\ i_t &= i_k k_t + \tilde{i}_\theta \theta_{t-1} + \sigma_i \epsilon_t, \end{aligned} \tag{15}$$

where $\tilde{x}_\theta = \phi\,(x_\omega + x_\theta)$ *and* $\sigma_x = \sigma_\omega x_\omega + x_\theta$ *for* $x \in \{u, c, i\}$. *Expressions of the steady state and the coefficients* x_k, x_ω, *and* x_θ *for* $x \in \{u, c, i\}$ *are found in Appendix B. The law of motion of the percentage deviations from the steady-state values of exogenous productivity, capital, and endogenous productivity are*

$$\begin{aligned} \theta_{t+1} &= \phi\,\theta_t + \epsilon_{t+1}, \\ k_{t+1} &= \frac{1-\delta}{e^\mu} k_t + \left(1 - \frac{1-\delta}{e^\mu}\right) i_t, \\ \omega_{t+1} &= \phi\,\theta_t + \sigma_\omega \epsilon_{t+1}, \end{aligned} \tag{16}$$

where $\epsilon \sim \mathbb{N}(0, \sigma^2)$. *Finally, the sensitivity of the endogenous productivity to exogenous shocks is given by*

$$\sigma_\omega = \frac{(1-\alpha)\nu + m_\theta}{(1-\alpha)(\nu - 1) - m_\omega}, \tag{17}$$

where the coefficients $m_\theta = -\gamma c_\theta - (\gamma - \frac{1}{\psi})(u_\theta - c_\theta)$ *and* $m_\omega = -\gamma c_\omega - (\gamma - \frac{1}{\psi})(u_\omega - c_\omega)$ *represent derivatives of the log stochastic discount factor with respect to* θ *and* ω, *respectively.*

In Proposition 1, the sensitivities with respect to ω, i.e., x_ω, represent the sensitivities with respect to the current level of productivity, whether this is endogenous or exogenous. Whereas, the sensitivities with respect to θ, i.e., x_θ, represent the sensitivities to the expected endogenous productivity and, thus, depend on the persistence of the exogenous shocks. If $\phi = 0$, then all sensitivities with respect to θ are zero. What differentiates the dynamics of the technology choice model from the dynamics of the standard RBC model is σ_ω, which represents the optimal technology choice in the log-linearized model and depends on risk aversion, γ. The steady state and the coefficients x_k, x_θ, and x_ω are independent of γ.

3.1 The standard RBC economy

The standard RBC economy corresponds to the case where $\nu = \infty$, in which case expressions simplify as $\sigma_\omega = 1$ and, hence, $\omega = \theta$. Since σ_ω does not depend on γ, macroeconomic

quantities are not risk aversion sensitive, as shown by Tallarini (2000).[9] Nevertheless, the following corollary shows that the economy without technology choice can be observationally identical to the economy with technology choice, provided that the exogenous productivity follows the same process as the endogenous productivity and risk aversion is identical in both economies.

Corollary 1. *The economy without technology choice ($\nu = \infty$), where the exogenous productivity process is given by*

$$\omega_{t+1} = \phi\,\theta_t + \tau_\omega \epsilon_{t+1}$$

is isomorphic in its pricing and macroeconomic implications with the technology choice economy provided that risk aversions are identical and $\tau_\omega = \sigma_\omega$.

3.2 The technology choice economy

We now show that with technology choice, risk aversion exerts a first-order effect on macroeconomic quantities. Proposition 1 shows that σ_ω depends on γ and enters in all elasticities with respect to exogenous shocks, i.e. σ_x. The other parameters and the deterministic steady state are independent of γ. The endogenous parameter σ_ω represents the technology choice, since it determines how endogenous productivity responds to exogenous shocks, and it depends on risk aversion through the marginal utility. The log-linear dynamics imply that (i) risk aversion, through technology choice, generates autocorrelation in the growth rates, which is a propagation mechanism and that (ii) risk aversion, through technology choice, breaks the (almost) perfect correlation between consumption, investment and output. First, we inspect how risk aversion affects technology choice in (17).

3.2.1 Technology choice, σ_ω

The representative firm shifts productivity across states depending on the tradeoff between maximizing average productivity and transferring productivity from low value states to high

[9] Risk aversion can exert second-order effects on macroeconomic quantities.

value states. On one hand, the firm maximizes productivity by shifting it to states with high exogenous productivity, where the transformation cost is lower. This mechanism is driven by the terms $(1-\alpha)\nu$ and $(1-\alpha)(\nu-1)$ in (17). When agents have risk-neutral CRRA utility ($\gamma = 1/\psi = 0$) then $m_\theta = m_\omega = 0$ and σ_ω takes the same value, $\nu/(\nu-1)$, as the exact solution in (14). For this case, a lower ν implies lower transformation cost and, thus, more productivity is shifted to high productivity states.

On the other hand, when agents are risk averse, the firm also wants to shift productivity to high value states, where the value of the state is determined by the value of the stochastic discount factor M. Thus, σ_ω in (17) depends on the elasticity of the marginal utility with respect to shocks to productivity, m_ω, and shocks to expected productivity, m_θ. From Proposition 1, we see that both elasticities depend on risk aversion, γ, and the EIS, ψ.

To understand the impact of risk aversion, we compute the derivatives of m_ω and m_θ with respect to γ, which are $-u_\omega$ and $-u_\theta$. For low values of ψ, both derivatives are typically negative. In this case, as risk aversion increases, the firm shifts productivity from high to low exogenous productivity states, driven by m_θ, and the absolute value of σ_ω declines, driven by m_ω. For high enough γ, σ_ω turns negative and declines below -1. Panel A of Figure 1 plots σ_ω as a function of γ, for various values of ψ, where all other parameters are held constant. Specifically, from the figure, we see that with $\psi = 0.35$, σ_ω turns negative for γ around 20 and declines below -1 for γ around 45.

When m_θ is negative, which is typically the case with low ψ, it is optimal to shift productivity to low exogenous productivity states, because it allows the firm to boost investment and reduce the impact of negative and persistent shocks to expected productivity. This is driven by the elasticity of investment to shocks to expected productivity, which for this case is negative.

Moving on to the EIS, we see that for high enough values of ψ both u_ω and u_θ are negative. In this case, an increase in risk aversion increases σ_ω as shown in Panel A of Figure 1 for ψ equal to 1.5 and 3.0. For $\psi = 3.0$ with γ around 30, technology choice amplifies the exogenous shocks with an σ_ω around 2.

Shocks to the exogenous productivity matter for technology choice only if $\phi > 0$. Otherwise,

when $\phi = 0$, then expected productivity is constant and independent of θ and, thus, m_θ is zero. In this case, where $\phi = 0$, and ψ is low so that u_ω is positive, $\sigma_\omega \to 0$ as $\gamma \to \infty$. That is, in the absence of shocks to expected productivity, typically the firm eliminates all one-period risk as risk aversion increases.[10]

In general, the optimal technology choice balances between maximizing productivity (through shifting productivity to high θ states), minimizing the cost of negative shocks to expected productivity (through shifting productivity to either low or high θ states, which depends on the EIS), and dampening the effect of shocks to productivity (which typically implies decreasing the absolute value of σ_ω, unless ψ is high enough). Obviously, the extent of transformation depends also on the cost of transformation ν. From Panel B of Figure 1, we see that as ν tends to infinity the optimal transformation tends to zero, through $\sigma_\omega \to 1$.

Further, technology choice also depends on the remaining structural parameters. For example, Panel B of Figure 1 shows its dependence on the capital adjustment cost, which decreases with χ, for an example with $\psi = 0.35$. In this case, decreasing capital adjustment costs leads to a steep decline in σ_ω, which is negative for ν below roughly 2.5. With low enough capital adjustment and transformation cost, the elasticity of investment to shocks to expected productivity is negative and large in absolute value, which makes optimal endogenous productivity also large and negative.

The rich structure of technology choice in Figure 1 implies certain relations between the endogenous and the exogenous productivity. For example, (i) when $\sigma_\omega > 1$, it is optimal to choose amplified shocks that comove with the underlying shocks; (ii) when $0 \leq \sigma_\omega \leq 1$, it is optimal to choose less volatile shocks that comove with the underlying shocks; (iii) when $-1 \leq \sigma_\omega \leq 0$, it is optimal to choose less volatile shocks that partly offset the underlying shocks; and, (iv) when $\sigma_\omega \leq -1$, it is optimal to choose amplified shocks that more than offset the underlying shocks.

[10] This corresponds to the case of utility smoothing discussed in Backus, Routledge, and Zin (2013).

3.2.2 Autocorrelation

From Proposition 1, it follows that the endogenous productivity in period $t+1$ is a weighted average between the expected productivity as of period t, $\phi\theta_t$, and the realized exogenous productivity θ_{t+1} (with the weights being $(1-\sigma_\omega)$ and σ_ω, respectively), i.e.,

$$\omega_{t+1} = \phi\,\theta_t + \sigma_\omega \epsilon_{t+1} = \phi(1-\sigma_\omega)\theta_t + \sigma_\omega \theta_{t+1}. \tag{18}$$

When σ_ω is close to zero, the endogenous productivity is close to its expected value and is little affected by the exogenous shock. Nevertheless, the exogenous shocks have a lagged effect on the endogenous productivity since they impact expected productivity, as long as $\phi > 0$. As a result, when $\sigma_\omega < 1$, technology choice generates a propagation mechanism in relation to the endogenous productivity shocks. Generally, when $\phi > 0$ and σ_ω differs from one, then the endogenous productivity follows an ARMA(1,1) process instead of an AR(1) process, which can generate significant autocorrelation in the growth rate of endogenous productivity. The following proposition summarizes the transformation from AR(1) to ARMA(1,1) and its effect on the growth rate of ω.

Proposition 2. *Endogenous technology follows an ARMA(1,1) process, where the AR(1) term originates from the exogenous productivity process,*

$$\omega_{t+1} = \phi\,\omega_t + \sigma_\omega \epsilon_{t+1} + \phi\,(1-\sigma_\omega)\epsilon_t. \tag{19}$$

The unconditional volatility and the first-order autocorrelation of the growth rate of the endogenous technology, denoted by $\Delta\omega_{t+1}$, are given by:

$$\sigma(\Delta\omega) = \sigma(\Delta\theta)\,\sqrt{\phi^2 + (1+\phi)\,(\sigma_\omega^2 - \phi\sigma_\omega)}, \tag{20}$$

where $\sigma(\Delta\theta)^2 = \frac{2\sigma^2}{1+\phi}$ *and*

$$ac_1(\Delta\omega) = -\frac{1}{2}\frac{(1-\phi)\sigma_\omega^2 + \phi(1-\sigma_\omega)(\phi - 2\sigma_\omega - \phi^2)}{\sigma_\omega^2 + \phi(1-\sigma_\omega)(\phi - \sigma_\omega)}. \tag{21}$$

From the case without technology choice, $\sigma_\omega = 1$*, we obtain* $ac_1(\Delta\theta) = \frac{\phi-1}{2}$*.*

Hence, the propagation mechanism is a result of the MA(1) term, $\phi\,(1-\sigma_\omega)\epsilon_t$. This term is generated because the expected endogenous productivity is driven by the exogenous productivity, that is, $\mathbb{E}_t(\omega_{t+1}) = \phi\theta_t$, and because ω_t differs from θ_t, when $\sigma_\omega \neq 1$. As a result, the expected endogenous productivity is driven by ω_t, as well as the difference between ω_t and θ_t, which equals $(1-\sigma_\omega)\epsilon_t$. Therefore, the MA(1) term is generated by the persistence of the exogenous shocks, ϕ, multiplied by the difference between the exogenous and the endogenous productivity.

In the standard model, when ϕ is close to 1 the autocorrelation of the TFP growth is close to 0. With technology choice, the autocorrelation can be either significantly negative or significantly positive, depending on σ_ω. This is shown in Panel B of Figure 1, where we plot (21) for various values of ϕ; similarly, we plot (20) in Panel B. When $\sigma_\omega = \frac{\phi}{1+\phi}$, $ac_1(\Delta\omega)$ reaches its maximum value of $\frac{\phi}{2}$. Also, when $|\sigma_\omega| \to \infty$ we have that $ac_1(\Delta\omega) \to -0.5$. Moreover, Panels A and B show that when the TFP growth is positively (negatively) autocorrelated, it is less (more) volatile than the exogenous productivity growth.

3.2.3 Correlations

Proposition 1 shows that production, investment, and consumption react differently to productivity shocks and shocks to expected productivity. With standard parameters, investment reacts negatively and consumption reacts positively to shocks to expected productivity, while both react positively to productivity shocks. The log-linear solution makes this distinction, through the dependence on ω, which is productivity, and θ, which determines expected productivity. However, in a standard RBC model without technology choice, there is no distinction between shocks to productivity and shocks to expected productivity and, thus, even though the

macro-economic variables react differently to such shocks, their correlations are almost perfect and their autocorrelations are almost identical to that of the TFP.

The introduction of technology choice creates a wedge between productivity ω and expected productivity $\phi\theta$, which breaks the perfect correlation between the macroeconomic variables. The less than perfect correlations has two consequences: (i) differing autocorrelations in the growth rates of macroeconomic variables and (ii) a higher volatility for the growth rate of investment given the volatility in the growth rates of consumption and output.

4 Solution method and asset prices

We log-linearize the economy and use log-normal pricing for the financial quantities similar to Jermann (1998). Modulo the log-linearized approximation, prices are closed-form since the stochastic discount factor, the cash flows, and the state variables are jointly conditionally log-normally distributed.

Starting with cash-flows, the following proposition presents the log consumption growth for the log-linearized approximation of the model's equilibrium.

Proposition 3. *Given the log-linear approximation of the equilibrium in Proposition 1, the log consumption growth is conditionally normal,* $\Delta c_{t+1} = \mu_t + \sigma_c\, \epsilon_{t+1}$. *Its conditional mean is given by*

$$\mu_t = \mu + \mu_k\, k_t + \mu_\theta\, \theta_{t-1} + \sigma_\mu\, \epsilon_t, \tag{22}$$

where $\mu_k = \delta c_k(i_k - 1)$, $\mu_\theta = \delta c_k i_\theta - c_\theta(1-\phi)$, *and* $\sigma_\mu = \delta c_k \sigma_i + c_\theta - \sigma_c$. *The coefficients* σ_c *and* σ_i *are defined in (15).*

The stochastic discount factor in (7) is also log-normally distributed in the log-linear approximation.

Proposition 4. *Given the log-linear approximation of equilibrium in Proposition 1, the log*

stochastic discount factor is conditionally normal:

$$\log M_{t,t+1} = \log \hat{\beta} - \frac{1}{\psi}\mu_t - \sigma_m \, \epsilon_{t+1}, \tag{23}$$

where

$$\log \hat{\beta} = \log \beta + \frac{1}{2}(1-\gamma)\left(\gamma - \frac{1}{\psi}\right)\sigma_u^2 \sigma^2, \tag{23a}$$

$$\sigma_m = \gamma \sigma_c + \left(\gamma - \frac{1}{\psi}\right)(\sigma_u - \sigma_c), \tag{23b}$$

where μ_t is given in Proposition 3, σ_u is defined in (15), and σ denotes the standard deviation of the exogenous shock ε defined in (1).

