What Does the Cross-Section Tell About Itself? Explaining Equity Risk Premia with Stock Return Moments

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Abstract

We derive a parsimonious equilibrium three-factor asset pricing model (cross-sectional CAPM, CS-CAPM) in which the realized cross-sectional second and third moments of long-short equity portfolio returns are the driving forces in terms of pricing cross-sectional equity risk premia. Stock market segmentation implies that these two (non-market) factors are priced in equilibrium. The three-factor model offers a large fit for the joint cross-sectional risk premia associated with 26 prominent CAPM anomalies, with explanatory ratios around or above 40%. The CS-CAPM compares favorably with multifactor models widely used in the asset pricing literature. The cross-sectional factors are not subsumed by traditional ICAPM risk factors.

Keywords: asset pricing, stock market anomalies, linear multifactor models, CAPM, cross-section of stock returns, realized return variance, realized return skewness, cross-sectional return moments, ICAPM

JEL classification: G10; G11; G12

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1 INTRODUCTION

In recent years, we have noticed a considerable increase in the number of CAPM or market anomalies, which correspond to patterns in cross-sectional equity risk premia left unexplained by the baseline CAPM of Sharpe (1964) and Lintner (1965). Specifically, earnings momentum stands for the evidence that stocks with high earnings surprises earn on average higher subsequent returns than stocks with low or negative earnings surprises (e.g., Foster, Olsen, and Shevlin 1984). The price momentum anomaly represents a pattern in which stocks that outperformed in the recent past continue to outperform in the near future, whereas stocks that have underperformed continue to perform poorly (Jegadeesh and Titman 1993). A related market anomaly is industry momentum, which refers to the evidence showing that stocks in past winning industries continue to outperform in the near future, while stocks in past losing industries continue to underperform (Moskowitz and Grinblatt 1999). Haugen and Baker (1996) and Hou, Xue, and Zhang (2015) show that more profitable stocks, those with higher return-on-equity (ROE), tend to offer higher average returns than less profitable stocks.[1] The asset growth or investment effect represents a cross-sectional pattern in which stocks of firms with low asset growth outperform stocks of firms with high asset growth (Cooper, Gulen, and Schill 2008; Hou, Xue, and Zhang 2015). The low-risk anomaly stems from the evidence that stocks with lower past volatility or market beta enjoy higher subsequent returns than very volatile or high beta stocks (Ang et al. 2006; Frazzini and Pedersen 2014).

All these patterns in the cross-section of average stock returns are not explained by the baseline CAPM and represent some of the most important challenges for existing asset pricing models. Specifically, the Fama-French three-factor model (FF3, Fama and French 1993), one of the traditional workhorses in the empirical asset pricing literature, is not able to explain these anomalies (see, for example, Fama and French 1996, 2015, Cochrane 2007, Maio 2013a, Hou, Xue, and Zhang 2015, among others). Perhaps more important, the recent five-factor model proposed by Fama and French (2015, 2016) (FF5), which adds an investment factor and a profitability factor to the three-factor model, is also unsuccessful in pricing portfolios associated with several of these anomalies (see Fama and French 2016, Maio and Philip 2018, Cooper and Maio 2019b, Cooper et al. 2021, among others).

[1] Novy-Marx (2013) and Fama and French (2015) use alternative measures of profitability in documenting a profitability premium in stock returns.

In this paper, we propose a new risk-based explanation for several key market anomalies. For this purpose, we derive a three-factor asset pricing model, denoted as cross-sectional CAPM (or CS-CAPM), which represents an extension of the baseline CAPM. In addition to the usual market factor, the other risk factors in the model are the realized cross-sectional second (RV) and third (RS) moments of long-short equity portfolio returns. In the underlying theoretical framework there is no single representative investor in the economy, and each investor holds an underdiversified equity portfolio that is tilted towards a segment of the stock market (by investing in a specific zero-cost portfolio strategy). By using a third-order Taylor approximation, it turns out that the average stochastic discount factor in such economy is a positive function of RV and a negative function of RS. Thus, high realizations of RV and low realizations of RS represent "bad" times on average; that is, periods with high marginal utility of consumption.

Consequently, in the expected return-beta (or covariance) representation of the model it follows that the risk prices associated with RV and RS are negative and positive, respectively. The intuition is that an asset positively correlated with RV will pay well when a greater number of portfolios (stocks) have very low returns (in result of higher return dispersion in the equity market). Thus, this asset provides a hedge against negative shocks in wealth, leading investors to require a lower risk premium to invest in it, compared to an asset that is uncorrelated with RV. In turn, this implies that the respective risk price is negative. Hence, since all investors hold undiversified portfolios that are associated with a segment of the stock market (a given equity factor), an increase in RV means possible negative shocks to their portfolios and thus to their wealth, which they want to hedge against. The intuition for the case of RS is similar: An asset that is positively correlated with RS pays well when a greater number of portfolios (stocks) have very high returns (in result of higher stock return skewness in the equity market). Thus, this asset does not provide a hedge against negative shocks in wealth, leading investors to require a higher risk premium to invest in it, compared to a stock that is uncorrelated with RS. In turn, this implies that the respective risk price is positive.

In the empirical estimation of the cross-sectional CAPM, we use (as testing assets) value-weighted portfolio deciles sorted on 26 equity characteristics, which are associated with important CAPM anomalies. These 26 anomalies are fairly representative of the broad cross-section of stock returns and display a large cross-sectional variation in risk premia. Following Hou, Xue, and Zhang

(2015, 2020), these portfolio groups can be generically classified into strategies related to trading frictions, momentum, investment, profitability, intangibles, and value-growth. The cross-sectional factors used in the CS-CAPM represent the cross-sectional second and third return moments of five long-short equity portfolios (or traded equity factors) that have been shown to describe the investment strategies of active portfolio managers in the stock market. These portfolios are related to the size, value-growth, beta, price momentum, and quality effects in average stock returns.

The results of the empirical asset pricing tests show that the cross-sectional CAPM offers a very good explanatory power for the cross-section of average stock returns. In the augmented test with the joint 26 anomalies (for a total of 156 decile portfolios), the OLS cross-sectional R^2 is 49% and such explanatory ratio is statistically greater than zero. This compares with a negative R^2 estimate in the case of the baseline CAPM, which shows that such model performs worse than a trivial model that predicts constant risk premia within the cross-section of the 156 portfolio returns. In the estimation with each major category of anomalies, the three-factor model explains more than 40% of the cross-sectional dispersion in portfolio risk premia among most categories. The estimates of the risk prices associated with RV are significantly negative in most cases, while the risk price estimates for RS are strongly significantly positive in all cases. Hence, these estimates are consistent with the qualitative (sign) restrictions of the theoretical model.

The performance of the three-factor model is robust to allowing for an unrestricted zero-beta rate. By using other equity portfolios in the construction of the cross-sectional factors, or by using the innovations in both RV and RS as factors, we obtain a similar performance of the three-factor model. By estimating nested two-factor models, we find that RS drives most of the performance of the three-factor model for the full cross-section of 26 anomalies. However, RV has also important pricing power for the momentum, profitability, and trading frictions broad categories.

We also find that CS-CAPM clearly dominates an alternative three-factor model containing stock-level cross-sectional return moments as factors when it comes to pricing a broad cross-section of equity risk premia. Indeed, the asset pricing implications of the two sets of measures of realized return second and third moments (based on portfolio and individual stock returns) seem quite different. In part, this stems from the fact that the cross-sectional variance of individual stock returns largely reflects idiosyncratic risk (see Garcia, Mantilla-Garcia, and Martellini 2014).

In what concerns the patterns in factors loadings, it turns out that value stocks and past momentum winners (both broadly defined) are more positively correlated with the third moment factor (RS) in comparison with growth stocks and past momentum losers, respectively. On the other hand, high investment-stocks tend to be less positively correlated with RS in comparison to low-investment stocks. Furthermore, more profitable stocks load less on the second-moment factor in comparison to less profitable stocks. These patterns in factor loadings also suggest that RS is relatively more important than RV for the overall pricing ability of the model. The reason is that the third-moment factor generates correct patterns in the factor loadings (i.e., with the correct sign) for more anomalies in comparison to RV.

The cross-sectional CAPM is compared with alternative multifactor models containing traded factors and that are widely used in the literature. The results show that the cross-sectional CAPM has a similar performance to the recently proposed four-factor model of Hou, Xue, and Zhang (2015) (HXZ4), five-factor model of Hou et al. (2019, 2020) (HMXZ5), six-factor model of Fama and French (2018) (FF6), and the four-factor model of Stambaugh and Yuan (2017). This is especially relevant when one takes into account the fact that the cross-sectional factors are not mechanically (linearly) related to the equity portfolios used in the asset pricing tests. In comparison, one should note that the performance of FF6 in terms of explaining the price and industry momentum portfolios is driven by the momentum (UMD) factor, which is (nearly) mechanically related to the momentum deciles. On the other hand, the performance of both HXZ4 and HMXZ5 in terms of explaining the return-on-equity portfolios is driven by their ROE factor.

We also compare the performance of the three-factor model with other multifactor "macro" models, in which some of the factors do not represent excess stock returns. The alternative models are mainly retrieved from the ICAPM literature, in which the factors (apart from the usual market factor) represent the innovations in state variables that forecast the changes in future investment opportunities (aggregate equity premium or stock market volatility). The cross-sectional CAPM dominates these alternative models both in economic and statistical terms. Critically, by estimating augmented multifactor models, we find that the cross-sectional factors are not subsumed by the ICAPM factors when it comes to explaining the 26 market anomalies.

In the last part of the paper, we examine the relationship between the loadings on both the RV and RS factors and individual stock returns. The results show that stocks that are more correlated with RV (RS) earn lower (higher) average returns than stocks that are less correlated with those same factors. This finding is exactly in line with the negative (positive) risk price estimates associated with RV (RS) in our main asset pricing tests.

To summarize, our results indicate that only two factors (the role of the market factor in our model is merely of picking the cross-sectional average risk premium) can explain an important fraction of the cross-sectional dispersion in risk premia among several prominent market anomalies (which are associated with all the major groups of CAPM anomalies, such as value-growth, investment, profitability, momentum, intangibles, and trading frictions). This is especially relevant as several of these anomalies are nearly uncorrelated or even negatively correlated, as are value-growth and momentum (see Asness, Moskowitz, and Pedersen 2013).

The rest of the paper is organized as follows. In Section 2, we derive the cross-sectional CAPM. Section 3 describes the variables and data, while Section 4 presents the main empirical results. In Section 5, we compare the performance of the three-factor model with alternative multifactor models widely used in the literature. Section 6 contains an analysis of the relationship between the loadings on the cross-sectional factors and individual stock returns. Finally, Section 7 concludes.

2 A THREE-FACTOR ASSET PRICING MODEL

In this section, we derive a three-factor asset pricing model, denoted by cross-sectional CAPM, which represents an extension of the standard CAPM of Sharpe (1964) and Lintner (1965). Additional details on the derivation are provided in the Internet Appendix.

A critical assumption in the model is that each agent specializes in a different market neutral investment strategy due to institutional constraints or information advantages. It is well known that asset managers, such as hedge funds, have different investment styles, which implies specializing in different types of stocks (e.g., Ackermann, McEnally, and Ravenscraft 1999). For example, Barberis and Shleifer (2003) write: "Most pension fund managers, as well as some mutual fund managers catering to the needs of individual investors, now identify themselves as following a particular investment style, such as growth, value, or technology." Mutual funds also tend to hold underdiversified portfolios (Didier, Rigobon, and Schmukler 2013).[2] This may arise as a

[2] On the other hand, there is strong evidence that U.S. individual investors hold underdiversified portfolios (Blume and Friend 1975; Goetzmann and Kumar 2008). Additionally, there is evidence that participants in 401(K) plans construct concentrated portfolios, often largely tilted toward the company's stock (e.g., Benartzi 2001; Agnew, Balduzzi, and Sundén 2003; Huberman and Sengmueller 2004).

consequence of different beliefs and/or information advantages related to one segment of the stock market. Betermier, Calvet, and Sodini (2017) show that over the life cycle, households progressively shift from growth to value as they become older and their balance sheets improve. Furthermore, investors with high human capital and high exposure to macroeconomic risk tilt their portfolios away from value. Some agents may also face liquidity constraints that prevent them from investing in small and/or illiquid stocks. Related to this, Frazzini and Pedersen (2014) claim that margin-constrained investors tilt their portfolios toward stocks with high market beta. On the other hand, Asness, Frazzini, and Pedersen (2019) show that the "quality" of a stock (measured in terms of profitability, growth, and safety) is a relevant stock characteristic, which is positively correlated with average returns. There is also widespread evidence that many active investors engage in so-called momentum strategies (e.g., Pedersen 2015). Calluzzo, Moneta, and Topaloglu (2019) show that there is a rise in anomaly-based trading (especially among hedge funds) following the public release of information about these cross-sectional patterns in stock returns.

Consider a financial market economy with L heterogeneous investors (hedge funds or other active funds) and $N-1$ risky assets. Each of the L investors can be interpreted as the "representative" investor among a group of homogeneous investors. Hence, each representative investor has idiosyncratic shocks in wealth due to the fact that he/she *mostly* invests (i.e., overweights) in a restricted segment of the stock market and does not hold a well diversified equity portfolio. This arises as a result of institutional or information constraints or advantages, as discussed above, which leads to a specialization in a given market neutral investment strategy. Agents also have private information sets and different beliefs. This implies that there is no global representative investor in this economy.

Each investor has power utility (with the same parameters) and solves the following dynamic portfolio choice problem,

$$\max_{\{C_{l,t+j}\}_{j=0}^{\infty},\{\omega_{i,t+j}^{l}\}_{j=0}^{\infty}} \mathrm{E}_{l,t}\left(\sum_{j=0}^{\infty}\delta^{j}\frac{C_{l,t+j}^{1-\gamma}}{1-\gamma}\right) \tag{1}$$

$$s.t. \begin{cases} W_{l,t+1}=R_{p,t+1}^{l}(W_{l,t}-C_{l,t}) \\ R_{p,t+1}^{l}=\sum_{i=1}^{N-1}\omega_{i,t}^{l}(R_{i,t+1}-R_{f,t+1})+R_{f,t+1} \end{cases}, \tag{2}$$

where $C_{l,t}$ is the level of real consumption for investor l; $W_{l,t}$ denotes the level of wealth for investor

l; $R^l_{p,t+1}$ is the gross return on investor l's reference portfolio between t and $t+1$; $\mathrm{E}_{l,t}$ is the expectation conditional on investor l's information set at time t; $R_{i,t+1}, (i = 1, ..., N-1)$ denotes the gross return on a risky asset; $R_{f,t+1}$ is the gross risk-free rate; $\omega^l_{i,t}$ is the weight for asset i in the investor l's portfolio; and δ is a time-subjective discount factor.

