What Drives Marginal Q Fluctuations?

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Abstract

Sharp swings in the shadow value of capital (marginal Q) characterize business cycle fluctuations. An important question is whether marginal Q fluctuates due to revisions in expected marginal profits or discount rates, and by how much of each. We infer marginal Q from the marginal cost of investment, derive a present-value relation, and conduct a VAR-based variance decomposition for marginal Q. We find that the bulk of fluctuations in marginal Q stems from expected investment return (discount rate) shocks. Yet, expected marginal profit shocks play a non-negligible role. Additionally, the entire variation of investment stems from discount rate shocks.

Keywords: Tobin's q; Present-value model; Investment return; Variance decomposition; VAR implied predictability; Aggregation bias; Marginal profit of capital; Long-horizon regressions

JEL classification: E22; E27; G10; G12; G17; G31

1 Introduction

Marginal Q is the present value of all future marginal profits entailed by installing an extra unit of capital. Thus, similar to stock prices, variations in marginal Q are driven by shocks to expected cash flows (expected marginal profits in the case of investment) as well as by discount rate shocks. That aggregate investment varies substantially over the business cycle implies that the marginal value of capital is also highly volatile. In this paper, we explore the roles of shocks to expected marginal profits and discount rate shocks in driving the variation in marginal Q.

Extensive literature studies the sources of fluctuations in (scaled) stock prices (for example, Cochrane, 2008, 2011). In contrast, the determinants of variations in marginal Q are largely unexplored perhaps partly because, unlike stock prices, marginal Q is not readily observable. In this paper, to achieve our goal we infer the marginal value of capital from the marginal adjustment cost of investment that we estimate from a structural model. Thus, our approach is the supply approach to valuation, as in Belo, Xue, and Zhang (2013) who use the supply side, that is, the firm's optimization conditions, to identify marginal Q. Whereas stock prices are determined by investors in the stock market, investment is undertaken by the firm's manager. Thus, our approach of using investment data to infer marginal Q from the marginal adjustment cost of investment implies that we are studying the marginal value that firms' managers, not necessarily investors, attribute to an extra unit of capital.

In order to study the roles of expected marginal profit shocks and discount rate shocks in driving marginal Q, we employ a classical model of optimal investment (as in Liu, Whited, and Zhang, 2009) and derive a present-value relation for the model-implied marginal Q. In the model, the firm's optimal investment behavior implies that marginal Q, that is, the marginal value of capital, is equal to the marginal cost of investment. Assuming a (standard) functional form for the adjustment cost function implies that the marginal cost of investment is a linear function of the investment-to-capital ratio. Belo, Xue, and Zhang (2013) use the same model to infer marginal Q from investment data. In the estimation of the model's parameters, namely the share of capital in profit and the adjustment cost parameter, we follow Gonçalves, Xue, and Zhang (2020) and correct for aggregation bias when conducting the GMM estimation (see also Belo, Gala, Salomao, and Vitorino, 2019). That is, we estimate the model's parameters by using firm-level data to match two moment conditions. First, we match portfolio-level stock returns to a weighted average of firm-level levered investment returns (the investment return moment). Second, we match weighted average Tobin's marginal Q in the data to a weighted average model-implied Tobin's Q (the valuation moment). Following the parameter estimation, we construct time series of marginal Q, marginal profits, and investment returns.

Armed with the estimated time-series for investment returns and its components, we derive a dynamic present-value relation for the log marginal Q (q), in which q is positively correlated with both future multi-period log marginal profits and the future q at some terminal date, and negatively correlated with future multi-period log investment returns. This present-value relation is analogous to the present-value relation associated with the log stock price derived in Campbell, Lo, and MacKinlay (1997) and is similar to the present-value relation derived in Lettau and Ludvigson (2002). Such relationship gives rise to a variance decomposition for q at each forecasting horizon, which contains the fractions of the variance of current q attributed to the predictability of future investment returns, marginal profits, and q.

We use two methods to estimate empirically the variance decomposition for q: first-order restricted VAR (as in Cochrane, 2008), and first-order unrestricted VAR (as in Larrain and Yogo, 2008 and Maio and Xu, 2020). The two methods produce qualitatively similar variance decompositions at both intermediate and long horizons. Specifically, the bulk of variation in q turns out to be investment return predictability, with predictability of the marginal profitability of capital assuming a secondary role (although statistically significant when the decomposition is based on the restricted VAR). On the other hand, predictability of future q only plays a relevant role at very short horizons.[1] To have an idea of these predictability patterns, the shares of long-run (infinite

[1] This finding is consistent with Eberly, Rebelo, and Vincent (2012) that lagged investment is a strong predictor of current investment.

horizon) investment return (marginal profits) predictability are 0.61 (0.37) and 0.80 (0.18) under the restricted and unrestricted VAR approaches, respectively. In fact, in the case of the variance decomposition based on the unrestricted VAR, we cannot reject the null that *all* the variation in q stems from long-run investment return predictability.

These findings are robust to a host of robustness checks. Specifically, the findings are robust to using median stock returns and investment returns instead of value-weighted stock returns and investment returns, as well as to conducting the GMM estimation of the structural investment model based on decile portfolios sorted by marginal Q. Our findings are also robust to using a bootstrap simulation (based on the restricted VAR), which represents an alternative statistical inference for the implied horizon-specific predictive slope estimates (that complements the standard asymptotic inference). Further, we obtain qualitatively similar results by estimating the variance decomposition for q based on long-horizon regressions (direct approach) rather than relying on the first-order VAR (indirect approach). The findings are also robust to varying the values of the capital depreciation parameter. As an additional robustness check, we re-estimate the technology parameters using the methodology in Liu, Whited, and Zhang (2009). This methodology does not account for aggregation bias. Reassuringly, the long-run predictability mix is qualitatively similar to that in our benchmark setting, even though the structural parameter estimates differ to some degree: the discount rate channel explains most (71%) of the variation in q.

Given that the share of capital in production and the adjustment cost parameters are estimated with errors, we conduct comparative statics by experimenting with several possible values of these parameters. We test several combinations of these parameters such that the fraction of adjustment costs in output varies in the range of 0% to 20% (the range surveyed in Bloom, 2009). Importantly, we use the same values of investment and sales as in the data, but varying the parameters yields different series of marginal profits, q, and investment returns. We find the following. First, the share of investment return (marginal profits) predictability declines (increases) with the share of capital parameter. Intuitively, if the share of capital is zero, then marginal profits are constant at zero and are not predictable at all. As the share of capital rises, the relative predictability of marginal profits versus investment returns rises. Second, the shares of return (marginal profits) predictability tend to increase (decrease) monotonically with the adjustment costs parameter. Intuitively, if adjustment costs are zero, marginal Q equals to one at each point in time, implying that investment returns are less time-varying.

The paper most closely related to ours is Abel and Blanchard (1986), who also analyze the empirical implications of a present-value relation for marginal Q (in levels). They conclude that variations in marginal Q are due more to discount rate variations than to variations in expected marginal profits. Among several other differences in the empirical designs of the two studies, there are several major differences. First, Abel and Blanchard extract from a VAR estimation both discount rates and expected marginal profits from which they compute a present value series (i.e. a series of the marginal values of capital). They then compute the standard deviations of the two components of Q and find that discount rates vary more. This leads them to conclude that discount rates contribute more to the variation of marginal Q than expected marginal profits. However, they do not conduct a proper variance decomposition in the sense that they do not calculate the weights associated with the variance and covariance terms, as well as the corresponding standard errors.[2] Thus, they are not able to formally quantify how important shocks to discount rates and expected profits are in terms of driving the dynamics of marginal Q. That is, their results cannot inform us of the statistical significance of the driving forces of Q. In contrast, our variance decomposition informs us how much of the variation in q is due to predictability of investment returns and how much is due to predictability of marginal profits, while also quantifying the statistical significance of these effects. Hence, our paper focuses on the predictive information in the log marginal Q for future investment returns and marginal profits, something that is not addressed in Abel and Blanchard (1986).

[2]See, for example, Campbell (1991), Campbell and Ammer (1993), Maio (2014), and Guo, Kontonikas, and Maio (2020), for properly defined variance decompositions (based on the variances and covariances of the components) associated with stock and bond returns.

Second, Abel and Blanchard (1986) do not compute the investment return from a structural investment model as we do. Instead, their definition of the investment return is an ad-hoc one, corresponding to a linear combination of stock and bond returns. Third, Abel and Blanchard find that the present value (i.e. marginal Q) series, although significantly related to investment, still leaves unexplained a large, serially correlated fraction of the movement in investment. This suggests a potential misspecification of their present value series. Fourth, our definition of marginal profits follows from the optimal investment model that we employ, whereas Abel and Blanchard assume that the profit function is linear in capital. Fifth, whereas Abel and Blanchard study only manufacturing firms for the period 1948 to 1979, we study the entire CRSP/Compustat universe (excluding financials and utilities) for the period 1964 through 2018.

Finally, we also examine the sources of variation in the (logarithm of) the investment-to-capital ratio (ik) by conducting a pseudo variance decomposition for this variable as well as forecasting regressions. We find that virtually the entire volatility of ik stems from investment return predictability. In contrast, Abel and Blanchard (1986) regress aggregate investment on these two components of their proxy for marginal Q, namely expected marginal profits and discount rates, and find that while both affect aggregate investment, the marginal profits component has a much larger and more significant effect on aggregate investment than the cost of capital component.[3] This finding stands in stark contrast to the convention in the asset pricing literature that the primary source of fluctuations in stock prices is discount rate variations (Cochrane, 2011).

Classic production-based asset pricing models (e.g. Cochrane, 1991; Restoy and Rockinger, 1994; Jermann, 1998; Liu, Whited, and Zhang, 2009; Kaltenbrunner and Lochstoer, 2010) predict that both investment and stock prices respond similarly to expected cash flows shocks and discount rate shocks.[4] For example, when stock prices rise on the expectations of higher future cash flows (due to, for example, a positive shock to expected productivity), the present value of marginal

[3]See their Table VI and their discussion on page 266.

[4]Cochrane (1991) shows that under complete markets firms will adjust their investment plans until investment returns equal stock returns. Equivalently, firms will remove arbitrage opportunities between stock returns and investment returns.

profits from an extra unit of capital, that is marginal Q, also rises, leading to an increase in optimal investment. These models, with value maximizing managers, would hence predict that the sources of fluctuations in stock prices and in real investment are the same. Abel and Blanchard's (1986) finding that investment is affected by expected marginal profits much more than by discount rates is therefore inconsistent with the predictions of these models. In contrast, our finding that investment variation is due entirely to expected investment returns (discount rates) fluctuations is consistent with the predictions of the leading production-based asset pricing models.

Cochrane (1991) shows that in univariate regressions the aggregate investment-to-capital ratio negatively predicts future stock market returns at quarterly and annual horizons.[5] This finding suggests that investment responds to discount rate shocks at least to some extent, implying that the marginal value of capital varies, at least to some extent, due to discount rate shocks. Arif and Lee (2014) confirm Cochrane's finding in a more updated sample. However, these studies do not explore the relative roles of expected marginal profit shocks and discount rate shocks in driving aggregate investment and marginal Q.

Lettau and Ludvigson (2002) use a dynamic present-value relation for q to motivate their empirical design, in which traditional predictors of the equity premium (such as the dividend yield, term spread, or default spread) are used to forecast future investment growth (controlling for investment-based variables). There are two key differences between the two papers. First, Lettau and Ludvigson (2002) do not compute a variance decomposition for q and hence they do not test the relative importance of discount rate shocks and expected marginal profits shocks in driving q. Second, the investment-based predictors that they employ in forecasting investment growth (such as the average Q or profit growth) are not directly obtained from a structural model of investment.

It is important to note that the discussion in the literature regarding the investment-cash flow sensitivity is irrelevant for our purpose. That is, the present value relation we derive holds regardless of whether or not firms' investment is sensitive to contemporaneous cash flows. According to our

[5] Cochrane (1991) also finds that the investment-to-capital ratio is not robust to the inclusion of well known predictors of returns in multiple regressions.

present-value relation, the model-implied marginal Q (which is a linear function of the investment to capital ratio) must always forecast future values of the marginal profit of capital, future investment returns, or both.

The rest of the paper is organized as follows. In Section 2, we present a model of a firm's optimal investment decisions. Section 3 describes the data and the econometric methodology for estimating the production and adjustment costs parameters and the components of investment returns. We derive a present-value relation for the log Q in Section 4, while Section 5 represents the main empirical analysis conducted in the paper. In Section 6, we provide a sensitivity analysis. Section 7 shows the results for a comparative statics exercise. In Section 8, we investigate the forecasting ability of the investment-to-capital ratio. The paper concludes in Section 9.

2 The Background Model

In this section, we provide the details of the underlying structural investment model.

2.1 Theoretical Model

We employ the model in Liu, Whited, and Zhang (2009) in order to derive our variables of interest. The firm is assumed to have linearly homogenous production function and adjustment cost function. The factors of production are capital, as well as costlessly adjustable inputs, such as labor. The firm is a price taker, and in each period chooses optimally the costlessly adjustable inputs to maximize operating profits, defined as revenues minus the cost of the costlessly adjustable inputs. Taking operating profits as given, the firm chooses optimal investment and debt to maximize the value of equity.

Let $\Pi(K_{i,t}, X_{i,t})$ denote the maximized operating profits of firm i at time t, where K is the stock of capital and X is a vector of aggregate and idiosyncratic shocks. The firm is assumed to have a Cobb-Douglas production function with constant returns to scale. The marginal operating profit of capital is given by

$$\frac{\partial \Pi \left(K_{i,t}, X_{i,t}\right)}{\partial K_{i,t}} = \alpha \frac{Y_{i,t}}{K_{i,t}}, \tag{1}$$

where $\alpha > 0$ is the share of capital and Y is sales.

We assume standard quadratic functional form for the adjustment cost function,

$$\Phi \left(I_{i,t}, K_{i,t}\right) = \frac{a}{2} \left(\frac{I_{i,t}}{K_{i,t}}\right)^2 K_{i,t}, \tag{2}$$

where $a > 0$ is the adjustment cost parameter. Taxable profits equal operating profits minus capital depreciation minus interest expenses.

The firm's investment return is given by

$$R_{i,t+1} = \frac{\left(1-\tau_{t+1}\right)\left[\alpha \frac{Y_{i,t+1}}{K_{i,t+1}} + \frac{a}{2}\left(\frac{I_{i,t+1}}{K_{i,t+1}}\right)^2\right] + \tau_{t+1}\delta + \left(1-\delta\right)\left[1 + \left(1-\tau_{t+1}\right) a \frac{I_{i,t+1}}{K_{i,t+1}}\right]}{\left[1 + \left(1-\tau_t\right) a \frac{I_{i,t}}{K_{i,t}}\right]}. \tag{3}$$

The marginal value of an additional unit of capital appears in the numerator, whereas the marginal cost of investment is in the denominator. $\left(1-\tau_{t+1}\right) \alpha Y_{i,t+1}/K_{i,t+1}$ is the after-tax marginal operating profit of capital, $\left(1-\tau_{t+1}\right)(a/2)\left(I_{i,t+1}/K_{i,t+1}\right)^2$ is the after-tax marginal reduction in adjustment costs in period $t+1$ that stems from the existence of an extra unit of capital installed in period t, $\tau_{t+1}\delta$ is the marginal depreciation tax shield, and $\left(1-\delta\right)\left[1+\left(1-\tau_{t+1}\right) a \left(I_{i,t+1}/K_{i,t+1}\right)\right]$ is the marginal value at $t+1$ of the undepreciated part of the unit of capital installed in period t (which under optimal investment at $t+1$ is equal to the marginal cost of investment at $t+1$).