Since technology choice changes the sensitivities to the exogenous shock, the σ_x's, the price of risk changes when technology choice is introduced. One concern is that the extra flexibility of the economy could significantly affect the model's ability to generate a high price of risk. Fortunately, this is not the case and the technology choice model is on the same footing as the standard RBC model in fitting the price of risk. The reason is as follows: The price of risk, which is approximately equal to σ_m, is determined by the conditional volatility of consumption, σ_c, and the conditional volatility of the utility to consumption ratio, $\sigma_u - \sigma_c$. The coefficient σ_c is effectively pinned down by the unconditional volatility of consumption growth and the technology choice model can fit it. The coefficient $\sigma_u - \sigma_c$ is determined by the long-run volatility of μ_t.[11] From Proposition 3 it follows that technology choice affects only σ_μ, but this coefficient has negligible effect on the long-run volatility of μ_t. Therefore, both the technology choice and the standard RBC model typically require a high γ to fit a high price of risk.

The technology choice model gives us, in addition, a connection between the price of risk and

[11] The log-linear approximation of the utility to consumption ratio is given as follows:

$$u_t - c_t = \sum_{\tau=1}^{\infty} \hat{\beta}^{\tau}(\mu_{t+\tau} - \mu),$$

where $\hat{\beta} = \beta e^{\mu(1-1/\psi)}$ and $\mu_{t+\tau} = \mathbb{E}_t(c_{t+\tau+1} - c_t)$.

production. Specifically, a high price of risk requires also a correspondingly high value for ν, as shown by the following relation (derived from equilibrium condition (13) and the log-linearized solution):

$$\sigma_m = (1-\alpha)\left[\nu\left(1-\sigma_\omega\right)+\sigma_\omega\right], \qquad \text{if } \nu < \infty. \tag{24}$$

The coefficient σ_ω is pinned down in the calibration by the volatility and autocorrelation of the TFP growth. Therefore, when risk aversion is high, then the cost to adjust productivity through technology choice is high.

Moving on to the asset prices, we note that log cash-flows, the conditional mean of consumption growth, and the log stochastic discount factor are jointly normally distributed. Therefore, the logarithm of the prices of zero-coupon bonds, $q_t^{(n)}$, that pay a unit of consumption n periods ahead and the natural logarithm of the price-dividend ratios of dividend strips, $p_t^{(n)}$, that pay a dividend n periods ahead are affine in the state variables and their coefficients can be computed recursively, as shown in Appendix C. We are interested in the one-period interest rate or risk-free rate and the price of the stock, which is a claim to the dividend stream defined in (9). They are given by

$$R_{f,t} = \exp\left(q_t^{(1)}\right) - 1 \qquad \text{and} \qquad P_t = D_t \sum_{n=1}^{\infty} \exp\left(p_t^{(n)}\right).$$

To guarantee convergence, we compute the stock price by the sum of the first 5000 terms.

Since risk aversion is constant, the price of risk, which is given by σ_m, is also constant. As a result, stock prices and bond prices vary only due to changes in cash-flow expectations as shown in the following proposition.

Proposition 5. *Given the stochastic discount factor in Proposition 4, the continuously compounded one-period risk-free rate is*

$$r_{f,t} = -\log\hat{\beta} + \frac{1}{\psi}\mu_t - \frac{1}{2}\sigma_m^2\sigma^2. \tag{25}$$

The log-linear approximation of the stock price-dividend ratio is given by

$$p_t - d_t \approx \overline{p-d} + \left(d_1 - \frac{1}{\psi}\right) \mathbb{E}_t \sum_{\tau=0}^{\infty} \hat{J}^{\tau} \left(\mu_{t+\tau} - \mu\right), \tag{26}$$

where d_1 is a constant coefficient in (9) and where $\hat{J}$ and $\overline{p-d}$ are defined in Appendix C.

We see that the dynamics of stock and bond prices depend on the dynamics of consumption growth expectations, μ_t, and, in general, the dynamics of the term-structure of consumption growth expectations, $\mu_{t+\tau}, \tau \geq 0$. These can be inferred from Proposition 3 and are determined by the dependence of μ_t on the state and the dynamics of the state variables.

5 Calibration

Since technology choice depends on all structural parameters, we calibrate the model to further analyze its implications with special focus on the effect of risk aversion on macroeconomic quantities. We judge the quantitative implications of the model through three calibrations: Models 1 and 3 have low EIS, while Model 2 has high EIS. While we focus on macroeconomic quantities, we also look at asset pricing implications and certain macro-finance linkages.

5.1 Data

We collect data for the period $1947Q1$ to $2012Q4$. Macroeconomic variables are from the NIPA tables. Output series are taken to be the total output reported, the consumption series is the consumption of non-durables and services, and the investment series is the non-residential fixed investments. All macroeconomic variables are deflated by inflation computed from the CPI index of the Bureau of Labor Statistics and normalized by the civilian noninstitutional population with age over 16, from the Current Population Survey (Serial ID LNU00000000Q). The total factor productivity (TFP) is inferred from the output series and the capital series constructed by Fernald (2014). We use quarterly CRSP value-weighted returns as the market return and the Fama 3-month T-bill rate as the risk-free rate from WRDS. Real returns equal

nominal returns deflated by inflation. The price-dividend ratio is inferred from the CRSP value-weighted returns with and without dividends.

5.2 Parameter selection

Certain parameters are fixed across model calibrations, which we discuss first. The long-term quarterly growth rate, μ, is set to 0.4%; this value is close to the average growth rates of output, consumption, and investment in the data. The capital share, α, is set to 0.36, which is similar to the capital share in Boldrin, Christiano, and Fisher (2001), while the quarterly depreciation rate, δ, is computed from the average investment to capital ratio over the data sample period from the NIPA tables as 0.026. Finally, the dividend process parameters, $d = (0.0025, 1.65, 0.04)$, are chosen to fit the mean and volatility of the annual growth in aggregate dividends as well as its correlation with the annual growth in consumption.

The technology choice parameter, ν, is set to fit the TFP autocorrelation with low EIS in Model 1 and with high EIS in Model 2. Model 3, instead, is calibrated to fit the volatility of investment growth for which the model requires a low EIS. Each model with technology choice is compared to a model without technology choice, $\nu = \infty$, referred to as the standard model throughout. To highlight the role of technology choice, the standard model has identical parameters, except for the exogenous volatility which is adjusted to fit the TFP growth volatility.

We focus on a set of models where we set γ equal to 5.[12] For the high EIS case, we set ψ equal to 1.5 and for the models with low EIS, we set ψ as low as possible requiring that the subjective discount factor, β, stays below one. The remaining parameters are determined as follows: the volatility of the exogenous shock, σ, is set to fit the volatility of TFP growth, the capital adjustment cost parameter, χ, is set to match the consumption growth volatility, the subjective discount factor is adjusted to fit the average price-to-dividend ratio of the stock market, and,

[12] In the Online appendix D, we consider a similar set of economies (Models 4 to 6) where we calibrate γ to fit the Sharpe ratio of the stock market. In these models, the parameter ν needs to be correspondingly higher and considerably higher than in Models 1 to 3. The implications of the model with technology choice are the same with the exception that with low γ the model cannot fit the stock market Sharpe ratio.

finally, the autocorrelation of the exogenous shock, ϕ, is chosen to fit the autocorrelation of the price-dividend ratio.

5.3 The macroeconomy

Panel A in Table 1 shows the following macroeconomic quantities: volatility of the log growth rate of consumption, investment, and output.[13] Each volatility is standardized by the volatility of the log growth rate of TFP, which we also report. In Panel A, all models with technology choice perfectly match the point estimate of the volatility of consumption, output, and TFP growth. Model 3 also matches perfectly the point estimate of the volatility of investment growth. The first two models produce lower investment growth volatility, nevertheless we cannot reject the hypothesis that the data, including the volatility of the growth rate of investment, are generated by the calibrated models with technology choice.

From Panel B, we see that Model 1 with technology choice matches almost perfectly the point estimates of the macroeconomic autocorrelations, despite being calibrated to match only the autocorrelation of the TFP growth. In the data, the autocorrelations of TFP and output are around 0.25, the autocorrelation of consumption growth is close to zero, 0.04, while the autocorrelation of investment equals 0.37. The hypothesis that the macroeconomic autocorrelations are generated by Model 1 with technology choice cannot be rejected for the four autocorrelations. Model 2, which exhibits high EIS is less successful in this regard as the autocorrelations of the macroeconomic growth rates are all close to 0.25. Model 3, which matches the investment growth volatility but cannot simultaneously match the autocorrelation of TFP growth, generates overall higher autocorrelations than in the data. Without technology choice all autocorrelations are effectively equal to zero in all our cases.

From Panel B of Figure 2, we gauge that to match the 0.25 autocorrelation in TFP growth in the data requires $\sigma_\omega < 1$. Indeed, σ_ω is close to 0.77 in Models 1 and 2, inferred from Panel

[13] To facilitate comparison between model calibrations and the data, each table shows the $t - statistic$ of the corresponding quantities with respect to the data estimate; i.e., a $t - statistic$ is computed as the difference between the data estimate and the model average scaled by the square root of the sum of the squared standard errors of the data estimate and the model average. Standard errors of the data estimates are Newey and West (1987) corrected, using 16 lags.

B of Table 3. Using maximum likelihood, we estimate an ARMA(1,1) process for the detrended TFP series yielding an estimate for σ_ω of 0.80. Panel A of Figure 2 shows that these estimates of σ_ω imply that the endogenous productivity is less volatile than the exogenous. Therefore, to fit the 1.81% volatility of TFP growth requires σ to be 2.25% for Model 1 and 2.27% for Model 2. These values are close to the 2.20% implied by the estimated ARMA(1,1) process.[14] Model 3, which is calibrated to fit the volatility of investment growth, instead, has a lower σ_ω of 0.51 and, thus, a higher σ of 2.61%. In the remaining analysis, we mostly focus attention on Model 1 because it explains the macroeconomic autocorrelations and the calibrated σ_ω is close to the estimated one; yet, Models 2 and 3 have their own merit.

The impulse response functions for Model 1 in Figure 3 show the propagation mechanism that generates the autocorrelations in the growth rates. In Panel A, we observe that the first period response of productivity to a one standard deviation shock in the standard model (dashed lines) is almost identical to that in the model with technology choice (solid lines). With technology choice, the positive shock propagates as a result of the MA(1) term and causes productivity to keep growing. With a σ_ω less than one, the productivity is expected to grow because the positive shock increases the expected productivity, $\phi\theta$, by more than the productivity, ω. The impulse response functions of output, shown in Panel B, exhibit very similar behavior in that the first period shock is almost identical for the two models but with technology choice the output shock propagates over to the second period.

Panel C of Table 1 shows that the correlations between the growth rates of macroeconomic variables in the standard model are perfect. Model 1 and 2 with technology choice produce almost perfect correlations; except that the correlation between consumption growth and investment growth in Model 1 is 0.95. Yet, from Model 3 we learn that technology choice can lead to low correlations between the macroeconomic growth rates. For instance, the correlation between consumption and investment growth, which at 0.43 is particularly low in the data, in Model 3 is 0.63.[15] While Model 3 fails to fit the point estimate in the data and while all but one

[14]The close match between the Model 1 values for σ_ω and σ and those implied by the estimated ARMA(1,1) process, implies that the TFP in the data is captured well by an ARMA(1,1) process.

[15]Obviously, in the data output does not equal to consumption plus investment, which contributes to the low

of the hypotheses that the data are generated by the model are rejected, we see a significant improvement relative to the standard model or the technology choice Models 1 and 2. This improvement, however, comes at the cost of somewhat higher autocorrelations.

We see that for Model 2, the volatilities of the macroeconomic variables are virtually identical with and without technology choice. This is because the high EIS in Model 2 generates almost perfect correlations between output, consumption, and investment, even with technology choice. With high EIS and technology choice, investments' response to expected productivity shocks are small and, thus, correlations are unaffected. This is also the reason why the autocorrelations in Model 2 are almost the same. For Models 1 and 3 with low EIS, the economies without technology choice show relatively smoother consumption and more volatile investment. Without technology choice, the representative firm cannot partly offset the negative expected productivity shocks with increasing investment and, thus, investment becomes more volatile. Since both the standard model and the model with technology choice fit output volatility, higher investment volatility in the standard model implies a smoother consumption process.

The impulse responses in Panels A and B of Figure 3, respectively, show how the behavior of consumption and investment change when we introduce technology choice in Model 1. The first thing we note is that technology choice generates propagation also for consumption and investment. Then in Panel A, we see that the first period reaction of investment weakens and that of consumption strengthens with the introduction of technology choice. This explains both the higher autocorrelation of investment, 0.36, compared to that of consumption, 0.12, and the less than perfect correlation between their growth rates. The reason behind the relatively weaker reaction of investment compared to consumption is, as we explained earlier, that investment in this case decreases when the expected productivity increases.

We close by noting that, to fit investment growth volatility, in addition to consumption and output volatility, Model 3 requires a relatively low correlation between consumption and invest-

correlation between output and investment and output and consumption. In the model, we have $Y = C + I$ at all times. If we measure output in the data as $Y = C + I$, then the point estimates for the correlation of output growth with investment growth increases from 0.62 to 0.77 and between output and consumption growth the correlation increases from 0.68 to 0.90. The model can produce a consumption-investment growth correlation at least as low as 0.43, as can be seen from Figure 4.

ment. This is because with perfect correlation, the volatility of output growth is a weighted average of the volatilities of consumption growth and investment growth.[16] Otherwise, when consumption and investment are less than perfectly correlated, $\sigma(\Delta y)$ is less than the weighted average of $\sigma(\Delta c)$ and $\sigma(\Delta i)$, which implies that $\sigma(\Delta y)$ and $\sigma(\Delta c)$ can be fitted with higher $\sigma(\Delta i)$. Model 3, generates this low correlation with a lower value for ν compared to Model 1. With higher flexibility in technology choice and low EIS, investment's responses to expected productivity shocks are larger, while moving counter to responses in consumption.

5.4 Asset prices

All Models are calibrated to match the mean and first-lag autocorrelation of the log price-to-dividend ratio of the exogenous dividend claim, where the dividend process is calibrated to match certain time-series properties of the aggregate stock market dividend in the data. What might seem surprising is that technology choice, which allows for consumption smoothing across states, does not restrict the performance of the model pertaining to asset pricing moments, as can be verified from the asset pricing moments shown in Table 5 in the Online Appendix D.[17] To the contrary, we see that the moments of the risk-free rate, shown in Table 2, improve: The technology choice model produces a higher volatility and a lower first-order autocorrelation of the risk-free rate, and lower absolute correlation between the log price-dividend ratio and the risk-free rate than the standard model.[18] Also, the model with technology choice generates the predictability of output growth by the risk-free rate.

All of the above improvements of the model with technology choice relative to the standard model result from the propagation of endogenous shocks that allows to fit the autocorrelation of output growth. From Proposition 5, we know that the risk-free rate in the model is entirely driven by the expected consumption growth, μ_t, given in Proposition 3. In the standard model,

[16] The weights are the fractions of consumption and investment of total output in the steady state, which are determined by the structural parameters.

[17] Table 5 shows the asset pricing moments of the models with $\gamma = 5$, whereas Table 7 shows the asset pricing moments of the models with γ calibrated to fit the Sharpe ratio of the dividend claim.