Each investor's reference portfolio is tilted toward a different segment of the stock market. More specifically, each portfolio assigns a weight of 100% to the market portfolio and also invests in a zero-cost strategy associated with a given segment of the equity market (e.g., value minus growth stocks):[3]

$$R^l_{p,t+1} = R_{m,t+1} + F_{l,t+1}, \tag{3}$$

where F_l is the return on the zero-cost portfolio held by investor l (factor l).[4] In equilibrium, the zero-cost portfolios cancel out at the aggregate level (zero net supply), that is, for each investor l, there is an investor $k \neq l$ who holds a zero-cost portfolio that is exactly symmetric to the zero-cost portfolio held by investor l, which implies that

$$F_{l,t+1} = -F_{k,t+1}. \tag{4}$$

Hence, the average portfolio across the L investors corresponds to the market portfolio, and the average portfolio return (across investors) equals the market return ($R_{m,t+1}$),

$$\frac{1}{L}\sum_{l=1}^{L} R^l_{p,t+1} = R_{m,t+1}. \tag{5}$$

As shown in the Internet Appendix, the SDF for investor l is given by

$$M_{l,t+1} = \delta \left(\frac{W_{l,t+1}}{W_{l,t}}\right)^{-\gamma}. \tag{6}$$

Following Brav, Constantinidies, and Geczy (2002) and Cogley (2002), the average stochastic

[3]To simplify the notation, we assume that the exposure to the zero-cost portfolio is one for all investors. However, this can be generalized to an arbitrary positive constant, $R^l_{p,t+1} = R_{m,t+1} + \vartheta F_{l,t+1}, \vartheta > 0$, given that the second component is a self-financing portfolio. Using $\vartheta = 1$ does not affect our main results. For some choices of ϑ and for some stock market compositions, such two-component portfolio (for a given investor) will be approximately equivalent to a simple portfolio that goes long in a subset of the stocks in the market (e.g., a portfolio of value stocks). The critical assumption here is specialized "style investing".

[4]Factor investing has become increasingly popular in the financial industry in recent years. See Ang (2014) for an introduction to the topic.

discount factor (SDF) in the economy is equal to

$$M_{t+1} = \frac{1}{L}\sum_{l=1}^{L} M_{l,t+1} = \frac{1}{L}\sum_{l=1}^{L}\delta\left(\frac{W_{l,t+1}}{W_{l,t}}\right)^{-\gamma} \equiv \frac{1}{L}\sum_{l=1}^{L}\delta WG_{l,t+1}^{-\gamma}, \tag{7}$$

where $WG_{l,t+1} \equiv \frac{W_{l,t+1}}{W_{l,t}}$ represents the gross growth rate in wealth for investor l.[5]

By using a third-order Taylor equation for $\delta WG_{l,t+1}^{-\gamma}$ and taking the average across the L investors, the average SDF in the economy can be rewritten as[6]

$$M_{t+1} = \frac{1}{L}\sum_{l=1}^{L}\delta WG_{l,t+1}^{-\gamma} \approx \delta WG_{t+1}^{-\gamma} + \frac{1}{2}\delta\gamma(\gamma+1)WG_{t+1}^{-\gamma-2}VW_{t+1} - \frac{1}{6}\delta\gamma(\gamma+1)(\gamma+2)WG_{t+1}^{-\gamma-3}SW_{t+1}, \tag{8}$$

where $WG_{t+1} \equiv \frac{1}{L}\sum_{l=1}^{L} WG_{l,t+1}$ represents the cross-sectional average of wealth growth; $VW_{t+1} \equiv \frac{1}{L}\sum_{l=1}^{L}(WG_{l,t+1} - WG_{t+1})^2$ denotes the cross-sectional variance of wealth growth; and $SW_{t+1} \equiv \frac{1}{L}\sum_{l=1}^{L}(WG_{l,t+1} - WG_{t+1})^3$ represents the cross-sectional skewness of wealth growth.

Given the intertemporal budget constraint in (2), it follows that the gross wealth growth for investor l is approximately equal to the gross return on his reference portfolio,[7]

$$WG_{l,t+1} \approx R_{p,t+1}^{l}, \tag{9}$$

leading to

$$WG_{t+1} \equiv \frac{1}{L}\sum_{l=1}^{L} WG_{l,t+1} \approx \frac{1}{L}\sum_{l=1}^{L} R_{p,t+1}^{l} = R_{m,t+1}. \tag{10}$$

Under these assumptions, the expression for VW becomes

$$VW_{t+1} \approx \frac{1}{L}\sum_{l=1}^{L}(R_{p,t+1}^{l} - R_{m,t+1})^2 = \frac{1}{L}\sum_{l=1}^{L} F_{l,t+1}^{2}, \tag{11}$$

which corresponds to the definition of portfolio return variance or portfolio second-moment (RV).

[5] The average SDF in the economy can price all assets because there are *approximate* segmented markets: each investor holds all stocks, although the portfolio weights associated with some stocks are very small in each portfolio (each investor has a portfolio that is largely concentrated in a given category of stocks).

[6] Brav, Constantinidies, and Geczy (2002), Cogley (2002), and Jacobs and Wang (2004) derive asset pricing models (based on heterogeneous investors) that rely on similar approximations of the SDF, which depends on individual consumption growth. Constantinides and Duffie (1996) propose a theoretical model in which the SDF depends on the cross-sectional distribution (across agents) of consumption growth.

[7] Since the investor has an infinite horizon, the level of consumption is much lower than the level of total wealth in each period, $C_{l,t} << W_{l,t}$. This implies that $W_{l,t+1}/(W_{l,t} - C_{l,t}) \approx W_{l,t+1}/W_{l,t}$.

Note that RV represents both the cross-sectional variance and second moment of portfolio returns since

$$\frac{1}{L}\sum_{l=1}^{L} F_{l,t+1} = 0. \tag{12}$$

Similarly, the expression for SW can be rewritten as

$$SW_{t+1} \approx \frac{1}{L}\sum_{l=1}^{L}(R^{l}_{p,t+1} - R_{m,t+1})^3 = \frac{1}{L}\sum_{l=1}^{L} F^3_{l,t+1}, \tag{13}$$

which corresponds to the definition of portfolio return skewness or portfolio return third moment (RS).

Therefore, the SDF can be rewritten as:

$$M_{t+1} \approx \delta R^{-\gamma}_{m,t+1} + \frac{1}{2}\delta\gamma(\gamma+1)R^{-\gamma-2}_{m,t+1}RV_{t+1} - \frac{1}{6}\delta\gamma(\gamma+1)(\gamma+2)R^{-\gamma-3}_{m,t+1}RS_{t+1}. \tag{14}$$

The SDF increases with RV,

$$\frac{\partial M_{t+1}}{\partial RV_{t+1}} \approx \frac{1}{2}\delta\gamma(\gamma+1)R^{-\gamma-2}_{m,t+1} > 0, \tag{15}$$

and declines with RS,

$$\frac{\partial M_{t+1}}{\partial RS_{t+1}} \approx -\frac{1}{6}\delta\gamma(\gamma+1)(\gamma+2)R^{-\gamma-3}_{m,t+1} < 0. \tag{16}$$

Thus, high realizations of RV and low realizations of RS represent "bad" times; that is, periods with high marginal utility of consumption.

Given the SDF in equation (14), and by applying the Stein's Lemma, it follows that the expected return-covariance equation can be represented approximately as

$$\mathrm{E}(R_{i,t+1} - R_{f,t+1}) \approx \gamma_M \operatorname{Cov}(R_{i,t+1}, R_{m,t+1}) + \gamma_{RV} \operatorname{Cov}(R_{i,t+1}, RV_{t+1}) + \gamma_{RS} \operatorname{Cov}(R_{i,t+1}, RS_{t+1}), \tag{17}$$

where the risk prices are equal to

$$\gamma_M \equiv \frac{\gamma\,\mathrm{E}\left(R^{-\gamma-1}_{m,t+1}\right) + \frac{1}{2}\gamma(\gamma+1)(\gamma+2)\,\mathrm{E}\left(R^{-\gamma-3}_{m,t+1}RV_{t+1}\right) - \frac{1}{6}\gamma(\gamma+1)(\gamma+2)(\gamma+3)\,\mathrm{E}\left(R^{-\gamma-4}_{m,t+1}RS_{t+1}\right)}{\mathrm{E}\left(R^{-\gamma}_{m,t+1}\right) + \frac{1}{2}\gamma(\gamma+1)\,\mathrm{E}\left(R^{-\gamma-2}_{m,t+1}RV_{t+1}\right) - \frac{1}{6}\gamma(\gamma+1)(\gamma+2)\,\mathrm{E}\left(R^{-\gamma-3}_{m,t+1}RS_{t+1}\right)}, \tag{18}$$

$$\gamma_{RV} \equiv -\frac{\frac{1}{2}\gamma(\gamma+1)\,\mathrm{E}\left(R_{m,t+1}^{-\gamma-2}\right)}{\mathrm{E}\left(R_{m,t+1}^{-\gamma}\right)+\frac{1}{2}\gamma(\gamma+1)\,\mathrm{E}\left(R_{m,t+1}^{-\gamma-2}RV_{t+1}\right)-\frac{1}{6}\gamma(\gamma+1)(\gamma+2)\,\mathrm{E}\left(R_{m,t+1}^{-\gamma-3}RS_{t+1}\right)}, \quad (19)$$

and

$$\gamma_{RS} \equiv \frac{\frac{1}{6}\gamma(\gamma+1)(\gamma+2)\,\mathrm{E}\left(R_{m,t+1}^{-\gamma-3}\right)}{\mathrm{E}\left(R_{m,t+1}^{-\gamma}\right)+\frac{1}{2}\gamma(\gamma+1)\,\mathrm{E}\left(R_{m,t+1}^{-\gamma-2}RV_{t+1}\right)-\frac{1}{6}\gamma(\gamma+1)(\gamma+2)\,\mathrm{E}\left(R_{m,t+1}^{-\gamma-3}RS_{t+1}\right)}. \quad (20)$$

By dividing and multiplying each covariance term in equation (17) by the variance of the respective factor, one can define the model in beta representation,

$$\mathrm{E}(R_{i,t+1}-R_{f,t+1}) \approx \lambda_M\beta_{i,M}+\lambda_{RV}\beta_{i,RV}+\lambda_{RS}\beta_{i,RS}, \quad (21)$$

where $\beta_{i,M}$, $\beta_{i,RV}$, and $\beta_{i,RS}$ denote the betas for asset i associated with the market, return variance, and return skewness factors, respectively. The corresponding prices of risk are given by $\lambda_M \equiv \gamma_M \operatorname{Var}(R_{m,t+1})$, $\lambda_{RV} \equiv \gamma_{RV} \operatorname{Var}(RV_{t+1})$, and $\lambda_{RS} \equiv \gamma_{RS} \operatorname{Var}(RS_{t+1})$.

This model is a generalization of the standard CAPM when there is cross-sectional dispersion in wealth across investors. If each investor holds the same portfolio (i.e., the market portfolio), then there is no cross-sectional dispersion and one obtains the standard CAPM as a special case:

$$\mathrm{E}(R_{i,t+1}-R_{f,t+1}) = \lambda_M\beta_{i,M}. \quad (22)$$

Turning to the signs of the factor risk prices above, it turns out that that the sign of the numerator dictates the sign of these expressions. This is because the denominator in each risk expression represents the mean SDF, which is positive by definition if we assume that there is a non-zero risk-free rate in the economy. Moreover, since the gross market return is always positive (due to limited liability of stocks), it follows that $\gamma_{RV} < 0$ and $\gamma_{RS} > 0$ (and the same holds for λ_{RV} and λ_{RS}).

On the other hand, for the market risk price to be positive, the following condition must hold:

$$\mathrm{E}\left(R_{m,t+1}^{-\gamma-1}\right)+\frac{1}{2}(\gamma+1)(\gamma+2)\,\mathrm{E}\left(R_{m,t+1}^{-\gamma-3}RV_{t+1}\right) > \frac{1}{6}(\gamma+1)(\gamma+2)(\gamma+3)\,\mathrm{E}\left(R_{m,t+1}^{-\gamma-4}RS_{t+1}\right). \quad (23)$$

The intuition for the sign of the risk prices is as follows. An asset that is positively correlated with the average stock in the economy, $\text{Cov}(R_{i,t+1}, R_{m,t+1}) > 0$, is not attractive since it pays well when the wealth of the average investor is also high. Thus risk-averse investors will require a higher risk premium to invest in such stock, compared to a stock that is less correlated (or nearly uncorrelated) with the market portfolio, implying that $\gamma_M > 0$. This represents the usual mechanism in the baseline CAPM.

Furthermore, an asset that is positively correlated with RV, $\text{Cov}(R_{i,t+1}, RV_{t+1}) > 0$, pays well when a greater number of portfolios (stocks) have very low returns (in result of higher stock return dispersion in the market). Thus, this asset provides a hedge against negative shocks in wealth, leading investors to require a lower risk premium to invest in it, compared to a stock that is uncorrelated with RV. In turn, this implies that the respective risk price is negative, given the assumption of a positive covariance with RV.[8] Since investors have undiversified portfolios, an increase in RV means possible negative shocks in their portfolios, and thus on their wealth, which they want to hedge against.

Finally, an asset that is positively correlated with RS, $\text{Cov}(R_{i,t+1}, RS_{t+1}) > 0$, pays well when a greater number of portfolios (stocks) have very high returns (in result of higher stock return skewness in the market). Thus, this asset does not provide a hedge against negative shocks in wealth, leading investors to require a higher risk premium to invest in it, compared to a stock that is uncorrelated with RS. This implies that the respective risk price is positive, given the assumption of a positive covariance with RS.

3 DATA AND VARIABLES

In this section, we describe the variables used in the empirical analysis conducted in the following sections.

3.1 Cross-Sectional Factors

The cross-sectional CAPM (denoted by CS-CAPM) is based on the second (RV) and third (RS) cross-sectional (uncentered) moments associated with the returns of five long-short equity portfolios

[8] The reasoning works in an analogous way if one assumes instead a negative covariance.

(or equity "factors"), F_j:

$$\begin{aligned}
\mathrm{E}(R_{i,t+1} - R_{f,t+1}) &= \lambda_M \beta_{i,M} + \lambda_{RV} \beta_{i,RV} + \lambda_{RS} \beta_{i,RS}, && (24)\\
RV_t &= \frac{1}{5} \sum_{j=1}^{5} F_{j,t}^2, && (25)\\
RS_t &= \frac{1}{5} \sum_{j=1}^{5} F_{j,t}^3. && (26)
\end{aligned}$$

The model is consistent with the theoretical model developed in the previous section in the sense that the non-market factors represent cross-sectional moments of the returns on long-short equity portfolios (rather than cross-sectional moments of individual stock returns). Hence, in line with the model, each fund invests into a specific segment of the stock market or a specific market-neutral investment strategy (e.g., value stocks versus growth stocks or past winners versus past losers), which is captured by a given equity portfolio return or factor. Moreover, to preserve consistency with the theoretical model, in which the cross-sectional mean of the long-short portfolio returns is zero, we use uncentered (rather than centered) cross-sectional moments.[9]

The choice of the long-short portfolios (included in the construction of the empirical cross-sectional factors defined above) is admittedly an ad-hoc one. However, based on the discussion in the previous section, it follows that zero-cost portfolios associated with the size, quality, value-growth, beta, and momentum effects in average stock returns represent natural candidates to be used in the construction of the empirical proxies for the cross-sectional return moments.[10] Accordingly, in the construction of both RV and RS, we use the size (SMB^{**}) factor of Stambaugh and Yuan (2017), the momentum factor (UMD) of Carhart (1997), the quality-minus-junk factor (QMJ) of Asness, Frazzini, and Pedersen (2019), the "low-beta" return spread (BAB^{*}) employed in Liu, Stambaugh, and Yuan (2018) and Barroso, Detzel, and Maio (2020), and the value factor (HML^{*}) of Asness and Frazzini (2013).[11] Each of these factors represents an excess return, that is, a zero-cost portfolio

[9]Several studies measure realized skewness as the third moment of returns standardized by the realized return variance (e.g., Albuquerque 2012; Amaya et al. 2015). We use the baseline third moment in order to be consistent with the theoretical model derived in the previous section.