We define marginal profit of capital, M, as follows:

$$M_{i,t+1} \equiv \left(1-\tau_{t+1}\right)\left[\alpha \frac{Y_{i,t+1}}{K_{i,t+1}} + \frac{a}{2}\left(\frac{I_{i,t+1}}{K_{i,t+1}}\right)^2\right] + \tau_{t+1}\delta. \tag{4}$$

Thus, the marginal profit of capital is the sum of the after-tax marginal operating profit of capital and reduction in adjustment costs due to the existence of the extra unit of capital, plus the depreciation shield. Optimal investment entails equating the marginal value of capital (Q) to the marginal cost of investment. Hence, optimal investment behavior implies that

$$Q_{i,t+1} = 1 + (1 - \tau_{t+1})\, a \frac{I_{i,t+1}}{K_{i,t+1}}. \tag{5}$$

Therefore, the investment return for firm i can be rewritten as

$$R_{i,t+1} = \frac{M_{i,t+1} + (1-\delta)Q_{i,t+1}}{Q_{i,t}}, \tag{6}$$

and the levered investment return $R^{Iw}_{i,t+1}$ depends on the investment return $R_{i,t+1}$, the after-tax corporate bond return $R^{Ba}_{i,t+1}$, and the market leverage $w_{i,t}$:

$$R^{Iw}_{i,t+1} = \frac{R_{i,t+1} - w_{i,t} R^{Ba}_{i,t+1}}{1 - w_{i,t}}. \tag{7}$$

2.2 A Model of Aggregation and Aggregate Investment Return

Given the firm-level parameters: the capital share (α) and the adjustment cost parameter (a), using Equation (4) to (6), for each firm, we can construct a time series of the firm's investment returns and the associated two components: the firm's marginal profitability of capital (M) and the firm's marginal value of capital (Q). At the aggregate market level, we need to construct the aggregate investment return and a similar representation as Equation (6) but with two aggregate components, namely aggregate marginal profitability of capital and aggregate marginal value of capital. This is needed in order to derive the present-value relation connecting the aggregate marginal value of capital to future aggregate marginal profitability of capital and future aggregate investment returns.

Let N be the number of firms in the market. Each firm optimizes by equating the marginal adjustment costs of investment to the marginal value of capital. Each firm makes an investment $I_{i,t}$ at time t, and exiting time t with a level of capital stock $K_{i,t+1}$. Given the constant returns to scale assumption, applying the result of Hayashi (1982) implies that the marginal value of capital is equal to the average value of capital, and hence the firm's value at the end of time t is given by $K_{i,t+1}Q_{i,t}$ where $Q_{i,t}$ is the marginal value of capital at the end of time t.[6] The aggregate market value is therefore $\sum_{i=1}^{N} K_{i,t+1}Q_{i,t}$. We measure the aggregate marginal value of capital at the end of time t, denoted by Q_t, by assuming that Q_t can price the total capital stock value at the end of time t if multiplied by the total capital stock at the end of time t, as follows:

$$\left(\sum_{i=1}^{N} K_{i,t+1}\right) Q_t = \sum_{i=1}^{N} K_{i,t+1}Q_{i,t}. \tag{8}$$

Equivalently, Q_t can be expressed as

$$Q_t = \sum_{i=1}^{N} \left(\frac{K_{i,t+1}}{\sum\limits_{j=1}^{N} K_{j,t+1}} \right) Q_{i,t}. \tag{9}$$

Thus, Q_t is a weighted average of individual firms' Q values where the weight of firm i is proportional to firm i's capital stock at the end of time t. Notice that an extra unit of capital in the economy invested according to the existing capital allocation in the economy, that is, invested proportionally to the fraction of capital of each firm from the total capital in the economy will indeed have a value of Q_t.[7]

For the same N firms at time $t+1$, we can measure the aggregate marginal Q at time $t+1$, Q_{t+1}, by assuming that Q_{t+1} can price the total firm value at $t+1$:

$$Q_{t+1} = \sum_{i=1}^{N} \left(\frac{K_{i,t+2}}{\sum_{j=1}^{N} K_{j,t+2}} \right) Q_{i,t+1}.$$

We assume that an extra unit of capital for the aggregate economy at time t is invested according

[6] This notation is consistent with the notation in Belo, Xue, and Zhang (2013).

[7] The allocation of an extra unit of capital in the aggregate economy according to firms' proportions of capital stocks keeps unchanged the distribution of capital in the economy.

to the firms' proportion of their capital stock at the end of time t. Therefore, the aggregate marginal profit of that extra unit of capital (M_{t+1}) is a capital stock weighted average of firms' marginal profits of capital. That is,

$$M_{t+1} = \sum_{i=1}^{N} \left(\frac{K_{i,t+1}}{\sum_{j=1}^{N} K_{j,t+1}} \right) M_{i,t+1}. \tag{10}$$

Finally, we define the aggregate investment return as the ratio of the aggregate marginal benefit of investment at time $t+1$ to the aggregate marginal cost of investment at time t:

$$R_{t+1} \equiv \frac{M_{t+1} + (1-\delta)\, Q_{t+1}}{Q_t}. \tag{11}$$

For an investor who holds the economy's stock of capital, an extra unit of capital at time t costs Q_t. This extra unit of capital generates profit M_{t+1} at time $t+1$ and depreciate to $1-\delta$ unit exiting time $t+1$ with a continuation value of $(1-\delta)\, Q_{t+1}$.

3 Estimating the Investment Return and its Components

In this section, we provide structural estimates of firm-level parameters and aggregate measures of the investment return and the respective components, which are based on the model presented in the last section.

3.1 Methodology

We follow Gonçalves, Xue, and Zhang (2020) and estimate the firm-level parameters, namely the capital share (α) and the adjustment cost parameter (a), using one-step GMM to fit the investment Euler equation moment for each testing portfolio jointly with an additional moment, namely the valuation moment as in Belo, Xue, and Zhang (2013). We include the valuation moment in the estimation because in the benchmark setting we consider the aggregate market portfolio as the testing portfolio. Hence, using only the investment Euler equation leads to an unidentified estimation with one moment but two parameters (the capital share and the adjustment cost parameter).[8] With two moments and two parameters, the estimation is exactly identified and the two moments fit perfectly.

Specifically, for a given set of testing portfolios (indexed by j), the first set of moment conditions corresponds to testing whether the average stock return equals the average levered investment return for each testing portfolio j,

$$e_j^r \equiv \mathrm{E}_T[R^S_{j,t+1} - R^{Iw}_{j,t+1}(\alpha, a)] = 0, \tag{12}$$

where $\mathrm{E}_T(\cdot)$ denotes the sample moment, $R^S_{j,t+1}$ is the portfolio stock return, and $R^{Iw}_{j,t+1}$ is the portfolio levered investment return that depends on parameters α and a.

The second set of moment conditions tests whether the average Tobin's Q in the data equals the average Q predicted by the model,

$$e_j^q \equiv \mathrm{E}_T\left[\widetilde{Q}_{j,t} - \left(1 + (1-\tau_t)a\frac{I_{j,t}}{K_{j,t}}\right)\frac{K_{j,t+1}}{A_{j,t}}\right] = 0, \tag{13}$$

where $A_{j,t}$ is the total assets and $\widetilde{Q}_{j,t}$ is the Tobin's Q in the data defined as the ratio of the sum of market equity and total debt to the total assets, $\widetilde{Q}_{j,t} \equiv (P_{j,t} + B_{j,t+1})/A_{j,t}$.

Firm-level accounting variables and, thus, firm-level investment returns are subject to the issue of outliers. The outliers in firm-level investment returns can contaminate the aggregate portfolio-level investment returns and lead to noisy parameter estimates from the GMM estimation. To alleviate the impact of outliers, we follow Gonçalves, Xue, and Zhang (2020) and construct firm-level investment returns using winsorized firm-level accounting variables, then compute value-weighted portfolio levered investment returns to match with value-weighted portfolio stock returns.[9] As a robustness check (see Section 6), instead of winsorization, we also follow an alternative approach

[8] Similarly, Belo, Xue, and Zhang (2013) base their tests on both the investment Euler equation and the valuation equation.

[9] We winsorize firm-level accounting variables at the 1-99% level.

that is employed in Belo, Gala, Salomao, and Vitorino (2019), where portfolio median is used to aggregate firm-level investment returns to portfolio level since the median is known to be robust to outliers. More details about the GMM estimation methodology are provided in the online appendix.

3.2 Data

We largely follow Gonçalves, Xue, and Zhang (2020) and Belo, Xue, and Zhang (2013) in measuring accounting variables and in aligning their timing with the timing of stock returns. Our sample consists of all common stocks on NYSE, Amex, and Nasdaq from 1963 to 2018. The firm-level data are from the merged CRSP and COMPUSTAT industrial database. We include all firms with fiscal year ending in the second half of the calendar year. We exclude firms with primary standard industrial classifications between 4900 and 4999 (utilities) and between 6000 and 6999 (financials). We also delete firm-year observations for which total assets, capital stock, or sales are either zero or negative.

Capital stock ($K_{i,t}$) is net property, plant, and equipment (Compustat annual item PPENT). Investment ($I_{i,t}$) is capital expenditures (Compustat annual item CAPX) minus sales of property, plant, and equipment (Compustat annual item SPPE, zero if missing). Total debt ($B_{i,t+1}$) is long-term debt (Compustat annual item DLTT, zero if missing) plus short-term debt (Compustat annual item DLC, zero if missing). $A_{i,t}$ is total assets (Compustat annual item AT). Market equity ($P_{i,t}$) is the stock price per share (CRSP item prc) times the number of shares outstanding (CRSP item shrout). Market leverage ($w_{i,t}$) is the ratio of total debt to the sum of total debt and market equity. We follow Cochrane (1991) and assume a depreciation rate (δ) equal to 0.1.[10] Output ($Y_{i,t}$) is sales (Compustat annual item SALE). Market leverage, $w_{i,t}$, is the ratio of total debt to the sum of total debt and the market value of equity. We measure the tax rate (τ_t) as the statutory corporate income tax (from the Commerce Clearing House, annual publications). The after-tax corporate bond returns ($R^{Ba}_{i,t+1}$) are computed from $R^{B}_{i,t+1}$ using the average of tax rates in year t

[10] In the data, the mean value of the annual depreciation rate is equal to 0.1089.

and $t+1$. For the pre-tax corporate bond returns ($R^B_{i,t+1}$) we use the ratio of total interest and related expenses (Compustat annual item XINT) scaled by the total debt ($B_{i,t+1}$).[11]

At the end of June of year t, we construct the aggregate "market" portfolio. That is, a portfolio whose value is the value of the aggregate capital stocks and whose return is the aggregate investment returns. Alternatively, we sort all stocks on Tobin's Q at the end of June of year t into deciles based on the NYSE breakpoints. For each testing portfolio, we compute annual value-weighted stock returns from July of year t to June of year $t+1$. We construct annual levered investment returns to match with annual stock returns and annual valuation ratios to match with annual Tobin's Q. To construct the matching annual levered investment returns, we use capital at the end of fiscal year $t-1$ ($K_{i,t}$), the tax rate, investment, and capital at the end of year t (τ_t, $I_{i,t}$, and $K_{i,t+1}$), as well as other variables at the end of year $t+1$ (τ_{t+1}, $Y_{i,t+1}$, and $I_{i,t+1}$). To match with $\widetilde{Q}_{i,t}$ for portfolios formed at the end of June of year t, we take $I_{i,t}$ from the fiscal year ending in calendar year t and $K_{i,t}$ from the fiscal year ending in year $t-1$.[12]

3.3 Structural Parameter Estimates

In the basecase GMM esitmation, the testing portfolio is the aggregate stock market portfolio and the portfolio returns are measured as the value-weighed returns. The estimation is exactly identified and the two moments fit perfectly. The estimate of capital share (α) is 0.08, which is similar to the results in Gonçalves, Xue, and Zhang (2020).[13] The estimate of adjustment cost parameter (a) is 15.18, which is higher than the corresponding estimates in Gonçalves, Xue, and Zhang (2020) based on testing portfolios different from our setup, but similar to the estimates in

[11] As shown in Gonçalves, Xue, and Zhang (2020), doing so increases the sample coverage by 12.7% as compared to the prior studies that use credit rating imputation such as Liu, Whited, and Zhang (2009).

[12] Compustat records both stock and flow variables at the end of year t. In the model, however, stock variables dated t are measured at the beginning of year t, and flow variables dated t are over the course of year t. To capture this timing difference, we follow Liu, Whited, and Zhang (2009) and take, for example, for the year 2003 the beginning-of-year capital ($K_{i,2003}$) from the 2002 balance sheet and any flow variable over the year, such as $I_{i,2003}$, from the 2003 income or cash flow statement.

[13] Gonçalves, Xue, and Zhang (2020) report estimates of capital share varying from 5.04% to 7.53% across different testing portfolios in Table 5 Panel B.

Liu, Whited, and Zhang (2009).[14] The estimated magnitude of the adjustment costs is 11.09% of sales, which is in line with those reported in prior studies.[15] Given the parameters α and a that are estimated using firm-level data, we compute the aggregate investment return and its components, by plugging these parameter estimates in Equations (9) to (11), together with firm-level accounting variables.

4 A Present-Value Relation

In this section, we derive a dynamic present-value relation for the log Q, which represents the basis for the empirical analysis conducted in the rest of the paper.

Our methodology relies on the definition of the realized gross investment return (R) presented in Equation (11). This definition is analogous to the usual definition of the gross stock return with Q playing the role of the stock price and M being the analog of dividends. By conducting a log-linear transformation of the investment return in Equation (11), and proceeding along the lines of Campbell and Shiller (1988) and Campbell, Lo, and MacKinlay (1997), we derive the following approximate difference equation in log Q,

$$q_t \approx const. + \rho q_{t+1} - r_{t+1} + (1-\rho) m_{t+1}, \tag{14}$$

where $q_t \equiv \ln(Q_t)$ is the log Q at time t; $r_{t+1} \equiv \ln(R_{t+1})$ represents the log investment return at time $t+1$; and $m_{t+1} \equiv \ln(M_{t+1})$ denotes the log marginal profit at time $t+1$. In this setting, variables denoted with lower-case letters represent the logs of the corresponding variables in upper-case letters.

ρ plays an important role in the analysis, representing a (log-linearization) discount coefficient

[14] Gonçalves, Xue, and Zhang (2020) report estimates of adjustment cost parameter varying from 0.72 to 5.66 in Table 5 (Panel B) and from 1.63 to 8.11 in Table 3 (Panel B). Liu, Whited, and Zhang (2009) report estimates varying from 11.5 to 28.9 when matching two moments: expected returns and variances.

[15] For example, Cooper and Priestley (2016) find that implied adjustment costs represent 12.21% of sales across a host of manufacturing industries. Bloom (2009) surveys the estimates of convex adjustment costs to be between zero and 20% of revenue.

that depends on the mean of the log marginal profits-to-Q ratio ($mq_t \equiv m_t - q_t$),

$$\rho \equiv \frac{e^{\ln(1-\delta)-\overline{mq}}}{1+e^{\ln(1-\delta)-\overline{mq}}},$$

where $\overline{mq}$ represents the average of mq_t.