[18] Many economists argue that the expected volatility of the real risk-free rate is only half of the realized volatility. If so, our model reproduces all point estimates involving the risk-free rate.

μ_t is mostly driven by slow movements in capital and θ, which explains the low volatility and high autocorrelation of the risk-free rate. With technology choice, the loading of μ_t on the exogenous shocks, σ_μ, is high. Panel B of Table 3 shows that in Model 1, σ_μ with technology choice is 0.096% compared to 0.003% in the standard model, 0.228% compared to 0.004% for Model 2, and 0.197% compared to 0.005% in Model 3. The hightened σ_μ increases the volatility of the risk-free rate and decreases its autocorrelation. This is shown in the impulse response of Panel C of Figure 3. At the same time, the high σ_μ decreases the correlation of the risk-free rate with the price-to-dividend ratio, because the price-to-dividend ratio is little affected by short-lived shocks to μ_t, since it is mostly driven by shocks to long-run expectations. Thus, the dynamics of $p - d$ are virtually unaffected by the increase in σ_μ, as can be verified by its impulse response function, shown in Panel D of Figure 3. Specifically, the first period response to the exogenous shock is very similar in both models.

Further, the output growth autocorrelation implies that much of the output growth variation is predictable. Therefore, since the risk-free rate in the technology choice model responds to shocks to expected growth in consumption, that are highly correlated with the expected growth shocks in output, it predicts output growth. Specifically, from Table 2, we see that in the data the risk-free rate predicts output growth with a correlation of 0.34. In Models 1 and 2 the same correlation is 0.28 and 0.29, respectively, which are statistically indistinguishable from the data. In Model 3, the higher flexibility in technology choice generates higher autocorrelation and predictiability that is too high compared to the data.

Finally, from Table 5 in the Online Appendix D we see that the model with or without technology choice cannot fit the Sharpe ratio with low risk aversion. In a model without technology choice, Kaltenbrunner and Lochstoer (2010) provide a model calibration that simultaneously fits the risk-free rate volatility and the Sharpe ratio using the same level of risk aversion as we do. They achieve this with permanent shocks and a volatility of the exogenous shocks that is twice as large as in our model. Since we calibrate the volatility of exogenous shocks to match the volatility of TFP growth and the autocorrelation to fit the first-lag autocorrelation of the price-to-dividend ratio, we cannot fit the Sharpe ratio without a much higher risk aversion.

5.5 The role of risk aversion

To see how risk aversion affects the model economy with technology choice, we perform a sensitivity analysis with respect to γ for Model 1, keeping all other parameters constant.[19] Figure 4 shows several quantities related to the macroeconomy as a function of risk aversion. Dashed lines correspond to the standard model and solid lines correspond to the model with technology choice. The first thing we note from all the graphs, is that the log-linear dynamics of the standard model are independent of risk aversion.

From Panel A we see that σ_ω decreases with risk aversion. In Panel B we see that, similar to σ_ω, the sensitivities of output, consumption, and investment to exogenous shocks also decrease with risk aversion. The investment sensitivity is the quantity that varies the most with respect to risk aversion, indicating how investment responds negatively to exogenous shocks for smoothing consumption over time. In fact, for values of γ beyond 15, σ_i turns negative, implying that a negative exogenous shock leads to an increase in investment.

The unconditional volatilities of the growth rates are shown in Panels C and D and all exhibit a U-shape. The volatilities initially decrease and reach their minima when the autocorrelations of the growth rates reach their maxima. These patterns reflect those observed in Figure 2. The autocorrelations of output, consumption, and investment are shown in Panel F. Initially, all autocorrelations increase with risk aversion, as the firm shifts more and more resources across states to lessen the impact of exogenous shocks, decreasing in this way σ_ω, and reach their maxima at around 0.5. Investment, once more is the quantity that reaches first its maximum reflecting the fact that with low EIS investment responses are more sensitive to risk aversion, compared to consumption and output.

The fact that output, consumption, and investment respond differently to technology choice, is then reflected by the less than perfect correlations in Panel E. As we discussed earlier, the less than perfect correlation is due to the fact that investment reacts negatively to shocks to expected productivity, while consumption reacts positively to such shocks and the fact that technology choice creates a wedge between productivity and expected productivity. As risk

[19] In the Online appendix, we present plots of the sensitivity analysis for Models 2 and 3.

aversion increases, this wedge increases and the correlations decrease. In particular, the correlation between investment and consumption decreases substantially and reaches a minimum slightly above 0.4.

Figure 5 shows how several asset pricing quantities vary with risk aversion. In Panel A, we plot the price of risk and its two components. We see that in both models the price of risk increases with risk aversion, but less so with technology choice. Since in this analysis ν is kept constant, increasing risk aversion causes more smoothing of the marginal utility across states. The same pattern is observed in Panel C showing the Sharpe ratio of investment returns, SR_i. Apparently, risk aversion and technology choice have a somewhat bigger impact on the Sharpe ratio of the dividend claim.

Finally, in Panels B and D we observe that risk aversion has a significant impact on the behavior of the risk-free rate in the model with technology choice. As risk aversion increases, the risk-free rate becomes more volatile and less autocorrelated, while the absolute correlation with the price-to-dividend ratio decreases. At the same time, risk aversion increases the power of the risk-free rate to predict output growth and reaches its maximum of close to 1 at around $\gamma = 20$. This is where σ_y, which reflects the one-period risk of output, reaches zero. For the standard model, risk aversion has no discernible effect on these aspects of the risk-free rate.

6 Conclusions

In this paper, we embark on a theoretical exploration of technology choice or state-contingent technology in a production-based economy. Our point of departure is that from a theoretical point of view, we have no reason to believe that risk aversion does not affect real life investment and production decisions. We are also not aware of empirical evidence that would unambiguously support such a hypothesis. On the contrary, recent evidence (Belo, 2010; Jermann, 2010) shows that the production side of the economy provides asset pricing related information through technology choice and suggests that risk aversion is also important for the macroeconomy. Unfortunately, the feature that risk aversion can have, at most, a second-order effect is

hardwired into the standard real business cycle model. In this respect, we find it important to analyze a general equilibrium model where risk aversion, not just the elasticity of intertemporal substitution, matters.

We solve the standard RBC model with technology choice and derive the first-order effects of risk aversion. The first important result is that technology choice explains the autocorrelation in the growth rates of consumption, investment, output, and TFP, exhibiting a propagation effect. This allows us to calibrate the model and analyze the effect of risk aversion: We find that risk aversion affects significantly the volatilities, correlations, and autocorrelations of the macroeconomic quantities. In addition, the model explains the output growth predictability by the risk-free rate and improves on certain moments of the risk-free rate. Also, technology choice can explain the low correlation between consumption growth and investment growth and the high investment growth volatility.

Given the aforementioned results, we feel that this line of research is promising for our understanding of how the macroeconomy operates as well as for its link with the financial side. A natural extension of the model is to incorporate time-varying risk aversion, since it can help explaining asset pricing moments. However, with the current form for technology choice the macroeconomic quantities are quite sensitive to risk aversion, as indicated by our sensitivity analysis. We infer that the extent of time-variation in risk aversion that is required to explain asset prices would have counter-factual implications for the macroeconomy. To incorporate time-varying risk aversion, we should develop a micro-founded model of technology choice. We leave these more ambitious goals for future research.

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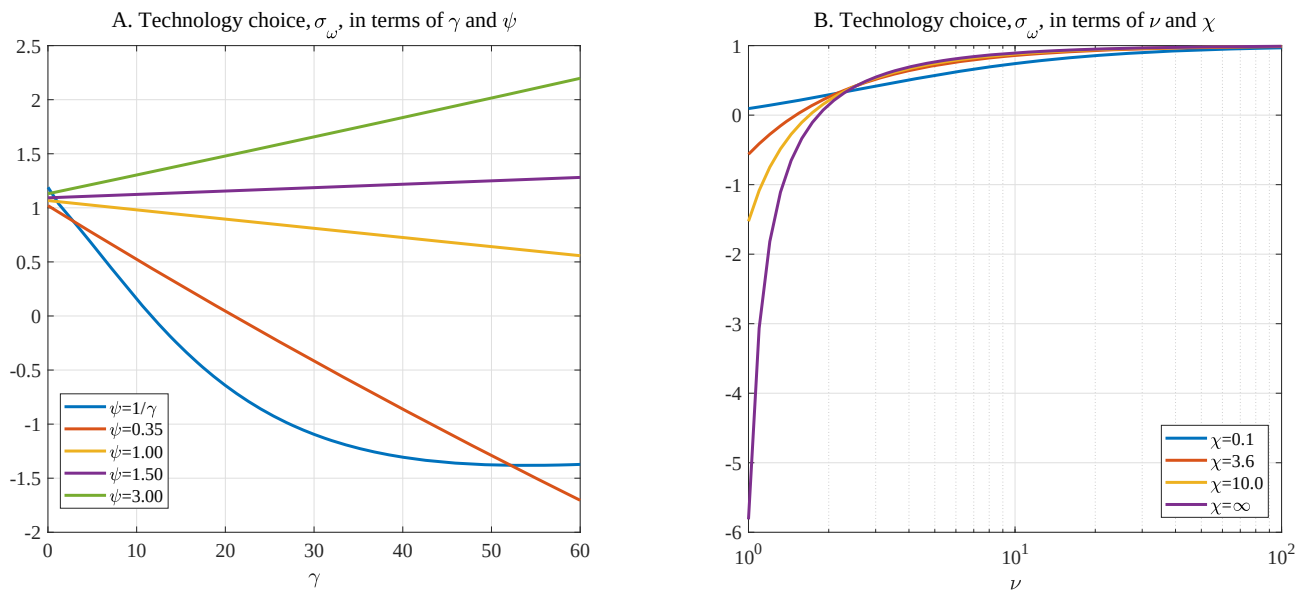
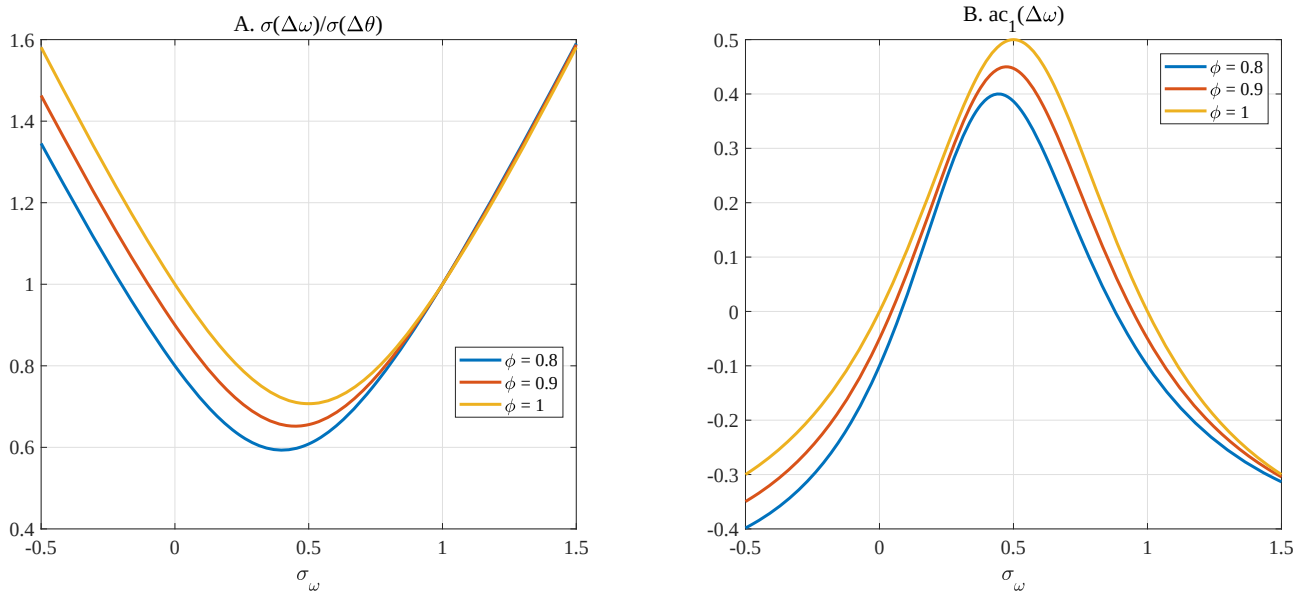


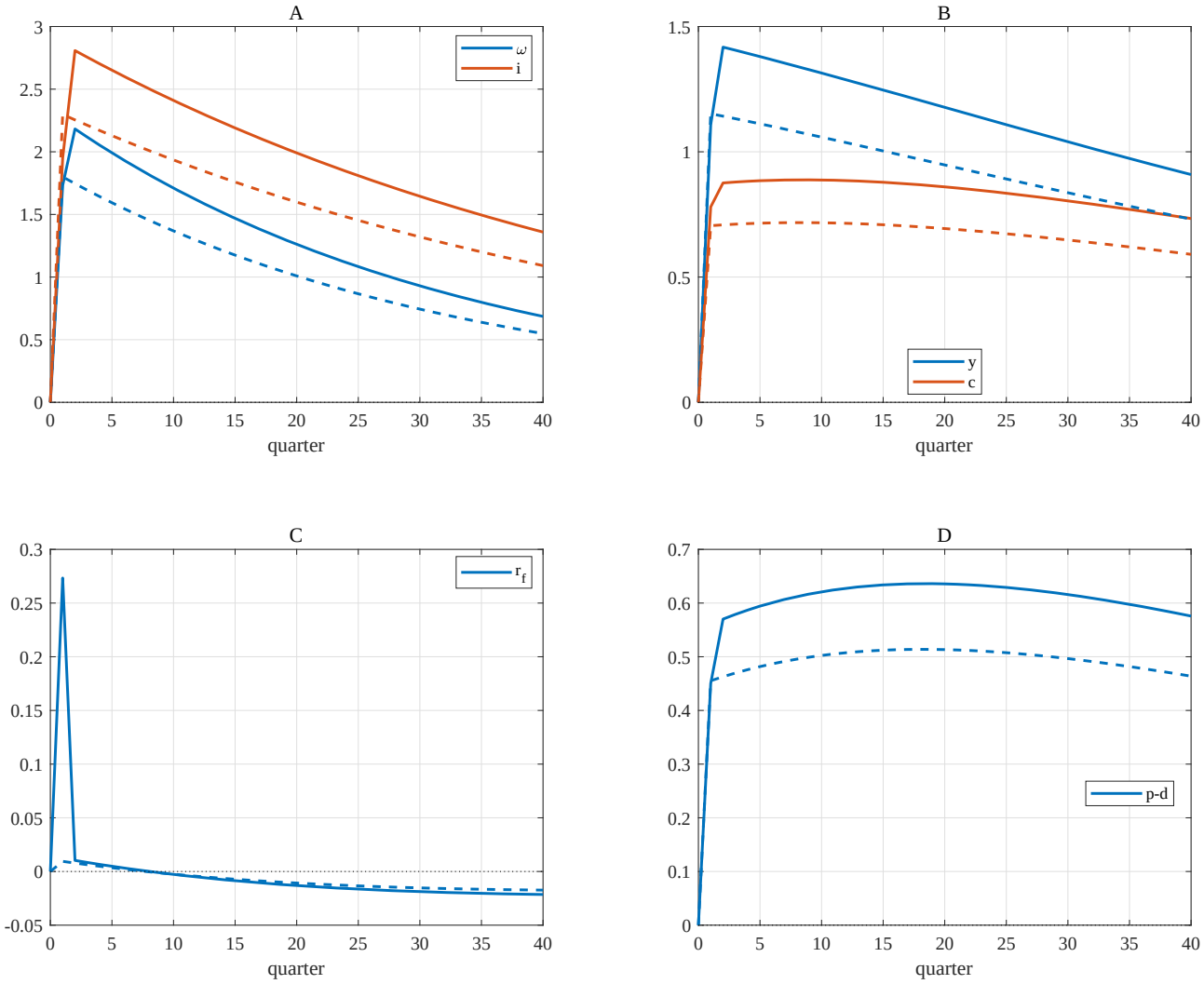
The figures plot the endogenous technology choice, σ_ω, as function of the risk aversion γ, the elasticity of intertemporal substitution ψ, the technology choice parameter ν, and the capital adjustment cost parameter χ. The remaining parameters are set as in Model 1, which uses $\gamma = 5$, $\psi = 0.35$, $\nu = 6.2$ and $\chi = 3.6$.