[10]Usually, the long-short portfolios are denoted as "equity factors" as they are included (as risk factors) in popular multifactor asset pricing models (such as those covered in Section 5). However, what concerns us here is that these factors represent long-short equity portfolios that proxy for a given market-neutral investment style or strategy (e.g., price momentum) that is implemented by investors.

[11]Using HML^{*} (instead of the traditional value factor, HML) is more consistent with the actual value-growth strategies pursued by investors (see Asness and Frazzini 2013). Employing BAB^{*}, instead of the BAB factor proposed by Frazzini and Pedersen (2014), stems from the fact that the latter long-short portfolio relies on unusual conventions

that goes long in stocks with a given score of a characteristic (e.g., stocks with high book-to-market ratio) and shorts stocks with the opposite score of that same characteristic (low book-to-market ratio).[12] The data on both UMD and the market factor (RM) are obtained from Kenneth French's data library. The data on both HML^* and QMJ are retrieved from AQR's data library. The time-series of SMB^{**} is obtained from Robert Stambaugh's web page. The data on BAB^* was provided by Andrew Detzel. The sample covers the period from 1973:01 to 2016:12.

Figure 1 plots the time-series of the cross-sectional factors, RV and RS. There are two important spikes in the time-series of RV—around 2000 and 2009. The first event is associated with the 2000 NASDAQ bear market following the bubble in technology-related stocks that took place in the 1990s. The second episode of high stock return dispersion is associated with the recent equity bear market related to the financial crisis that began in late 2007. On the other hand, RS is also especially volatile around these two key events. Additionally, both series exhibit peaks in volatility around the late 1970s.

Figure 1 also suggests that RV is a countercyclical variable, as some of the most pronounced spikes in this variable occur around recession periods. By running a simple time-series regression of RV on the National Bureau of Economic Research (NBER) business cycle dummy (which takes the value of one in an economic expansion and the value of zero in recessions), we get the following results (GMM-based t-ratios in parentheses):

$$\begin{aligned} RV_t &= 0.003 - 0.001 NBER_t, R^2 = 0.02, \\ & \quad (3.96)(-1.96). \end{aligned}$$

These results confirm that RV tends to rise around recessions, which is in line with previous theoretical and empirical evidence for other measures of stock return dispersion (e.g., Gomes, Kogan, and Zhang 2003; Stivers 2003). On the other hand, Figure 1 suggests that RS is not substantially correlated with the business cycle.[13]

in its construction, such as using rank-weighted (instead of value-weighted) portfolio returns (see Novy-Marx and Velikov 2016, 2019 for details). On the other hand, Stambaugh and Yuan (2017) claim that their size factor is less subject to mispricing bias in comparison to the traditional size factor (SMB) of Fama and French (1993).

[12] Loungani, Rush, and Tave (1990), Stivers and Sun (2010), and Maio (2016) represent other studies that employ measures of stock return variance based on equity portfolios. In related work, Bessembinder, Chan, and Seguin (1996) and Morck, Yeung, and Yu (2000) use alternative proxies of return dispersion.

[13] A regression of RS on the NBER dummy yields a positive, but largely insignificant, slope estimate.

Table 1 presents the descriptive statistics for the cross-sectional factors as well as the basis factors used in their construction. RV has an autocorrelation coefficient of 0.16, which makes this factor only marginally more persistent than the excess market return. On the other hand, RS is not serially correlated, as indicated by the slightly negative autoregressive slope (-0.02). Hence, the third-moment factor tends to revert to the mean, as suggested by the graph in Figure 1. The mean of RV is 0.16% per month, while RS has a mean around zero. The corresponding volatilities are 0.38% and 0.07%, respectively.

The pairwise correlations between the cross-sectional factors and the equity factors (last two columns in Table 1) show that the market factor is basically uncorrelated with the cross-sectional factors, as indicated by the correlations around zero. It turns out that RV and RS are negatively correlated (correlation of -0.70). On the other hand, the correlations between both cross-sectional factors and the basis factors are relatively small (below 0.30 in magnitude) in most cases. The most relevant estimate is the positive correlation between the beta factor and RS (0.59). On the other hand, BAB^* shows a weak negative correlation with RV (-0.33), whereas RS is weakly positively correlated with the momentum factor (0.31). We can also see that the correlation between both RV and RS and the remaining equity factors have rather small magnitudes in most cases. Among the exceptions, we observe a weak negative (positive) correlation between RV (RS) and the profitability (ROE) factor of Hou, Xue, and Zhang (2015). Therefore, to a very large degree, the cross-sectional factors proxy for different dimensions of systematic risk in comparison to the traditional equity factors existing in the literature.

3.2 Testing Portfolios

In the asset pricing tests conducted in the following sections, we use single-sorted portfolios corresponding to 26 alternative market or CAPM anomalies. These anomalies represent patterns in the cross-section of average stock returns that are not explained by the baseline CAPM. These 26 anomalies are fairly representative of the broad cross-section of stock returns and display a large cross-sectional variation in risk premia (e.g., Hou, Xue, and Zhang 2015; Fama and French 2016). The description of the full list of anomalies is presented in the Internet Appendix. Following Hou, Xue, and Zhang (2015, 2020), these anomalies can be generically classified in strategies related to six broad categories: Investment, trading frictions, value-growth, momentum, profitability, and intangibles. Each of these categories includes four CAPM anomalies, with the exception of the more heterogeneous "intangibles" group, which includes a total of six anomalies, as detailed in the Internet Appendix.

For all anomalies, we form value-weighted decile portfolios with NYSE breakpoints and rebalance these portfolios monthly. For most of the anomalies, we follow the same procedure of portfolio construction used in Green, Hand, and Zhang (2017). The exception applies to four anomalies that use quarterly earnings/sales information—earnings announcement return (*ear*); return on assets (*roaq*); return on equity (*roeq*); and revenue surprise (*rsup*). For these anomalies, we use earnings/sales data from Compustat quarterly files in the months immediately after the most recent public earnings announcement dates (Compustat item RDQ) when forming portfolio sorts of stocks. Furthermore, for a firm to be included in the portfolio sorts, we require the end of the fiscal quarter (corresponding to the most recently announced earnings/sales) to be within six months prior to the portfolio formation, to exclude stale earnings/sales information. This procedure is consistent with Hou, Xue, and Zhang (2015, 2020). To construct portfolio excess returns, we subtract the one-month Treasury bill rate, available from Kenneth French's website.

Table 2 presents the descriptive statistics for high-minus-low return spreads between the last and first deciles among each of the 26 portfolio classes. The anomaly with the largest spread in average returns is 12-month momentum (*mom*), with a premium above 1% per month. The return spreads associated with asset growth (*agr*), *ear*, earnings-to-price ratio (*ep*), capital expenditures and inventory (*invest*), *roaq*, *roeq*, and sales-to-price ratio (*sp*) are also quite pervasive in economic terms, with (absolute) means above 0.50% per month. The anomalies with lower average returns (in magnitude) are market beta (*beta*), beta squared (*betasq*), industry-adjusted growth in employees (*chempia*), organizational capital (*orgcap*), % change in the quick ratio (*pchquick*), and tax income-to-book income (*tb*), all with average return spreads below 0.20% (in absolute value). However, most of the high-minus-low return spreads produce statistically significant (at the 10% level) CAPM risk-adjusted returns (alphas) by using single-sided p-values.[14] This does not hold for four anomalies: *chempia*, debt-to-equity ratio (*lev*), *pchquick*, and *tb*. However, untabulated results show that the return spreads involving the three (rather than one) extreme portfolios on each leg produce

[14]Employing single-sided p-values is justified by the fact that the "signs" of the anomalies (i.e., the sign of the correlation between stock returns and a given equity characteristic) is well established in the literature.

significant CAPM alphas (at the 10% level) for each of those four anomalies.

beta and *betasq* are the anomalies with the most volatile return spreads (standard deviations above 8% per month), followed by *mom* and stock return volatility (*retvol*), with volatilities above 7%. At the other end of the spectrum, it turns out that *chempia*, *pchquick*, % change in sales minus % change in inventory (*pchsale_pchinvt*), and *tb* have volatilities of return spreads below 3% per month. As usually with stock or portfolio returns, the return spreads are typically not persistent over time, as indicated by the first-order autoregressive slopes below 0.15 in the majority of the cases. The most persistent return spreads are those associated with operating profitability (*operprof*), *roaq*, *roeq*, and *sp*, all with autoregressive coefficients above 0.15, which are still quite modest.

4 MAIN RESULTS

In this section, we present the main results for the cross-sectional asset pricing tests of our three-factor model.

4.1 Econometric Methodology

To test the three-factor model, we use the two-pass regression approach employed in the empirical asset pricing literature (see Black, Jensen, and Scholes 1972, Jagannathan and Wang 1998, Cochrane 2005, Brennan, Wang, and Xia 2004, Cooper and Maio 2019a, among others). Specifically, in the first step, the factor betas are estimated from the time-series regressions for each testing portfolio,

$$R_{i,t} - R_{f,t} = \delta_i + \beta_{i,M} RM_t + \beta_{i,RV} RV_t + \beta_{i,RS} RS_t + \varepsilon_{i,t}, \tag{27}$$

and in the second step, the expected return-beta representation is estimated through an OLS cross-sectional regression,

$$\mu_i = \lambda_M \widehat{\beta}_{i,M} + \lambda_{RV} \widehat{\beta}_{i,RV} + \lambda_{RS} \widehat{\beta}_{i,RS} + \alpha_i, \tag{28}$$

where μ_i represents the average (time-series) excess return for asset i; α_i denotes the respective pricing error; and $\widehat{\beta}_i$s represent the factor loadings for asset i.[15]

One can test the null hypothesis that the pricing errors for the N testing assets are jointly equal to zero (that is, the model is perfectly specified) with the following Wald test,

$$\widehat{\boldsymbol{\alpha}}' \widehat{\mathrm{Var}} \left(\widehat{\boldsymbol{\alpha}} \right)^{\dagger} \widehat{\boldsymbol{\alpha}} \sim \chi^2 (N - K), \tag{29}$$

where K denotes the number of risk factors in the model ($K = 3$ in the cross-sectional CAPM); $\widehat{\boldsymbol{\alpha}}$ is the $(N \times 1)$ vector of estimated pricing errors; and $\dagger$ denotes a pseudo inverse.[16] The t-statistics associated with the factor risk price estimates, as well as the specification test above, are based on Shanken's standard errors (Shanken 1992). These standard errors account for the "error-in-variables" bias in the cross-sectional regression that arises from estimation error in the factor loadings (see Cochrane 2005, chapter 12 for details).[17]

As an alternative to the asymptotic standard errors, we conduct a bootstrap simulation to produce more robust p-values for the tests of individual significance of the factor risk prices. This bootstrap simulation consists of 10,000 replications in which the excess portfolio returns and factor realizations are simulated (with replacement from the original sample) independently and without imposing the model's restrictions. Hence, the artificial data are simulated under the assumption that the factors are independent from the testing assets ("useless factors" as in Kan and Zhang 1999). The full details of the bootstrap simulation algorithm are available in the Internet Appendix.[18]

In comparison to the Wald-statistic, a simpler and more robust measure of the fit of a given model

[15] We do not include an intercept in the cross-sectional regression, which implies that an asset that has zero betas against all factors should earn a zero risk premium (relative to the risk-free rate). Several studies follow this practice (e.g., Brennan, Wang, and Xia 2004, Campbell and Vuolteenaho 2004, Cochrane 2005, Jagannathan and Wang 2007, and Kan, Robotti, and Shanken 2013 (see their Section B.4)). Excluding the intercept also enables to avoid the multicollinearity problem (between the intercept and some of the factor betas) arising from small cross-sectional variation in those betas. This often leads to economically implausible factor risk price estimates (Jagannathan and Wang 2007). Moreover, the focus in this paper (as well as in most of the related literature) is in explaining the cross-section of equity risk premia rather than in fitting the risk-free rate, which assumes secondary relevance. We show below that this empirical choice has no effect on the fit of our model.

[16] From now onwards, N denotes the number of testing assets.

[17] As a robustness check, we compute the t-ratios employed in Fama and MacBeth (1973), Jagannathan and Wang (1998), and Kan, Robotti, and Shanken (2013). The corresponding statistical significance of the risk price estimates in our model is qualitatively similar to that based on the Shanken's t-ratios.

[18] Other studies that employ the bootstrap method in order to obtain "robust" standard errors for the risk price estimates include Campbell and Vuolteenaho (2004), Maio and Santa-Clara (2017), and Guo and Maio (2020).

for the cross-section of average stock returns is the cross-sectional OLS coefficient of determination,

$$R^2_{OLS} = 1 - \frac{\mathrm{Var}_N(\widehat{\alpha}_i)}{\mathrm{Var}_N(\mu_i)}, \tag{30}$$

where $\mathrm{Var}_N(\cdot)$ stands for the cross-sectional variance. R^2_{OLS} represents the fraction of the cross-sectional variation in the average excess returns (on the testing assets) that is explained by the factor loadings associated with a given model.[19] Since an intercept is not included in the cross-sectional regression, this explanatory ratio can assume negative values. A negative estimate indicates that the cross-sectional regression (including the betas) does worse than a trivial regression containing only the intercept, that is, the factor betas underperform the cross-sectional average risk premium in terms of explaining cross-sectional dispersion in average excess returns.

Following Lewellen, Nagel, and Shanken (2010), Maio and Santa-Clara (2017), and Guo and Maio (2020), to address the statistical uncertainty associated with the sample cross-sectional R^2 estimates, we compute empirical p-values based on the bootstrap simulation described above. The empirical p-value represents the fraction of artificial samples in which the pseudo explanatory ratio is higher than the corresponding sample estimate. By computing the empirical p-values we account for the sampling error associated with the sample R^2_{OLS} estimates. More specifically, under the assumption of independence between portfolio excess returns and risk factors, the empirical p-value indicates how likely it is that we obtain the fit observed in the data.

4.2 Testing the Three-Factor Model

Since the three-factor model nests the standard CAPM, we first present results for the single-factor model in terms of pricing the 26 market anomalies. For each anomaly, only the extreme three portfolios on each leg are considered (i.e., six portfolios for each portfolio group) in the estimation. The reason for this choice hinges on the fact that most of the cross-sectional dispersion in portfolio risk premia is concentrated on the extreme deciles in each portfolio group. Hence, excluding the middle deciles can produce more powerful and sharp asset pricing tests. Following Hou et al. (2020), the asset pricing tests are conducted for each of the six major anomaly categories described in the previous section—investment, trading frictions, value-growth, momentum, profitability, and

[19] Jagannathan and Wang (1996), Lettau and Ludvigson (2001), Campbell and Vuolteenaho (2004), Kan, Robotti, and Shanken (2013), and Lioui and Maio (2014) use similar R^2 metrics in cross-sectional asset pricing tests.

intangibles.

The results in Table 3 (Panel A) confirm previous evidence showing that the CAPM cannot price each of those six major categories. Indeed the estimates for the OLS coefficient of determination are negative in all cases, which means that the model has less explanatory power than a trivial model that predicts constant expected excess returns within the cross-section of risk premia in each major class of anomalies. In the augmented, and more challenging, asset pricing test including all six categories (for a total of 156 portfolios) the explanatory ratio is -69%, which confirms a rather negative performance. Moreover, the CAPM fails to pass (at the 5% level) the formal specification test in most cases, as indicated by the p-values below 5%. The sole exception holds for the estimation with the profitability group of anomalies, in which case the model is not rejected at the 5% level (despite being rejected at the 10% level). However, such formal non-rejection has no value, as the model produces large pricing errors (as indicated by the negative R^2_{OLS} estimate).

