

# Risk Aversion Sensitive Real Business Cycles \*

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## Abstract

Technology choice allows for substitution of production across states of nature and depends on state dependent risk aversion. In equilibrium, endogenous technology choice can counter a persistent negative productivity shock with an increase in investment. An increase in risk aversion intensifies transformation across states, which directly leads to higher investment volatility. In our model and the data, the conditional volatility of investment correlates negatively with the price-dividend ratio and predicts excess stock market returns. In addition, the same mechanism generates predictability of consumption and produces fluctuations in the risk-free rate.

**Keywords:** State-contingent technology; Time-varying risk aversion; Conditional volatility of investment; Predictability of returns

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# 1 Introduction

In the standard real business cycle model, production plans are made one period ahead implying that current period capital is fixed across states of nature. Thus, only exogenous shocks but no endogenous current period choices within the representative firm drive output across states of nature. In such an economic environment, the output risk is completely exogenous and independent of the firm's technology or the representative agent's preferences through risk aversion. Further, risk aversion has only small second-order effects on the dynamics of the macroeconomy, as shown in Tallarini (2000).<sup>1</sup> Any (time series) variations in risk aversion should have unnoticeable effects on quantities such as consumption or investment. Yet, cyclical variations in risk aversion play a prominent role in explaining the variations in expected excess returns of the stock market in many theoretical works within the consumption based asset pricing framework, in which consumption is exogenous.<sup>2</sup> In this paper, we show that with a more plausible state dependent production technology, cyclical variations in risk aversion can jointly drive variations in asset prices and the macroeconomy. In our model, the conditional volatility of investment growth evolves pro-cyclically relative to risk aversion, correlates negatively with the price-dividend ratio of the stock market, and predicts excess stock market returns. Consistent with the model, we see in the data a negative correlation between the conditional volatility of investment and the price-dividend ratio of the stock market and that the conditional volatility of investment predicts (excess) stock market returns.

Ideally, we would like to provide micro-foundations for the stylized production technology employed in the model, which we borrow from Cochrane (1993). While we do provide a sketch for how an aggregate state dependent production technology can emerge from aggregation of technologies per good and then aggregation of goods to total output; this, however, is only one step in that direction.<sup>3</sup> Instead, we entertain the hypothesis that if variations in risk aversion

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<sup>1</sup>Cochrane (2008) calls this defect of standard real business cycle models the divorce between asset pricing and macroeconomics.

<sup>2</sup>See, for example, Campbell and Cochrane (1999), Chan and Kogan (2002), Xiouros and Zapatero (2010), and Ehling and Heyerdahl-Larsen (2017).

<sup>3</sup>A practicable way to substitute productivity across states is through investing in different production technologies. In the Online appendix A, we provide a theoretical connection between the reduced-form approach

drive asset prices and macroeconomic quantities as in our model, then we should find empirical evidence for such a relation. Specifically, in the model, the representative firm chooses the endogenous productivity optimally, conditional on the exogenous time-varying risk aversion and the exogenous persistent productivity risk. In equilibrium, technology choice and time-varying risk aversion induce the conditional volatility of investments to vary. In contrast, otherwise comparable production-based asset pricing models such as Kaltenbrunner and Lochstoer (2010) typically do not produce variation in the conditional volatility of investment and, hence, cannot speak to our empirical finding that the estimated conditional volatility of investment growth is counter-cyclical.

Conditional on risk aversion, technology choice allows choosing the risk of the total factor productivity (TFP) growth. Hence, one could conjecture that an increase in risk aversion decreases the volatility of output and investment. However, in the data the volatility of the growth rate of output and investment increases in recessions. In our preferred specification of the model, the endogenous technology choice moves counter to a persistent productivity shock. This implies that investment declines less than in an economy without technology choice or that it even increases when facing a negative productivity shock. As shocks are persistent in the model, this can imply that investment first moves counter to a negative exogenous shock and only declines with a lag relative to investment in a benchmark economy. With an increase in risk aversion, investment reacts even more positively to a negative exogenous shock. Thereby, the endogenous technology choice increases the volatility of investment.