Figure 1: Technology choice



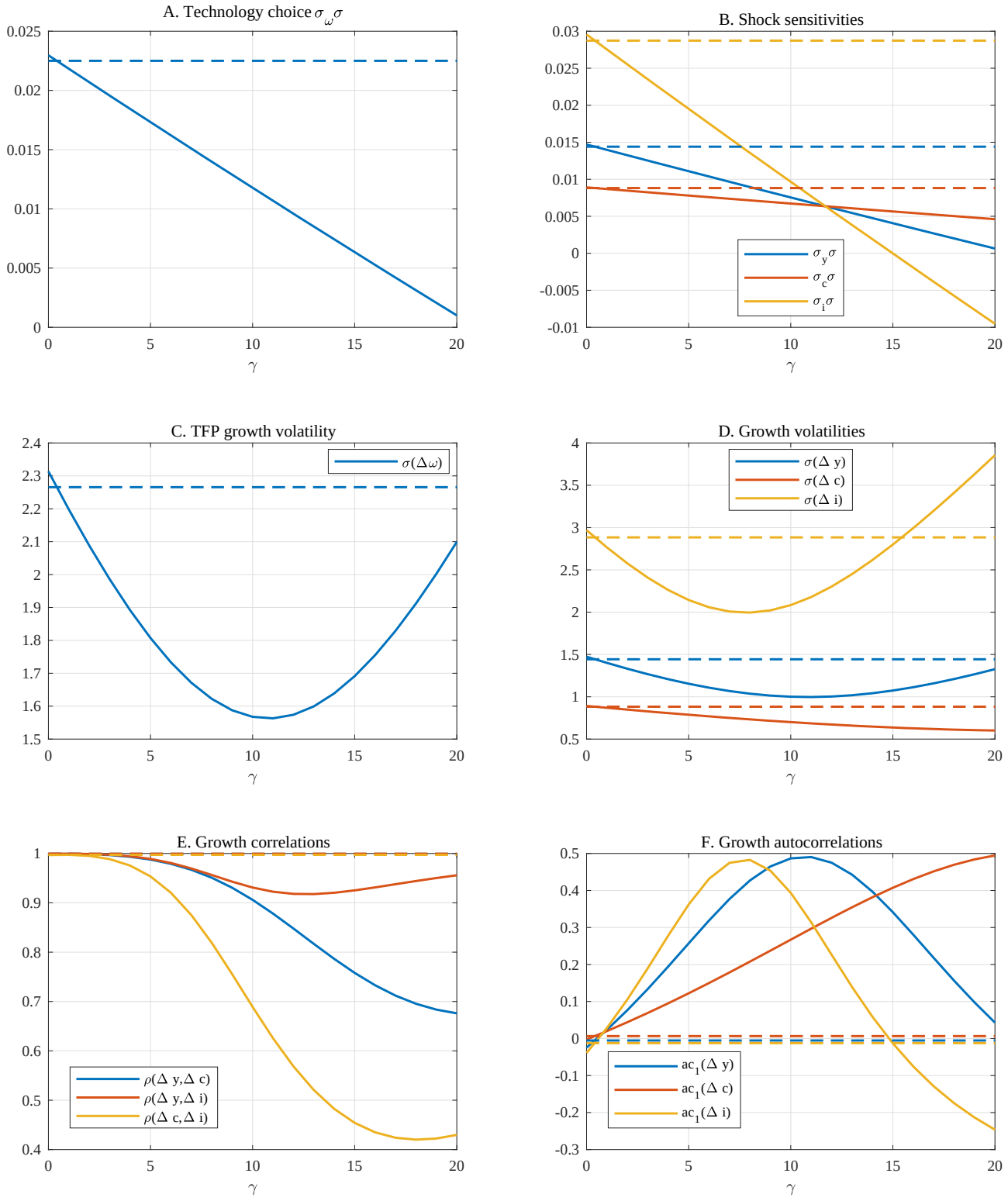
The figures plot the unconditional volatility and first-order autocorrelation of endogenous productivity shocks with respect to the optimal technology choice (σ_ω), as provided by Proposition 2, assuming different persistence (ϕ) of exogenous shocks.

Figure 2: Unconditional volatility and autocorrelation of endogenous shocks



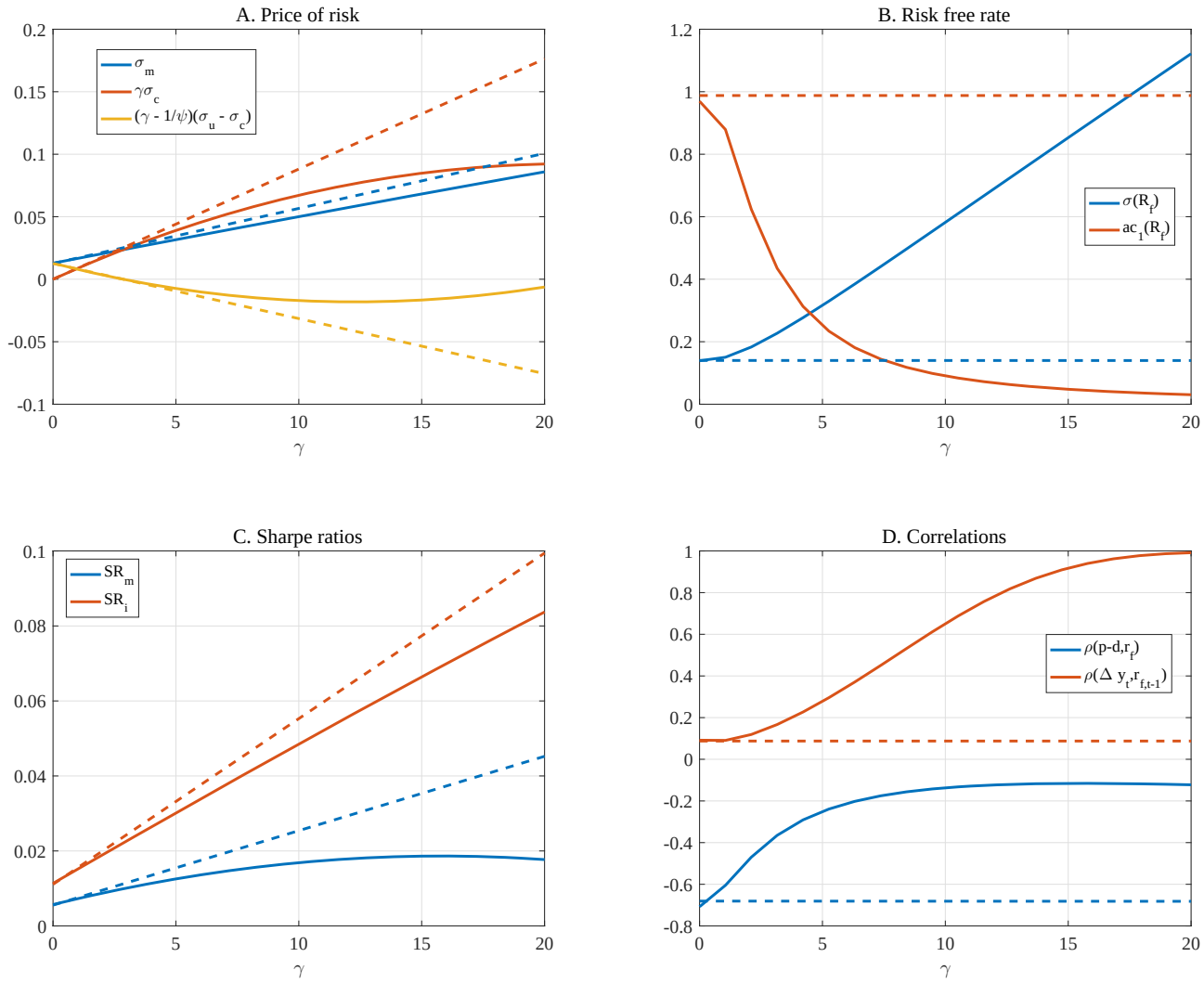
The figure plots impulse responses to a one standard deviation exogenous shock, for Model 1 with technology choice (solid lines) and without (dashed lines).

Figure 3: Impulse response functions (Model 1)



The figure plots several macro quantities by varying the γ parameter, for Model 1 with technology choice (solid lines) and without (dashed lines).

Figure 4: Sensitivity analysis of macro quantities in relation to γ (Model 1)



The figure plots several asset pricing quantities by varying the γ parameter, for Model 1 with technology choice (solid lines) and without (dashed lines).

Figure 5: Sensitivity analysis of asset pricing quantities in relation to γ (Model 1)

Table 1: Macroeconomic moments

The data statistics are computed from quarterly macroeconomic data obtained from the NIPA tables over 1947Q1-2012Q4. The model statistics are averages of 1000 simulated paths of 300 quarters with a burn-in of 100 quarters. The parentheses next to the data estimates show the standard errors (*s.e.*), which are Newey and West (1987) corrected with 16 lags. The parentheses next to the model statistics show the t-statistics (*t.st.*) of the hypotheses that the data estimates are different from the model averages. Δx denotes the first-difference of the natural logarithm of a variable X. y denotes (the natural logarithm of) total output; c denotes total consumption; i denotes total investment and ω denotes total factor productivity. For a variable x, $\sigma(x)$ denotes its volatility; $ac_1(x)$ is its first-order autocorrelation and $\rho(x, z)$ is its correlation with variable z. The models with technology choice are calibrated to fit the TFP growth volatility, the consumption growth volatility, the first-order autocorrelation of the stock market log price-to-dividend ratio and the first-order autocorrelation of the TFP growth (Models 1 and 2) or the investment growth volatility (Model 3). The exogenous volatility (σ) of the models without technology choice ($\nu = \infty$) is adjusted to fit the TFP growth volatility. For all models the average growth rate in the economy is $\mu = 0.4\%$, the capital share is $\alpha = 0.36$, the capital depreciation is $\delta = 0.026$ and the dividend process parameters are $d = (-0.0026, 1.65, 0.04)$. With technology choice the σ_ω is 0.77, 0.76, and 0.51 for Models 1, 2, and 3, respectively.

			Model 1				Model 2				Model 3			
			$\psi = 0.35, \gamma = 5, \chi = 3.6$				$\psi = 1.5, \gamma = 5, \chi = 4.5$				$\psi = 0.3, \gamma = 5, \chi = 4.5$			
			$\beta = 0.9990, \phi = 0.970$				$\beta = 0.9903, \phi = 0.960$				$\beta = 0.9989, \phi = 0.970$			
			$\nu = \infty$		$\nu = 6.20$		$\nu = \infty$		$\nu = 2.60$		$\nu = \infty$		$\nu = 2.95$	
	Data		$\sigma = 1.80$		$\sigma = 2.25$		$\sigma = 1.79$		$\sigma = 2.27$		$\sigma = 1.80$		$\sigma = 2.61$	
	est.	*s.e.*	*avg.*	*t.st.*	*avg.*	*t.st.*	*avg.*	*t.st.*	*avg.*	*t.st.*	*avg.*	*t.st.*	*avg.*	*t.st.*
A. Volatilities														
$\sigma(\Delta y)/\sigma(\Delta\omega)$	0.64	(0.09)	0.64	(0.01)	0.64	(0.02)	0.64	(0.01)	0.64	(0.03)	0.64	(0.01)	0.64	(0.02)
$\sigma(\Delta c)/\sigma(\Delta\omega)$	0.44	(0.07)	0.39	(0.74)	0.44	(0.06)	0.43	(0.10)	0.44	(0.02)	0.37	(0.98)	0.44	(0.04)
$\sigma(\Delta i)/\sigma(\Delta\omega)$	1.40	(0.22)	1.27	(0.58)	1.19	(0.97)	1.17	(1.06)	1.16	(1.08)	1.32	(0.39)	1.40	(0.01)
$\sigma(\Delta\omega)$	1.81	(0.14)	1.81	(0.01)	1.81	(0.05)	1.81	(0.05)	1.81	(0.04)	1.81	(0.01)	1.81	(0.01)
B. Autocorrelations														
$ac_1(\Delta y)$	0.24	(0.07)	-0.00	(3.73)	0.26	(0.24)	-0.01	(3.82)	0.26	(0.33)	-0.00	(3.72)	0.49	(3.77)
$ac_1(\Delta c)$	0.04	(0.07)	0.01	(0.48)	0.12	(1.18)	0.01	(0.43)	0.28	(3.46)	0.01	(0.43)	0.24	(2.92)
$ac_1(\Delta i)$	0.37	(0.08)	-0.01	(4.67)	0.36	(0.13)	-0.02	(4.81)	0.25	(1.45)	-0.01	(4.67)	0.36	(0.11)
$ac_1(\Delta\omega)$	0.25	(0.07)	-0.02	(3.92)	0.25	(0.06)	-0.02	(4.01)	0.25	(0.01)	-0.02	(3.92)	0.48	(3.36)
C. Correlations														
$\rho(\Delta y, \Delta i)$	0.68	(0.05)	1.00	(6.36)	0.99	(6.13)	1.00	(6.34)	1.00	(6.32)	1.00	(6.36)	0.88	(4.00)
$\rho(\Delta y, \Delta c)$	0.62	(0.04)	1.00	(9.99)	0.99	(9.71)	1.00	(9.95)	1.00	(9.92)	1.00	(9.98)	0.92	(7.98)
$\rho(\Delta c, \Delta i)$	0.43	(0.06)	1.00	(8.78)	0.95	(8.11)	1.00	(8.70)	0.99	(8.64)	1.00	(8.77)	0.63	(3.11)

Table 2: Asset pricing moments

The empirical data statistics are computed from quarterly return data obtained from CRSP and WRDS Federal Reserve data (3-month T-bill yields) over 1947Q1-2012Q4. The model statistics are computed as averages from 1000 simulated paths, where each path has 300 quarters with a burn-in of 100 quarters. The parentheses next to the data estimates show the standard errors ($s.e.$), which are Newey and West (1987) corrected with 16 lags. The parentheses next to the model statistics show the t-statistics ($t.st.$) of the hypotheses that the data estimates are different from the model averages. $p-d$ denotes the log price-to-dividend ratio of the dividend claim and R_f is the quarterly risk-free rate. For a variable x $\mu(x)$ is the mean; $\sigma(x)$ is the standard deviation and $ac_1(x)$ is the first-order autocorrelation; whereas $\rho(p-d, r_f)$ denotes the correlation between $p-d$ and the log risk-free rate. For the model calibrations and other parameter values see caption of Table 1.