The results for the three-factor model (CS-CAPM) are presented in Table 3, Panel B. We can see that the cross-sectional CAPM offers a relatively large explanatory power for each of the six broad categories, as indicated by the R^2_{OLS} estimates varying between 39% (estimation with the investment group of anomalies) and values around 70% (estimation with the momentum, profitability, and value-growth groups of anomalies). Clearly, the model's explanatory ratios in the cross-sectional tests associated with each of those six major groups are considerably higher than the negative estimates produced by the baseline CAPM discussed above. Critically, the explanatory ratios produced by the three-factor model are strongly statistically significant (1% level) in most cases. The exceptions occur in the estimation with the investment and trading frictions categories, in which cases the estimates of R^2_{OLS} are statistically significant at the 10% level. The three-factor model does not pass the formal specification test (at the 5% level) in the estimation with the investment and momentum groups, with p-values below 1%. This likely results from a miss-measurement of the inverse of the covariance matrix of the pricing errors given the relatively small magnitudes of the pricing errors. However, the model is not formally rejected (at the 5% level) in the estimation with the other four groups of anomalies. Thus, the large fit of the model (as indicated by the high R^2_{OLS} estimates) tends to be confirmed by a formal statistical validation of the model (as indicated by passing the Wald test).

The risk price estimates associated with RV are negative in all cases, varying between -0.01

(estimation with the value-growth group) and -0.28 (trading frictions group). These estimates are statistically significant (10% or better level) in the cross-sectional tests associated with the trading frictions, momentum, and profitability groups, based on both types of p-values. By considering single-sided p-values, it turns out that the estimate of λ_{RV} is also significant (at the 10% level) in the estimation with the intangibles category (based on the asymptotic inference).[20] On the other hand, the risk price estimates associated with RS are positive and strongly significant (1% or 5% level, based on both types of inference) in all six cross-sectional tests. Therefore, the risk price estimates associated with both RV and RS are generally consistent with the theoretical model presented in Section 2.

In the more challenging asset pricing test that includes simultaneously the six major categories (156 portfolios), the explanatory ratio produced by CS-CAPM is 49%, which indicates a large economic significance. Moreover, this estimate is strongly statistically significant (at the 1% level), as shown by the p-value around zero. The model does not pass the specification test, which is related to a poor inversion of the covariance matrix of the pricing errors in a relatively large cross-section (156 testing portfolios). Consistent with the results for the single-category tests, the risk price estimates associated with RV and RS are negative and positive, respectively. The estimate of λ_{RS} is strongly significant (1% level) according to both types of p-values. However, the estimate of λ_{RV} is significant only based on the asymptotic inference and using single-sided p-values (t-ratio of -1.32). The weaker significance of λ_{RV} stems from the inclusion of the four value-growth anomalies in the menu of testing assets. Indeed, untabulated results show that when we exclude those four anomalies from the augmented asset pricing estimation, the risk price estimate for RV is -0.09 and significant at the 5% level (t-ratio of -2.01). Finally, the risk price estimate for the market factor (0.51%) is quite close to the corresponding sample mean reported in Table 1 (0.53%).

Overall, the results of this subsection show that the three-factor model has a considerable explanatory power for the 26 market anomalies that cover a comprehensive cross-section of stock returns.

[20]Using single-sided p-values is validated by the fact that the signs of the factor risk prices in our model are restricted by theory.

4.3 Sensitivity Analysis

We conduct several robustness checks to the main results discussed above. Most of the results are associated with the augmented asset pricing test including all 26 anomalies. To save space and keep the focus, the results are tabulated in the Internet Appendix.

First, we estimate the cross-sectional CAPM by specifying an alternative second-pass OLS cross-sectional regression. Following Kan, Robotti, and Shanken (2013), we employ portfolio returns (instead of excess returns) and include an intercept in the cross-sectional regression,

$$\overline{R_i} = \lambda_0 + \lambda_M \widehat{\beta}_{i,M} + \lambda_{RV} \widehat{\beta}_{i,RV} + \lambda_{RS} \widehat{\beta}_{i,RS} + \alpha_i, \tag{31}$$

where $\overline{R_i}$ denotes the time-series mean of $R_{i,t}$ and λ_0 represents the zero-beta rate. The t-ratio associated with λ_0 tests the null hypothesis that the zero-beta rate in excess of the average risk-free rate (one-month T-bill rate) is equal to zero.

We estimate the regression above in the augmented asset pricing test with all 26 anomalies. The results show that the estimate of the excess zero-beta rate is clearly insignificant (t-ratio of 0.75). Moreover, the explanatory ratio is very close to the corresponding value in the benchmark estimation with no intercept (0.51 versus 0.49). This suggests that allowing for an unrestricted zero-beta rate does not have an effect on the model's performance. In other words, the model is able to match the zero-beta rate, and thus, there is no associated misspecification.[21] On the other hand, while the estimate for λ_{RS} is strongly significant, it turns out that the risk price estimates for both the market factor and RV are clearly insignificant (even by using single-sided p-values). This stems from the multicolinearity induced by the presence of the intercept in the cross-sectional regression (e.g., Jagannathan and Wang 2007), which legitimates our choice of employing a restricted zero-beta rate in our main empirical analysis.

Second, we estimate alternative versions of CS-CAPM, which are based on different definitions of the cross-sectional factors. First, we replace QMJ by the operating profitability factor (RMW) of Fama and French (2015) in the construction of both RV and RS. Second, we substitute HML^* by the traditional value factor (HML) of Fama and French (1993). Furthermore, instead of using

[21] In comparison, the baseline CAPM is clearly misspecified, as the excess zero-beta rate estimate is strongly significant (1% level).

SMB^{**}, we employ the size factors of Fama and French (2015) (SMB^*) or Hou, Xue, and Zhang (2015) (ME). The data on ME are obtained from Lu Zhang's data library. The data on SMB^*, HML, and RMW are retrieved from Kenneth French's library.

The results displayed in the Internet Appendix show that the performance of these alternative four versions of the cross-sectional CAPM is quite similar to that of the benchmark version, as indicated by the explanatory ratios on the 0.46-0.51 range. Moreover, the risk price estimates for the non-market factors have the usual signs in nearly all cases. The sole exception is the slightly positive estimate for λ_{RV} in the version of the model based on HML, yet such estimate is largely insignificant. Similarly to the benchmark model, the risk price estimates for RS are strongly significant (1% level) in all cases.

Third, we estimate another version of the three-factor model, which is also based on alternative definitions of the cross-sectional factors. Specifically, we employ the 26 long-short portfolios (or high-minus-low return spreads) analyzed in Table 2 above. This version of the cross-sectional factors is less parsimonious than that in the benchmark model in the sense that it is based on significantly more equity portfolios. Moreover, the theoretical justification for including these 26 portfolios is substantially weaker in comparison to the benchmark cross-sectional factors, as discussed in Section 2. It is also more likely that there is some relevant overlapping across long-short portfolios (e.g., momentum versus industry momentum return spreads).

The performance of the new CS-CAPM version is quite poor, as indicated by the negative explanatory ratio (-0.22). This implies that such model does not improve the baseline CAPM in terms of pricing the 26 anomalies. Further, the third-moment factor has a slightly negative risk price estimate (-0.01), which is significant at the 10% level. Such point estimate is clearly inconsistent with our theoretical model. Therefore, these results indicate that this alternative version of the three-factor model performs substantially worse than the benchmark model, both in economic and statistical terms. This legitimates our choice of the five basis equity factors (employed in the construction of both RV and RS in the benchmark CS-CAPM), which is based on simple investment intuition and empirical evidence, as discussed in Section 2.

Fourth, we estimate the following alternative specification of the three-factor model,

$$\mathrm{E}(R_{i,t+1} - R_{f,t+1}) = \lambda_M \beta_{i,M} + \lambda_{\Delta RV} \beta_{i,\Delta RV} + \lambda_{\Delta RS} \beta_{i,\Delta RS}, \tag{32}$$

where $\lambda_{\Delta RV}$ and $\lambda_{\Delta RS}$ represent the risk prices for the innovations in RV and RS, respectively, while $\beta_{i,\Delta RV}$ and $\beta_{i,\Delta RS}$ denote the respective factor loadings.[22] This specification follows the Intertemporal CAPM (ICAPM) literature, in which the "hedging" or non-market factors represent the innovations in state variables that drive future aggregate investment opportunities (Merton 1973).

Following the ICAPM literature, the hedging factors are obtained from an AR(1) process,

$$\Delta z_t \equiv \varpi_t = z_t - \psi - \phi z_{t-1}, \tag{33}$$

where $z \equiv RV, RS$.

The pricing performance of this alternative specification is quite close to that of the baseline model, as indicated by the explanatory ratio of 0.47. Moreover, the risk price estimates associated with ΔRV and ΔRS are -0.07 and 0.03, respectively, which are nearly the same as the estimates corresponding to the baseline factors (RV and RS). As in the benchmark model, the estimate of $\lambda_{\Delta RV}$ is significant at the 10% level (by using single-sided p-values) while the estimate of $\lambda_{\Delta RS}$ is strongly significant (1% level).

Fifth, we assess the performance of the three-factor model for two other important CAPM anomalies that are not included in our main analysis: Accruals (*acc*, Sloan 1996) and book-to-market ratio (*bm*, Rosenberg, Reid, and Lanstein 1985). We use the same procedures as Green, Hand, and Zhang (2017) in defining these two equity characteristics. For each anomaly, we form value-weighted decile portfolios with NYSE breakpoints and rebalance these portfolios monthly. We conduct an augmented asset pricing estimation that includes those two anomalies in addition to the 26 anomaly groups employed in the benchmark tests. We have a total of 168 testing portfolios in such augmented asset pricing test.

The model produces a marginally lower fit than in the benchmark case (0.47 versus 0.49). Moreover, the risk price estimate associated with RV, despite having the correct sign (-0.05), is largely insignificant (t-ratio of -1.15). This means that by adding the two additional CAPM anomalies, the performance of the model deteriorates in economic terms, as judged by the sign (and significance) of some of the risk price estimates. Therefore, these results suggest that, although

[22] We thank an anonymous referee for suggesting this analysis.

the CS-CAPM has a sounder theoretical background than alternative multifactor models (as those covered in the following section), it cannot be used as a workhorse factor model to price a very broad cross-section of risk premia. In particular, the model is less successful in pricing the *acc* and *bm* anomalies. These results also suggest that there are some relevant differences in risk premia among anomalies grouped in the same broad category (e.g., *agr* and *acc*), in line with the evidence provided in Hou, Xue, and Zhang (2015) and Cooper and Maio (2019b).

Sixth, we estimate two-factor models containing either RV or RS as the sole non-market factor. Both of these models are nested in our benchmark CS-CAPM. According to our theoretical framework, both non-market factors should be included in the SDF so that the key model for empirical tests is the CS-CAPM. Nonetheless, such analysis enables to quantify the empirical contribution of each of those two factors for the overall pricing ability of the three-factor model.

The results in the Internet Appendix show that the nested model based on RS produces an explanatory ratio of 41% when it comes to pricing all 26 anomalies, which is only slightly lower than the corresponding fit for the benchmark model (49%). This shows that RS is the main factor in terms of driving the pricing performance of CS-CAPM for the average anomaly in our cross-section. However, such result masks the important role played by RV in terms of explaining several segments of cross-sectional risk premia. In particular, the nested model based on RV produces a large fit for the anomalies in the momentum (R^2_{OLS} of 66%), profitability (63%), and trading frictions (47%) major categories. In this last case, the model based on RV clearly outperforms the model associated with RS, with a gap in R^2_{OLS} around 30 percentage points. Hence, RV dominates RS when it comes to pricing anomalies in the trading frictions category. In all three cases mentioned above, the negative estimates for λ_{RV} are strongly significant (1% level). It turns out that the poor performance of the model based on RV in the augmented estimation (with all 26 anomalies) stems from the inability of such factor in explaining the anomalies in the investment, intangibles, and value-growth categories, as indicated by the negative estimates of R^2_{OLS}. In comparison, the nested model associated with RS generates positive performance in the asset pricing tests for each of the six broad categories, as indicated by the positive explanatory ratios in all cases.

Finally, following Jiang (2010) and Garcia, Mantilla-Garcia, and Martellini (2014), we estimate an alternative three-factor model in which both the cross-sectional return second-moment (CSV) and the cross-sectional return third-moment (CSS) factors are constructed from individual stock returns (rather than zero-cost equity portfolios). Specifically, we estimate the following three-factor model,

$$\mathrm{E}(R_{i,t+1} - R_{f,t+1}) = \lambda_M \beta_{i,M} + \lambda_{CSV} \beta_{i,CSV} + \lambda_{CSS} \beta_{i,CSS}, \tag{34}$$

where $\beta_{i,CSV}$ and $\beta_{i,CSS}$ denote the loadings associated with CSV and CSS, respectively, while λ_{CSV} and λ_{CSS} represent the corresponding risk prices.[23] In contrast to CS-CAPM, this alternative three-factor model is not consistent with the theoretical model derived in Section 2, since the cross-sectional factors are based on the returns of individual stocks, rather than long-short portfolios.

The estimation results show that the new three-factor model clearly underperforms CS-CAPM when it comes to explaining simultaneously the portfolios associated with the 26 market anomalies. Indeed, the explanatory ratio is negative (-0.08), which indicates that the alternative model performs worse than a trivial model containing only an intercept. On the other hand, the estimate of λ_{CSS} is significantly (at the 5% level) negative, which is inconsistent with the theoretical model. Therefore, these results indicate that cross-sectional factors based on portfolio returns clearly dominate stock-level cross-sectional factors when it comes to pricing a broad cross-section of equity risk premia. In other words, the asset pricing implications of the two measures of cross-sectional return moments (based on portfolio and individual stock returns) seem quite different. This arises from small correlations among the two sets of factors. Specifically, the correlation between RV and CSV is relatively modest (0.45), which stems from the fact that CSV largely reflects idiosyncratic risk (see Garcia, Mantilla-Garcia, and Martellini 2014).[24] On the other hand, the correlation between RS and CSS is around zero, showing that the two proxies for the third moment of stock returns are not related.[25]

[23] Chang, Cheng, and Khorana (2000) and Garcia, Mantilla-Garcia, and Martellini (2014) also employ measures of return dispersion that are based on individual stock returns. Goyal and Santa-Clara (2003), Bali et al. (2005), Ang et al. (2006), Guo and Savickas (2008), and Guo, Kassa, and Ferguson (2014) use measures of idiosyncratic stock volatility that are related to the return dispersion based on individual stocks (see Garcia, Mantilla-Garcia, and Martellini 2014).

[24] In related work, Maio (2016) compares the forecasting performance of portfolio- and stock-based measures of return dispersion for the aggregate equity premium and finds that the first metric has significantly higher predictive power.

[25] This is consistent with the evidence provided in Albuquerque (2012).