By iterating the equation above forward, we obtain the following present-value dynamic relation for q at each forecasting horizon H:

$$q_t \approx const. - \sum_{h=1}^{H} \rho^{h-1} r_{t+h} + \sum_{h=1}^{H} \rho^{h-1}(1-\rho) m_{t+h} + \rho^H q_{t+H}. \tag{15}$$

Under this present-value relation, current q (q_t) is positively correlated with both future multi-period log profits (m_{t+h}) and the future q at terminal date $t + H$ (q_{t+H}), while being negatively correlated with future multi-period log investment returns (r_{t+h}). This present-value relation is analogous to the present-value relation associated with the stock price derived in Campbell, Lo, and MacKinlay (1997), where q replaces the log stock price (p), m is analogous to the log dividend (d), and r replaces the log stock return.

At an infinite horizon, by assuming the following transversality (or no-bubbles) condition,

$$\lim_{H\to\infty} \rho^H q_{t+H} = 0,$$

we obtain the following long-run present-value relation:

$$q_t \approx const. - \sum_{h=1}^{\infty} \rho^{h-1} r_{t+h} + \sum_{h=1}^{\infty} \rho^{h-1}(1-\rho) m_{t+h}. \tag{16}$$

Hence, at very long horizons, only predictability of future investment returns and predictability of marginal profits drives the variation in the current q.[16] Which of these two components matters

[16] Lettau and Ludvigson (2002) derive a related dynamic accounting decomposition for the log Q. However, their present-value relation is based on a second-order Taylor expansion.

most in terms of driving the dynamics of q remains an empirical question, which will be addressed in the following sections.

Table 1 (Panel A) presents the descriptive statistics for the variables in the present-value relation for q. Both the investment return and q have a volatility of 10%. On the other hand, both q (with a first-order autocorrelation of 0.61) and m (0.54) are considerably more persistent than r (0.12). Figure 1 plots the time series of r, m, and q. All three variables appear to be mean-reverting to a large degree, and hence, stationary. Both r and m seem to be procyclical variables, as they tend to decline around most recession periods and rise during economic booms. On the other hand, q appears to be less correlated with the business cycle.

Panel B of Table 1 contains the correlations among the three variables. The correlation between m and q is very high, at 0.95. Thus, times of high marginal profitability of capital correspond to times when both the marginal value of capital and investment are high. We can also see that the investment return is positively correlated with both m and q, albeit to a lower degree, as indicated by the correlations around 0.60. The reason is that shocks to both marginal profits and Q are also shocks to contemporaneous returns.

Regarding the critical parameter ρ, which is a function of the average mq ratio, we obtain an estimate of 0.85 for our sample period (1964–2018). This estimate is somewhat smaller than corresponding estimates for the analogue parameter values in present-value relations associated with stock returns, which are typically above 0.90 (see Campbell and Vuolteenaho, 2004; Cochrane, 2008, 2011; Maio, 2013; Maio and Santa-Clara, 2015, among others).[17]

5 Variance Decomposition for q

In this section, we evaluate the forecasting performance of q for both future investment returns and marginal profits by deriving and estimating a variance decomposition for q. The objective is

[17] In those studies, the value of ρ is either calibrated or estimated as a function of the average log dividend-to-price ratio.

to assess what are the sources of predictability that drive the variation in q over time.

5.1 Restricted VAR

Following Cochrane (2008), we specify the following first-order restricted VAR,

$$r_{t+1} = \pi_r + \lambda_r q_t + \varepsilon^r_{t+1}, \quad (17)$$

$$m_{t+1} = \pi_m + \lambda_m q_t + \varepsilon^m_{t+1}, \quad (18)$$

$$q_{t+1} = \pi_q + \phi q_t + \varepsilon^q_{t+1}, \quad (19)$$

where the εs represent forecasting errors. This VAR system is estimated by multiple-equation OLS (see Hayashi, 2000), with Newey and West (1987) t-statistics (computed with one lag).[18]

By combining the VAR above with the present-value relation in Equation (15), we obtain an approximate identity involving the predictability coefficients associated with q_t, at each forecasting horizon H:

$$\begin{aligned} 1 &\approx b^H_m - b^H_r + b^H_q, \quad (20) \\ b^H_r &\equiv \lambda_r \frac{1-\rho^H\phi^H}{1-\rho\phi}, \\ b^H_m &\equiv (1-\rho)\lambda_m \frac{1-\rho^H\phi^H}{1-\rho\phi}, \\ b^H_q &\equiv \rho^H\phi^H. \end{aligned}$$

This equation can be interpreted as a variance decomposition for q. The predictive coefficients, $-b^H_r$, b^H_m, and b^H_q, represent the fraction of the variance of current q attributed to the predictability of future investment returns, marginal profits, and q, respectively. Hence, these slopes measure the weight (of the predictability) of each of these variables ($\sum_{h=1}^{H} \rho^{h-1} r_{t+h}$, $\sum_{h=1}^{H} \rho^{h-1}(1-\rho) m_{t+h}$, and $\rho^H q_{t+H}$) in terms of driving the variation in the current q. This relation also imposes a

[18] Using zero lags (White, 1980) leads to similar statistical inference.

quantitative constraint on the predictability associated with q in the sense that the slopes need to add (approximately) to one. Hence, if at some forecasting horizon H, q_t forecasts neither future investment returns nor future marginal profits, then it must forecast its own future value at time $t+H$. Otherwise q would not vary over time, something that is counterfactual, as discussed above.

In this variance decomposition, the predictive slopes at each forecasting horizon H are obtained from the one-period VAR slopes. Cochrane (2008, 2011) specifies a similar variance decomposition for the dividend yield. The expressions above imply that the relative shares of predictability (e.g., b_r^H/b_m^H) are invariant with the forecasting horizon. Further, the multi-horizon slopes represent mechanical transformations of the one-year VAR slopes, which means that the short-run VAR dynamics dictate all the implied long-run dynamics. The first-order VAR addresses the concern of the lack of statistical power at long horizons associated with long-horizon regressions.

We can also compute the variance decomposition for an infinite horizon ($H \to \infty$):

$$\begin{aligned} 1 &\approx b_m^{lr} - b_r^{lr}, \\ b_r^{lr} &\equiv \frac{\lambda_r}{1-\rho\phi}, \\ b_m^{lr} &\equiv \lambda_m \frac{1-\rho}{1-\rho\phi}. \end{aligned} \tag{21}$$

In the long-run decomposition, all the variation in current q is associated with either return or marginal profits predictability. The VAR approach enables one to estimate this long-run decomposition, something that is not feasible under the direct method. The t-statistics associated with both the multi-horizon and long-run predictive coefficients are computed from the t-statistics corresponding to the VAR slopes by using the delta method. The full details on the derivation of the variance decomposition are available in the online appendix.

Following Cochrane (2008), we compute t-statistics for two joint null hypotheses of long-run predictability: the first null assumes that there is only marginal profits predictability,

$$H_0 : b_r^{lr} = 0, b_m^{lr} = 1,$$

while the second null hypothesis assumes that there is only return predictability:

$$H_0 : b_r^{lr} = -1, b_m^{lr} = 0.$$

The results for the baseline variance decomposition are shown in Table 2 (Panel A) and Figure 2 (Panels A and C). The return channel plays a dominant role at nearly all forecasting horizons. Apart from $H = 1$, predictability of future investment returns is the major source of variation in q. Indeed, the return slope estimates are strongly significant (1% level) at all horizons beyond one year, with magnitudes around or above 0.60 at most horizons. Marginal profits predictability assumes a secondary role, with weights close to 0.40 at intermediate and long horizons. Despite the smaller magnitudes, it turns out that the m coefficient estimates are also strongly significant (1% or 5% level) at all horizons. This arises from the fact that the corresponding one-year VAR slope estimate is largely significant (t-ratio of 4.58). In fact, the VAR equation for m has a larger fit than the corresponding return equation, as indicated by the explanatory ratio of 0.28 (versus 0.08 for r). However, there is a downward effect caused by the term $1 - \rho$, which is substantially smaller than one (0.15). Therefore, the larger magnitude of the m VAR coefficient (1.24 versus -0.29) is not enough to generate larger multi-horizon predictive slopes than the coefficients associated with future investment returns (in magnitude). The shares associated with predictability of future (single-date) q are economically and statistically significant at very short horizons. However, such estimates decay to zero, and become insignificant, at a relatively rapid pace ($H > 3$).

We observe that the sum of the variance decomposition is very close to one (above 0.98) at all forecasting horizons. This shows that the present-value relation for q is quite accurate. In other words, ignoring the higher-order terms, which are absent from the first-order Taylor approximation underlying the present-value relation for q, does not have a significant impact on the variance decomposition.

At very long horizons, the results in Table 2 (Panel A) indicate that the return and marginal profits coefficients are -0.61 and 0.37, respectively. We clearly reject the null that $b_r^{lr} = 0, b_m^{lr} = 1$ (t-ratios around 4 in magnitude). Yet, we also reject the null that $b_r^{lr} = -1, b_m^{lr} = 0$, with t-ratios around 2.50. Hence, in statistical terms, both the return and marginal profits channels matter in terms of driving q. However, the size of the return channel is nearly twice as large as the size of the marginal profits channel.

5.2 Simulation

Next, we conduct a bootstrap simulation of the restricted VAR model estimated above. The objective is to account for the relatively poor small-sample properties of long-horizon predictability and the question of whether the asymptotic inference is valid when assessing the statistical significance of the implied multi-horizon slopes (see Valkanov, 2003; Torous, Valkanov, and Yan, 2004; Boudoukh, Richardson, and Whitelaw, 2008, among others for a discussion on this issue). In related work, Cochrane (2008) and Maio and Santa-Clara (2015) conduct VAR-based Monte-Carlo simulations to assess the predictive ability of the dividend yield for future stock returns and dividend growth. One key advantage of a bootstrap simulation relative to a monte-carlo simulation is that we can skip the normally-distributed assumption for the variables in the system.

To assess predictability of future returns, we impose a null hypothesis where q does not forecast the future investment return. Under this null, all the variation in q comes from predicting future marginal profits. Thus, we simulate the first-order VAR by imposing the restrictions, both in the predictive slopes and residuals, associated with this null hypothesis,

$$\begin{pmatrix} r_{t+1} \\ m_{t+1} \\ q_{t+1} \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{1-\rho\phi}{1-\rho} \\ \phi \end{pmatrix} q_t + \begin{pmatrix} \rho\varepsilon_{t+1}^q + (1-\rho)\varepsilon_{t+1}^m \\ \varepsilon_{t+1}^m \\ \varepsilon_{t+1}^q \end{pmatrix}, \tag{22}$$

where all the variables in the VAR are demeaned.

To assess predictability of future marginal profits, we simulate an alternative VAR specification under the null hypothesis that q does not forecast future m. This means that all the variation in q emanates from predicting future investment returns:

$$\begin{pmatrix} r_{t+1} \\ m_{t+1} \\ q_{t+1} \end{pmatrix} = \begin{pmatrix} \rho\phi - 1 \\ 0 \\ \phi \end{pmatrix} q_t + \begin{pmatrix} \rho\varepsilon^q_{t+1} + (1-\rho)\varepsilon^m_{t+1} \\ \varepsilon^m_{t+1} \\ \varepsilon^q_{t+1} \end{pmatrix}. \tag{23}$$

We conduct a bootstrap experiment associated with each of the VARs specified above. We draw the VAR residuals (10,000 times) with replacement from the original VAR estimates. The realization of q for the base period is chosen randomly from the original time-series of q_t. We compute the pseudo p-values associated with the implied VAR return slopes at each horizon, which represent the fractions of simulated estimates of the return coefficients (from the simulations associated with the first VAR above) that are lower than the corresponding estimates found in the data. Similarly, the pseudo p-values associated with the marginal profits coefficients represent the fractions of pseudo estimates of the profitability slopes (obtained from the simulations under the second VAR presented above) that are higher than the corresponding sample estimates. The full details of the bootstrap simulation are available in the online appendix.

We note that our bootstrap simulation does not account for the fact that the variables in the predictive system are nested variables, that is, they are estimated with error (rather than observed). In principle, we could simulate the structural model (described in Section 2) inside the bootstrap experiment in order to account for the estimation error in those variables. However, such procedure is problematic in our case, since we have to employ non-linear GMM estimation of the structural model, as described in Section 3. Specifically, it is likely that for many of the pseudo samples, the numerical optimization underlying the GMM estimation does not converge properly, which would lead to a problematic or infeasible bootstrap simulation. Perhaps, more important, the bootstrap simulation conducted in this subsection produces p-values for the return slopes that are very small. Hence, it is unlikely that incorporating such additional source of statistical uncertainty would turn the most relevant predictive coefficients (in the variance decomposition) insignificant.

The results associated with the bootstrap simulation are presented in Figure 3. It turns out that the p-values associated with the return slopes are below 1% at all forecasting horizons beyond the two-year horizon. In the case of the marginal profit coefficient estimates, the corresponding p-values are below 1% at all horizons beyond the four-year horizon. At very short horizons, both r and m coefficient estimates are significant at the 10% or 5% level. Therefore, these results are consistent with the asymptotic inference presented in the previous subsection, that is, both the return and marginal profits slope estimates are strongly statistically significant at nearly all forecasting horizons.

5.3 Unrestricted VAR

In this subsection, we estimate an alternative variance decomposition for q, based on a less restrictive first-order VAR.

Specifically, we consider an unrestricted VAR(1):

$$r_{t+1} = \pi_r + \gamma_r r_t + \theta_r m_t + \lambda_r q_t + \varepsilon^r_{t+1}, \quad (24)$$

$$m_{t+1} = \pi_m + \gamma_m r_t + \theta_m m_t + \lambda_m q_t + \varepsilon^m_{t+1}, \quad (25)$$

$$q_{t+1} = \pi_q + \gamma_q r_t + \theta_q m_t + \phi q_t + \varepsilon^q_{t+1}. \quad (26)$$

This specification accounts for relevant predictability of lagged returns and marginal profits on all three variables in the system, something that the benchmark VAR misses. Indeed, Maio and Xu (2020) show that the restricted VAR(1), in a similar context, can be severely misspecified. This originates an implausible long-run variance decomposition for aggregate stock price ratios, such as the earnings yield or dividend yield.

The VAR above can be presented in matrix form,

$$\begin{pmatrix} r_{t+1} \\ m_{t+1} \\ q_{t+1} \end{pmatrix} = \begin{pmatrix} \pi_r \\ \pi_m \\ \pi_q \end{pmatrix} + \begin{pmatrix} \gamma_r & \theta_r & \lambda_r \\ \gamma_m & \theta_m & \lambda_m \\ \gamma_q & \theta_q & \phi \end{pmatrix} \begin{pmatrix} r_t \\ m_t \\ q_t \end{pmatrix} + \begin{pmatrix} \varepsilon^r_{t+1} \\ \varepsilon^m_{t+1} \\ \varepsilon^q_{t+1} \end{pmatrix}. \tag{27}$$

Equivalently, the VAR can be defined as

$$\mathbf{z}_{t+1} = \boldsymbol{\pi} + \mathbf{A}\mathbf{z}_t + \boldsymbol{\varepsilon}_{t+1}, \tag{28}$$

where the last equation defines the variables of interest.