			Model 1				Model 2				Model 3			
			$\psi = 0.35, \gamma = 5, \chi = 3.6$				$\psi = 1.5, \gamma = 5, \chi = 4.5$				$\psi = 0.3, \gamma = 5, \chi = 4.5$			
			$\beta = 0.9990, \phi = 0.970$				$\beta = 0.9903, \phi = 0.960$				$\beta = 0.9989, \phi = 0.970$			
			$\nu = \infty$		$\nu = 6.20$		$\nu = \infty$		$\nu = 2.60$		$\nu = \infty$		$\nu = 2.95$	
	Data		$\sigma = 1.80$		$\sigma = 2.25$		$\sigma = 1.79$		$\sigma = 2.27$		$\sigma = 1.80$		$\sigma = 2.61$	
	$est.$	$s.e..$	$avg.$	$t.st.$	$avg.$	$t.st.$	$avg.$	$t.st.$	$avg.$	$t.st.$	$avg.$	$t.st.$	$avg.$	$t.st.$
$\mu(p-d)$	4.87	(0.10)	4.87	(0.03)	4.88	(0.04)	4.87	(0.01)	4.87	(0.00)	4.86	(0.10)	4.86	(0.08)
$ac_1(p-d)$	0.98	(0.01)	0.98	(0.23)	0.98	(0.61)	0.98	(0.16)	0.98	(0.55)	0.98	(0.39)	0.99	(0.88)
$\sigma(R_f)$	0.94	(0.12)	0.11	(7.16)	0.32	(5.38)	0.04	(7.78)	0.16	(6.71)	0.12	(7.12)	0.60	(2.92)
$ac_1(R_f)$	0.34	(0.10)	0.99	(6.69)	0.25	(0.97)	1.01	(6.72)	0.13	(2.22)	0.98	(6.61)	0.13	(2.27)
$\rho(p-d, r_f)$	0.04	(0.13)	-0.68	(5.68)	-0.25	(2.29)	0.77	(5.76)	0.18	(1.08)	-0.58	(4.89)	-0.14	(1.45)
$\rho(\Delta y_t, r_{f,t-1})$	0.34	(0.10)	0.09	(2.67)	0.28	(0.66)	0.11	(2.46)	0.29	(0.53)	0.07	(2.81)	0.67	(3.33)

Table 3: Log-linear solution

The table provides the parameters of the log-linear solution of the three calibrated models:

$$x_t = x_k\, k_t + x_\theta\, \theta_t + x_\omega\, \omega_t \qquad \Rightarrow \qquad x_t = x_k\, k_t + \tilde{x}_\theta\, \theta_{t-1} + \sigma_x\, \epsilon_t,$$

where x_t is the log-deviation of variable X from the steady-state, and where $\tilde{x}_\theta = \phi(x_\omega + x_\theta)$ and $\sigma_x = \sigma_\omega x_\omega + x_\theta$. When $x = k$, x_t refers to k_{t+1}, which is determined at the end of period t. See Proposition 1 for further information. For each model configuration, the standard model differs from the technology choice model only in the parameters ν and σ, which affect only σ_ω and, hence, the σ_x's. The parameters in Panel A and the parameters $\tilde{x}_\theta$ in Panel B are the same for the standard and the technology choice model. Panel B also provides the coefficients of the exogenous shocks, σ_x's, in percentages, and after being multiplied by the volatility of the exogenous shocks, σ. The coefficients are provided for TFP (ω), capital (k), output (y), consumption (c), investment (i), expected consumption growth (μ), and utility (u). The last row of Panel B shows the coefficient of the stochastic discount factor (m) to the exogenous shock, whose absolute value is the price of risk, i.e. the maximum conditional Sharpe ratio.

	Model 1			Model 2			Model 3		
	$\psi = 0.35, \gamma = 5, \chi = 3.6$			$\psi = 1.5, \gamma = 5, \chi = 4.5$			$\psi = 0.3, \gamma = 5, \chi = 4.5$		
	$\beta = 0.9990, \phi = 0.970$			$\beta = 0.9903, \phi = 0.960$			$\beta = 0.9989, \phi = 0.970$		
	A. Coefficients to state vector $(k_t, \theta_t, \omega_t)$								
x	x_k	x_θ	x_ω	x_k	x_θ	x_ω	x_k	x_θ	x_ω
κ	0.981	-0.015	0.053	0.966	-0.000	0.035	0.981	-0.016	0.055
y	0.360	0.000	0.640	0.360	0.000	0.640	0.360	0.000	0.640
c	0.356	0.196	0.196	0.557	0.007	0.428	0.362	0.208	0.168
i	0.370	-0.503	1.780	-0.146	-0.018	1.182	0.354	-0.535	1.856
μ	-0.007	0.179	-0.177	-0.019	0.411	-0.409	-0.007	0.151	-0.147
u	0.110	0.188	0.007	0.110	0.148	0.007	0.111	0.190	0.008
	B. Coefficients of θ_{t-1} and ϵ_t								
		σ_x $(\times 100\sigma)$			σ_x $(\times 100\sigma)$			σ_x $(\times 100\sigma)$	
		$\nu = \infty$	$\nu = 6.20$		$\nu = \infty$	$\nu = 2.60$		$\nu = \infty$	$\nu = 2.95$
x	$\tilde{x}_\theta$	$\sigma = 1.80$	$\sigma = 2.25$	$\tilde{x}_\theta$	$\sigma = 1.79$	$\sigma = 2.27$	$\tilde{x}_\theta$	$\sigma = 1.80$	$\sigma = 2.61$
ω	0.970	1.800	1.732	0.960	1.790	1.724	0.970	1.800	1.328
κ	0.037	0.069	0.058	0.033	0.062	0.060	0.038	0.071	0.032
y	0.621	1.152	1.108	0.614	1.146	1.104	0.621	1.152	0.850
c	0.380	0.705	0.780	0.418	0.779	0.754	0.364	0.676	0.766
i	1.238	2.298	1.951	1.118	2.085	1.998	1.281	2.377	1.067
μ	0.002	0.003	0.096	0.002	0.004	0.228	0.003	0.005	0.197
u	0.190	0.352	0.437	0.149	0.278	0.348	0.191	0.355	0.505
m		-2.770	-3.164		-1.723	-2.012		-2.693	-3.271

A Online appendix (Not for publication)

A Technology choice

The following section provides some economic intuition for the reduced-form formulation of technology choice, which we borrow from Cochrane (1993), in our economy. Suppose that the central planner can choose to invest in a complete set of different technologies as in Jermann (2010). With a complete set we mean that there are as many independent technologies, indexed by $i = [1, \ldots, I]$, as there are states of nature denoted by $s = [1, \ldots, S]$. The productivity of a technology i is denoted by $\Theta_i(s)$ for state s. Without loss of generality, let also $\Theta_1(s)$ be the productivity next period for the exogenous benchmark technology which is log-normally distributed,

$$\log \Theta_1 = \mu + \epsilon, \tag{A1}$$

where $\epsilon \sim N(0, \sigma^2)$. Define

$$\vartheta_i(s) = \frac{\Theta_i(s)}{\Theta_1(s)}, \forall i = 1, ..., I,$$

where by definition $\vartheta_1(s) = 1$.

Each technology produces the same good and the production of a technology i is given by

$$Y_i(s) = K_i^{\alpha} \Theta_i(s)^{1-\alpha},$$

where K_i is the capital invested in technology i at the beginning of the current period. The central planner has a total of K capital to allocate over the set of technologies. Let w_i be the fraction invested in technology i, i.e.,

$$w_i = \frac{K_i}{K}.$$

Then, total production can be expressed as follows:

$$Y = K^{\alpha} \Theta_1^{1-\alpha} \sum_{i=1}^{I} w_i^{\alpha} \vartheta_i(s)^{1-\alpha}.$$

Let us now define

$$T(\mathbf{w}, s) = \sum_{i=1}^{I} w_i^{\alpha} \vartheta_i(s)^{1-\alpha} \qquad \text{and} \qquad \Omega(s) = \Theta_1(s) T(\mathbf{w}, s)^{1/1-\alpha}.$$

Then, aggregate output can be rewritten as

$$Y = K^{\alpha}\Omega^{1-\alpha},$$

where Ω becomes the endogenously chosen productivity or technology next period through the choice of the portfolio of technologies $\mathbf{w} = [w_1, \ldots, w_I]$. Since, the production technology market is complete, instead of choosing $\mathbf{w}$ the social planner can directly choose Ω (or T) in all future states given, of course, the joint productivity distribution of the technologies. Instead of specifying, however, the joint productivity distribution of the available technologies, we adopt the reduced-form assumption by which we can choose T given the constraint

$$\mathbb{E}\left[T^{\nu}\right] \leq 1, \tag{A2}$$

for some constant ν. This implies that the endogenously chosen productivity Ω can have any conditional distribution as long as (A2) holds. Since we log-linearize the economy, the endogenous productivity next period Ω can be expressed as

$$\log \Omega = \log X + \sigma_{\omega}\epsilon + \sigma_u u, \tag{A3}$$

where $u \sim N(0,1)$ is an innovation to productivity orthogonal to ϵ. The central planner can therefore choose, σ_{ω}, σ_u and X according to a certain objective and subject to the constraint (A3). Choosing $\sigma_{\omega} = 1$, $\sigma_u = 0$ and $\log X = \mu$ ensures that $\Omega = \Theta$.

To understand the role of the parameter ν, we can derive the optimal choice for σ_{ω}, σ_u, and $\log X$ from maximizing average production next period, which is given by

$$\mathbb{E}\left[\Omega^{1-\alpha}\right] = X^{1-\alpha}\exp\left[\frac{1}{2}(1-\alpha)^2(\sigma^2\sigma_{\omega}^2 + \sigma_u^2)\right].$$

Then, we can investigate the cost to average production from deviating from such a choice. Note, first, that the productivity choice constraint (A2) implies that

$$X^{1-\alpha} \leq \exp\left\{(1-\alpha)\mu + \frac{1}{\nu} - \frac{1}{2}\nu(1-\alpha)^2\left[(\sigma_{\omega}-1)^2 + \sigma_u^2\right]\right\}.$$

Assuming, therefore, that the above constraint is binding at the optimum, we have that the average productivity next period is given by

$$\mathbb{E}\left[\Omega^{1-\alpha}\right] = \exp\left\{(1-\alpha)\mu + \frac{1}{\nu} + \frac{1}{2}(1-\alpha)^2\left[\sigma_{\omega}^2\sigma^2 - \nu(\sigma_{\omega}-1)^2\sigma^2 + (1-\nu)\sigma_u^2\right]\right\}.$$

Maximizing next period's average production would then mean that

$$\max_{\sigma_\omega,\sigma_u} \qquad \sigma_\omega^2\sigma^2 - \nu(\sigma_\omega - 1)^2\sigma^2 + (1-\nu)\sigma_u^2.$$

Given this maximization problem, if ν was less than one then increasing σ_u as much as possible would be the optimal decision. To avoid such examples, we restrict to cases where $\nu > 1$ and, therefore, the optimal solution is $\sigma_u^* = 0$. The optimal exposure to the exogenous productivity of the benchmark technology becomes then

$$\sigma_\omega^* = \frac{\nu}{\nu - 1},$$

which ensures the maximum average production next period. If any other exposure $\sigma_\omega = \sigma_\omega^* - \Delta$ is chosen, then the cost to the average production is proportional to $(\nu - 1)\Delta^2$. Therefore, the larger the parameter ν is, the larger is the cost to average production from a deviation Δ from the growth optimal choice. When $\nu \to \infty$, then it becomes infinitely costly to deviate from the exogenous benchmark productivity and $\sigma_\omega^* \to 1$.

B Loglinearization

B.1 Equilibrium conditions

With a slight abuse of notation, all variables below are normalized by the time trend. The equilibrium conditions for recursive preferences with technology choice are summarized as follows:

$$\lambda_t^1 = (1-\beta)U_t^{\frac{1}{\psi}} C_t^{-\frac{1}{\psi}}, \tag{B4}$$

$$\lambda_t^1 = \lambda_t^2\, g_{I_t}^{'}, \tag{B5}$$

$$\lambda_t^2 e^{\mu} = \mathbb{E}_t\left[\frac{\partial U_t}{\partial U_{t+1}}\frac{\partial U_{t+1}}{\partial K_{t+1}}\right], \tag{B6}$$

$$0 = \frac{\partial U_t}{\partial U_{t+1}}\frac{\partial U_{t+1}}{\partial \Omega_{t+1}} - \lambda_t^3 \nu \left(\frac{\Omega_{t+1}^{1-\alpha}}{\Theta_{t+1}^{1-\alpha}}\right)^{\nu-1} \frac{1-\alpha}{\Theta_{t+1}^{1-\alpha}}\Omega_{t+1}^{-\alpha}, \tag{B7}$$

$$\Omega_t^{1-\alpha}K_t^{\alpha} = C_t + I_t, \tag{B8}$$

$$K_{t+1}e^{\mu} = (1-\delta)K_t + g_t, \tag{B9}$$

$$1 = \mathbb{E}_t\left[\frac{\Omega_{t+1}^{1-\alpha}}{\Theta_{t+1}^{1-\alpha}}\right]^{\nu}, \tag{B10}$$

$$U_t = \left\{(1-\beta)\, C_t^{1-1/\psi} + \beta e^{\mu(1-\gamma)/\theta}\mathbb{E}_t\left[U_{t+1}^{1-\gamma}\right]^{\frac{1-1/\psi}{1-\gamma}}\right\}^{\frac{1}{1-1/\psi}}. \tag{B11}$$

The key variables in the deterministic steady-state of the economy are described by

$$
\begin{aligned}
K &= \left[\frac{e^{\mu/\psi} - \beta(1-\delta)}{\alpha\beta}\right]^{\frac{1}{\alpha-1}}, \\
C &= K^{\alpha} - (e^{\mu} - 1 + \delta)K, \\
I &= (e^{\mu} - 1 + \delta)K, \\
U &= C\left[\frac{1-\beta}{1-\beta e^{\mu(1-1/\psi)}}\right]^{\frac{1}{1-1/\psi}}, \\
\lambda^1 &= (1-\beta)\left[\frac{1-\beta}{1-\beta e^{\mu(1-1/\psi)}}\right]^{\frac{1}{\psi-1}}, \\
\lambda^2 &= (1-\beta)\left[\frac{1-\beta}{1-\beta e^{\mu(1-1/\psi)}}\right]^{\frac{1}{\psi-1}}, \\
\lambda^3 &= \frac{\beta K^{\alpha} e^{\mu(1-1/\psi)}\lambda^1}{\nu},
\end{aligned}
$$

where variables without subscripts indicate steady-state values. Clearly, the deterministic state is independent of risk aversion γ, and only λ^3 depends on the technology choice curvature ν.

B.2 Loglinearization: Recursive preferences with technology choice

By convention, the percentage deviation of variable X_t from its detrended steady-state value (X) is defined as $x_t = logX_t - logX$. For example, the exogenous technology shock process can be rewritten as $\theta_t = \phi\,\theta_{t-1} + \epsilon_t$ where $\epsilon \sim \mathbb{N}(0, \sigma^2)$. The loglinearized model depends on the three state variables θ_t, k_t, and ω_t, which measure the percentage deviation from the steady-state values of the detrended variables Θ_t, K_t, and Ω_t.