4.4 Factor Loadings

We analyze the patterns in the loadings associated with the cross-sectional factors. Given the large explanatory power of the cross-sectional CAPM documented above, it follows that the cross-sectional dispersion of the betas associated with either RV or RS has to match the cross-sectional dispersion in risk premia associated with each anomaly portfolio group. This is because the dispersion in market betas (the other factor in the model) does not match the raw dispersion in risk premia, as evidenced by the clear failure of the CAPM in explaining those 26 anomalies.

Table 4 contains the factor loading estimates, and respective GMM-based t-ratios, for the return spreads associated with each of the 26 portfolio groups. There are two types of high-minus-low return spreads. The first gap represents the return difference between the extreme tenth and first deciles ($S_1 \equiv D10 - D1$), which are analyzed in Section 3. The second return spread involves the extreme three deciles on each leg, $S_3 \equiv \frac{1}{3}(D8 + D9 + D10 - D1 - D2 - D3)$.

The factor loading estimates show that the return spreads associated with several anomalies (*ep*; industry momentum, *indmom*; *lev*; *mom*; *orgcap*; *pchsale_pchinvt*; *rsup*; sales-to-receivables ratio, *salerec*; and *sp*) have significant positive loadings on RS, that is, the upper deciles have higher betas than the lower deciles. By considered single-sided p-values, we also find significant positive loadings in the cases of the cash-flow-to-price ratio (*cfp*), *ear*, and *operprof* anomalies. Such positive differences in betas scaled by the positive risk price associated with the third-moment factor generate the positive gap in risk premia that helps matching the original positive spreads in average returns for all those anomalies. On the other hand, the return spreads associated with several other anomalies (*agr*; growth in shareholder equity, *egr*; growth in long-term net operating assets, *grltnoa*; *invest*; *pchquick*; and long-term return reversal, *rev*) have significant negative loadings on RS, that is, the upper deciles have lower betas than the lower deciles. These negative estimates in factor loadings generate negative gaps in risk premia that help explaining the original negative average return spreads.

In the case of RV, we obtain significant positive betas for the return spread associated with the *chempia* anomaly and significant negative betas for the return spreads corresponding to the *operprof* and *roeq* (in this last case, by using single-sided p-values) portfolio groups. These estimates scaled by the negative risk price associated with such factor produce the positive (in the case of both *operprof* and *roeq*) or negative (in the case of *chempia*) spreads in risk premia that help explaining the corresponding gaps in raw average returns.

These results suggest that value stocks and past momentum winners (both broadly defined) are more positively correlated with the third moment factor (RS) in comparison with growth stocks and past momentum losers, respectively. On the other hand, high investment-stocks tend to be less positively correlated with RS in comparison to low-investment stocks. Furthermore, more profitable stocks (broadly defined) load less on the second-moment factor in comparison to less profitable stocks. These patterns in factor loadings also suggest that RS is relatively more important than RV for the overall pricing ability of the three-factor model. The reason is that the third-moment factor generates correct patterns in the factor loadings across portfolio in a given group (i.e., with the correct sign) for substantially more anomalies in comparison to RV.

5 ALTERNATIVE FACTOR MODELS

In this section, we compare the performance of CS-CAPM with alternative multifactor models widely used in the literature in terms of pricing the 26 CAPM anomalies.

5.1 Equity Models

The first set of models analyzed contain only traded factors, that is, factors that represent excess stock returns.

First, we estimate the four-factor model of Carhart (1997) (C4, henceforth), which adds a momentum factor (UMD, up-minus-down short-term past returns) to the Fama and French (1993, 1996) three-factor model (FF3),

$$\mathrm{E}(R_{i,t+1} - R_{f,t+1}) = \lambda_M \beta_{i,M} + \lambda_{SMB} \beta_{i,SMB} + \lambda_{HML} \beta_{i,HML} + \lambda_{UMD} \beta_{i,UMD}, \tag{35}$$

where λ_{SMB}, λ_{HML}, and λ_{UMD} denote the risk prices associated with the size, value, and momentum factors, respectively, and the βs stand for the respective factor loadings associated with asset i.

The second model is the four-factor model of Pástor and Stambaugh (2003) (PS4), which adds a stock liquidity factor (LIQ) to FF3:

$$\mathrm{E}(R_{i,t+1} - R_{f,t+1}) = \lambda_M \beta_{i,M} + \lambda_{SMB} \beta_{i,SMB} + \lambda_{HML} \beta_{i,HML} + \lambda_{LIQ} \beta_{i,LIQ}. \tag{36}$$

The third model is the four-factor model of Hou, Xue, and Zhang (2015) (HXZ4, see also Hou et al. 2019). In addition to the market and size (ME) factors, this model contains an investment factor (IA, low-minus-high investment-to-assets ratio) and a profitability factor (ROE, high-minus-low return on equity):

$$\mathrm{E}(R_{i,t+1} - R_{f,t+1}) = \lambda_M \beta_{i,M} + \lambda_{ME} \beta_{i,ME} + \lambda_{IA} \beta_{i,IA} + \lambda_{ROE} \beta_{i,ROE}. \tag{37}$$

Next, we consider the five-factor model of Fama and French (2015, 2016) (FF5), in which the investment (CMA) and profitability (RMW) factors are added to FF3:[26]

$$\mathrm{E}(R_{i,t+1} - R_{f,t+1}) = \lambda_M \beta_{i,M} + \lambda_{SMB^*} \beta_{i,SMB^*} + \lambda_{HML} \beta_{i,HML} + \lambda_{CMA} \beta_{i,CMA} + \lambda_{RMW} \beta_{i,RMW}. \tag{38}$$

The fifth model is the six-factor model of Fama and French (2018), which augments FF5 by the momentum factor (FF6):[27]

$$\mathrm{E}(R_{i,t+1} - R_{f,t+1}) = \lambda_M \beta_{i,M} + \lambda_{SMB^*} \beta_{i,SMB^*} + \lambda_{HML} \beta_{i,HML} + \lambda_{CMA} \beta_{i,CMA} + \lambda_{RMW} \beta_{i,RMW} + \lambda_{UMD} \beta_{i,UMD}. \tag{39}$$

Next, we estimate the three-factor model proposed by Asness and Frazzini (2013) (AF3), in which a more "timely" version of HML is employed (denoted by HML^*):

$$\mathrm{E}(R_{i,t+1} - R_{f,t+1}) = \lambda_M \beta_{i,M} + \lambda_{SMB} \beta_{i,SMB} + \lambda_{HML^*} \beta_{i,HML^*}. \tag{40}$$

[26] SMB^* is defined differently than SMB (see Fama and French 2015 for details). However, their correlation is quite large, as reported in the Internet Appendix.

[27] This stems from strong evidence that FF5 is not successful in pricing momentum-based anomalies (e.g., Fama and French 2016; Maio and Philip 2018).

The seventh model is the six-factor model proposed by Barillas and Shanken (2018) (BS6),

$$\begin{aligned} \mathrm{E}(R_{i,t+1} - R_{f,t+1}) &= \lambda_M \beta_{i,M} + \lambda_{SMB^*}\beta_{i,SMB^*} + \lambda_{HML^*}\beta_{i,HML^*} + \lambda_{UMD}\beta_{i,UMD} \\ &+ \lambda_{IA}\beta_{i,IA} + \lambda_{ROE}\beta_{i,ROE}. \end{aligned} \quad (41)$$

which combines some of the factors contained in the HXZ4, FF5, AF3, and C4 models.[28]

The eighth model is the five-factor model proposed by Hou et al. (2019, 2020) (denoted by HMXZ5), which augments HXZ4 by an expected growth factor (EG):

$$\mathrm{E}(R_{i,t+1} - R_{f,t+1}) = \lambda_M \beta_{i,M} + \lambda_{ME}\beta_{i,ME} + \lambda_{IA}\beta_{i,IA} + \lambda_{ROE}\beta_{i,ROE} + \lambda_{EG}\beta_{i,EG}. \quad (42)$$

Finally, we estimate the four-factor model of Stambaugh and Yuan (2017) (SY4):

$$\mathrm{E}(R_{i,t+1} - R_{f,t+1}) = \lambda_M \beta_{i,M} + \lambda_{SMB^{**}}\beta_{i,SMB^{**}} + \lambda_{MGMT}\beta_{i,MGMT} + \lambda_{PERF}\beta_{i,PERF}. \quad (43)$$

In addition to the size factor, this model contains two "mispricing" factors ($MGMT$ and $PERF$). $MGMT$ is constructed from six anomalies that are associated with firms' management actions, while $PERF$ is constructed from five anomalies associated with firms' performance.

Following Maio and Santa-Clara (2017) and Cooper and Maio (2019b) (see also Cochrane 2005 and Lewellen, Nagel, and Shanken 2010 for a related discussion), we compute the "constrained" cross-sectional R^2 to assess the performance of the equity multifactor models presented above:

$$R_C^2 = 1 - \frac{\mathrm{Var}_N(\widehat{\alpha}_{i,C})}{\mathrm{Var}_N(\mu_i)}. \quad (44)$$

This metric is similar to R^2_{OLS}, but it is based on the estimated pricing errors ($\widehat{\alpha}_{i,C}$) from a "constrained cross-sectional regression" that restricts the risk price estimates to be equal to the respective factor means. This stems from the fact that when all the factors in a specific linear asset pricing model represent excess stock returns, the factor risk price estimates should be equal to the corresponding factor sample means.

[28] Barillas and Shanken (2018) employ HML^* instead of HML.

For example, in the case of C4, those pricing errors are obtained from the following equation,

$$\mu_i = \overline{RM}\widehat{\beta}_{i,M} + \overline{SMB}\widehat{\beta}_{i,SMB} + \overline{HML}\widehat{\beta}_{i,HML} + \overline{UMD}\widehat{\beta}_{i,UMD} + \widehat{\alpha}_{i,C}, \tag{45}$$

where $\overline{RM}$, $\overline{SMB}$, $\overline{HML}$, and $\overline{UMD}$ denote the sample means of the market, size, value, and momentum factors, respectively. It is important to note that such restriction does not apply to the cross-sectional CAPM since both RV and RS, despite representing functions of traded factors, do not represent holding-period returns on a traded portfolio. Hence, the risk price estimates for both RV and RS need not be equal to the respective means, which implies that R^2_{OLS} is the correct metric to evaluate the performance of CS-CAPM. To assess the statistical significance of R^2_C, we compute empirical p-values obtained from a bootstrap simulation, which is similar to the simulation used in the case of R^2_{OLS} (see the Internet Appendix for details).

The data on SMB and CMA are obtained from Kenneth French's data library. The data on IA, ROE, and EG are retrieved from Lu Zhang's data library, while the data on LIQ, $MGMT$, and $PERF$ are obtained from Robert Stambaugh's webpage. The pairwise correlations of the equity factors presented in the Internet Appendix indicate that some factors are (by construction) strongly correlated. This is the case of the four size factors, and also the case of both IA and CMA, with correlations around or above 0.90 in all cases. On the other hand, the two profitability factors (ROE and RMW) are substantially less correlated (0.67).[29] Moreover, HML is positively correlated with both investment factors (correlations around 0.70), but we observe a weaker pattern for HML^* (correlations around 0.50). This stems from the fact that both value factors are not very strongly correlated (0.77). Consequently, HML^* is significantly negatively correlated with the momentum factor (−0.65), while we do not detect a similar pattern for HML (correlation of −0.18). We also observe that $MGMT$ is positively correlated with both HML and the two investment factors (correlations above 0.70). Finally, $PERF$ exhibits correlations above 0.60 with both UMD and ROE.

The asset pricing results for the equity factor models are displayed in Table 5. To save space and keep the focus, we only present the results for the joint estimation with the 26 anomalies (156 portfolios), which represents the most challenging and interesting asset pricing test. Each

[29] Fama and French (2015, 2016) use operating profitability (rather than ROE) in the construction of their RMW factor.

of the nine rows presents the results for a different model, with the order being the same as the order of presentation above. At a first glance, one would be tempted to conclude that several of these alternative multifactor models have a considerable joint explanatory power for the 26 portfolio groups, as judged by the R^2_{OLS} estimates. In fact, these estimates are relatively large for the C4, HXZ4, FF5, FF6, BS6, HMXZ5, and SY4 models, assuming values in the 48-72% range. In comparison, the performance of the other two traded models (PS4 and AF3) is quite poor, as indicated by the negative explanatory ratios. This means that these two multifactor models perform worse than a trivial model that predicts constant risk premia within the cross-section of the 156 portfolios.[30]

However, this seemingly large fit associated with the seven models indicated above is either spurious or exaggerated, as it comes at the cost of implausible risk price estimates for several of the factors in those models. Specifically, the estimates for λ_{SMB} and λ_{HML^*} are negative in several cases, and thus far away from the respective factor means (which are positive by construction). In other cases ($\lambda_{SMB^{**}}$, λ_{HML}, λ_{IA}, λ_{RMW}, λ_{EG}, or λ_{MGMT}) the signs of the risk price estimates are correct, but the magnitudes are substantially different from the corresponding factor means. Consequently, when we impose the restriction of equality between the risk price estimates and the corresponding factor sample means, the explanatory power of several models declines in a substantial way. This is the case of the C4 and FF5 models, whose fit drops sharply (by more than 20 percentage points) after imposing the restriction on the factor risk price estimates. The decline in fit is even stronger in the case of BS6, as the estimate of R^2_C declines by more than 50 percentage points (relative to the corresponding R^2_{OLS} estimate).

In the end, out of the nine multifactor models, only the HXZ4, FF6, HMXZ5, and SY4 models offer a positive, and economically significant, explanatory power for the 26 portfolio groups: the R^2_C estimates vary (in a close range) between 56% (SY4) and 63% (FF6), being strongly statistically significant (1% level) in all four cases. However, and more importantly to our case, such level of performance is only slightly higher than that registered for the CS-CAPM (49%).

To better gauge the relative performance of the cross-sectional CAPM against the equity models, we compute the pairwise spreads, $S_{OLS,C} \equiv R^2_{OLS} - R^2_C$, in which R^2_{OLS} is associated with CS-CAPM

[30]These results are consistent with previous evidence showing that the Fama-French three-factor model is not able to explain several anomalies, in particular price momentum (see, for example, Fama and French 1996, Maio and Santa-Clara 2012, and Maio 2013a, among others).

and R_C^2 is associated with one of the equity models. To evaluate the statistical significance of these spreads in R^2, we compute empirical p-values based on two alternative bootstrap simulations. The first simulation (denoted by Bootstrap I) is similar to the bootstrap described in Section 4, and assumes that the portfolio returns are independent from the factors of both models under analysis. The second simulation (denoted by Bootstrap II) assumes that the portfolio returns are correlated with the factors of the traded model (e.g., FF5). The full details of these simulations are provided in the Internet Appendix.

To save space, the results for the model pairwise comparisons are presented in the Internet Appendix. The results show that CS-CAPM dominates statistically (at the 5% level) the C4, PS4, FF5, AF3, and BS6 models, as the corresponding p-values (from both types of simulation) are below 5% in all cases. In fact, in the cases of PS4 and AF3, such dominance occurs at the 1% level when using both p-values. On the other hand, the CS-CAPM is equivalent to the HXZ4, HMXZ5, FF6, and SY4 models from a statistical viewpoint, as indicated by the p-values (associated with $S_{OLS,C}$) substantially above 10%. In other words, our three-factor model is not dominated (in statistical terms) by any of the multifactor models widely used in the empirical asset pricing literature.