The benchmark restricted VAR(1) is nested in this general specification, with

$$\mathbf{A} = \begin{pmatrix} 0 & 0 & \lambda_r \\ 0 & 0 & \lambda_m \\ 0 & 0 & \phi \end{pmatrix}.$$

Consider the indicator vectors, $\mathbf{e}_r \equiv (1, 0, 0)'$, $\mathbf{e}_m \equiv (0, 1, 0)'$, and $\mathbf{e}_q \equiv (0, 0, 1)'$, which represent the position of each state variable in the VAR. As in the benchmark case, the VAR is estimated by applying multiple-equation OLS, with Newey-West t-ratios. The t-ratios of the implied horizon-specific coefficients are produced by employing the delta method. The covariance matrix of the state variables is given by $\boldsymbol{\Sigma} \equiv \mathrm{Cov}(\mathbf{z}_t, \mathbf{z}'_t)$. Given these definitions, and following Larrain and Yogo (2008) and Maio and Xu (2020), we derive the following variance decomposition for q at each horizon H,

$$
\begin{aligned}
1 &\approx b_m^H - b_r^H + b_q^H, \qquad (29)\\
b_r^H &\equiv \frac{\mathbf{e}_r'\mathbf{A}(\mathbf{I}-\rho^H\mathbf{A}^H)(\mathbf{I}-\rho\mathbf{A})^{-1}\boldsymbol{\Sigma}\mathbf{e}_q}{\mathbf{e}_q'\boldsymbol{\Sigma}\mathbf{e}_q},\\
b_m^H &\equiv \frac{(1-\rho)\mathbf{e}_m'\mathbf{A}(\mathbf{I}-\rho^H\mathbf{A}^H)(\mathbf{I}-\rho\mathbf{A})^{-1}\boldsymbol{\Sigma}\mathbf{e}_q}{\mathbf{e}_q'\boldsymbol{\Sigma}\mathbf{e}_q},\\
b_q^H &\equiv \frac{\rho^H\mathbf{e}_q'\mathbf{A}^H\boldsymbol{\Sigma}\mathbf{e}_q}{\mathbf{e}_q'\boldsymbol{\Sigma}\mathbf{e}_q},
\end{aligned}
$$

where $\mathbf{I}$ represents a conformable identity matrix.

Further details on the derivation of this variance decomposition are available in the online appendix. The expressions above show that the relative shares of predictability (e.g., b_r^H/b_m^H) change with the forecasting horizon, in contrast to the restricted VAR case. In other words, the unrestricted VAR enables for a decoupling between the short-run and implied long-run forecasting dynamics.

At an infinite horizon, it turns out that $\lim_{H\to\infty}\rho^H\mathbf{A}^H$ approaches to a matrix of zeros. Thus, the corresponding long-run VAR-based variance decomposition for q is given by

$$
\begin{aligned}
1 &\approx b_m^{lr} - b_r^{lr}, \qquad (30)\\
b_r^{lr} &\equiv \frac{\mathbf{e}_r'\mathbf{A}(\mathbf{I}-\rho\mathbf{A})^{-1}\boldsymbol{\Sigma}\mathbf{e}_q}{\mathbf{e}_q'\boldsymbol{\Sigma}\mathbf{e}_q},\\
b_m^{lr} &\equiv \frac{(1-\rho)\mathbf{e}_m'\mathbf{A}(\mathbf{I}-\rho\mathbf{A})^{-1}\boldsymbol{\Sigma}\mathbf{e}_q}{\mathbf{e}_q'\boldsymbol{\Sigma}\mathbf{e}_q}.
\end{aligned}
$$

As in the restricted VAR case, the t-ratios for the implied infinite-horizon slope estimates are obtained by using the delta method.

The estimation of the unrestricted VAR above yields the following results,

$$\begin{pmatrix} r_{t+1} \\ m_{t+1} \\ q_{t+1} \end{pmatrix} = \widehat{\boldsymbol{\pi}} + \begin{pmatrix} 0.41(\underline{2.43}) & -0.06(-0.29) & -0.39(-0.91) \\ 0.59(\textit{1.65}) & 0.13(0.31) & 0.63(0.65) \\ 0.39(\mathbf{2.98}) & -0.08(-0.56) & 0.59(\textit{1.74}) \end{pmatrix} \begin{pmatrix} r_t \\ m_t \\ q_t \end{pmatrix} + \begin{pmatrix} \widehat{\varepsilon}^r_{t+1} \\ \widehat{\varepsilon}^m_{t+1} \\ \widehat{\varepsilon}^q_{t+1} \end{pmatrix},$$

with R^2 estimates of 0.18, 0.33, and 0.45, respectively. The numbers in parentheses represent the t-ratios, with bold, underlined, and italic numbers denoting significance at the 1%, 5%, and 10%, respectively.

These results show that r_t helps to forecast a rise in all three variables in the system, as the respective coefficient estimates are positive and statistically significant in all cases (marginally so in the equation for marginal profits). On the other hand, the slope estimates associated with m_t are largely insignificant in all cases. Moreover, the estimate of λ_m is cut to about half the magnitude of the corresponding estimate in the restricted VAR (0.63 versus 1.24) and becomes largely insignificant. In comparison, the estimate of λ_r increases in magnitude (-0.39 versus -0.29), but also with no statistical significance. The estimate of ϕ is similar to that obtained in the baseline VAR (0.59 versus 0.61), albeit the statistical significance becomes relatively weak (10% level). These results suggest that the short-run dynamics associated with the restricted and unrestricted VARs can differ by a good deal. In particular, there is a kind of a "substitution effect" in predictive power among some of the variables: Restricting to zero the slopes associated with lagged r, magnifies the forecasting role of q for future marginal profits.

The horizon-specific variance decompositions based on the unrestricted VAR(1) are presented in Figure 2 (Panels B and D). The results point to an even more dominant role of return predictability in comparison to the benchmark VAR. Specifically, the long-run (infinite horizon) return and marginal profit slope estimates are -0.80 and 0.18, respectively. Further, while the return coefficient estimates are strongly significant at all forecasting horizons, there is significance for the m slopes estimates only at very short horizons ($H < 3$). Untabulated results indicate that we cannot reject the null (at the 10% level, t-ratio of 1.20) that all the variation in q stems from long-run return predictability. As in the restricted VAR, predictability of future q is the major driving force at $H = 1$, but this effect dies off at a faster pace.

All in all, the punch line of this section is that predictability of future investment returns (the discount-rate channel) is the major driving force of variation in q. Predictability of future marginal profits (the cash-flow channel) plays a secondary role at most. This pattern is even more evident under the more robust unrestricted VAR.

6 Sensitivity Analysis

In this section, we provide a sensitivity analysis to the empirical results discussed in the previous section. To save space and keep the focus, most results are based on the restricted VAR method.

6.1 Alternative Investment Series

We conduct the variance decomposition for q by using alternative time series of the investment variables.

First, the data is generated from GMM estimation of the structural investment model based on ten Tobin's Q-sorted portfolios, as in Belo, Xue, and Zhang (2013). The resulting GMM estimates are similar to the base case. The estimate of capital share (α) is 0.07. The estimate of the adjustment cost parameter (a) is 20.31 and the corresponding ratio of adjustment-cost-to-sales is equal to 14.84%. The model is not rejected by the χ^2-test, with a p-value around 0.44. The results tabulated in Table 2 (Panel B) show that the predictability mix is very similar to that estimated with the benchmark data. Specifically, the long-run return and profits slopes are -0.62 and 0.36, respectively. The statistical significance is also very close to that obtained in the benchmark case.

Second, the investment data are associated with the median firm, rather than the value-weighted average. This is in line with Belo, Gala, Salomao, and Vitorino (2019) who use the portfolio median to aggregate firm-level investment returns to portfolio level since the median is robust to outliers. In particular, for a given portfolio of firms, they compute the portfolio median of firm-level investment returns to match with the portfolio median of stock returns. Using artificial data simulated from known firm-level parameters, they show that matching the portfolio median in the GMM estimation can recover the true firm-level parameters without bias. The estimate of α is 0.08, while the estimate of a is 20.89 (with the corresponding ratio of adjustment-cost-to-sales being equal to 15.27%). As shown in Table 2 (Panel C), the long-run predictability mix is identical to that estimated under the benchmark case.

6.2 Higher-Order VAR

Next, we estimate a variance decomposition for q, based on a restricted second-order VAR. The rationale is that the second lag of q might provide useful information for predicting the three variables in the system.

The restricted VAR(2) specification is given by

$$r_{t+1} = \pi_r + \lambda_{r1} q_t + \lambda_{r2} q_{t-1} + \varepsilon^r_{t+1}, \tag{31}$$

$$m_{t+1} = \pi_m + \lambda_{m1} q_t + \lambda_{m2} q_{t-1} + \varepsilon^m_{t+1}, \tag{32}$$

$$q_{t+1} = \pi_q + \phi_1 q_t + \phi_2 q_{t-1} + \varepsilon^q_{t+1}. \tag{33}$$

The VAR(2) is estimated by multiple-equation OLS, with Newey–West t-statistics (computed with two lags). We can write the VAR above as a VAR(1) in the companion form:

$$\begin{pmatrix} r_{t+1} \\ m_{t+1} \\ q_{t+1} \\ r_t \\ m_t \\ q_t \end{pmatrix} = \begin{pmatrix} \pi_r \\ \pi_m \\ \pi_q \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & \lambda_{r1} & 0 & 0 & \lambda_{r2} \\ 0 & 0 & \lambda_{m1} & 0 & 0 & \lambda_{m2} \\ 0 & 0 & \phi_1 & 0 & 0 & \phi_2 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} r_t \\ m_t \\ q_t \\ r_{t-1} \\ m_{t-1} \\ q_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon^r_{t+1} \\ \varepsilon^m_{t+1} \\ \varepsilon^q_{t+1} \\ 0 \\ 0 \\ 0 \end{pmatrix}, \tag{34}$$

or equivalently,

$$\mathbf{z}_{t+1} = \boldsymbol{\pi} + \mathbf{A}\mathbf{z}_t + \boldsymbol{\varepsilon}_{t+1}. \tag{35}$$

The indicator vectors are defined as follows,

$$\mathbf{e}_r \equiv (1,0,0,0,0,0)', \tag{36}$$

$$\mathbf{e}_m \equiv (0,1,0,0,0,0)', \tag{37}$$

$$\mathbf{e}_q \equiv (0,0,1,0,0,0)', \tag{38}$$

while the covariance matrix of $\mathbf{z}_t$ corresponds to

$$\boldsymbol{\Sigma} = \mathrm{Cov}(\mathbf{z}_t, \mathbf{z}_t') = \begin{pmatrix} \mathrm{var}(r_t) & \mathrm{Cov}(r_t, m_t) & \mathrm{Cov}(r_t, q_t) & 0 & 0 & 0 \\ \mathrm{Cov}(r_t, m_t) & \mathrm{var}(m_t) & \mathrm{Cov}(m_t, q_t) & 0 & 0 & 0 \\ \mathrm{Cov}(r_t, q_t) & \mathrm{Cov}(m_t, q_t) & \mathrm{var}(q_t) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \tag{39}$$

In order to obtain the predictive slopes at each forecasting horizon H, we use the definitions above into the same formulas presented in the previous section for the case of the unrestricted VAR(1). Similar to the unrestricted VAR case, the t-ratios for the long-horizon coefficients are obtained by applying the delta method.

The estimation of the VAR(2) above yields the following results,

$$\begin{pmatrix} r_{t+1} \\ m_{t+1} \\ q_{t+1} \end{pmatrix} = \widehat{\boldsymbol{\pi}} + \begin{pmatrix} -0.02(-0.16) & -0.42(-\mathbf{2.75}) \\ 1.67(\mathbf{6.36}) & -0.67(\underline{-2.01}) \\ 0.86(\mathbf{7.82}) & -0.39(-\mathbf{3.31}) \end{pmatrix} \begin{pmatrix} q_t \\ q_{t-1} \end{pmatrix} + \begin{pmatrix} \widehat{\varepsilon}^{r}_{t+1} \\ \widehat{\varepsilon}^{m}_{t+1} \\ \widehat{\varepsilon}^{q}_{t+1} \end{pmatrix},$$

with R^2 estimates of 0.18, 0.33, and 0.45, respectively. The numbers in parentheses represent the

t-ratios, with bold, underlined, and italic numbers denoting significance at the 1%, 5%, and 10%, respectively.

These results show that q_{t-1} helps to forecast (a decline in) all three variables in the system, as the respective slope estimates are negative and statistically significant in all cases. On the other hand, the estimate of λ_{r1} becomes close to zero and largely insignificant. We also observe that the estimates for both λ_{m1} and ϕ_1 register larger magnitudes than the corresponding estimates under the restricted VAR(1). Notably, there is a large increase in the fit of the forecasting regressions associated with r (from 0.08 to 0.18) and q (from 0.35 to 0.45), in comparison to the baseline VAR.

Untabulated results show that the long-run predictive slopes associated with future r and m are -0.69 and 0.29, respectively, and both of these estimates are statistically significant (at the 5% or 1% level). This indicates a slightly larger share of return predictability (69% versus 61%) in comparison to the restricted VAR(1) case.

6.3 Alternative δ

We employ an alternative value for the depreciation rate ($\delta = 0.1219$). This value represents the capital-weighted average firm-level depreciation rate in the sample, where the depreciation rate has the difference between Compustat item DP and Compustat item AM in the numerator, and Compustat item PPENT in the denominator.

The VAR estimation results are displayed in Table 2 (Panel D). We can see that the results are very similar to those in the benchmark case. Specifically, the long-run r and m coefficient estimates are -0.60 and 0.38, respectively. Regarding the statistical significance of these estimates, we get the same qualitative inference than in the benchmark setting.

6.4 Alternative Structural Estimation

We conduct the variance decomposition for q by using other time series of the investment variables. In contrast with the rest of the paper, we rely on the structural estimation method employed in Liu, Whited, and Zhang (2009). Using the aggregate market portfolio as the testing portfolio and matching the value-weighted portfolio returns, the estimated capital share (α) is 0.23, which is higher than the basecase estimate. On other hand, the estimated adjustment costs are similar to the basecase, as indicated by both the estimated parameter (a) of 16.10 and the estimated adjustment-costs-to-sales ratio (Φ/Y) of 11.76%.

The VAR estimation results are displayed in Table 2 (Panel E). The predictability mix is qualitatively similar to that estimated under the benchmark case, with a somewhat higher degree of return predictability. Specifically, the long-run return and marginal profits coefficient estimates are -0.71 and 0.30, respectively. This means that return predictability accounts for more than twice the share of marginal profits profitability in terms of explaining the variation in q. Both of these estimates are strongly significant (1% level), which means that both channels are important from a statistical viewpoint. In comparison to the benchmark case, the larger role for the discount-rate channel stems from both a larger magnitude of the VAR return slope estimate (-0.35 versus -0.29) and a lower magnitude of the m coefficient estimate (0.82 versus 1.24).

6.5 Direct Approach

We estimate the variance decomposition for q by using the direct approach.

Following Cochrane (2008, 2011) and Maio and Santa-Clara (2015), we estimate weighted long-horizon regressions of future multiperiod log investment returns, future multiperiod log marginal profits, and future q on the current q:

$$\sum_{h=1}^{H} \rho^{h-1} r_{t+h} = a_r^H + b_r^H q_t + \varepsilon_{t+H}^r, \tag{40}$$

$$\sum_{h=1}^{H} \rho^{h-1} (1-\rho) m_{t+h} = a_m^H + b_m^H q_t + \varepsilon_{t+H}^m, \tag{41}$$

$$\rho^H q_{t+H} = a_q^H + b_q^H q_t + \varepsilon_{t+H}^q. \tag{42}$$

The estimation is conducted by equation-by-equation OLS and the t-statistics for the direct predictive slopes are based on Newey and West (1987) standard errors with $H-1$ lags (i.e., the Bartlett Kernel with a bandwidth of H). These standard errors incorporate a correction of the bias induced by using overlapping observations in the regressions presented above.