The percentage deviations of consumption, investment, and utility can be summarized as follows

$$c_t = c_k k_t + c_\omega \omega_t + c_\theta \theta_t, \tag{B12}$$

$$i_t = i_k k_t + i_\omega \omega_t + i_\theta \theta_t, \tag{B13}$$

$$u_t = u_k k_t + u_\omega \omega_t + u_\theta \theta_t, \tag{B14}$$

where c_k, c_ω, c_θ, i_k, i_ω, i_θ,u_k, u_ω, and u_θ are coefficients to be determined.

Log-linearizing the above equilibrium conditions (B7) and (B10) gives the optimal technology choice

$$\omega_{t+1} = \phi(1-\sigma_\omega)\theta_t + \sigma_\omega \theta_{t+1} = \phi\theta_t + \sigma_\omega \epsilon_{t+1}, \tag{B15}$$

where

$$\sigma_\omega = \frac{(1-\alpha)\nu - \frac{1}{\psi}c_\theta + (\frac{1}{\psi}-\gamma)u_\theta}{(1-\alpha)(\nu-1) + \frac{1}{\psi}c_\omega + (\gamma - \frac{1}{\psi})u_\omega}. \tag{B16}$$

Loglinearizing the equilibrium conditions gives the coefficients. For example, c_k is the positive root from the following quadratic equation

$$0 = B\Big[\frac{\alpha(C+I)k_2}{I} + k_1\Big] - \frac{\alpha(C+I)-I}{\chi I} - \Big(\frac{Bk_2C}{I} - \frac{1}{\psi} - \frac{C}{\chi I}\Big)c_k, \tag{B17}$$

where

$$B = \frac{\alpha K^{\alpha-1}(\alpha-1)}{\alpha K^{\alpha-1}+1-\delta} - \frac{c_k}{\psi} + \frac{1}{\chi(\alpha K^{\alpha-1}+1-\delta)}\left[\frac{\alpha(C+I)}{I} - \frac{C}{I}c_k - 1\right], \tag{B18}$$

$$k_1 = \frac{1-\delta}{e^\mu}, \tag{B19}$$

$$k_2 = \frac{e^\mu - 1 + \delta}{e^\mu}. \tag{B20}$$

The other coefficients are given by:

$$c_\omega = \frac{\frac{(\alpha-1)(C+I)}{\chi I} + B\frac{k_2}{I}(1-\alpha)(C+I)}{Bk_2\frac{C}{I} - \frac{1}{\psi} - \frac{C}{\chi I}}, \tag{B21}$$

$$c_\theta = \frac{\phi\left\{\frac{\alpha K^{\alpha-1}(1-\alpha)}{\alpha K^{\alpha-1}+1-\delta} - \frac{c_\omega}{\psi} + \frac{1}{\chi(\alpha K^{\alpha-1}+1-\delta)}\left[\frac{(1-\alpha)(C+I)}{I} - \frac{C}{I}c_\omega\right]\right\}}{\phi\left(\frac{1}{\psi} + \frac{C}{(\alpha K^{\alpha-1}+1-\delta)\chi I}\right) + \frac{Bk_2C}{I} - \frac{1}{\psi} - \frac{C}{\chi I}}, \tag{B22}$$

$$i_k = \frac{\alpha(C+I)}{I} - \frac{C}{I}c_k, \tag{B23}$$

$$i_\omega = \frac{(1-\alpha)(C+I)}{I} - \frac{C}{I}c_\omega, \tag{B24}$$

$$i_\theta = -\frac{C}{I}c_\theta, \tag{B25}$$

$$u_k = \frac{u_1c_k}{1-u_2k_1-u_2k_2i_k}, \tag{B26}$$

$$u_\omega = u_1c_\omega + u_2k_2u_ki_\omega, \tag{B27}$$

$$u_\theta = \frac{u_1c_\theta + u_2k_2u_ki_\theta + \phi u_2u_\omega}{1-\phi u_2}, \tag{B28}$$

where

$$u_1 = 1 - \beta e^{\mu(1-\frac{1}{\psi})}, \tag{B29}$$

$$u_2 = \beta e^{\mu(1-\frac{1}{\psi})}. \tag{B30}$$

From the above equations, we see that coefficients $u_k, u_\omega, u_\theta, c_k, c_\omega, c_\theta, i_k, i_\omega, i_\theta$ are dependent on EIS (ψ) but independent of the risk aversion (γ) and technology choice curvature (ν). Moreover, from equation (B16), σ_ω depends on the risk aversion (γ) and technology choice curvature (ν). Thus, in a standard RBC economy without technology choice, macroeconomic quantities are not risk aversion sensitive. Introducing technology choice makes macroeconomic quantities sensitive to the risk aversion. Proposition 1 concludes the above subsection.

C Stock and bond prices

The stochastic discount factor, M, is log-normally distributed, as shown in Proposition 4, and can be expressed in the following form:

$$\log M_{t,t+1} = \log \hat{\beta} - \frac{1}{\psi}\mu_t - \sigma_m \epsilon_{t+1} = m_0 + m_1\, z_t - \sigma_m\, \epsilon_{t+1},$$

where $z_t = [k_{t+1}, \theta_t, c_t]'$ is a sufficient state vector, which is also normally distributed, and whose law of motion can be expressed as,

$$z_{t+1} = Z\, z_t + \Sigma_z\, \epsilon_{t+1}.$$

The coefficients Z and Σ_z are inferred from equations (15) and (16), while the coefficients of the stochastic discount factor are $m_0 = \log \hat{\beta} - \mu/\psi$ and $m_1 = -[c_k, c_\theta, -1]/\psi$.

C.1 Zero coupon bonds

The natural logarithm of the prices of the zero-coupon bonds $q_t^{(n)}$, that pay a unit of consumption n periods ahead, satisfy recursively the standard Euler equation

$$q_t^{(n)} = \log \mathbb{E}_t \left[M_{t,t+1} \exp\left(q_{t+1}^{(n-1)} \right) \right]. \tag{C31}$$

As a result $q_t^{(n)}$ are affine in the state z_t, that is,

$$q_t^{(n)} = q_0^{(n)} + q_1^{(n)}\, z_t, \tag{C32}$$

and the above coefficients are defined recursively as follows,

$$q_0^{(n)} = m_0 + q_0^{(n-1)} + \frac{1}{2}\sigma^2 \left(q_1^{(n-1)}\, \Sigma_z - \sigma_m \right)^2, \tag{C33}$$

$$q_1^{(n)} = m_1 + q_1^{(n-1)} Z, \tag{C34}$$

for $n \geq 1$ where $q_0^{(0)} = 0,\ q_1^{(0)} = [0, 0, 0]$. The one-period risk-free rate is given by

$$r_{f,t} = -m_0 - \frac{1}{2}(\sigma_n \sigma)^2 - m_1 z_t = -\log \hat{\beta} + \frac{1}{\psi}\mu_t - \frac{1}{2}\sigma_m^2 \sigma^2. \tag{C35}$$

C.2 Zero coupon stocks

Similarly, the natural logarithm of the price-dividend ratios of the zero-coupon stocks $p_t^{(n)}$, that pay the dividend D_{t+n}, n periods ahead, satisfy recursively the Euler equation,

$$p_t^{(n)} = \log \mathbb{E}_t \left[M_{t,t+1} \exp\left(\Delta d_{t+1} + p_{t+1}^{(n-1)} \right) \right], \tag{C36}$$

and as such are also affine in the state z_t,

$$p_t^{(n)} = p_0^{(n)} + p_1^{(n)}\, z_t. \tag{C37}$$

Given the definition of the dividend process specified in (9), the coefficients of the coefficients of the zero-coupon stock price-dividend ratios are defined recursively according to,

$$p_0^{(n)} = m_0 + d_0 + d_1\, \mu + \frac{1}{2}\, d_2^2 + p_0^{(n-1)} + \frac{1}{2}\sigma^2 \left(d_1\, \sigma_c - \sigma_m + p_1^{(n-1)}\, \Sigma_z \right)^2, \tag{C38}$$

$$p_1^{(n)} = (1 - d_1\, \psi)\, m_1 + p_1^{(n-1)} Z, \tag{C39}$$

for $n \geq 1$ where $p_0^{(0)} = p_1^{(0)} = 0$.

C.3 Log-linear approximation of the stock price-dividend ratio

The Euler equation of the stock is given as follows

$$e^{p_t - d_t} = \mathbb{E}_t \left[J_{t,t+1} \left(e^{p_{t+1} - d_{t+1}} + 1 \right) \right], \tag{C40}$$

where $J_{t,t+1} = M_{t,t+1} D_{t+1}/D_t$. The log of the price-dividend ratio is approximated to be linear in the state z_t,

$$p_t - d_t \approx \overline{p - d} + b\, z_t, \tag{C41}$$

where $\overline{p - d}$ is the average log price-dividend ratio, since the mean of the vector z_t is zero, and we need to solve for the coefficients b. If $z_t = 0$, then $p_{t+1} - d_{t+1} = \overline{p - d} + b\Sigma_z \epsilon_{t+1}$. Solving the Euler equation when the state is $z_t = 0$, we obtain the following:

$$e^{\overline{p-d}} = \hat{J} e^{\overline{p-d}} + J, \tag{C42}$$

where

$$\log\, J = \log \hat{\beta} + d_0 + \left(d_1 - \frac{1}{\psi}\right)\mu + \frac{1}{2}\left[d_2^2 + \sigma^2(d_1\sigma_c - \sigma_m)^2\right], \tag{C43}$$

$$\log\, \hat{J} = \log\, J + (d_1\sigma_c - \sigma_m)\, b\Sigma_z + \frac{1}{2}(b\Sigma_z)^2, \tag{C44}$$

and, therefore, $\overline{p-d} = \log\left(J/(1-\hat{J})\right)$. Solving the Euler equation for a general state and applying a first-order approximation, we obtain the following:

$$(p_t - d_t) - \overline{p-d} \approx \hat{J}\,\mathbb{E}_t\left[(p_{t+1} - d_{t+1}) - \overline{p-d}\right] + \left(d_1 - \frac{1}{\psi}\right)(\mu_t - \mu)\,. \tag{C45}$$

Solving forward the above equation, we obtain the following expression:

$$p_t - d_t \approx \overline{p-d} + \left(d_1 - \frac{1}{\psi}\right)\sum_{\tau=0}^{\infty}\hat{J}^{\tau}\mathbb{E}_t\left(\mu_{t+\tau} - \mu\right). \tag{C46}$$

Finally, using the proposed approximation (C41) in equation (C45), we obtain that

$$b = (1 - d_1\psi)\, m_1\left(\mathbf{I} - \hat{J}Z\right)^{-1}, \tag{C47}$$

where $\mathbf{I}$ indicates the unit matrix, which in this case is three dimensional. In the above equation, b is solved numerically since $\hat{J}$ also includes b.

D Additional tables and figures

Table 4: Log-linear solution

The table provides the parameters of the log-linear solution of the three calibrated models:

$$x_t = x_k\, k_t + x_\theta\, \theta_t + x_\omega\, \omega_t \qquad \Rightarrow \qquad x_t = x_k\, k_t + \tilde{x}_\theta\, \theta_{t-1} + \sigma_x\, \epsilon_t,$$

where x_t is the log-deviation of variable X from the steady-state, and where $\tilde{x}_\theta = \phi(x_\omega + x_\theta)$ and $\sigma_x = \sigma_\omega x_\omega + x_\theta$. When $x = k$, x_t refers to k_{t+1}, which is determined at the end of period $t+1$. See Proposition 1 for further information. For each model configuration, the standard model differs from the technology choice model only in the parameters ν and σ, which affect only σ_ω and, hence, the σ_x's. The parameters in Panel A and the parameters $\tilde{x}_\theta$ in Panel B are the same for the standard and the technology choice model. Panel B also provides the coefficients of the exogenous shocks, σ_x's, in percentages, and after being multiplied by the volatility of the exogenous shocks, σ. The coefficients are provided for TFP (ω), capital (k), output (y), consumption (c), investment (i), expected consumption growth (μ), and utility (u). The last row of Panel B shows the coefficient of the stochastic discount factor (m) to the exogenous shock, whose absolute value is the price of risk, i.e. the maximum conditional Sharpe ratio.

	Model 4			Model 5			Model 6		
	$\psi = 0.3, \gamma = 110, \chi = 3.6$			$\psi = 1.5, \gamma = 200, \chi = 4.5$			$\psi = 0.3, \gamma = 100, \chi = 6.5$		
	$\beta = 0.9997, \phi = 0.970$			$\beta = 0.9935, \phi = 0.980$			$\beta = 0.9995, \phi = 0.970$		
	A. Coefficients to state vector $(k_t, \theta_t, \omega_t)$								
x	x_k	x_θ	x_ω	x_k	x_θ	x_ω	x_k	x_θ	x_ω
k	0.982	-0.017	0.056	0.967	-0.001	0.034	0.981	-0.019	0.061
y	0.360	0.000	0.640	0.360	0.000	0.640	0.360	0.000	0.640
c	0.342	0.210	0.179	0.563	0.013	0.416	0.354	0.236	0.118
i	0.408	-0.562	1.875	-0.100	-0.029	1.148	0.376	-0.637	2.047
μ	-0.006	0.161	-0.159	-0.018	0.406	-0.396	-0.007	0.101	-0.097
u	0.121	0.206	0.008	0.077	0.186	0.005	0.123	0.209	0.009
	B. Coefficients of θ_{t-1} and ϵ_t								
		σ_x $(\times 100\sigma)$			σ_x $(\times 100\sigma)$			σ_x $(\times 100\sigma)$	
		$\nu = \infty$	$\nu = 155$		$\nu = \infty$	$\nu = 260$		$\nu = \infty$	$\nu = 80$
x	$\tilde{x}_\theta$	$\sigma = 1.80$	$\sigma = 2.26$	$\tilde{x}_\theta$	$\sigma = 1.80$	$\sigma = 2.25$	$\tilde{x}_\theta$	$\sigma = 1.80$	$\sigma = 2.55$
ω	0.970	1.800	1.729	0.980	1.800	1.743	0.970	1.800	1.488
k	0.038	0.071	0.059	0.033	0.060	0.058	0.041	0.076	0.042
y	0.621	1.152	1.106	0.627	1.152	1.116	0.621	1.152	0.953
c	0.377	0.699	0.783	0.420	0.771	0.753	0.344	0.638	0.779
i	1.274	2.364	1.972	1.096	2.013	1.935	1.367	2.537	1.421
μ	0.002	0.003	0.089	0.010	0.018	0.224	0.004	0.008	0.114
u	0.208	0.386	0.480	0.187	0.343	0.426	0.211	0.391	0.545
m		-43.503	-53.827		-68.868	-85.462		-39.955	-55.303

Table 5: Asset pricing moments

The empirical data statistics are computed from quarterly return data obtained from CRSP and WRDS Federal Reserve data (3-month T-bill yields) over 1947Q1-2012Q4. The model statistics are computed as averages from 1000 simulated paths, where each path has 300 quarters with a burn-in of 100 quarters. The parentheses next to the data estimates show the standard errors (*s.e.*), which are Newey and West (1987) corrected with 16 lags. The parentheses next to the model statistics show the t-statistics (*t.st.*) of the hypotheses that the data estimates are different from the model averages. $p-d$ denotes the log price-to-dividend ratio of the dividend claim; R_f is the quarterly risk-free rate, R_m is the quarterly return to the dividend claim and R_i is the quarterly investment return. For a variable x $\mu(x)$ is the mean; $\sigma(x)$ is the standard deviation and $ac_1(x)$ is the first-order autocorrelation. The Sharpe ratio, SR, equals $\mu(R-R_f)/\sigma(R-R_f)$ and $\rho(p-d, r_f)$ denotes the correlation between $p-d$ and the log risk-free rate. For the model calibrations and other parameter values see caption of Table 1.