Next, we estimate the following additional six-factor traded model,

$$\begin{aligned} \mathrm{E}(R_{i,t+1} - R_{f,t+1}) &= \lambda_M \beta_{i,M} + \lambda_{SMB^{**}} \beta_{i,SMB^{**}} + \lambda_{HML^*} \beta_{i,HML^*} + \lambda_{UMD} \beta_{i,UMD} \\ &+ \lambda_{QMJ} \beta_{i,QMJ} + \lambda_{BAB^*} \beta_{i,BAB^*}, \end{aligned} \tag{46}$$

in which λ_{QMJ} and λ_{BAB^*} represent the risk prices associated with QMJ and BAB^*, respectively. This multifactor model contains the five basis equity factors employed in the construction of the cross-sectional factors in the CS-CAPM. According to our theoretical model in Section 2, it should be the cross-sectional moments of these equity factors (RV and RS), rather than the equity factors themselves, that are priced in equilibrium.

Results tabulated in the Internet Appendix indicate that the six-factor model above produces an R_{OLS}^2 estimate of 0.69 (in the estimation with all 26 anomalies), which represents a somewhat larger fit than that obtained for the benchmark three-factor model (0.49). However, such level of performance is illusory, as it comes at the cost of implausible risk price estimates for several of the factors in the model above. Indeed, the risk price estimates deviate substantially from the corresponding factor means for some of the factors (e.g., HML^* and QMJ). It follows that when we restrict the risk price estimates to exactly match the corresponding factor means, the model generates a substantially lower fit, as indicated by the R^2_C estimate of only 0.29. Such performance clearly lags behind the fit produced by CS-CAPM. Therefore, this evidence suggests that the good performance of our model is not driven by possible mechanical effects due to the inclusion of equity factors, which are strongly correlated with some of the testing assets (e.g., UMD in relation with the price momentum deciles), in the construction of both RV and RS.[31]

In sum, the results of this subsection suggest that the performance of the three-factor model, when compared to the best performing multifactor models in the literature, seems quite favorable. This is especially relevant when one takes into account the fact that the cross-sectional factors are not mechanically related to the equity portfolios used in the asset pricing tests. In comparison, one should note that the performance of both C4 and FF6 in terms of explaining the price and industry momentum portfolios is driven by the UMD factor, which is (nearly) mechanically related to the price momentum deciles.[32] On the other hand, the good performance of both HXZ4 and HMXZ5 in terms of explaining the *roeq* portfolios is driven by the presence of the ROE factor.

5.2 ICAPM Models

We compare the performance of the cross-sectional CAPM with other multifactor "macro" models, in which some of the factors do not represent excess stock returns. The alternative models are mainly retrieved from the ICAPM literature, in which the factors (apart from the traditional market factor) represent the innovations in state variables that forecast the changes in future aggregate investment opportunities (aggregate equity premium or stock market volatility). Most candidate state variables in empirical applications of the Merton's ICAPM (Merton 1973) represent variables

[31] RV and RS represent the cross-sectional second- and third-moments (i.e., non-linear transformation) of the equity factor returns, respectively, rather than a cross-sectional first-moment (i.e., linear transformation). Hence, a priori, the large explanatory power of the cross-sectional factors for the risk premia associated with e.g. the momentum portfolios should not arise from a large mechanical correlation between those factors and UMD. Indeed, the results tabulated in Table 1 indicate that the cross-sectional factors are at most weakly correlated with some of the basis factors used in their construction. Furthermore, by conducting Monte Carlo simulations, we further explore the statistical significance of the correlations between both RV and RS and the five basis factors. Untabulated results from these simulations show that we cannot reject the null hypothesis of no linear relationship between the five basis factors and the CS-CAPM factors. Details are available upon request.

[32] This is not the case for the factors in the HXZ4 model, and the fact that this model helps to explain the price and industry momentum portfolios represents one of its major empirical successes (see Hou, Xue, and Zhang 2015 and Cooper and Maio 2019b).

commonly used as predictors of the excess market return.[33] As in the sensitivity analysis conducted in Section 4, the innovations in the state variables are constructed from an AR(1) process.

The first model considered is the three-factor model of Hahn and Lee (2006) (HL3),

$$\mathrm{E}(R_{i,t+1} - R_{f,t+1}) = \lambda_M \beta_{i,M} + \lambda_{TERM} \beta_{i,TERM} + \lambda_{DEF} \beta_{i,DEF}, \tag{47}$$

where λ_{TERM} and λ_{DEF} denote the risk prices associated with the innovations in the term spread ($\Delta TERM$) and default spread (ΔDEF), respectively.

Next, we estimate the five-factor ICAPM proposed by Petkova (2006) (P5), which adds to HL3 the innovations in the log market dividend yield (ΔDY) and T-bill rate (ΔRF):

$$\begin{aligned}\mathrm{E}(R_{i,t+1} - R_{f,t+1}) = \lambda_M \beta_{i,M} + \lambda_{TERM} \beta_{i,TERM} + \lambda_{DEF} \beta_{i,DEF} + \\ \lambda_{DY} \beta_{i,DY} + \lambda_{RF} \beta_{i,RF}.\end{aligned} \tag{48}$$

The third model is an unrestricted version of the ICAPM proposed by Campbell and Vuolteenaho (2004) (CV4),

$$\mathrm{E}(R_{i,t+1} - R_{f,t+1}) = \lambda_M \beta_{i,M} + \lambda_{TERM} \beta_{i,TERM} + \lambda_{PE} \beta_{i,PE} + \lambda_{VS} \beta_{i,VS}, \tag{49}$$

in which λ_{PE} and λ_{VS} represent the risk prices associated with the innovations in the aggregate price-earnings ratio (ΔPE) and the value spread (ΔVS), respectively.[34]

The fourth model represents an unrestricted version of the ICAPM proposed by Campbell et al. (2018) (CGPT6), which adds to CV4 the innovations in DEF and in the stock market variance

[33] Another important literature of macro models concerns the cross-sectional tests of the Consumption-CAPM from Breeden (1979), or extensions of this model, in which a key risk factor is aggregate consumption growth. An incomplete list of papers that estimate this class of models on the cross-section of stock returns includes Breeden, Gibbons, and Litzenberger (1989), Aït-Sahalia, Parker, and Yogo (2004), Parker and Julliard (2005), Yogo (2006), Delikouras (2017), and Maio and Silva (2020). We do not include these models in our analysis because they rely on aggregate consumption data, which is only available at a quarterly frequency. Another class of macro asset pricing models are production- or investment-based models, which also rely on annual or quarterly data (e.g., Cochrane 1996; Belo 2010).

[34] The factors in Campbell and Vuolteenaho (2004) are aggregate cash-flow and discount rate news, which represent linear functions of the innovations in the original state variables used in a first-order VAR. Hence, the two specifications are related to the extent that they both depend on the same state variables (see Campbell 1996 and Maio 2013b for details).

($\Delta SVAR$), and replaces $\Delta TERM$ by ΔRF:

$$\mathrm{E}(R_{i,t+1} - R_{f,t+1}) = \lambda_M \beta_{i,M} + \lambda_{RF}\beta_{i,RF} + \lambda_{PE}\beta_{i,PE} + \lambda_{VS}\beta_{i,VS} + \lambda_{DEF}\beta_{i,DEF} + \lambda_{SVAR}\beta_{i,SVAR}. \tag{50}$$

$TERM$ is computed as the yield spread between the ten-year and the one-year Treasury bonds, and DEF denotes the yield spread between BAA and AAA corporate bonds from Moody's. The bond yield data are available from the St. Louis Fed web page. RF represents the three-month T-bill rate, available from Amit Goyal's website. DY is computed as the log ratio of annual dividends to the price level of the S&P 500 index, while PE is the log price-earnings ratio associated with the same index, where the earnings metric represents a 10-year moving average of annual earnings. The data on PE are retrieved from Robert Shiller's website. The data on the components of DY are from Amit Goyal's website. VS represents the difference in the log book-to-market ratios of small-value and small-growth portfolios, where the book-to-market portfolio data are retrieved from Kenneth French's data library.[35] $SVAR$ represents the realized stock market variance employed in Welch and Goyal (2008) (see also Andersen et al. 2003 and Guo, Wang, and Yang 2013).

The results for the four multifactor models described above are presented in Table 6. We can see that the performance of these models lags substantially behind the CS-CAPM, with explanatory ratios that are either negative (HL3 and CV4) or zero (P5). Only in the case of CGPT6 is the R^2_{OLS} estimate positive, albeit quite modest (0.07) and largely insignificant in statistical terms (p-value of 0.25). Further, all four ICAPM models are rejected (at the 5% level) by the formal specification test.

We compute the pairwise differences in R^2_{OLS} between the cross-sectional CAPM and each of the ICAPM models, $S_{OLS} \equiv R^2_{OLS} - R^2_{OLS,j}$, where $R^2_{OLS,j}$ denotes the explanatory ratio of one of the models among HL3, P5, CV4, and CGPT6. To assess the statistical significance of this metric, we conduct similar bootstrap simulations to those associated with $S_{OLS,C}$, with the full details explained in the Internet Appendix. Results tabulated in the Internet Appendix show that the estimates for S_{OLS} are positive and economically significant in all cases, with values between 42 (comparison with CGPT6) and 80 percentage points (HL3). Such incremental performance

[35] See Campbell and Vuolteenaho (2004) for details on the construction of VS.

associated with CS-CAPM is not statistically significant in the case of HL3, when relying on the inference from Bootstrap I, which assumes that portfolio returns are independent from the factors of both the CS-CAPM and the macro model. However, the gaps in R^2_{OLS} are significant (at the 10% or 5% level, based on Bootstrap I) in the cases of the remaining three ICAPM models. Moreover, when using p-values from Bootstrap II, which assumes that portfolio returns are correlated with the alternative macro model (e.g., P5), it follows that the cross-sectional CAPM dominates statistically by a large degree (at the 1% level) all four macro models.

Next, to further investigate the linkages between the cross-sectional and the ICAPM factors, we estimate augmented macro models that contain both RV and RS. For example, the augmented HL3 model is given by

$$\mathrm{E}(R_{i,t+1} - R_{f,t+1}) = \lambda_M \beta_{i,M} + \lambda_{TERM} \beta_{i,TERM} + \lambda_{DEF} \beta_{i,DEF} + \lambda_{RV} \beta_{i,RV} + \lambda_{RS} \beta_{i,RS}, \quad (51)$$

and similarly for the other three ICAPM models. The objective of this exercise is to check whether RV and RS remain priced in the presence of the ICAPM factors. In other words, we want to assess whether the other macro factors subsume the pricing power of the cross-sectional factors.

The results for the asset pricing estimation containing all 26 anomalies are tabulated in Table 7. We can see that the risk price estimates for both RV and RS have the correct signs in all cases. The estimates for λ_{RS} are strongly significant (1% level) in all four cases, in line with the estimates obtained for the baseline CS-CAPM. On the other hand, the estimates for λ_{RV} are significant (at the 10% or 5% level, based on double-sided asymptotic p-values) in the cases of the augmented HL3, P5, and CV4 models. When we compare with the estimate of λ_{RV} obtained in the benchmark model, it follows that adding the hedging factors to the CS-CAPM pricing equation clarifies the pricing performance of RV. The sole exception to such pattern is the augmented CGPT6 model, in which case the risk price estimate for RV is not significant at the 10% level.

The R^2_{OLS} estimates of the augmented models (including RV and RS) are concentrated on the small 50-53% range, and thus only marginally above the fit obtained for the CS-CAPM (49%). This indicates that the non-market factors on each of these original ICAPM models add little explanatory power to a model that already contains the market and the two cross-sectional factors. In other words, the joint pricing ability of those ICAPM factors is subsumed by the pricing ability of both

RV and RS and this pattern is robust in the four cases.

Overall, the results of this subsection indicate that the CS-CAPM clearly outperforms several ICAPM models from the empirical asset pricing literature.

6 FACTOR EXPOSURE AND STOCK RETURNS

In this section, we examine the relationship between the loadings on the RV and RS factors and individual stock returns.[36]

More specifically, we estimate the sensitivity of individual stock returns to the RV and RS factors by running the following regression,

$$R_{i,\tau} - R_{f,\tau} = \alpha_i + \beta_{i,M} RM_\tau + \beta_{i,RV} RV_\tau + \beta_{i,RS} RS_\tau + \varepsilon_{i,\tau}, \tag{52}$$

where $R_{i,\tau} - R_{f,\tau}$ is stock i's excess return in month τ, and RM_τ, RV_τ, and RS_τ denote the market excess return, RV, and RS, in month τ, respectively.[37]

For each stock i in each month t, we run the regression above over the rolling 60-month window from $t-60$ to $t-1$, and we require at least 36 months of non-missing return data for stock i to have valid values of both $\beta_{i,RV}$ and $\beta_{i,RS}$. The sample period for this analysis is from 1978:01 to 2016:12, which leaves the initial five-year window for estimation. As standard in the literature, we include all common stocks traded on NYSE, Amex, and NASDAQ, and also exclude financial firms.

We conduct a double-sorting analysis to examine the return spread associated with β_{RV} (β_{RS}) after controlling for β_{RS} (β_{RV}). We sort stocks independently into 3×3 groups based on β_{RS} and β_{RV}. We form value-weighted portfolios and examine monthly portfolio returns. The CAPM alphas of these portfolios are presented in Table 8.

As shown in Table 8, in each of the low, middle, and high terciles of β_{RS}, the difference in CAPM alphas between the high and low β_{RV} terciles is negative. It is -0.61%, significant at the 5% level ($t = -2.30$), for the high β_{RS} tercile. Untabulated results show that averaged across the three β_{RS} terciles, the difference in CAPM alphas between the high and low β_{RV} terciles is -0.35%, with significance at the 5% level ($t = -2.04$).

[36] We thank an anonymous referee for suggesting this analysis.

[37] Unlike the previous sections, in this section, i refers to an individual stock.

In each of the low, middle, and high terciles of β_{RV}, the difference in CAPM alphas between the high and low β_{RS} terciles is positive. It is 0.64%, significant at the 5% level ($t = 2.25$), for the low β_{RV} tercile, and is 0.65%, significant at the 1% level ($t = 2.69$), for the middle β_{RV} tercile. Untabulated results show that averaged across the three β_{RV} terciles, the difference in CAPM alphas between the high and low β_{RS} terciles is 0.49%, with significance at the 5% level ($t = 2.41$).

Overall, this set of empirical results using individual stock returns show that the return spread associated with β_{RV} (β_{RS}) as a stock characteristic is negative (positive). In other words, stocks that are more correlated with RV (RS) earn lower (higher) average returns than stocks that are less correlated with those same factors. This finding is exactly in line with the negative (positive) risk price estimates associated with RV (RS) that we obtain in Section 4.

7 CONCLUSIONS

In this paper, we propose a new risk-based explanation for several key market anomalies. For this purpose, we derive a three-factor asset pricing model, denoted as cross-sectional CAPM (or CS-CAPM), which represents an extension of the baseline CAPM. In addition to the usual market factor, the other risk factors in the model are the realized cross-sectional second (RV) and third (RS) moments of long-short equity portfolio returns. In the underlying theoretical framework there is no single representative investor in the economy, and each investor holds an underdiversified equity portfolio that is tilted towards a segment of the stock market (by investing in a specific zero-cost portfolio strategy). By using a third-order Taylor approximation, it turns out that the average stochastic discount factor in the economy is a positive function of RV and a negative function of RS. Thus, high realizations of RV and low realizations of RS represent "bad" times on average; that is, periods with high marginal utility of consumption. Consequently, in the expected return-beta (or covariance) representation of the model it follows that the risk prices associated with RV and RS are negative and positive, respectively.