Similarly to Cochrane (2011), by combining the present-value relation for q in Equation (15) with the predictive regressions presented above, we obtain an approximate identity involving the predictability coefficients associated with q_t, at each forecasting horizon H:

$$1 \approx -b_r^H + b_m^H + b_q^H. \tag{43}$$

If the first-order VAR does not fully capture the dynamics of the data generating process for r, q, and m, it follows that the corresponding variance decomposition will be a poor approximation of the true decomposition for q, as discussed in Cochrane (2008) and Maio and Xu (2020).[19] This problem does not exist under the direct approach, which a priori should yield the most correct estimates for the variance decomposition (see Cochrane, 2008, 2011; Maio and Santa-Clara, 2015). The minus side of the direct approach is that with small or moderate samples, the statistical power of the long-horizon regressions is negatively affected at very long horizons, given the substantial decline in the number of usable observations. For example at $H = 20$, 20 observations are lost by running the corresponding long-horizon regression.

The term structure of direct variance decompositions are presented in Figure 4. At the one-year horizon, the dominant source of variation in current q is its own predictability, with a share of 52%. This result emanates from the existence of some short-run persistence in this variable, as indicated in Table 1. Yet, such effect dies off quickly. Indeed, for forecasting horizons beyond one year, the key driving force of variation in q becomes return predictability, with shares above 65% at most horizons. At very long horizons, there is a slightly lower share of return predictability,

[19] In a broader scope, there are several examples in the economics literature showing an inconsistency between the short-run dynamics of stochastic processes and the implied long-run dynamics (e.g., Campbell and Shiller, 1988; Bandi, Perron, Tamoni, and Tebaldi, 2019; Gonçalves, 2020).

with weights below 56%. The negative return slope estimates are strongly statistically significant (at the 1% or 5% level) at all horizons. In comparison, the positive coefficient estimates associated with future marginal profits are significantly smaller in magnitude, with shares around or below 30% at most forecasting horizons. At very long horizons ($H > 18$), we obtain a slightly larger role for m predictability, with weights around 40%. These slope estimates are statistically significant (at the 5% level) at most forecasting horizons, with the few exceptions occurring at some short and intermediate horizons.

Overall, the results from the direct approach are largely consistent with those based on the indirect or VAR approach used in our main empirical analysis.

7 Comparative Statics

In this section, we conduct a comparative statics exercise.

Specifically, we estimate a range of long-run variance decompositions associated with q for a set of artificial series of the key investment variables in the system (r, m, and q). The artificial time-series are obtained from calibration of the two key structural parameters of the theoretical model presented in Section 2 (α and a).[20] The goal of this analysis is to assess if the predictability mix associated with q, that we obtained in the previous sections, holds for a reasonable range of those two underlying parameters.

The simulation results when the variance decomposition is based on the restricted VAR are presented in Table 3. Table 4 displays the simulation results based on the unrestricted VAR. We calibrate five different values for α (0.05, 0.15, 0.30, 0.50, and 0.70) and five values for a (1.37, 6.85, 13.69, 20.53, and 27.38). In the case of a, these values are associated with a calibration of the adjustment cost-to-output ratio (Φ/Y) of 0.01, 0.05, 0.10, 0.15, and 0.20, respectively. Hence, we have a total of 25 (5×5) different artificial data sets, which are used in the computation of the

[20]Importantly, we use the same series of investment and sales as in the data. However, different parameter values will yield different series of r, m, and q.

variance decomposition.[21] To save space and keep the focus, we report only the long-run (infinite horizon) variance decompositions for q.

The first key pattern that emerges from Tables 3-4 is that the share of return (marginal profits) predictability declines (increases) with α. Such pattern is especially predominant in the case of the unrestricted VAR: at high values of this parameter, that is $\alpha = 0.50, 0.70$, it turns out that the return slopes (that have the correct sign) are below 60% (in magnitude) in most cases, despite the large statistical significance. At the other end of the spectrum, for $\alpha = 0.05$, the estimates of b_r^{lr} are around or above 0.80 (in magnitude) in most cases. In the case of the restricted VAR, the difference in return slope estimates is less severe. On the other hand, we observe exactly the opposite pattern for the m slopes, and this holds for both VAR specifications. For example, we obtain estimates of b_m^{lr} around 0.60 for $\alpha = 0.05, a = 1.37$, while the corresponding estimates are around 2 for $\alpha = 0.70, a = 1.37$, and this holds under both VAR frameworks. We also observe across the board that the weights of return predictability tend to be larger under the unrestricted VAR than under the restricted VAR method. This pattern is especially evident among low values of α and is consistent with the evidence from Section 5.

Higher values of α entail higher volatilities of both marginal profits and investment returns. However, the volatility of marginal profits rises substantially more. For example, for a value of a of 1.37, a rise in the share of capital in production from 0.05 to 0.7, implies that the volatility of investment returns is six times higher, whereas the volatility of marginal profits is about 12 times higher. Consequently, the rise in the covariance of marginal profits and lagged q is substantially higher than the rise in the covariance of investment returns and lagged q.

The second key result from Tables 3-4 is that, for a given value of α, the share of return (marginal profits) predictability tends to increase (decrease) monotonically with a. Indeed, apart from the extreme case of $\alpha = 0.05$, the long-run return slope estimates have the wrong sign (positive) at very low values of a (1.37), albeit most of these estimates not being significant at the 10% level.

[21] These ratios are consistent with Bloom (2009). Bloom (2009, Table IV) surveys the estimates of convex adjustment costs to be between zero and 20% of revenues.

Consequently, at those pairs of calibrated structural parameters, we obtain shares for long-run marginal profit predictability above 100%, which are strongly significant. This means that, in economic terms, marginal profits predictability explains all the variation in q, for extreme low values of a. However, for $a \geq 13.69$, it follows that the discount rate channel is dominant for most choices of α. Intuitively, in the extreme, if the value of the adjustment cost is zero, marginal Q is always one, implying no investment return predictability.

All in all, the results of this section show that the dominant role of return predictability in terms of driving variation in q is robust to a plausible range of the key parameters in the structural investment model. However, these simulation results also show that it is possible to find a relevant (and even dominant) share of marginal profits predictability under less plausible values for those structural parameters.

8 The Linkage to the Investment-Capital Ratio

The motivation for the analysis of the drivers of marginal Q is that it should be the major determinant of the rate of investment. Therefore, we turn to examining the drivers of aggregate investment directly, more specifically, the investment-capital ratio (I/K).

According to Equation (5), marginal Q is closely connected to I/K. A simple regression of q on the log investment-capital ratio ($ik \equiv \ln(I/K)$) yields the following results, with heteroskedasticity-robust t-ratios (White, 1980) presented in parenthesis:

$$\begin{aligned} q_t &= 1.60 + 0.34 ik_t, R^2 = 0.31, \\ & \quad (18.77)(6.94). \end{aligned}$$

These estimates show that q and ik are significantly correlated on a contemporaneous basis. Yet, this correlation is far from perfect, as indicated by the R^2 of 31% and the correlation coefficient of

0.56.[22] This stems from the fact that q represents a non-linear function of ik, as shown by Equation (5). Nonetheless, it is instructive to conduct a pseudo variance decomposition for ik in order to check what are the driving forces of this variable, as a first-order approximation.

The first-order restricted VAR associated with ik is given by

$$r_{t+1} = \pi_r + \lambda_r ik_t + \varepsilon^r_{t+1}, \tag{44}$$

$$m_{t+1} = \pi_m + \lambda_m ik_t + \varepsilon^m_{t+1}, \tag{45}$$

$$ik_{t+1} = \pi_q + \phi ik_t + \varepsilon^{ik}_{t+1}, \tag{46}$$

and the corresponding long-run variance decomposition is as follows:

$$\begin{aligned} 0.34 &\approx b^{lr}_m - b^{lr}_r, \\ b^{lr}_r &\equiv \frac{\lambda_r}{1-\rho\phi}, \\ b^{lr}_m &\equiv \lambda_m \frac{1-\rho}{1-\rho\phi}. \end{aligned} \tag{47}$$

We note that the long-run coefficient estimates should add up to 0.34 rather than 1.[23]

The estimate of ρ is the same as in the benchmark decomposition associated with q. The VAR estimation results are presented in Table 5. ik predicts a significant decline in the one-period ahead investment return, with a slope estimate of -0.19. This is in line with the evidence for q in Section 5. The main difference relative to q is that ik does not forecast a significant increase in future m, as indicated by the insignificant slope estimate (t-ratio of 1.41) and the corresponding very low explanatory ratio (R^2 of only 3%). ik is only marginally more persistent than q, with an autoregressive coefficient of 0.69. Consequently, the implied long-run return and marginal profit coefficient estimates are -0.45 and 0.08, respectively. While the first estimate is strongly significant

[22] In our sample period, ik has a mean of -1.67 and a standard deviation of 0.17.

[23] For any three variables y, x, and z, the identity $y = x + z$ implies a variance decomposition $1 = \beta(x, y) + \beta(z, y)$, where $\beta(x, y)$ denotes the slope of a univariate regression of x onto y. Consider a fourth variable v, such that $y = \kappa + \varphi v$. The corresponding variance decomposition for v is given by $\varphi = \beta(x, v) + \beta(z, v)$.

(t-ratio of -4.32), the second estimate is not even significant at the 10% level (t-ratio of 1.19). Indeed, we cannot reject the null that all the variation in ik arises from return predictability, that is, $b_r^{lr} = -0.34$ (t-ratio of -1.05). This indicates an even larger role for investment return predictability than in the decomposition associated with q.

However, we cannot interpret these estimates as being strictly associated with a variance decomposition for ik, as the sum of the long-run slope estimates is far from the correct value (0.53 versus 0.34). As already discussed above, this stems from the fact that the relationship between q and ik is non-linear. Yet, this exercise provides preliminary evidence that ik has much stronger forecasting power for future investment returns than future marginal profits.

We conduct a less parametric approach to gauge the forecasting ability of ik for future r and m. Specifically, we run simple long-horizon regressions of future multiperiod log investment returns and future multiperiod log marginal profits on the current ik (e.g., Keim and Stambaugh 1986; Fama and French 1988, 1989):[24]

$$\sum_{h=1}^{H} r_{t+h} = \varpi_r^H + \eta_r^H ik_t + u_{t+H}^r, \tag{48}$$

$$\sum_{h=1}^{H} m_{t+h} = \varpi_m^H + \eta_m^H ik_t + u_{t+H}^m. \tag{49}$$

We use forecasting horizons of 1, 5, 10, 15, and 20 years ahead. As a reference point, we also show the results for forecasting regressions in which q is the predictor. The results for the regressions associated with future marginal profits and investment returns are reported respectively in Tables 6 and 7. We can see that ik forecasts a decline in future m for horizons beyond one year, albeit there is only marginal significance (10% level) at $H = 5, 20$. In contrast, q predicts a significant increase in future marginal profits at the one- and 20-year horizons, while such effect is marginally significant at $H = 10$.

[24] Note that these long-horizon regressions differ from those presented in Section 6, as the left-hand side variables do not contain the terms involving ρ. This means that distant ahead returns and marginal profits are not scaled downwards.

When it comes to forecast future investment returns, the results show a different pattern: Both variables forecast a significant decline in r at most horizons, as the corresponding negative slope estimates are significant at the 1% or 5% level. The sole exception occurs at $H = 20$ when the predictor is q. In terms of forecasting performance, it turns out that the R^2 estimates in the regressions containing ik are clearly above the corresponding estimates in the regressions containing q for horizons beyond one year. This is especially relevant at $H = 20$. On the other hand, ik has stronger forecasting power for future r than for future m, as indicated by the substantially larger explanatory ratios for horizons beyond $H = 1$.

The conclusion from the results in Tables 6-7 is that ik helps forecasting a significant decline in future investment returns, while the predictive power for future marginal profits is much more modest. This evidence is consistent with the results obtained from the pseudo variance decomposition for ik discussed above.

Leading production-based asset pricing models (e.g., Cochrane 1991) predict that the sources of fluctuations in stock prices and investment are the same. Our result that investment is driven mostly by future returns on investment (discount rates) is consistent with the predictions of these models. In contrast, Abel and Blanchard (1986) find that, although expected marginal profits and discount rates affect investment, the marginal profit component has a much larger and significant effect, which is the opposite of the drivers of stock prices. Their result is therefore at odds with the predictions of leading production-based asset pricing models.

9 Conclusion

This paper explores the sources of fluctuations in aggregate marginal Q. The extant literature studies intensively the sources of variation in (scaled) stock prices but the drivers of marginal Q are largely unexplored. The approach we undertake is the supply approach to valuation. That is, we infer marginal Q from the marginal adjustment cost of investment, as in Belo, Xue, and Zhang (2013).

We employ a parsimonious model of optimal investment behavior with standard production and adjustment cost technologies. The model's optimal investment condition, namely that the marginal value of capital equals the marginal cost of investment, implies that marginal Q is a linear function of the investment-to-capital ratio. Subsequently, we estimate the share of capital and adjustment cost parameters by employing GMM estimation for the aggregate of firms on the Compustat database. Following Gonçalves, Xue, and Zhang (2020), we correct for aggregation bias. We then derive a present-value relation to show that variations in the log of marginal Q (q) (and hence in investment) must reflect shocks to expected future marginal profits of capital, shocks to expected investment returns, or the future value of q at some terminal date, or any combination of these three variables.

We conduct variance decomposition for q using alternative methodologies, namely a first-order restricted VAR and an unrestricted VAR. We find that the bulk of variations in q is due to investment return predictability, whereas predictability of the marginal profits of capital assumes a secondary role (albeit statistically significant in some cases). We conduct several robustness checks, namely using portfolios sorted by Tobin's Q as well as using portfolio medians in the GMM estimation of the structural model; conducting simulation exercises in the variance decomposition; employing weighted long horizon regressions to obtain the decomposition; estimating the variance decomposition from a second-order VAR; varying the value of the depreciation rate; and conducting the GMM estimation without correcting for aggregation bias. Our main qualitative results are robust in all those checks.

We also conduct a pseudo variance decomposition for aggregate investment and find that the bulk of fluctuations in investment stems from variations in expected investment returns. This result is confirmed with direct long horizon regressions of future investment returns on current investment. These results are consistent with the predictions of leading production-based asset pricing models.

Table 1: Descriptive Statistics

This table reports descriptive statistics for the log investment return (r), log marginal profits (m), and log Q (q). The sample is 1964–2018. AR(1) designates the first-order autocorrelation. The correlations between the variables are presented in Panel B.

	Panel A				
	Mean	S.D.	Min.	Max.	AR(1)
r	0.06	0.10	−0.19	0.31	0.12
m	−0.85	0.24	−1.30	0.03	0.54
q	1.03	0.10	0.85	1.29	0.61
	Panel B (Correl.)				
	r	m	q		
r	1.00	0.62	0.57		
m		1.00	0.95		
q			1.00		

Table 2: VAR Estimates

This table reports the VAR(1) estimation results when the predictor is log Q (q). The variables in the VAR are the log investment return (r), log marginal profits (m), and q. The results in Panel A correspond to the baseline set of investment variables. Results in Panels B to E correspond to alternative sets of the investment variables. λ, ϕ denote the VAR slopes associated with lagged q, while t denotes the respective Newey and West (1987) t-statistics (calculated with one lag). R^2 is the coefficient of determination for each equation in the VAR. b^{lr} denote the long-run coefficients (infinite horizon). $t(b_r^{lr} = 0)$ and $t(b_r^{lr} = -1)$ denote the t-statistics associated with the null hypotheses ($b_r^{lr} = 0, b_m^{lr} = 1$) and ($b_r^{lr} = -1, b_m^{lr} = 0$), respectively. The original sample is 1964–2018. Italic, underlined, and bold numbers denote statistical significance at the 10%, 5%, and 1% levels, respectively.