			Model 1				Model 2				Model 3			
			$\psi=0.35, \gamma=5, \chi=3.6$				$\psi=1.5, \gamma=5, \chi=4.5$				$\psi=0.3, \gamma=5, \chi=4.5$			
			$\beta=0.9990, \phi=0.970$				$\beta=0.9903, \phi=0.960$				$\beta=0.9989, \phi=0.970$			
			$\nu=\infty$		$\nu=6.20$		$\nu=\infty$		$\nu=2.60$		$\nu=\infty$		$\nu=2.95$	
	Data		$\sigma=1.80$		$\sigma=2.25$		$\sigma=1.79$		$\sigma=2.27$		$\sigma=1.80$		$\sigma=2.61$	
	est.	*s.e..*	*avg.*	*t.st.*	*avg.*	*t.st.*	*avg.*	*t.st.*	*avg.*	*t.st.*	*avg.*	*t.st.*	*avg.*	*t.st.*
$\mu(p-d)$	4.87	(0.10)	4.87	(0.03)	4.88	(0.04)	4.87	(0.01)	4.87	(0.00)	4.86	(0.10)	4.86	(0.08)
$\sigma(p-d)$	0.42	(0.06)	0.03	(6.74)	0.04	(6.62)	0.03	(6.79)	0.04	(6.66)	0.03	(6.75)	0.04	(6.53)
$ac_1(p-d)$	0.98	(0.01)	0.98	(0.23)	0.98	(0.61)	0.98	(0.16)	0.98	(0.55)	0.98	(0.39)	0.99	(0.88)
$\mu(R_f)$	0.23	(0.12)	1.22	(8.18)	1.21	(8.11)	1.24	(8.36)	1.24	(8.35)	1.23	(8.28)	1.22	(8.19)
$\sigma(R_f)$	0.94	(0.12)	0.11	(7.16)	0.32	(5.38)	0.04	(7.78)	0.16	(6.71)	0.12	(7.12)	0.60	(2.92)
$ac_1(R_f)$	0.34	(0.10)	0.99	(6.69)	0.25	(0.97)	1.01	(6.72)	0.13	(2.22)	0.98	(6.61)	0.13	(2.27)
$\mu(R_m)$	2.03	(0.44)	1.27	(1.73)	1.27	(1.73)	1.26	(1.75)	1.26	(1.75)	1.27	(1.72)	1.28	(1.71)
$\mu(R_i)$			1.26		1.26		1.25		1.25		1.27		1.26	
$\mu(R_m-R_f)$	1.79	(0.42)	0.05	(4.20)	0.06	(4.17)	0.01	(4.27)	0.02	(4.26)	0.04	(4.20)	0.05	(4.18)
$\sigma(R_m)$	8.26	(0.55)	4.37	(7.11)	4.42	(7.01)	4.12	(7.56)	4.13	(7.54)	4.34	(7.17)	4.41	(7.04)
$\sigma(R_i)$			1.38		1.47		0.62		0.65		1.30		1.40	
$\sigma(R_m-R_f)$	8.19	(0.55)	4.36	(6.97)	4.41	(6.89)	4.12	(7.41)	4.13	(7.40)	4.33	(7.03)	4.36	(6.97)
SR_m	0.22	(0.06)	0.01	(3.43)	0.01	(3.39)	0.00	(3.54)	0.00	(3.53)	0.01	(3.44)	0.01	(3.40)
SR_i			0.03		0.03		0.02		0.02		0.03		0.03	
$\rho(p-d, r_f)$	0.04	(0.13)	-0.68	(5.68)	-0.25	(2.29)	0.77	(5.76)	0.18	(1.08)	-0.58	(4.89)	-0.14	(1.45)
$\rho(\Delta y_t, r_{f,t-1})$	0.34	(0.10)	0.09	(2.67)	0.28	(0.66)	0.11	(2.46)	0.29	(0.53)	0.07	(2.81)	0.67	(3.33)

Table 6: Macroeconomic moments

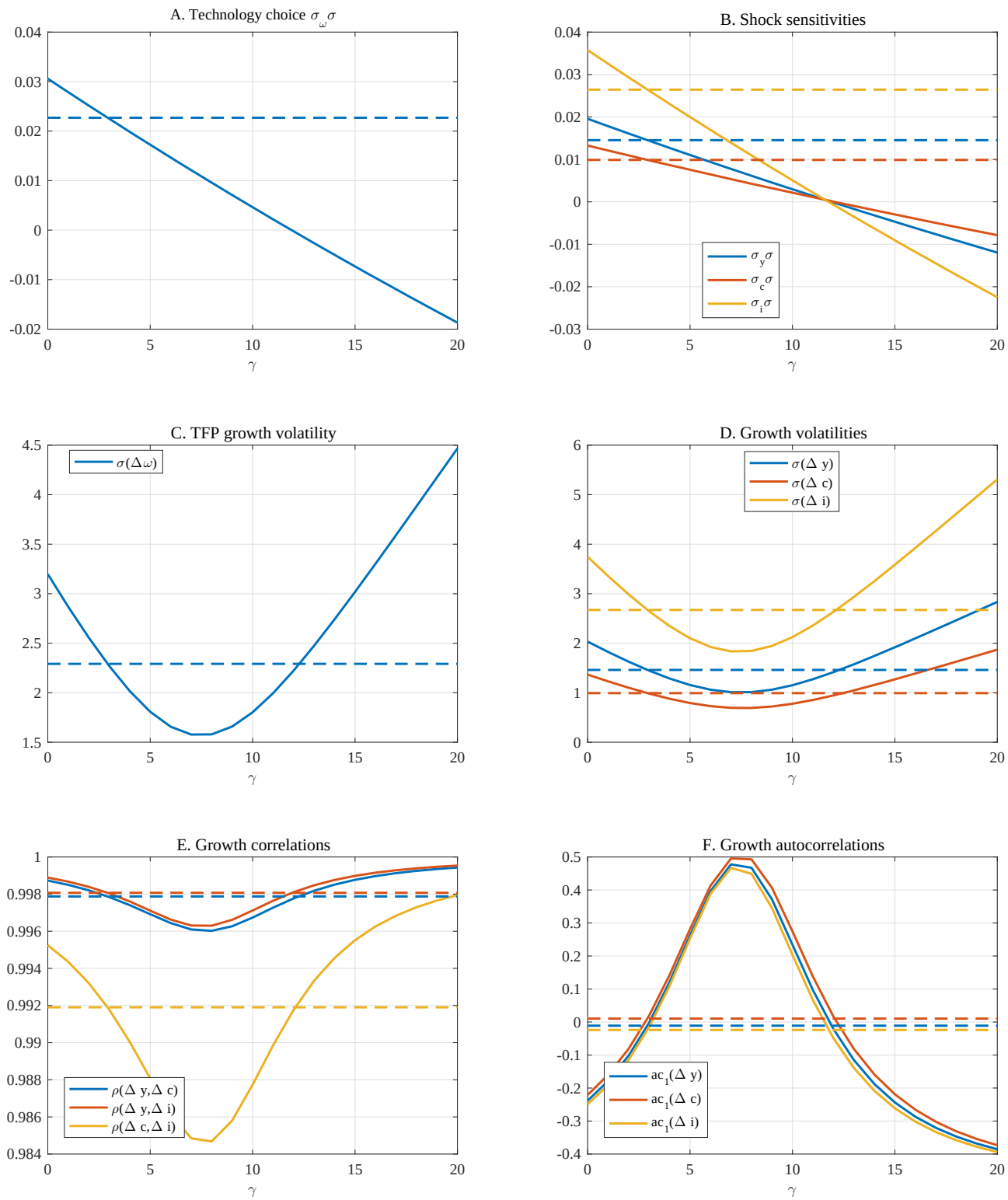
The data statistics are computed from quarterly macroeconomic data obtained from the NIPA tables over 1947Q1-2012Q4. The model statistics are averages of 1000 simulated paths of 300 quarters with a burn-in of 100 quarters. The parentheses next to the data estimates show the standard errors (*s.e.*), which are Newey and West (1987) corrected with 16 lags. The parentheses next to the model statistics show the t-statistics (*t.st.*) of the hypotheses that the data estimates are different from the model averages. Δx denotes the first-difference of the natural logarithm of a variable X. y denotes (the natural logarithm of) total output; c denotes total consumption; i denotes total investment and ω denotes total factor productivity. For a variable x, $\sigma(x)$ denotes its volatility; $ac_1(x)$ is its first-order autocorrelation and $\rho(x, z)$ is its correlation with variable z. The models with technology choice are calibrated to fit the TFP growth volatility, the consumption growth volatility, the first-order autocorrelation of the stock market log price-to-dividend ratio and the first-order autocorrelation of the TFP growth (Models 4 and 5) or the investment growth volatility (Model 6). The CRRA was chosen to fit the Sharpe ratio of the stock market portfolio. The exogenous volatility (σ) of the models without technology choice ($\nu = \infty$) is adjusted to fit the TFP growth volatility. For all models the average growth rate in the economy is $\mu = 0.4\%$, the capital share is $\alpha = 0.36$, the capital depreciation is $\delta = 0.026$ and the dividend process parameters are $d = (-0.0026, 1.65, 0.04)$. With technology choice the σ_ω is 0.77, 0.78, and 0.58 for Models 4, 5, and 6, respectively.

			Model 4				Model 5				Model 6			
			$\psi = 0.3, \gamma = 110, \chi = 3.6$				$\psi = 1.5, \gamma = 200, \chi = 4.5$				$\psi = 0.3, \gamma = 100, \chi = 6.5$			
			$\beta = 0.9997, \phi = 0.970$				$\beta = 0.9935, \phi = 0.980$				$\beta = 0.9995, \phi = 0.970$			
			$\nu = \infty$		$\nu = 155$		$\nu = \infty$		$\nu = 260$		$\nu = \infty$		$\nu = 80$	
	Data		$\sigma = 1.80$		$\sigma = 2.26$		$\sigma = 1.80$		$\sigma = 2.25$		$\sigma = 1.80$		$\sigma = 2.55$	
	est.	*s.e.*	*avg.*	*t.st.*	*avg.*	*t.st.*	*avg.*	*t.st.*	*avg.*	*t.st.*	*avg.*	*t.st.*	*avg.*	*t.st.*
A. Volatilities														
$\sigma(\Delta y)/\sigma(\Delta\omega)$	0.64	(0.09)	0.64	(0.01)	0.64	(0.02)	0.64	(0.02)	0.64	(0.05)	0.64	(0.00)	0.64	(0.03)
$\sigma(\Delta c)/\sigma(\Delta\omega)$	0.44	(0.07)	0.39	(0.79)	0.44	(0.04)	0.43	(0.17)	0.44	(0.06)	0.35	(1.29)	0.44	(0.04)
$\sigma(\Delta i)/\sigma(\Delta\omega)$	1.40	(0.22)	1.31	(0.42)	1.21	(0.85)	1.12	(1.27)	1.11	(1.30)	1.40	(0.01)	1.40	(0.00)
$\sigma(\Delta\omega)$	1.81	(0.14)	1.81	(0.01)	1.81	(0.05)	1.81	(0.04)	1.81	(0.00)	1.81	(0.01)	1.81	(0.05)
B. Autocorrelations														
$ac_1(\Delta y)$	0.24	(0.07)	-0.00	(3.73)	0.26	(0.33)	0.00	(3.63)	0.26	(0.32)	-0.00	(3.70)	0.46	(3.29)
$ac_1(\Delta c)$	0.04	(0.07)	0.01	(0.49)	0.11	(1.05)	0.03	(0.15)	0.28	(3.50)	0.01	(0.37)	0.15	(1.52)
$ac_1(\Delta i)$	0.37	(0.08)	-0.01	(4.66)	0.38	(0.07)	-0.01	(4.69)	0.25	(1.46)	-0.01	(4.67)	0.45	(0.90)
$ac_1(\Delta\omega)$	0.25	(0.07)	-0.02	(3.92)	0.25	(0.03)	-0.01	(3.86)	0.25	(0.04)	-0.02	(3.92)	0.45	(2.89)
C. Correlations														
$\rho(\Delta y, \Delta i)$	0.68	(0.05)	1.00	(6.36)	0.99	(6.08)	1.00	(6.32)	1.00	(6.30)	1.00	(6.35)	0.90	(4.36)
$\rho(\Delta y, \Delta c)$	0.62	(0.04)	1.00	(9.99)	0.99	(9.63)	1.00	(9.94)	1.00	(9.92)	1.00	(9.98)	0.93	(8.11)
$\rho(\Delta c, \Delta i)$	0.43	(0.06)	1.00	(8.79)	0.94	(7.94)	0.99	(8.67)	0.99	(8.60)	1.00	(8.75)	0.67	(3.74)