In the empirical estimation of the cross-sectional CAPM, we use (as testing assets) value-weighted portfolio deciles sorted on 26 stock characteristics, which are associated with important CAPM anomalies. These 26 anomalies are fairly representative of the broad cross-section of stock returns and display a large cross-sectional variation in risk premia. Following Hou, Xue, and Zhang (2015, 2020), these portfolio groups can be generically classified in strategies related to trading frictions, momentum, investment, profitability, intangibles, and value-growth. The cross-sectional factors used in the CS-CAPM represent the cross-sectional second and third moments of five long-short equity portfolio returns (or traded equity factors) that have been shown to describe the investment strategies of active portfolio managers in the stock market. These portfolios are related to the size, value-growth, beta, momentum, and quality effects in average stock returns.

The results of the empirical asset pricing tests show that the cross-sectional CAPM offers a very good explanatory power for the cross-section of average stock returns. In the augmented test with the joint 26 anomalies (for a total of 156 decile portfolios), the OLS cross-sectional R^2 is 49% and such explanatory ratio is statistically greater than zero. This compares with a negative R^2 estimate for the baseline CAPM, which shows that such model performs worse than a trivial model that predicts constant risk premia within the cross-section of the 156 portfolio returns. In the estimation with each major category of anomalies, the three-factor model explains more than 40% of the cross-sectional dispersion in portfolio risk premia among most categories. The estimates for the risk prices associated with RV are significantly negative in most cases, while the risk price estimates for RS are strongly significantly positive in all cases. Hence, these estimates are consistent with the qualitative restrictions of the theoretical model.

In what concerns the patterns in factors loadings, it turns out that value stocks and past momentum winners (both broadly defined) are more positively correlated with the third moment factor (RS) in comparison with growth stocks and past momentum losers, respectively. On the other hand, high investment-stocks tend to be less positively correlated with RS in comparison to low-investment stocks. Furthermore, more profitable stocks load less on the second-moment factor in comparison to less profitable stocks.

The performance of the cross-sectional CAPM is compared with alternative multifactor models containing traded factors, which are widely used in the literature. The results show that the cross-sectional CAPM has a similar performance to the recently proposed four-factor model of Hou, Xue, and Zhang (2015), five-factor model of Hou et al. (2019, 2020), six-factor model of Fama and French (2018), and the four-factor model of Stambaugh and Yuan (2017). We also compare the performance of the three-factor model with other multifactor ICAPM models, in which some of the factors do not represent excess stock returns. The cross-sectional CAPM dominates these alternative models both in economic and statistical terms. Critically, by estimating augmented factor models, we find that the cross-sectional factors are not subsumed by the ICAPM factors when it comes to explaining the 26 market anomalies.

In the last part of the paper, we examine the relationship between the loadings on both the RV and RS factors and individual stock returns. The results show that stocks that are more correlated with RV (RS) earn lower (higher) average returns than stocks that are less correlated with those same factors. This finding is exactly in line with the negative (positive) risk price estimates associated with RV (RS) in our main asset pricing tests.

To summarize, our results indicate that only two factors (the role of the market factor in the model is merely of picking the cross-sectional average risk premium) can explain an important fraction of the cross-sectional dispersion in risk premia among several prominent market anomalies, which are associated with all the major groups of CAPM anomalies. This is especially relevant as several of these anomalies are nearly uncorrelated or even negatively correlated.

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Table 1

DESCRIPTIVE STATISTICS FOR EQUITY FACTORS

	Mean (%)	Stdev. (%)	Min. (%)	Max. (%)	ϕ	Corr(RV,F)	Corr(RS,F)
RM	0.53	4.57	−23.24	16.10	0.07	0.01	0.02
RV	0.16	0.38	0.00	5.16	0.16	1.00	
RS	0.00	0.07	−1.19	0.20	−0.02	−0.70	1.00
SMB	0.19	3.10	−16.88	21.71	−0.02	0.16	−0.08
HML	0.39	2.94	−11.10	12.90	0.16	0.01	0.10
UMD	0.66	4.44	−34.39	18.36	0.07	−0.28	0.31
LIQ	0.42	3.43	−12.49	11.08	0.09	0.06	−0.07
ME	0.30	3.10	−14.39	22.13	0.00	0.16	−0.04
IA	0.41	1.85	−7.16	9.25	0.09	−0.01	0.15
ROE	0.55	2.59	−13.85	10.37	0.10	−0.30	0.30
EG	0.56	1.84	−6.19	9.34	0.11	−0.05	0.14
SMB^*	0.23	3.03	−14.85	18.27	0.00	0.16	−0.05
RMW	0.27	2.35	−18.72	13.51	0.15	−0.14	0.22
CMA	0.35	1.98	−6.87	9.58	0.13	0.02	0.13
HML^*	0.38	3.59	−17.96	26.87	0.16	0.25	−0.14
SMB^{**}	0.42	2.79	−11.10	16.04	−0.02	0.11	−0.02
$MGMT$	0.65	2.82	−8.93	14.58	0.13	−0.04	0.13
$PERF$	0.64	3.92	−21.45	18.52	0.11	−0.16	0.23
BAB^*	0.70	5.89	−38.65	21.29	0.06	−0.33	0.59
QMJ	0.40	2.36	−9.10	12.39	0.19	−0.15	0.18

NOTE: This table reports descriptive statistics for the risk factors associated with the three-factor model and alternative factor models. RM, SMB/SMB^*, HML, and UMD denote the traditional market, size, value, and momentum factors, respectively. ME, IA, ROE, and EG represent the Hou–Mo–Xue–Zhang size, investment, profitability, and expected growth factors, respectively. RMW and CMA denote the Fama–French profitability and investment factors. SMB^{**}, $MGMT$, and $PERF$ represent the Stambaugh–Yuan size, management, and performance factors, respectively. HML^* is the Asness–Frazzini value factor. BAB^* denotes the Liu–Stambaugh–Yuan betting-against-beta factor. QMJ stands for the Asness–Frazzini–Pedersen quality-minus-junk factor. RV and RS represent the cross-sectional second moment and third moment factors, respectively. The sample is 1973:01–2016:12. ϕ designates the first-order autocorrelation coefficient. The column labeled Corr(RV,F) (Corr(RS,F)) represents the correlations between RV (RS) and each of the other factors.

Table 2

Descriptive Statistics for Return Spreads

	Mean (%)	Stdev. (%)	Min. (%)	Max. (%)	ϕ	α
agr	−0.54	3.18	−19.01	10.75	0.04	−4.44
beta	−0.01	8.30	−30.79	38.26	0.11	−2.63
betasq	0.00	8.30	−30.66	38.29	0.11	−2.60
cfp	0.45	4.45	−23.49	24.95	0.10	2.99
chempia	−0.05	2.87	−14.04	13.49	−0.01	−1.22
ear	0.64	3.08	−15.35	14.14	−0.06	5.07
egr	−0.48	3.27	−18.91	16.73	−0.05	−4.18
ep	0.64	5.17	−28.73	22.82	0.10	3.97
grltnoa	−0.36	3.07	−10.12	18.75	−0.04	−2.78
indmom	0.45	6.62	−39.87	21.63	0.13	1.84
invest	−0.56	3.05	−14.66	10.53	0.11	−4.69
lev	0.21	4.74	−23.34	16.65	0.10	0.87
maxret	−0.35	6.43	−31.01	33.41	0.06	−3.53
mom	1.05	7.12	−45.08	22.73	0.05	4.03
operprof	0.26	3.21	−10.13	11.67	0.19	1.86
orgcap	0.15	3.88	−11.47	11.47	0.12	1.43
pchquick	−0.12	2.09	−10.22	7.08	0.08	−1.13
pchsale_pchinvt	0.32	2.51	−8.81	11.55	0.08	3.00
retvol	−0.40	7.59	−35.10	38.23	0.08	−3.71
rev	−0.35	5.22	−32.04	15.26	0.07	−1.41
roaq	0.54	4.60	−18.53	22.51	0.15	3.62
roeq	0.59	4.79	−24.70	25.83	0.17	3.75
rsup	0.38	3.77	−13.65	19.73	0.02	2.36
salerec	0.21	3.56	−14.03	15.60	0.14	2.01
sp	0.61	4.44	−16.06	20.39	0.16	3.02
tb	0.09	2.83	−12.55	8.69	0.10	1.22

Note: This table reports descriptive statistics for the "high-minus-low" return spreads associated with different portfolio classes. See the Internet Appendix for a description of the different portfolio sorts. The sample is 1973:01–2016:12. ϕ designates the first-order autocorrelation coefficient. α denotes the t-ratio associated with the CAPM alpha.

Table 3

Factor Risk Premia Estimates

	λ_M	λ_{RV}	λ_{RS}	χ^2	R^2_{OLS}
Panel A: CAPM					
Investment	0.56***			64.33	−0.43
	(**2.79**)			(0.000)	(0.309)
Trading frictions	0.44***			40.15	−2.42
	(2.17)			(0.015)	(0.677)
Momentum	0.53***			55.44	−0.40
	(**2.64**)			(0.000)	(0.371)
Intangibles	0.53***			76.27	−0.61
	(**2.67**)			(0.000)	(0.214)
Profitability	0.46***			32.42	−0.67
	(2.32)			(0.092)	(0.406)
Value-growth	0.60***			57.60	−0.57
	(**2.99**)			(0.000)	(0.311)
All	0.52**			379.02	−0.69
	(**2.60**)			(0.000)	(0.338)
Panel B: CS-CAPM					
Investment	0.50***	−0.06	0.04***	41.25	0.39
	(2.51)	(−1.01)	(**3.19**)	(0.005)	(0.086)
Trading frictions	0.67***	−0.28***	0.03***	20.74	0.53
	(**3.18**)	(**−2.87**)	(2.48)	(0.475)	(0.094)
Momentum	0.58***	−0.13*	0.03***	40.31	0.68
	(**2.86**)	(*−1.84*)	(**3.80**)	(0.007)	(0.009)
Intangibles	0.51***	−0.09	0.04***	37.44	0.57
	(2.54)	(−1.60)	(**3.58**)	(0.272)	(0.000)
Profitability	0.51***	−0.20**	0.04***	18.89	0.68
	(2.53)	(−2.21)	(**3.52**)	(0.592)	(0.008)
Value-growth	0.51***	−0.01	0.03***	28.82	0.70
	(2.56)	(−0.14)	(**2.93**)	(0.118)	(0.004)
All	0.51**	−0.06	0.03***	275.03	0.49
	(2.55)	(−1.32)	(**4.47**)	(0.000)	(0.001)

Note: This table reports the estimation and evaluation results for both the baseline CAPM (Panel A) and the three-factor model (CS-CAPM, Panel B). The estimation procedure is the two-pass regression approach. The testing portfolios are decile portfolios associated with 26 CAPM anomalies. Only the extreme three deciles on each leg (for each portfolio group) are considered. "All" refers to an asset pricing test containing all 156 portfolios. λ_M, λ_{RV}, and λ_{RS} denote the risk price estimates (in %) for the market, second moment, and third moment factors, respectively. Below the risk price estimates are displayed t-statistics based on Shanken's standard errors (in parentheses). Risk price estimates marked with *, **, *** represent statistical significance at the 10%, 5%, and 1% levels, respectively, based on the empirical p-values from a bootstrap simulation. The column labeled χ^2 presents the Wald statistic (first line), and associated asymptotic p-values (in parentheses), for the test on the joint significance of the pricing errors. The column labeled R^2_{OLS} denotes the cross-sectional OLS R^2 with the corresponding empirical p-value shown in parentheses. The sample is 1973:01–2016:12. Italic, underlined, and bold t-ratios denote statistical significance at the 10%, 5%, and 1% levels, respectively.

Table 4

Factor Loadings for Return Spreads

	$D10 - D1$			$\frac{1}{3}(D8 + D9 + D10 - D1 - D2 - D3)$		
Anomaly	RM	RV	RS	RM	RV	RS
agr	0.16	−2.42	−15.89	0.16	−1.16	−9.84
	(**4.33**)	(**−3.24**)	(**−4.21**)	(**6.17**)	(*−1.78*)	(**−2.97**)
beta	1.28	0.81	−19.45	0.84	0.30	−16.25
	(**17.60**)	(0.40)	(−1.08)	(**17.64**)	(0.20)	(−1.23)
betasq	1.28	0.82	−19.20	0.84	0.29	−16.25
	(**17.62**)	(0.40)	(−1.08)	(**17.64**)	(0.20)	(−1.23)
cfp	−0.24	1.07	7.46	−0.18	1.30	8.60
	(**−4.07**)	(0.51)	(0.97)	(**−4.33**)	(0.96)	(1.45)
chempia	0.18	1.36	2.20	0.12	1.09	2.72
	(**5.89**)	(*1.82*)	(0.56)	(**6.34**)	(2.17)	(1.23)
ear	−0.07	−0.15	4.37	0.04	−0.02	1.12
	(*−1.80*)	(−0.14)	(1.35)	(*1.94*)	(−0.04)	(0.61)
egr	0.20	−2.80	−12.51	0.16	−1.37	−7.88
	(**5.31**)	(**−3.65**)	(*−1.83*)	(**6.24**)	(−2.57)	(−2.14)
ep	−0.38	1.69	20.08	−0.31	1.83	16.74
	(**−6.17**)	(0.80)	(2.21)	(**−7.39**)	(1.43)	(**2.96**)
grltnoa	0.03	−1.07	−14.13	−0.01	−0.87	−11.06
	(1.06)	(−1.37)	(**−2.77**)	(−0.54)	(−1.62)	(**−3.10**)
indmom	−0.15	−0.65	23.53	−0.08	−0.93	9.10
	(−2.04)	(−0.28)	(**2.62**)	(*−1.84*)	(−0.64)	(*1.83*)
invest	0.14	0.00	−2.83	0.08	−0.99	−9.04
	(**4.17**)	(0.00)	(−1.31)	(**2.64**)	(−1.26)	(−2.43)
lev	0.05	2.26	13.71	−0.01	1.29	9.23
	(0.82)	(1.36)	(2.46)	(−0.20)	(1.11)	(2.34)
maxret	0.85	2.11	−11.13	0.60	0.67	−11.08
	(**12.59**)	(1.12)	(−0.71)	(**13.09**)	(0.53)	(−0.97)
mom	−0.29	−2.58	18.97	−0.21	−1.83	16.16
	(**−3.43**)	(−0.91)	(1.62)	(**−3.43**)	(−0.95)	(*1.92*)
operprof	−0.02	−0.42	3.54	0.05	−0.78	−3.22
	(−0.60)	(−0.75)	(1.44)	(*1.67*)	(*−1.73*)	(−1.33)
orgcap	−0.17	0.39	6.02	−0.15	0.55	7.05
	(**−3.98**)	(0.53)	(1.60)	(**−4.58**)	(0.90)	(2.15)
pchquick	−0.02	−1.67	−5.80	−0.01	−0.50	−0.19
	(−0.62)	(**−3.32**)	(−2.57)	(−0.68)	(−1.39)	(−0.15)
pchsale_pchinvt	−0.03	0.67	6.86	−0.03	0.37	3.35
	(−0.91)	(1.10)	(2.18)	(*−1.87*)	(0.92)	(2.25)
retvol	1.05	2.19	−14.35	0.74	1.01	−11.25
	(**13.57**)	(1.03)	(−0.77)	(**15.66**)	(0.78)	(−0.86)
rev	−0.05	−5.38	−12.13	−0.04	−3.15	−4.64
	(−0.72)	(**−3.72**)	(*−1.72*)	(−0.94)	(**−3.49**)	(−1.16)
roaq	−0.31	−0.91	8.73	−0.23	−0.70	4.20
	(**−5.71**)	(−0.72)	(1.04)	(**−6.03**)	(−0.84)	(1.05)
roeq	−0.30	−1.16	11.52	−0.09	−0.98	3.05
	(**−4.47**)	(−0.66)	(1.12)	(**−2.83**)	(−1.47)	(0.70)
rsup	−0.02	0.25	11.01	0.00	0.18	8.79
	(−0.42)	(0.27)	(2.12)	(0.04)	(0.36)	(**3.10**)
salerec	−0.19	0.36	4.51	−0.21	0.66	6.87
	(**−4.53**)	(0.42)	(1.50)	(**−8.83**)	(1.12)	(*1.94*)
sp	0.03	2.94	16.10	0.07	1.55	8.49
	(0.57)	(2.12)	(**3.00**)	(*1.75*)	(*1.89*)	(2.39)
tb	−0.11	0.09	0.76	−0.13	−0.22	3.67
	(**−4.11**)	(0.12)	(0.32)	(**−5.65**)	(−0.41)	(1.15)

Note: This table reports the beta estimates (and respective t-ratios) associated with the factors in the CS-CAPM. The testing portfolios are the high-minus-low return spreads for decile portfolios associated with 26 CAPM anomalies. D_j refers to the jth decile in a given portfolio group. The sample is 1973:01–2016:12. Italic, underlined, and bold t-ratios denote statistical significance at the 10%, 5%, and 1% levels, respectively.