	λ, ϕ	t	R^2	b^{lr}	$t(b_r^{lr} = 0)$	$t(b_r^{lr} = -1)$
Panel A						
r	−0.29	−2.32	0.08	−0.61	**−3.96**	2.52
m	1.24	**4.58**	0.28	0.37	**−4.24**	2.54
q	0.61	**5.91**	0.35			
Panel B (Q Deciles)						
r	−0.30	−2.38	0.09	−0.62	**−4.16**	2.54
m	1.21	**4.63**	0.28	0.36	**−4.48**	2.55
q	0.61	**5.93**	0.35			
Panel C (Median firm)						
r	−0.29	−2.31	0.08	−0.61	**−3.99**	2.51
m	1.27	**4.59**	0.28	0.37	**−4.30**	2.52
q	0.61	**5.91**	0.35			
Panel D (Alternative δ)						
r	−0.30	−2.33	0.08	−0.60	**−3.90**	2.56
m	1.17	**4.53**	0.28	0.38	**−4.14**	2.57
q	0.61	**5.91**	0.35			
Panel E (Alternative series)						
r	−0.35	**−3.30**	0.13	−0.71	**−6.52**	**2.71**
m	0.82	**4.38**	0.23	0.30	**−6.49**	**2.73**
q	0.61	**6.46**	0.36			

Table 3: Simulation with Restricted VAR

This table reports the simulation results for the long-run variance decomposition associated with log Q (q). The simulated series for the log investment return (r), log marginal profits (m), and q are based on different pairs of the calibrated structural parameters α and a from the theoretical model. The implied long-run predictive statistics are based on a restricted VAR(1). b^{lr} denote the long-run coefficients (infinite horizon), while t represent the corresponding t-statistics. The original sample is 1964–2018. Italic, underlined, and bold numbers denote statistical significance at the 10%, 5%, and 1% levels, respectively.

	b_r^{lr}	t	b_m^{lr}	t
$\alpha = 0.05, a = 1.37$	−0.39	(−1.37)	0.60	(2.15)
$\alpha = 0.05, a = 6.85$	−0.58	(**−3.38**)	0.40	(2.43)
$\alpha = 0.05, a = 13.69$	−0.61	(**−3.95**)	0.37	(2.50)
$\alpha = 0.05, a = 20.53$	−0.63	(**−4.20**)	0.36	(2.53)
$\alpha = 0.05, a = 27.38$	−0.63	(**−4.34**)	0.35	(2.54)
$\alpha = 0.15, a = 1.37$	0.08	(0.15)	1.09	(2.13)
$\alpha = 0.15, a = 6.85$	−0.52	(**−2.66**)	0.48	(2.49)
$\alpha = 0.15, a = 13.69$	−0.59	(**−3.66**)	0.40	(**2.58**)
$\alpha = 0.15, a = 20.53$	−0.61	(**−4.06**)	0.38	(**2.60**)
$\alpha = 0.15, a = 27.38$	−0.62	(**−4.27**)	0.37	(**2.61**)
$\alpha = 0.30, a = 1.37$	0.50	(0.71)	1.53	(2.15)
$\alpha = 0.30, a = 6.85$	−0.44	(−1.99)	0.56	(2.48)
$\alpha = 0.30, a = 13.69$	−0.56	(**−3.27**)	0.44	(**2.61**)
$\alpha = 0.30, a = 20.53$	−0.59	(**−3.84**)	0.40	(**2.65**)
$\alpha = 0.30, a = 27.38$	−0.61	(**−4.13**)	0.39	(**2.67**)
$\alpha = 0.50, a = 1.37$	0.82	(0.98)	1.86	(2.18)
$\alpha = 0.50, a = 6.85$	−0.38	(−1.50)	0.63	(2.45)
$\alpha = 0.50, a = 13.69$	−0.53	(**−2.88**)	0.48	(**2.60**)
$\alpha = 0.50, a = 20.53$	−0.57	(**−3.56**)	0.43	(**2.67**)
$\alpha = 0.50, a = 27.38$	−0.59	(**−3.94**)	0.41	(**2.70**)
$\alpha = 0.70, a = 1.37$	1.02	(1.11)	2.07	(2.21)
$\alpha = 0.70, a = 6.85$	−0.34	(−1.23)	0.68	(2.43)
$\alpha = 0.70, a = 13.69$	−0.50	(**−2.60**)	0.50	(**2.58**)
$\alpha = 0.70, a = 20.53$	−0.56	(**−3.32**)	0.45	(**2.66**)
$\alpha = 0.70, a = 27.38$	−0.58	(**−3.75**)	0.42	(**2.70**)

Table 4: Simulation with Unrestricted VAR

This table reports the simulation results for the long-run variance decomposition associated with log Q (q). The simulated series for the log investment return (r), log marginal profits (m), and q are based on different pairs of the calibrated structural parameters α and a from the theoretical model. The implied long-run predictive statistics are based on an unrestricted VAR(1). b^{lr} denote the long-run coefficients (infinite horizon), while t represent the corresponding t-statistics. The original sample is 1964–2018. Italic, underlined, and bold numbers denote statistical significance at the 10%, 5%, and 1% levels, respectively.

	b_r^{lr}	t	b_m^{lr}	t
$\alpha = 0.05, a = 1.37$	−0.37	(−0.83)	0.62	(1.42)
$\alpha = 0.05, a = 6.85$	−0.78	(**−4.14**)	0.19	(1.11)
$\alpha = 0.05, a = 13.69$	−0.82	(**−5.19**)	0.16	(1.10)
$\alpha = 0.05, a = 20.53$	−0.83	(**−5.60**)	0.15	(1.10)
$\alpha = 0.05, a = 27.38$	−0.83	(**−5.83**)	0.14	(1.10)
$\alpha = 0.15, a = 1.37$	0.33	(0.79)	1.35	(**3.24**)
$\alpha = 0.15, a = 6.85$	−0.50	(*−1.78*)	0.49	(*1.77*)
$\alpha = 0.15, a = 13.69$	−0.70	(**−3.19**)	0.29	(1.40)
$\alpha = 0.15, a = 20.53$	−0.76	(**−4.15**)	0.23	(1.30)
$\alpha = 0.15, a = 27.38$	−0.79	(**−4.74**)	0.20	(1.26)
$\alpha = 0.30, a = 1.37$	0.59	(1.41)	1.64	(**3.88**)
$\alpha = 0.30, a = 6.85$	−0.39	(*−1.79*)	0.62	(**2.90**)
$\alpha = 0.30, a = 13.69$	−0.55	(<u>−2.42</u>)	0.45	(<u>1.99</u>)
$\alpha = 0.30, a = 20.53$	−0.64	(**−2.94**)	0.35	(*1.65*)
$\alpha = 0.30, a = 27.38$	−0.70	(**−3.46**)	0.30	(1.51)
$\alpha = 0.50, a = 1.37$	0.80	(*1.84*)	1.86	(**4.25**)
$\alpha = 0.50, a = 6.85$	−0.36	(<u>−2.05</u>)	0.66	(**3.74**)
$\alpha = 0.50, a = 13.69$	−0.50	(**−2.76**)	0.50	(**2.79**)
$\alpha = 0.50, a = 20.53$	−0.56	(**−2.86**)	0.44	(<u>2.23</u>)
$\alpha = 0.50, a = 27.38$	−0.61	(**−3.01**)	0.39	(*1.91*)
$\alpha = 0.70, a = 1.37$	0.94	(<u>2.14</u>)	2.01	(**4.49**)
$\alpha = 0.70, a = 6.85$	−0.34	(<u>−2.11</u>)	0.68	(**4.16**)
$\alpha = 0.70, a = 13.69$	−0.49	(**−3.19**)	0.52	(**3.36**)
$\alpha = 0.70, a = 20.53$	−0.54	(**−3.24**)	0.46	(**2.75**)
$\alpha = 0.70, a = 27.38$	−0.58	(**−3.18**)	0.42	(<u>2.35</u>)

Table 5: VAR Estimates: ik

This table reports the VAR(1) estimation results when the predictor is the log investment-to-capital ratio (ik). The variables in the VAR are the log investment return (r), log marginal profits (m), and ik. λ, ϕ denote the VAR slopes associated with lagged ik, while t denotes the respective Newey and West (1987) t-statistics (calculated with one lag). R^2 is the coefficient of determination for each equation in the VAR. b^{lr} denote the long-run coefficients (infinite horizon). $t(b_r^{lr} = 0)$ and $t(b_r^{lr} = -0.34)$ denote the t-statistics associated with the null hypotheses $(b_r^{lr} = 0, b_m^{lr} = 0.34)$ and $(b_r^{lr} = -0.34, b_m^{lr} = 0)$, respectively. The original sample is 1964–2018. Italic, underlined, and bold numbers denote statistical significance at the 10%, 5%, and 1% levels, respectively.

	λ, ϕ	t	R^2	b^{lr}	$t(b_r^{lr} = 0)$	$t(b_r^{lr} = -0.34)$
r	−0.19	<u>−2.56</u>	0.09	−0.45	**−4.32**	−1.05
m	0.23	1.41	0.03	0.08	**−3.86**	1.19
q	0.69	**7.85**	0.47			

Table 6: Long-horizon Regressions: m

This table reports the results for univariate long-horizon regressions associated with future log marginal profits (m) at horizons (H) of 1, 5, 10, 15, and 20 years ahead. The forecasting variables are either the log Q (q) or log investment-to-capital ratio (ik). The original sample is 1964–2018. In each regression, η_m^H denotes the slope estimate. The corresponding t-ratios, which are based on Newey and West (1987) standard errors (computed with $H-1$ lags), are displayed in parentheses. R^2 denotes the coefficient of determination of a given regression. Bold, underlined, and italics t-ratios indicate statistical significance at the 1%, 5%, and 10% level, respectively.

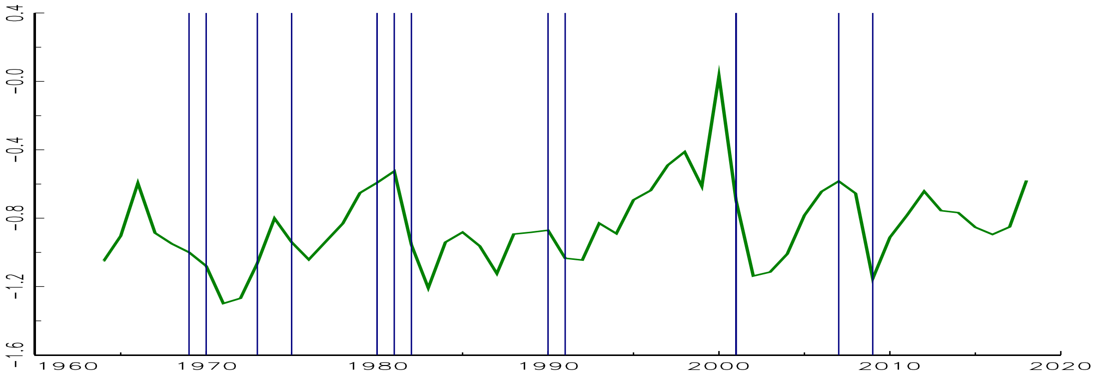
	$H=1$	$H=5$	$H=10$	$H=15$	$H=20$
			Panel A (q)		
η_m^H	1.24	1.62	1.78	1.39	5.40
	(**4.40**)	(1.23)	(*1.65*)	(0.66)	(**2.98**)
R^2	0.28	0.05	0.03	0.01	0.12
			Panel B (ik)		
η_m^H	0.23	−1.05	−2.83	−3.84	−3.07
	(−1.56)	(*−1.74*)	(**−2.90**)	(**−4.28**)	(*−1.83*)
R^2	0.03	0.05	0.18	0.27	0.10

Table 7: Long-horizon Regressions: r

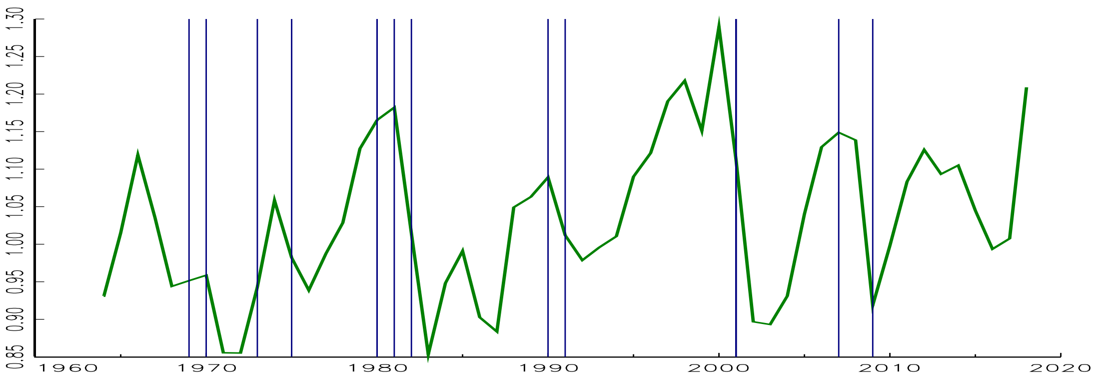
This table reports the results for univariate long-horizon regressions associated with future log investment returns (r) at horizons (H) of 1, 5, 10, 15, and 20 years ahead. The forecasting variables are either the log Q (q) or log investment-to-capital ratio (ik). The original sample is 1964–2018. In each regression, η_r^H denotes the slope estimate. The corresponding t-ratios, which are based on Newey and West (1987) standard errors (computed with $H-1$ lags), are displayed in parentheses. R^2 denotes the coefficient of determination of a given regression. Bold, underlined, and italics t-ratios indicate statistical significance at the 1%, 5%, and 10% level, respectively.