Table 7: Asset pricing moments

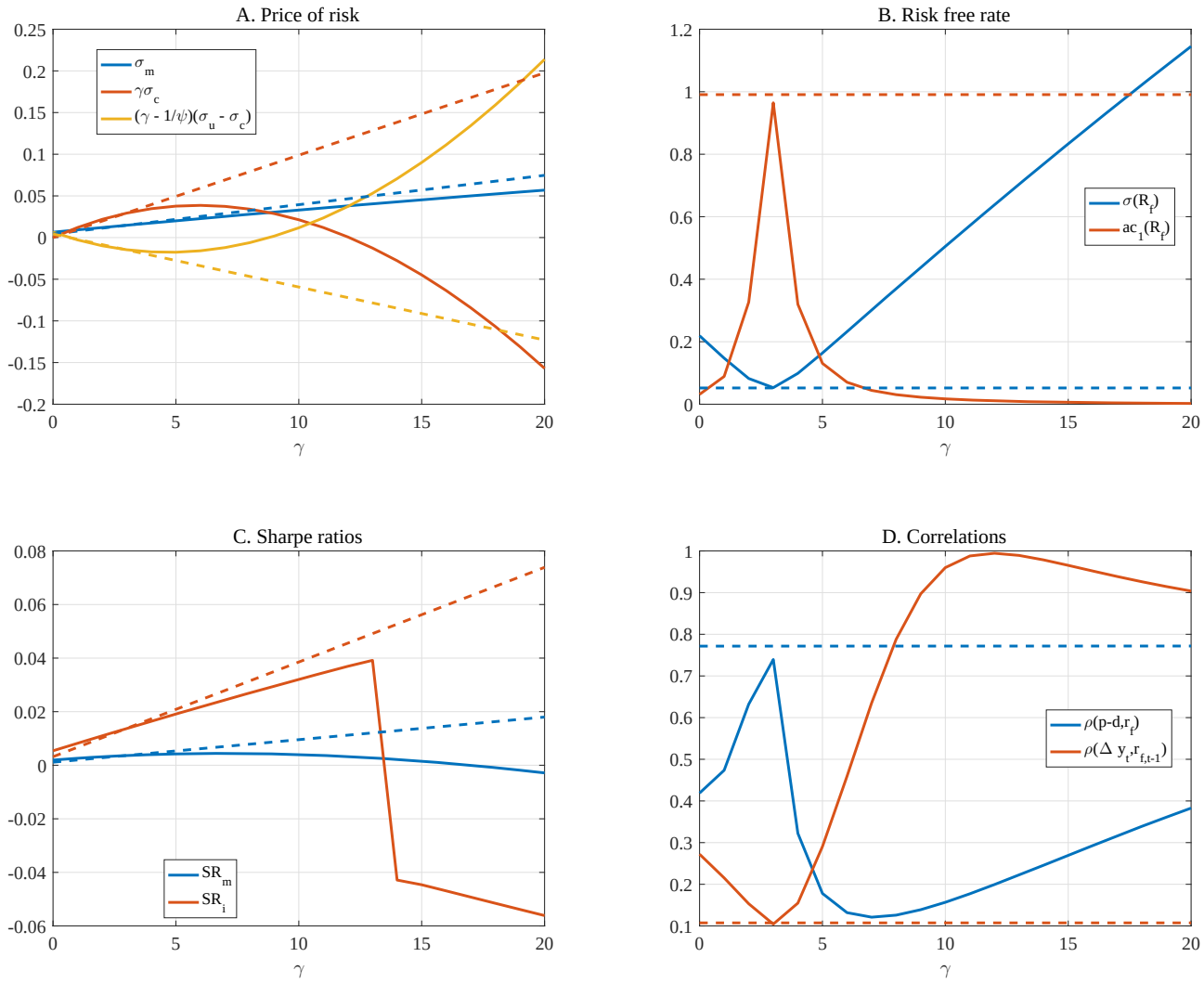
The empirical data statistics are computed from quarterly return data obtained from CRSP and WRDS Federal Reserve data (3-month T-bill yields) over 1947Q1-2012Q4. The model statistics are computed as averages from 1000 simulated paths, where each path has 300 quarters with a burn-in of 100 quarters. The parentheses next to the data estimates show the standard errors (*s.e.*), which are Newey and West (1987) corrected with 16 lags. The parentheses next to the model statistics show the t-statistics (*t.st.*) of the hypotheses that the data estimates are different from the model averages. $p-d$ denotes the log price-to-dividend ratio of the dividend claim; R_f is the quarterly risk-free rate, R_m is the quarterly return to the dividend claim and R_i is the quarterly investment return. For a variable x $\mu(x)$ is the mean; $\sigma(x)$ is the standard deviation and $ac_1(x)$ is the first-order autocorrelation. The Sharpe ratio, SR, equals $\mu(R-R_f)/\sigma(R-R_f)$ and $\rho(p-d, r_f)$ denotes the correlation between $p-d$ and the log risk-free rate. For the model calibrations and other parameter values see caption of Table 6.

			Model 4				Model 5				Model 6			
			$\psi=0.3, \gamma=110, \chi=3.6$				$\psi=1.5, \gamma=200, \chi=4.5$				$\psi=0.3, \gamma=100, \chi=6.5$			
			$\beta=0.9997, \phi=0.970$				$\beta=0.9935, \phi=0.980$				$\beta=0.9995, \phi=0.970$			
			$\nu=\infty$		$\nu=155$		$\nu=\infty$		$\nu=260$		$\nu=\infty$		$\nu=80$	
	Data		$\sigma=1.80$		$\sigma=2.26$		$\sigma=1.80$		$\sigma=2.25$		$\sigma=1.80$		$\sigma=2.55$	
	est.	*s.e..*	*avg.*	*t.st.*	*avg.*	*t.st.*	*avg.*	*t.st.*	*avg.*	*t.st.*	*avg.*	*t.st.*	*avg.*	*t.st.*
$\mu(p-d)$	4.87	(0.10)	4.81	(0.57)	4.87	(0.03)	5.06	(1.93)	4.88	(0.06)	4.78	(0.87)	4.87	(0.02)
$\sigma(p-d)$	0.42	(0.06)	0.04	(6.60)	0.05	(6.43)	0.04	(6.62)	0.05	(6.49)	0.04	(6.61)	0.05	(6.34)
$ac_1(p-d)$	0.98	(0.01)	0.98	(0.31)	0.99	(0.68)	0.99	(0.71)	1.01	(1.04)	0.99	(0.65)	0.99	(0.97)
$\mu(R_f)$	0.23	(0.12)	0.57	(2.77)	0.28	(0.43)	0.53	(2.46)	0.43	(1.66)	0.73	(4.15)	0.32	(0.74)
$\sigma(R_f)$	0.94	(0.12)	0.12	(7.05)	0.34	(5.16)	0.05	(7.72)	0.16	(6.73)	0.14	(6.92)	0.43	(4.38)
$ac_1(R_f)$	0.34	(0.10)	0.99	(6.69)	0.26	(0.84)	0.95	(6.35)	0.21	(1.36)	0.97	(6.52)	0.28	(0.63)
$\mu(R_m)$	2.03	(0.44)	1.32	(1.62)	1.28	(1.71)	1.12	(2.06)	1.25	(1.76)	1.34	(1.57)	1.27	(1.72)
$\mu(R_i)$			1.20		1.12		0.96		0.99		1.23		1.10	
$\mu(R_m-R_f)$	1.79	(0.42)	0.75	(2.51)	1.01	(1.92)	0.59	(2.88)	0.82	(2.33)	0.60	(2.86)	0.95	(2.02)
$\sigma(R_m)$	8.26	(0.55)	4.41	(7.04)	4.47	(6.93)	4.13	(7.54)	4.16	(7.48)	4.33	(7.18)	4.42	(7.01)
$\sigma(R_i)$			1.48		1.60		0.64		0.67		1.27		1.48	
$\sigma(R_m-R_f)$	8.19	(0.55)	4.40	(6.90)	4.45	(6.81)	4.13	(7.39)	4.16	(7.34)	4.32	(7.05)	4.40	(6.90)
SR_m	0.22	(0.06)	0.17	(0.80)	0.22	(0.07)	0.14	(1.24)	0.20	(0.34)	0.14	(1.30)	0.22	(0.04)
SR_i			0.43		0.54		0.69		0.85		0.40		0.55	
$\rho(p-d, r_f)$	0.04	(0.13)	-0.69	(5.75)	-0.27	(2.45)	0.27	(1.82)	0.11	(0.52)	-0.49	(4.17)	-0.21	(2.01)
$\rho(\Delta y_t, r_{f,t-1})$	0.34	(0.10)	0.09	(2.68)	0.28	(0.61)	0.04	(3.12)	0.27	(0.74)	0.06	(2.98)	0.53	(1.92)



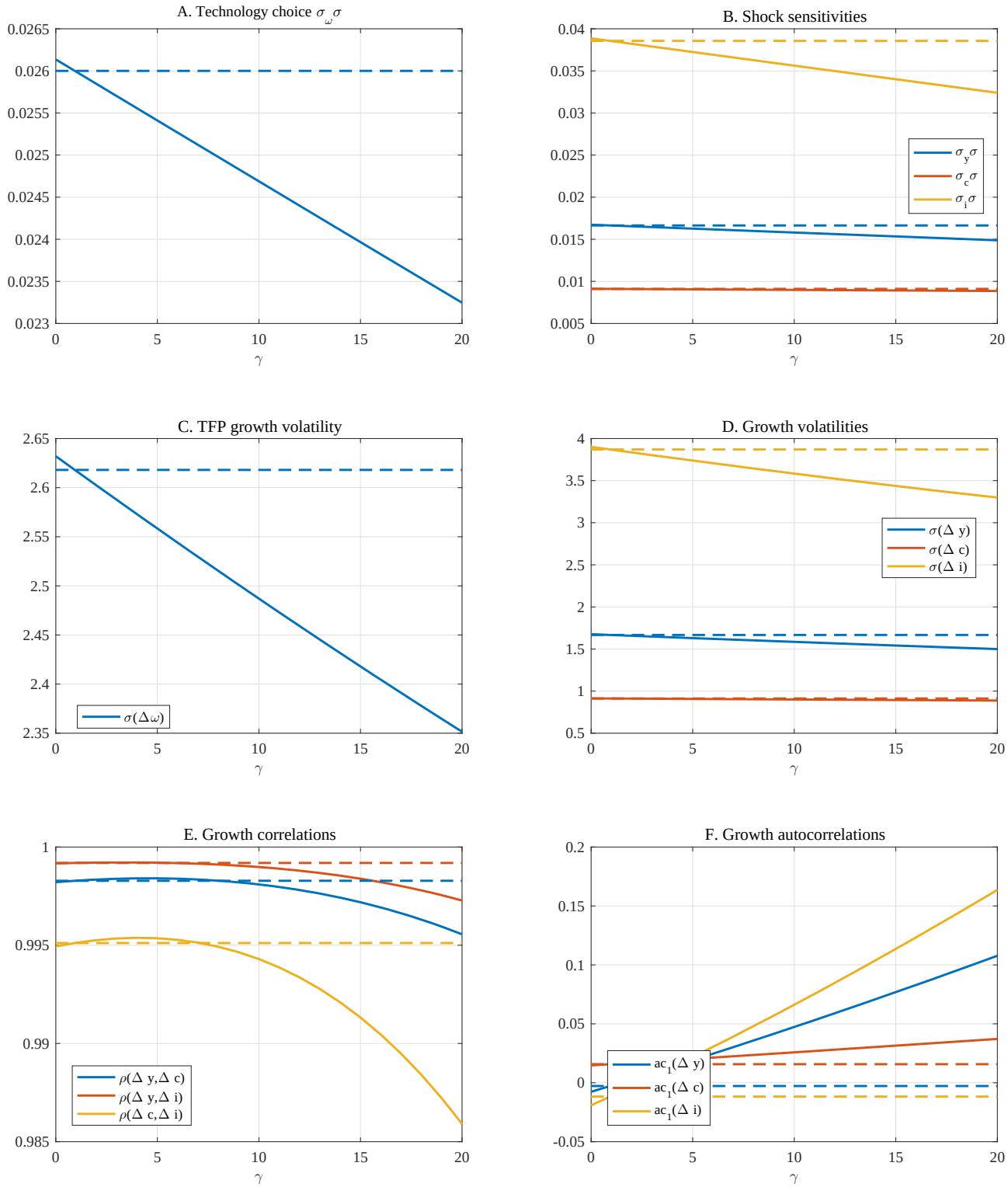
The figure plots several macro quantities by varying the γ parameter, for Model 2 with technology choice (continuous lines) and without (dashed lines).

Figure 6: Sensitivity analysis of macro quantities in relation to γ (Model 2)



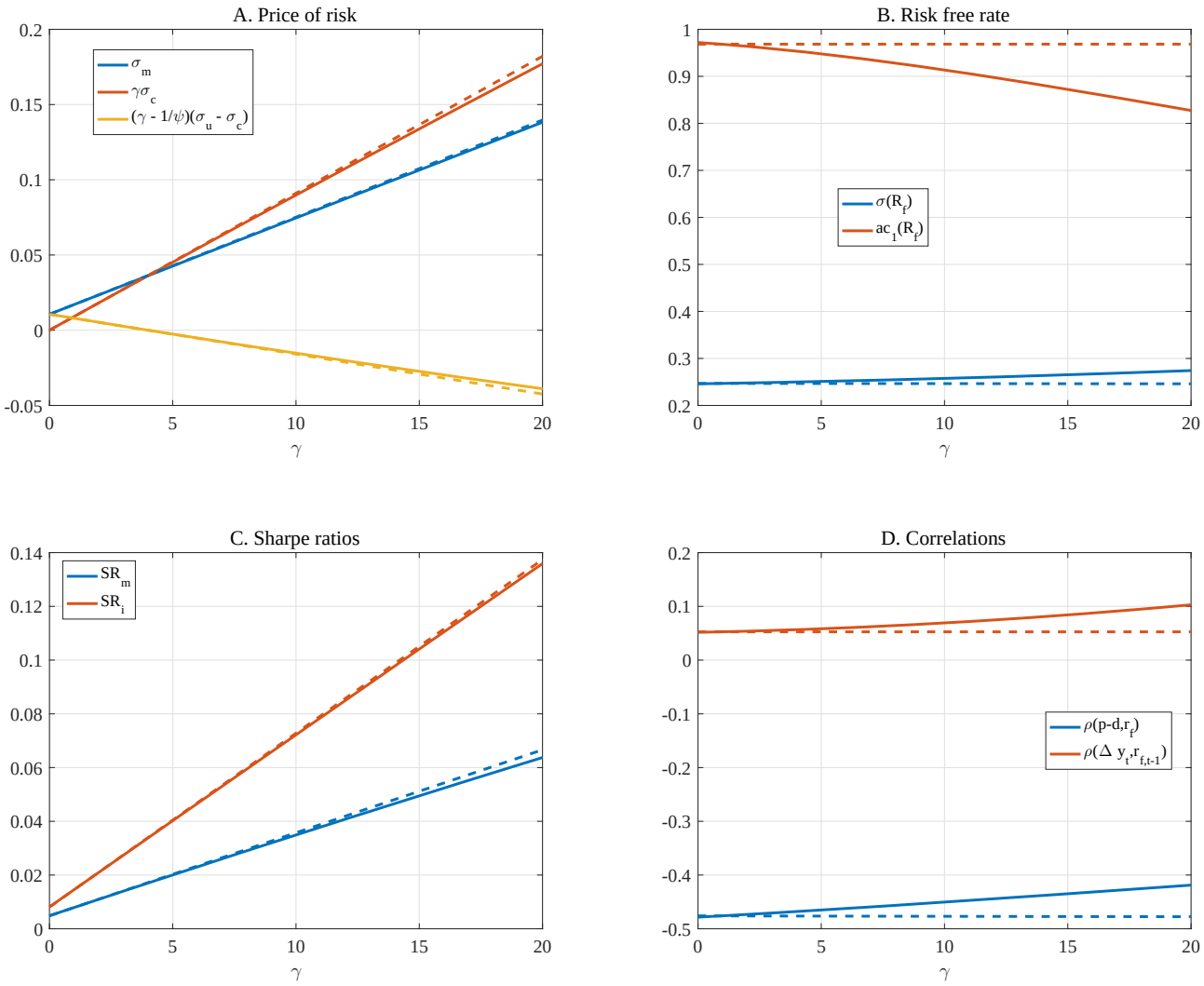
The figure plots several asset pricing quantities by varying the γ parameter, for Model 2 with technology choice (continuous lines) and without (dashed lines).

Figure 7: Sensitivity analysis of asset pricing quantities in relation to γ (Model 2)



The figure plots several macro quantities by varying the γ parameter, for Model 3 with technology choice (continuous lines) and without (dashed lines).

Figure 8: Sensitivity analysis of macro quantities in relation to γ (Model 3)



The figure plots several asset pricing quantities by varying the γ parameter, for Model 3 with technology choice (continuous lines) and without (dashed lines).

Figure 9: Sensitivity analysis of asset pricing quantities in relation to γ (Model 3)