Table 5

Factor Risk Premia for Alternative Models: Traded Factors

	λ_M	λ_{SMB} λ_{SMB^*} $\lambda_{SMB^{**}}$	λ_{HML} λ_{HML^*}	λ_{UMD}	λ_{LIQ}	λ_{ME}	λ_{IA}	λ_{ROE}	λ_{RMW}	λ_{CMA}	λ_{EG}	λ_{MGMT}	λ_{PERF}	χ^2	R^2_{OLS}	R^2_C
1	0.56**	−0.19	0.37**	0.67***										327.23	0.48	0.26
	(**2.81**)	(−1.03)	(2.53)	(**3.25**)										(0.000)	(0.002)	(0.001)
2	0.54**	−0.41*	0.27		0.15									351.98	−0.05	−0.69
	(**2.73**)	(−2.18)	(*1.87*)		(0.43)									(0.000)	(0.198)	(1.000)
3	0.51**					0.33	0.30**	0.54***						299.40	0.64	0.59
	(2.57)					(*1.78*)	(**2.77**)	(**3.85**)						(0.000)	(0.000)	(0.000)
4	0.49**	0.27	0.05						0.46***	0.34***				288.25	0.56	0.27
	(2.48)	(1.60)	(0.32)						(**3.75**)	(**3.22**)				(0.000)	(0.002)	(0.000)
5	0.52**	0.25	0.18	0.60**					0.37***	0.27**				293.44	0.70	0.63
	(**2.59**)	(1.50)	(1.23)	(**2.89**)					(**3.16**)	(**2.64**)				(0.000)	(0.000)	(0.000)
6	0.57**	−0.47**	0.04											364.79	−0.21	−1.20
	(**2.84**)	(−2.47)	(0.21)											(0.000)	(0.233)	(1.000)
7	0.52**	0.22	−0.15	0.60**			0.33***	0.51***						298.04	0.66	0.11
	(**2.61**)	(1.30)	(−0.84)	(**2.89**)			(**3.09**)	(**3.85**)						(0.000)	(0.000)	(0.081)
8	0.53**					0.34	0.29**	0.47***			0.36**			265.51	0.72	0.60
	(**2.68**)					(*1.81*)	(**2.69**)	(**3.48**)			(**2.61**)			(0.000)	(0.000)	(0.000)
9	0.55**	0.27										0.41**	0.60***	290.14	0.70	0.56
	(**2.74**)	(1.57)										(**2.77**)	(**3.05**)	(0.000)	(0.000)	(0.000)

Note: This table reports the estimation and evaluation results for equity multifactor models. The estimation procedure is the two-pass regression approach. The testing portfolios are decile portfolios associated with 26 CAPM anomalies (for a total of 156 portfolios). λ_M, $\lambda_{SMB}/\lambda_{SMB^*}$, λ_{HML}, λ_{UMD}, and λ_{LIQ} denote the risk price estimates (in %) for the market, size, value, momentum, and liquidity factors, respectively. λ_{ME}, λ_{IA}, λ_{ROE}, and λ_{EG} represent the risk prices associated with the Hou–Mo–Xue–Zhang size, investment, profitability, and expected growth factors, respectively. λ_{RMW} and λ_{CMA} denote the risk price estimates for the Fama–French profitability and investment factors, respectively. λ_{HML^*} is the risk price estimate for the Asness–Frazzini value factor. $\lambda_{SMB^{**}}$, λ_{MGMT}, and λ_{PERF} represent the risk price estimates for the Stambaugh–Yuan size, management, and performance factors, respectively. Below the risk price estimates are displayed t-statistics based on Shanken's standard errors (in parentheses). Risk price estimates marked with *, **, *** represent statistical significance at the 10%, 5%, and 1% levels, respectively, based on the empirical p-values from a bootstrap simulation. The column labeled χ^2 presents the Wald statistic (first line), and associated asymptotic p-values (in parentheses), for the test on the joint significance of the pricing errors. The column labeled R^2_{OLS} denotes the cross-sectional OLS R^2 with the corresponding empirical p-value shown in parentheses. R^2_C represents the constrained cross-sectional R^2, with the respective empirical p-value displayed in parentheses. The sample is 1973:01–2016:12. Italic, underlined, and bold t-ratios denote statistical significance at the 10%, 5%, and 1% levels, respectively.

Table 6

FACTOR RISK PREMIA FOR ALTERNATIVE MODELS: MACRO FACTORS

	λ_M	λ_{TERM}	λ_{DEF}	λ_{DP}	λ_{TB}	λ_{PE}	λ_{VS}	λ_{SVAR}	χ^2	R^2_{OLS}
1	0.56**	−0.13	0.07**						251.29	−0.31
	(**2.80**)	(−1.34)	(<u>2.57</u>)						(0.000)	(0.298)
2	0.57**	0.02	0.00	−0.65	−0.15***				293.83	0.00
	(**2.86**)	(0.16)	(0.02)	(**−3.17**)	(−1.60)				(0.000)	(0.239)
3	0.53**	−0.14				−0.44	3.63**		196.35	−0.25
	(**2.66**)	(−1.42)				(−1.02)	(<u>2.50</u>)		(0.009)	(0.352)
4	0.56**		0.04		−0.18	−0.94	1.56	−0.19**	190.34	0.07
	(**2.82**)		(*1.70*)		(*−1.82*)	(−1.48)	(*1.70*)	(**−2.66**)	(0.014)	(0.246)

NOTE: This table reports the estimation and evaluation results for alternative multifactor models. The estimation procedure is the two-pass regression approach. The testing portfolios are decile portfolios associated with 26 CAPM anomalies (for a total of 156 portfolios). λ_M denotes the risk price estimate (in %) for the market factor. λ_{TERM}, λ_{DEF}, λ_{DP}, λ_{TB}, λ_{PE}, λ_{VS}, and λ_{SVAR} represent the risk prices associated with the innovations in the term spread, default spread, dividend yield, T-bill rate, price-earnings ratio, value spread, and stock market variance, respectively. Below the risk price estimates are displayed t-statistics based on Shanken's standard errors (in parentheses). Risk price estimates marked with *, **, *** represent statistical significance at the 10%, 5%, and 1% levels, respectively, based on the empirical p-values from a bootstrap simulation. The column labeled χ^2 presents the Wald statistic (first line), and associated asymptotic p-values (in parentheses), for the test on the joint significance of the pricing errors. The column labeled R^2_{OLS} denotes the cross-sectional OLS R^2 with the corresponding empirical p-value shown in parentheses. The sample is 1973:01–2016:12. Italic, underlined, and bold t-ratios denote statistical significance at the 10%, 5%, and 1% levels, respectively.

Table 7

FACTOR RISK PREMIA FOR AUGMENTED MODELS

	λ_M	λ_{TERM}	λ_{DEF}	λ_{DY}	λ_{TB}	λ_{PE}	λ_{VS}	λ_{SVAR}	λ_{RV}	λ_{RS}	χ^2	R^2_{OLS}
1	0.51**	0.07	0.01						−0.08	0.04***	264.24	0.50
	(<u>2.57</u>)	(0.86)	(0.40)						(<u>−2.08</u>)	(**4.78**)	(0.000)	(0.002)
2	0.51**	0.05	−0.00	−0.56**	−0.05				−0.07	0.03***	263.53	0.52
	(<u>2.57</u>)	(0.63)	(−0.11)	(**−2.72**)	(−0.66)				(*−1.74*)	(**4.87**)	(0.000)	(0.007)
3	0.52**	0.08				−0.21	0.72		−0.07	0.03***	251.94	0.52
	(**2.58**)	(1.09)				(−0.62)	(0.78)		(*−1.66*)	(**4.51**)	(0.000)	(0.005)
4	0.52**		−0.01		−0.08	−0.30	0.71	−0.01	−0.05	0.03***	249.38	0.53
	(**2.61**)		(−0.34)		(−1.05)	(−0.84)	(0.97)	(−0.30)	(−1.08)	(**4.31**)	(0.000)	(0.010)

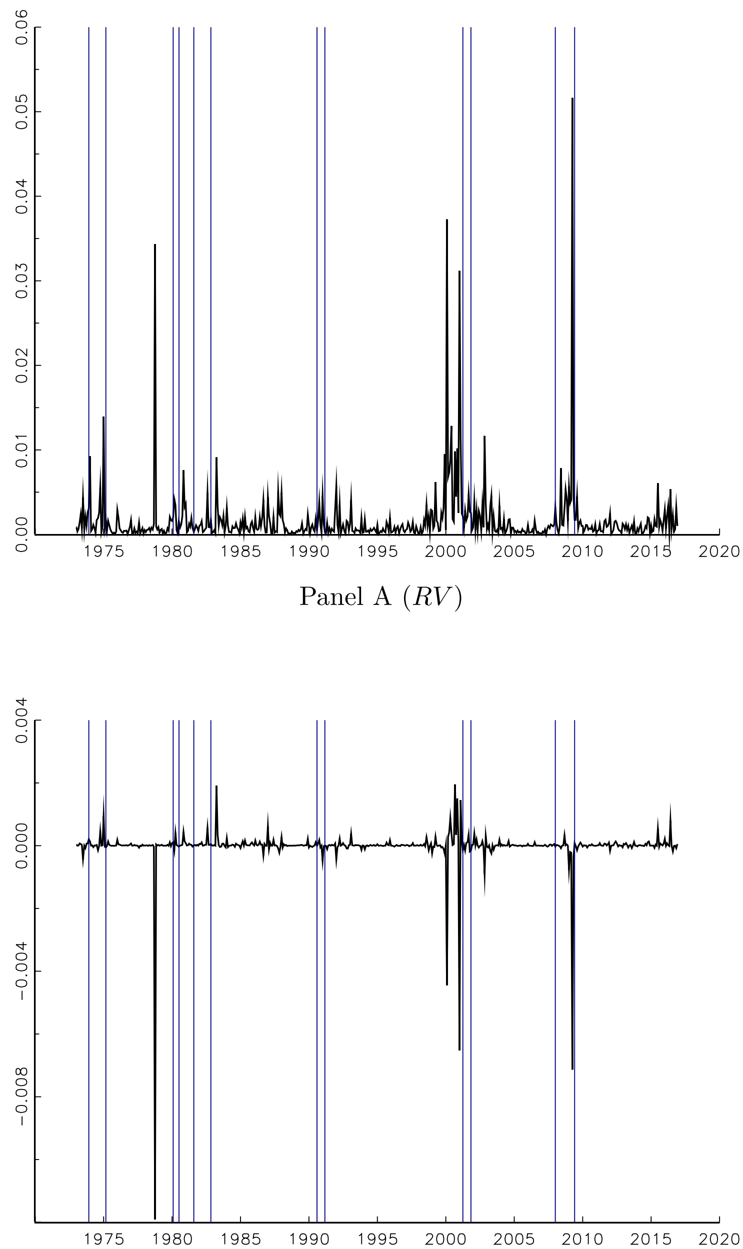
NOTE: This table reports the estimation and evaluation results for augmented multifactor models. The estimation procedure is the two-pass regression approach. The testing portfolios are decile portfolios associated with 26 CAPM anomalies (for a total of 156 portfolios). λ_M denotes the risk price estimate (in %) for the market factor. λ_{TERM}, λ_{DEF}, λ_{DY}, λ_{TB}, λ_{PE}, λ_{VS}, and λ_{SVAR} represent the risk prices associated with the innovations in the term spread, default spread, dividend yield, t-bill rate, price-earnings ratio, value spread, and stock market variance, respectively. λ_{RV} and λ_{RS} represent the risk price estimates for the cross-sectional second and third moment factors, respectively. Below the risk price estimates are displayed t-statistics based on Shanken's standard errors (in parentheses). Risk price estimates marked with *, **, *** represent statistical significance at the 10%, 5%, and 1% levels, respectively, based on the empirical p-values from a bootstrap simulation. The column labeled χ^2 presents the Wald statistic (first line), and associated asymptotic p-values (in parentheses), for the test on the joint significance of the pricing errors. The column labeled R^2_{OLS} denotes the cross-sectional OLS R^2 with the corresponding empirical p-value shown in parentheses. The sample is 1973:01–2016:12. Italic, underlined, and bold t-ratios denote statistical significance at the 10%, 5%, and 1% levels, respectively.

Table 8

CAPM ALPHAS ON PORTFOLIOS SORTED ON β_{RV} AND β_{RS}

β_{RS} tercile	β_{RV} tercile			
	Low	Middle	High	High-Low
Low	-0.23	-0.40**	-0.39	-0.16
	(-1.33)	(-2.32)	(-1.46)	(-0.57)
Middle	0.06	0.02	-0.20	-0.26
	(0.51)	(0.26)	(-1.29)	(-1.27)
High	0.41*	0.25**	-0.20	-0.61**
	(1.81)	(2.21)	(-1.43)	(-2.30)
High-Low	0.64**	0.65***	0.19	
	(2.25)	(2.69)	(0.61)	

NOTE: This table reports monthly CAPM alphas (in percentage) for portfolios formed by sorting independently on β_{RV} and β_{RS}. All portfolios are value-weighted. For each month, a stock's β_{RV} and β_{RS} are estimated by regressing the stock's excess return on the market excess return and the RV and RS factors, using a rolling five-year window. We sort stocks independently into 3×3 groups based on β_{RS} and β_{RV}. In each tercile of β_{RS} (β_{RV}), we report the CAPM alphas (with robust t-statistics displayed in parentheses) for different β_{RV} (β_{RS}) terciles and the difference in CAPM alphas between the high and low terciles. The sample period is from 1978:01 to 2016:12. *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels, respectively.



Panel A (RV)

Panel B (RS)

Figure 1: Time-Series of Factors

This figure shows the time-series for the factors in the CS-CAPM (RV and RS). The sample is 1973:01–2016:12. The vertical lines indicate the NBER recession periods.