	$H=1$	$H=5$	$H=10$	$H=15$	$H=20$
Panel A (q)					
η_r^H	−0.29	−0.91	−0.76	−0.72	−0.26
	(<u>−2.21</u>)	(**−4.83**)	(**−4.19**)	(<u>−2.21</u>)	(−1.41)
R^2	0.08	0.29	0.22	0.18	0.02
Panel B (ik)					
η_r^H	−0.19	−0.63	−0.70	−0.69	−0.63
	(**−2.74**)	(**−5.16**)	(**−9.66**)	(**−5.22**)	(**−2.72**)
R^2	0.09	0.37	0.46	0.40	0.31

Panel A (r)



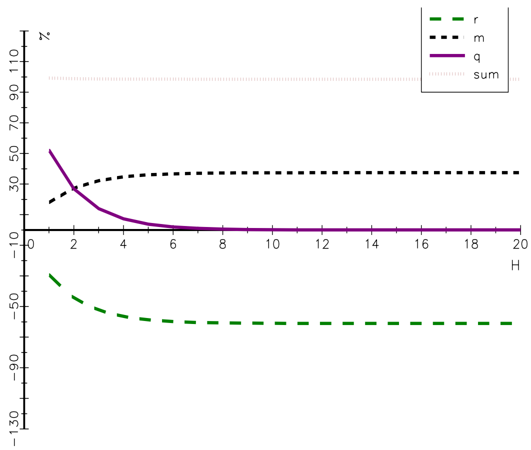
Panel B (m)



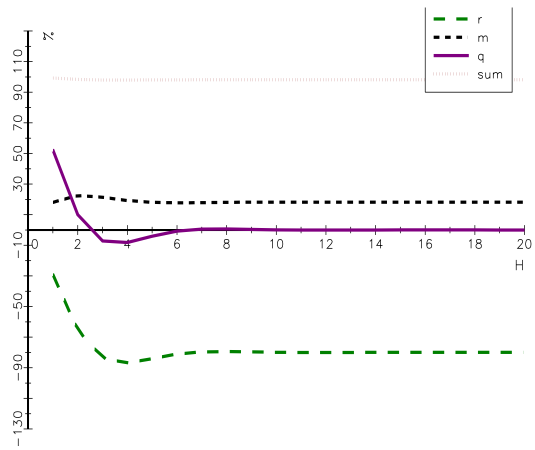
Panel C (q)

Figure 1: Time-Series for r, m, and q

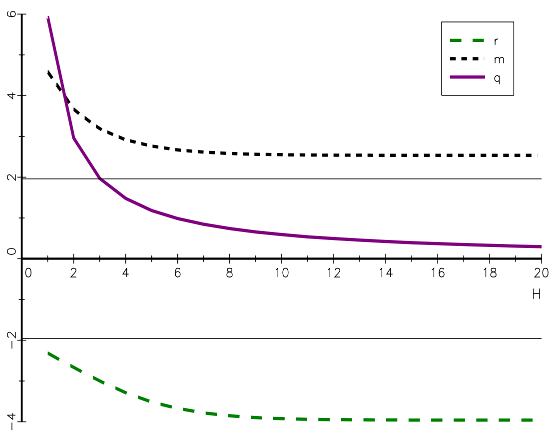
This figure plots the time-series for the log investment return (r), log marginal profit (m), and log Q (q). The bars contain the years with NBER recessions (the 1980 and 2001 recessions are indicated by a single line). The sample is 1964 to 2018.



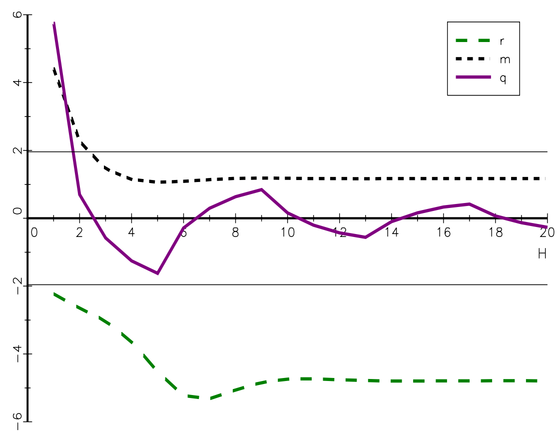
Panel A (Rest. VAR, slopes)



Panel B (Unrest. VAR, slopes)



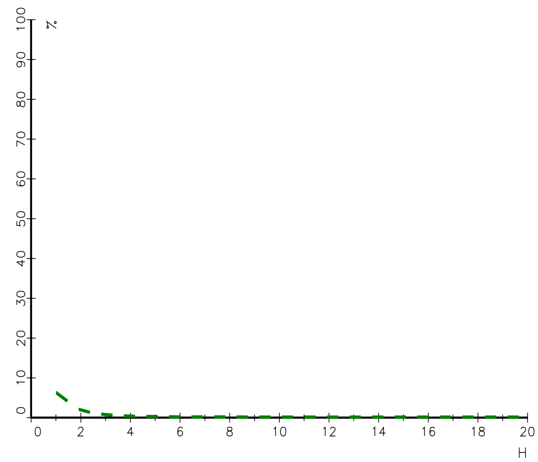
Panel C (Rest. VAR, *t*-stats)



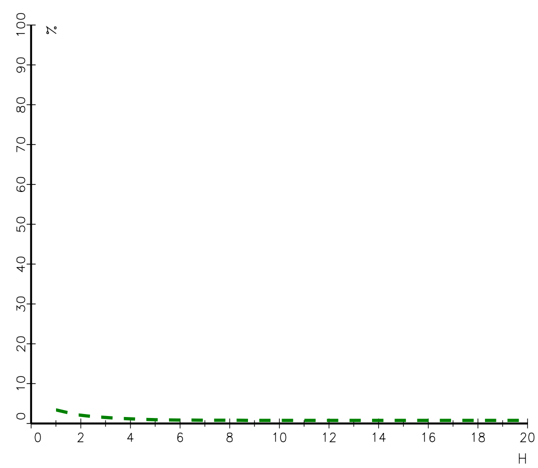
Panel D (Unrest. VAR, *t*-stats)

Figure 2: Variance Decomposition

This figure plots the term structure of multiple-horizon predictive coefficients (in %), and respective t-statistics, corresponding to the variance decompositions for log Q (q). The predictive slopes are obtained from either a restricted or an unrestricted first-order VAR. The coefficients are associated with the log investment return (r), log marginal profits (m), and future q. The forecasting variable is q in all three cases. "Sum" denotes the value of the variance decomposition. H represents the number of years ahead. The horizontal lines represent the 5% critical values (-1.96, 1.96). The original sample is 1964 to 2018.



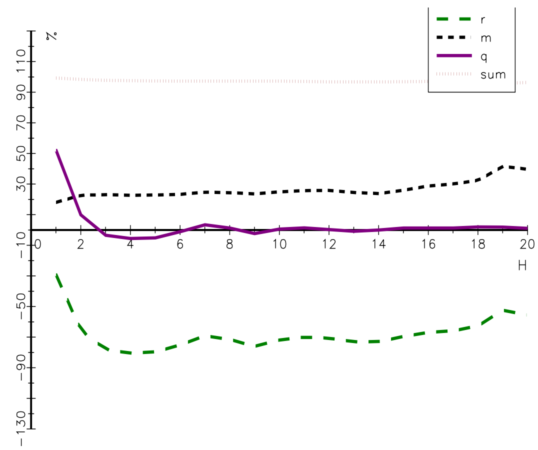
Panel A (r)



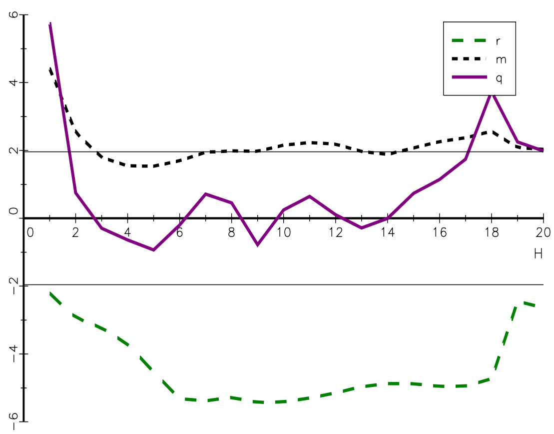
Panel B (m)

Figure 3: Bootstrap Simulation

This figure plots the simulated p-values for the restricted VAR-based return (r) and profitability (m) slopes from a Bootstrap simulation with 10,000 replications. The predictive variable is log Q (q). The numbers indicate the fraction of pseudo samples under which the return (profitability) coefficient is lower (higher) than the corresponding estimates from the original sample. H represents the number of years ahead. The original sample is 1964 to 2018.



Panel A (slopes)



Panel B (t-stats)

Figure 4: Variance Decomposition: Direct Approach

This figure plots the term structure of multiple-horizon predictive coefficients (in %), and respective t-statistics, corresponding to the variance decompositions for log Q (q). The predictive slopes are obtained from weighted long-horizon regressions. The coefficients are associated with the log investment return (r), log marginal profits (m), and future q. The forecasting variable is q in all three cases. "Sum" denotes the value of the variance decomposition. H represents the number of years ahead. The horizontal lines represent the 5% critical values (−1.96, 1.96). The original sample is 1964 to 2018.

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A Online Appendix: Not for Publication

A.1 Estimating the Investment Return and its Components

A.1.1 GMM Estimation Methodology

We estimate the parameters, $\mathbf{b} \equiv (\alpha, a)$, by minimizing a weighted average of the sample moments e_i^r and e_i^q, denoted by $\mathbf{g}_T$. The GMM objective function is a weighted sum of squares of the model errors, that is, $\mathbf{g}_T' \mathbf{W} \mathbf{g}_T$, in which $\mathbf{W}$ is the (adjusted) identity matrix.[25] Following Belo *et al.* (2013), we adjust the weighting matrix $\mathbf{W}$ such that the two sets of errors, e_i^r and e_i^q, are comparable in magnitude.[26] Let $\mathbf{D} = \partial \mathbf{g}_T / \partial \mathbf{b}$ and $\mathbf{S}$ equal a consistent estimate of the variance-covariance matrix of the sample errors $\mathbf{g}_T$. We estimate $\mathbf{S}$ using a standard Bartlett kernel with a window length of two.[27] The estimate of $\mathbf{b}$, denoted $\widehat{\mathbf{b}}$, is asymptotically normal with variance-covariance matrix given by

$$\text{var}(\widehat{\mathbf{b}}) = \frac{1}{T}(\mathbf{D}'\mathbf{W}\mathbf{D})^{-1}\mathbf{D}'\mathbf{W}\mathbf{S}\mathbf{W}\mathbf{D}(\mathbf{D}'\mathbf{W}\mathbf{D})^{-1}. \tag{A.1}$$

To construct standard errors for the model errors, we use

$$\text{var}(\mathbf{g}_T) = \frac{1}{T}[\mathbf{I} - \mathbf{D}(\mathbf{D}'\mathbf{W}\mathbf{D})^{-1}\mathbf{D}'\mathbf{W}]\mathbf{S}[\mathbf{I} - \mathbf{D}(\mathbf{D}'\mathbf{W}\mathbf{D})^{-1}\mathbf{D}'\mathbf{W}], \tag{A.2}$$

which is the variance-covariance matrix for the model errors, $\mathbf{g}_T$.

To evaluate the investment Euler equation moment for each testing portfolio, Liu *et al.* (2009) aggregate firm-level accounting variables to portfolio-level variables, from which portfolio levered investment returns are constructed and matched with portfolio stock returns. Gonçalves *et al.* (2020) argue this approach has important drawbacks as follows. First, this approach assumes that

[25] As in Liu *et al.* (2009), we conduct a numerical optimization based on the simplex search method of Lagarias *et al.* (1998).

[26] We multiply the valuation moments by a factor of $|e_i^r| / |e_i^q|$, in which $|e_i^q|$ is the mean absolute valuation error from estimating only the valuation moments and $|e_i^r|$ is the mean absolute return error from estimating only the expected return moments across the same testing assets.

[27] The GMM estimation is robust to the choice of window length.

all firms within a portfolio have the same investment returns. Second, the approach ignores substantial amount of heterogeneity across firms' accounting variables that can help identify structural parameters. We also note that non-linearities in the firm-level production and adjustment costs functions imply that the estimated share of capital and adjustment cost parameters do not represent the true firm-level parameters. Intuitively, the aggregate production, for example, depends on the cross sectional distribution of capital. By using artificial data simulated from known firm-level parameters, Belo *et al.* (2019) show that the parameter estimates obtained from the aggregation procedure in Liu *et al.* (2009) and Belo *et al.* (2013) are subject to an aggregation bias, and hence do not have a structural interpretation. Thus, portfolio-level and aggregate investment returns implied by the aggregation approach can potentially be different from a value-weighted investment returns of individual firms. Existing studies of the sources of fluctuations in aggregate investment, for example Abel and Blanchard (1986) abstract from the issue of aggregation bias. To treat aggregation bias, we follow Gonçalves *et al.* (2020) and Belo *et al.* (2019) in the benchmark estimation, where firm-level accounting variables are used to construct firm-level levered investment returns, which are then aggregated to the portfolio level to match with portfolio stock returns.

Firm-level accounting variables and, thus, firm-level investment returns are subject to the issue of outliers. The outliers in firm-level investment returns can contaminate the aggregate portfolio-level investment returns and lead to noisy parameter estimates from the GMM estimation. To alleviate the impact of outliers in the firm-level GMM estimation, Gonçalves *et al.* (2020) construct firm-level investment returns using winsorized firm-level accounting variables, then compute equal- or value-weighted portfolio levered investment returns to match with equal- or value-weighted portfolio stock returns. Instead of winsorization, Belo *et al.* (2019) use the portfolio median to aggregate firm-level investment returns to portfolio level since the median is robust to outliers. In particular, for a given portfolio of firms, they compute the portfolio median of firm-level levered investment returns to match with the portfolio median of stock returns. Using artificial data simulated from known firm-level parameters, they show that matching the portfolio median in the

GMM estimation can recover the true firm-level parameters without bias. For robustness, we try both the winsorization approach and the median approach in our GMM estimations. In the winsorization approach, we winsorize firm-level accounting variables at the 1–99% level, then construct value-weighted portfolio levered investment returns to match with value-weighted portfolio stock returns.[28] In the median approach, we do not winsorize data but use portfolio median to aggregate both firm-level investment returns and stock returns to the portfolio level.

For the valuation moment, we need to aggregate firm-level Q to the portfolio level, which is similar to the task of aggregating firm-level investment returns to the portfolio level. Again, aggregation bias may be introduced if one aggregates firm-level accounting data to portfolio-level variables, from which portfolio Q is constructed and matched with portfolio Q in the data. To avoid aggregation bias, we construct firm-level Q using firm-level accounting variables, then aggregate firm-level Q to portfolio level. That is, the portfolio's Q is a weighted average of the individual firms' Q values. When aggregating firm-level Q to portfolio level, we use each firm's total assets as weight and calculate weighted average portfolio Q both in the model and in the data. This aggregation approach is equivalent to a valuation moment that matches the total portfolio firm value in the data with the total portfolio firm value predicted by the model.[29]

For robustness, we also conduct the GMM estimation following Liu *et al.* (2009), where firm-level accounting variables are aggregated to portfolio-level variables, from which portfolio-level levered investment returns are constructed to match with portfolio-level stock returns, either equal- or value-weighted. For the valuation moment, we follow Belo *et al.* (2013) and construct both the portfolio Q in the data ($\widetilde{Q}$) and the portfolio Q predicted by the model using portfolio-level accounting variables.

[28] Equal-weighted portfolio mean are still sensitive to outliers even after winsorizing firm-level data at 1-99% level, thus the results are not tabulated.

[29] At firm level, the market value of firm i satisfies: $P_{i,t}+B_{i,t+1} = \left(1+(1-\tau_t)a\left(\frac{I_{i,t}}{K_{i,t}}\right)\right)K_{i,t+1}$, thus $\frac{P_{i,t}+B_{i,t+1}}{A_{i,t+1}} = \left(1+(1-\tau_t)a\left(\frac{I_{i,t}}{K_{i,t}}\right)\right)\frac{K_{i,t+1}}{A_{i,t+1}}$. Aggregating individual firms in portfolio j using weights $\frac{A_{i,t+1}}{A_{j,t+1}}$, where $A_{j,t+1} = \sum_{i\in j} A_{i,t+1}$, we get $\sum_{i\in j}\left(\frac{A_{i,t+1}}{A_{j,t+1}}\frac{P_{i,t}+B_{i,t+1}}{A_{i,t+1}}\right) = \sum_{i\in j}\left[\left(1+(1-\tau_t)a\left(\frac{I_{i,t}}{K_{i,t}}\right)\right)\frac{K_{i,t+1}}{A_{i,t+1}}\frac{A_{i,t+1}}{A_{j,t+1}}\right]$. It can be simplified as $\sum_{i\in j}(P_{i,t}+B_{i,t+1}) = \sum_{i\in j}\left[\left(1+(1-\tau_t)a\left(\frac{I_{i,t}}{K_{i,t}}\right)\right)K_{i,t+1}\right]$.

A.2 Variance Decompositions

In this section, we provide details on the derivations of the VAR-based variance decompositions for q.

A.2.1 Restricted VAR(1)

By multiplying both sides of the present-value relation for q by $q_t - \mathrm{E}(q_t)$, and taking unconditional expectations, we obtain the following variance decomposition for q_t,

$$\mathrm{var}(q_t) \approx (1-\rho)\,\mathrm{Cov}\left(\sum_{h=1}^{H} \rho^{h-1} m_{t+h}, q_t\right) - \mathrm{Cov}\left(\sum_{h=1}^{H} \rho^{h-1} r_{t+h}, q_t\right) + \mathrm{Cov}\left(\rho^H q_{t+H}, q_t\right). \quad \text{(A.3)}$$

By dividing both sides by $\mathrm{var}(q_t)$, we have,

$$1 \approx \beta\left[(1-\rho)\sum_{h=1}^{H} \rho^{h-1} m_{t+h}, q_t\right] - \beta\left(\sum_{h=1}^{H} \rho^{h-1} r_{t+h}, q_t\right) + \beta\left(\rho^H q_{t+H}, q_t\right), \quad \text{(A.4)}$$

where $\beta(y, x)$ denotes the slope from a regression of y on x. This represents the variance decomposition for q based on the direct approach.

By using the property of regression coefficients, $\beta(y + z, x) = \beta(y, x) + \beta(z, x)$, we have:

$$1 \approx (1-\rho)\sum_{h=1}^{H} \rho^{h-1} \beta(m_{t+h}, q_t) - \sum_{h=1}^{H} \rho^{h-1} \beta(r_{t+h}, q_t) + \rho^H \beta(q_{t+H}, q_t). \quad \text{(A.5)}$$

Under the restricted first-order VAR, we have,

$$q_{t+h-1} = \phi^{h-1} q_t + \phi^{h-1} \sum_{l=1}^{h-1} \phi^{-l}(\pi_q + \varepsilon^q_{t+l}), \quad \text{(A.6)}$$

and by combining with the VAR equation for the currency return,

$$r_{t+h} = \pi_r + \lambda_r q_{t+h-1} + \varepsilon^r_{t+h}, \quad \text{(A.7)}$$

implies the following equation for r_{t+h}:

$$r_{t+h} = \pi_r + \phi^{h-1}\lambda_r q_t + \phi^{h-1}\lambda_r \sum_{l=1}^{h-1} \phi^{-l}(\pi_q + \varepsilon^q_{t+l}) + \varepsilon^r_{t+h}. \tag{A.8}$$

Since $\text{Cov}(\varepsilon^q_{t+l}, q_t) = 0, l > 0$ and $\text{Cov}(\varepsilon^r_{t+h}, q_t) = 0$, by construction, it follows that

$$\beta(r_{t+h}, q_t) = \phi^{h-1}\lambda_r. \tag{A.9}$$

Similarly, we have,

$$\beta(m_{t+h}, q_t) = \phi^{h-1}\lambda_m. \tag{A.10}$$

On the other hand, given the expanded expression for q_{t+H},

$$q_{t+H} = \phi^H q_t + \phi^H \sum_{l=1}^{H} \phi^{-l}(\pi_q + \varepsilon^q_{t+l}), \tag{A.11}$$

we have

$$\beta(q_{t+H}, q_t) = \phi^H, \tag{A.12}$$

which leads to

$$1 \approx (1-\rho)\sum_{h=1}^{H} \rho^{h-1}\phi^{h-1}\lambda_m - \sum_{h=1}^{H} \rho^{h-1}\phi^{h-1}\lambda_r + \rho^H\phi^H. \tag{A.13}$$

By simplifying the sums above, we obtain the VAR-based variance decomposition associated with q:

$$\begin{aligned} 1 &\approx b^H_m - b^H_r + b^H_q, \\ b^H_m &\equiv (1-\rho)\frac{\lambda_m(1-\rho^H\phi^H)}{1-\rho\phi}, \\ b^H_r &\equiv \frac{\lambda_r(1-\rho^H\phi^H)}{1-\rho\phi}, \\ b^H_q &\equiv \rho^H\phi^H. \end{aligned} \tag{A.14}$$

To compute the t-statistics for the predictive coefficients, $\mathbf{b}^H \equiv \left(b_m^H, b_r^H, b_q^H\right)'$, we use the delta method. From the standard errors associated with the VAR slopes, $\mathbf{b} \equiv (\lambda_m, \lambda_r, \phi)'$, we have:

$$\mathrm{var}\left(\mathbf{b}^H\right) = \frac{\partial \mathbf{b}^H}{\partial \mathbf{b}'} \mathrm{var}\left(\mathbf{b}\right) \frac{\partial \mathbf{b}^H}{\partial \mathbf{b}}. \tag{A.15}$$

The matrix of derivatives is given by

$$\frac{\partial \mathbf{b}^H}{\partial \mathbf{b}'} \equiv \begin{bmatrix} \frac{\partial b_m^H}{\partial \lambda_m} & \frac{\partial b_m^H}{\partial \lambda_r} & \frac{\partial b_m^H}{\partial \phi} \\ \frac{\partial b_r^H}{\partial \lambda_m} & \frac{\partial b_r^H}{\partial \lambda_r} & \frac{\partial b_r^H}{\partial \phi} \\ \frac{\partial b_q^H}{\partial \lambda_m} & \frac{\partial b_q^H}{\partial \lambda_r} & \frac{\partial b_q^H}{\partial \phi} \end{bmatrix} = \begin{bmatrix} (1-\rho)\frac{1-\rho^H\phi^H}{1-\rho\phi} & 0 & \frac{-H\lambda_m(1-\rho)\rho^H\phi^{H-1}(1-\rho\phi)+\rho\lambda_m(1-\rho)(1-\rho^H\phi^H)}{(1-\rho\phi)^2} \\ 0 & \frac{1-\rho^H\phi^H}{1-\rho\phi} & \frac{-H\lambda_r\rho^H\phi^{H-1}(1-\rho\phi)+\rho\lambda_r(1-\rho^H\phi^H)}{(1-\rho\phi)^2} \\ 0 & 0 & H\rho^H\phi^{H-1} \end{bmatrix}. \tag{A.16}$$

A.2.2 Unrestricted VAR(1)

After recursive substitution, the vector of state variables at $t+h$ can be written as,

$$\mathbf{z}_{t+h} = (\mathbf{I} + \mathbf{A} + ... + \mathbf{A}^{h-1})\boldsymbol{\pi} + \mathbf{A}^h \mathbf{z}_t + \mathbf{A}^{h-1}\boldsymbol{\varepsilon}_{t+1} + ... + \mathbf{A}\boldsymbol{\varepsilon}_{t+h-1} + \boldsymbol{\varepsilon}_{t+h}, \tag{A.17}$$

or equivalently,

$$\mathbf{z}_{t+h} = \mathbf{A}^h \mathbf{z}_t + \mathbf{A}^h \sum_{l=1}^{h} \mathbf{A}^{-l}(\boldsymbol{\pi} + \boldsymbol{\varepsilon}_{t+l}). \tag{A.18}$$

This implies that the regression coefficient of r_{t+h} on q_t is given by

$$\beta(r_{t+h}, q_t) = \frac{\mathrm{Cov}(\mathbf{e}_r'\mathbf{z}_{t+h}, \mathbf{e}_q'\mathbf{z}_t)}{\mathrm{var}(\mathbf{e}_q'\mathbf{z}_t)} = \frac{\mathrm{Cov}(\mathbf{e}_r'\mathbf{A}^h\mathbf{z}_t, \mathbf{e}_q'\mathbf{z}_t)}{\mathrm{var}(\mathbf{e}_q'\mathbf{z}_t)} = \frac{\mathbf{e}_r'\mathbf{A}^h\boldsymbol{\Sigma}\mathbf{e}_q}{\mathbf{e}_q'\boldsymbol{\Sigma}\mathbf{e}_q}, \tag{A.19}$$

where we use the fact that $\mathrm{Cov}(\boldsymbol{\varepsilon}_{t+l}, \mathbf{z}_t) = \mathbf{0}$ for $l > 0$.

By using the result above, it follows that the H-period return slope is given by

$$\begin{aligned}
\sum_{h=1}^{H}\rho^{h-1}\beta(r_{t+h},q_t) &= \sum_{h=1}^{H}\frac{\mathbf{e}_r'\rho^{h-1}\mathbf{A}^h\mathbf{\Sigma}\mathbf{e}_q}{\mathbf{e}_q'\mathbf{\Sigma}\mathbf{e}_q} \\
&= \frac{\mathbf{e}_r'}{\mathbf{e}_q'\mathbf{\Sigma}\mathbf{e}_q}\left(\sum_{h=1}^{H}\rho^{h-1}\mathbf{A}^h\right)\mathbf{\Sigma}\mathbf{e}_q \\
&= \frac{\mathbf{e}_r'}{\mathbf{e}_q'\mathbf{\Sigma}\mathbf{e}_q\rho}\left(\sum_{h=1}^{H}\rho^{h}\mathbf{A}^h\right)\mathbf{\Sigma}\mathbf{e}_q \\
&= \frac{\mathbf{e}_r'(\rho\mathbf{A}-\rho^{H+1}\mathbf{A}^{H+1})(\mathbf{I}-\rho\mathbf{A})^{-1}\mathbf{\Sigma}\mathbf{e}_q}{\rho\mathbf{e}_q'\mathbf{\Sigma}\mathbf{e}_q} \\
&= \frac{\mathbf{e}_r'\mathbf{A}(\mathbf{I}-\rho^{H}\mathbf{A}^{H})(\mathbf{I}-\rho\mathbf{A})^{-1}\mathbf{\Sigma}\mathbf{e}_q}{\mathbf{e}_q'\mathbf{\Sigma}\mathbf{e}_q}. \qquad \text{(A.20)}
\end{aligned}$$

The H-period m slope is defined in an analogous way. The slope associated with future q at $t+H$ is derived as follows:

$$\beta(q_{t+H},q_t)=\frac{\text{Cov}(\mathbf{e}_q'\mathbf{z}_{t+H},\mathbf{e}_q'\mathbf{z}_t)}{\text{var}(\mathbf{e}_q'\mathbf{z}_t)}=\frac{\text{Cov}(\mathbf{e}_q'\mathbf{A}^H\mathbf{z}_t,\mathbf{e}_q'\mathbf{z}_t)}{\text{var}(\mathbf{e}_q'\mathbf{z}_t)}=\frac{\mathbf{e}_q'\mathbf{A}^H\mathbf{\Sigma}\mathbf{e}_q}{\mathbf{e}_q'\mathbf{\Sigma}\mathbf{e}_q}, \qquad \text{(A.21)}$$

which implies that

$$\rho^H\beta(q_{t+H},q_t)=\frac{\rho^H\mathbf{e}_q'\mathbf{A}^H\mathbf{\Sigma}\mathbf{e}_q}{\mathbf{e}_q'\mathbf{\Sigma}\mathbf{e}_q}. \qquad \text{(A.22)}$$

In the case of the unrestricted VAR(1), the t-ratios associated with the horizon-specific coefficients $\mathbf{b}^H \equiv \left(b_m^H, b_r^H, b_q^H\right)'$, are obtained by using the delta method,

$$\text{var}\left(\mathbf{b}^H\right)=\frac{\partial\mathbf{b}^H}{\partial\mathbf{b}'}\,\text{var}\left(\mathbf{b}\right)\frac{\partial\mathbf{b}^H}{\partial\mathbf{b}}, \qquad \text{(A.23)}$$

where $\mathbf{b}\equiv(\gamma_m,\theta_m,\lambda_m,\gamma_r,\theta_r,\lambda_r,\gamma_q,\theta_q,\phi)'$. The derivatives are obtained from numerical methods.[30]

[30] We use the statistical package *Gauss*.

A.3 Bootstrap Simulation

The bootstrap simulation associated with the (restricted VAR-based) decomposition for q consists of the following steps.

1. We estimate the first-order restricted VAR,

$$
\begin{aligned}
r_{t+1} &= \pi_r + \lambda_r q_t + \varepsilon^r_{t+1}, \\
m_{t+1} &= \pi_m + \lambda_m q_t + \varepsilon^m_{t+1}, \\
q_{t+1} &= \pi_q + \phi q_t + \varepsilon^q_{t+1},
\end{aligned}
$$

   and save the time-series of residuals (ε^r_{t+1}, ε^m_{t+1}, and ε^q_{t+1}), as well as the estimates of ϕ and ρ.

2. In each replication ($s = 1, ..., 10,000$), we construct pseudo VAR innovations by drawing with replacement from the original VAR residuals:

$$(\varepsilon^{r,s}_{t+1}, \varepsilon^{m,s}_{t+1}, \varepsilon^{q,s}_{t+1})', t = v^s_1, ..., v^s_T,$$

   where the time indices $v^s_1, ..., v^s_T$—which are common for all the three VAR innovations—are created randomly from the original time sequence $1, ..., T$.

3. For each replication, we construct pseudo-samples by imposing the data generating process for r (no-return predictability null),

$$r_{s,t+1} = \rho\varepsilon^{q,s}_{t+1} + (1-\rho)\varepsilon^{m,s}_{t+1},$$

   for m (no-profit predictability null),

$$m_{s,t+1} = \varepsilon^{m,s}_{t+1},$$

and for q:

$$q_{s,t+1} = \phi q_{s,t} + \varepsilon_{t+1}^{q,s}.$$

The log Q for the base period (q_1) is picked at random from one of the observations of q_t.

4. In each replication, we use the artificial data to estimate the VAR (1),

$$\begin{aligned} r_{s,t+1} &= \pi_{r,s} + \lambda_{r,s} q_{s,t} + \upsilon_{t+1}^{r,s}, \\ m_{s,t+1} &= \pi_{m,s} + \lambda_{m,s} q_{s,t} + \upsilon_{t+1}^{m,s}, \end{aligned}$$

and estimate the implied long-horizon slopes,

$$\begin{aligned} b_{r,s}^H &\equiv \lambda_{r,s} \frac{1-\rho_s^H \phi_s^H}{1-\rho_s \phi_s}, \\ b_{m,s}^H &\equiv (1-\rho)\lambda_{m,s} \frac{1-\rho_s^H \phi_s^H}{1-\rho_s \phi_s}, \end{aligned}$$

where ρ_s is the estimate of ρ based on the artificial sample. In result, we have a distribution of the VAR implied slope estimates, $\left\{b_{r,s}^H, b_{m,s}^H\right\}_{s=1}^{10,000}$ for each forecasting horizon H.

5. The p-values associated with the implied VAR slope estimates are calculated as

$$\begin{aligned} p(b_r^H) &= \#\left\{b_{r,s}^H < b_r^H\right\}/10000, \\ p(b_m^H) &= \#\left\{b_{m,s}^H > b_m^H\right\}/10000, \end{aligned}$$

where $\#\left\{b_{m,s}^H > b_m^H\right\}$ denotes the number of simulated slope estimates that are higher than the original slope estimate.