Inspecting the log-linear solution of our model, we see that technology choice directly depends on risk aversion. However, the Tallarini (2000) result still holds that there are no first-order effects of risk aversion on macroeconomic quantities. This is why we look for effects in conditional volatilities of macroeconomic quantities. Specifically, when risk aversion is time-varying, then the conditional volatility of investment evolves with risk aversion. When risk

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to technology choice that we adopt and investing in several technologies as in Jermann (2010). For example, it seems plausible, that the different technologies of generating electricity, e.g., coal, natural gas, nuclear, oil, solar, wind, etc., are broadly consistent with Jermann (2010) and, therefore, also with our reduced-form approach. Specifically, since each technology has its own risk characteristics combining them allows choosing the risk profile of energy generation.

aversion is constant in a variant of our model with technology choice, then the conditional volatility of investment is also constant. When there is no technology choice and no variations in risk aversion, then the model collapses to the model of Kaltenbrunner and Lochstoer (2010) with recursive preferences and an exogenous productivity that follows an AR(1) process. Calibrating the models, we see that they perform equally well on the chosen macroeconomic quantities and they all match the Sharpe ratio of the stock market.

Technology choice is governed by a parameter, which determines how costly it is to transform productivity. The parameter is pinned down by calibrating the model to the volatility of the risk-free rate, which the model without technology cannot match. Since in our preferred calibration technology choice and risk aversion move counter to an exogenous shock, it delays the reaction not only of investment but also of consumption to a shock. As a result, technology choice generates predictability in consumption growth. The predictability generates fluctuations in the risk-free rate but since it is short lived it does not affect the dividend and consumption claims. Consequently, technology choice increases the volatility of the risk-free rate and reduces the correlation of the risk-free rate with the price-dividend ratio, making it statistically indistinguishable from the correlation in the data.

Regressing excess stock market returns on the log price-dividend ratio, we see that the models with time-varying risk aversion and with and without technology choice produce predictability that is statistically indistinguishable from the data. Further, without targeting it, our model with technology choice and time-varying risk aversion reproduces the conditional volatility of investment and the correlation between the log price-dividend ratio of the stock market and the volatility of investment, but only with a high elasticity of intertemporal substitution (EIS). Finally, when regressing excess stock market returns on the conditional volatility of investment growth, we see that the model with time-varying risk aversion, technology choice, and high EIS produces predictability that is statistically indistinguishable from the data.

Our paper speaks to the literature that explores the asset pricing implications of production transformation across states or technologies. To allow for production transformation across states, Cochrane (1993) proposes to allow firms to choose state-contingent productivity en-

dogenously subject to a constraint set. In closely related works, Cochrane (1988) and Jermann (2010) back out the stochastic discount factor from producers' first-order conditions assuming complete technologies, i.e., that there are as many technologies as states of nature. In similar spirit, Belo (2010) applies state-contingent productivity to derive a pure production-based pricing kernel in a partial equilibrium setting, which gives rise to a macro-factor asset pricing model that explains the cross-sectional variation in average stock returns. The takeaway from these papers is that state-contingent technology can explain asset prices in both the time-series and the cross-section and that the way the economy substitutes productivity across states is related to asset prices, suggesting that risk aversion matters for the macroeconomy. However, these studies do not look at the joint implications of state-contingent technology for asset prices and the macroeconomy. Our paper fills this gap in the literature.<sup>4</sup>

Seminal contributions to the literature on investment- or production-based asset pricing include Jermann (1998) who introduces habit formation and capital adjustment costs and Boldrin, Christiano, and Fisher (2001) who introduce, in addition, two sectors in the standard real business cycle (RBC) model to explain the equity premium and the stock return volatility. Kaltenbrunner and Lochstoer (2010) introduce Epstein-Zin (EZ) preferences in the standard RBC model with capital adjustment costs, in which the persistence in capital generates long-run risk à la Bansal and Yaron (2004). Kaltenbrunner and Lochstoer (2010) explain the stock market Sharpe ratio, with high EIS, or also the stock market equity premium and stock return volatility, with low EIS. With high EIS, they also explain the stock market return volatility for a dividend claim that resembles the stock market dividends.<sup>5</sup> We build on Kaltenbrunner and Lochstoer (2010) by introducing technology choice and time-varying risk aversion. Time-varying risk-aversion generates excess return predictability and, through

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<sup>4</sup>In a contemporaneous contribution, Bretscher, Hsu, and Tamoni (2018) show that endogenous macroeconomic responses to uncertainty shocks are amplified through higher level of risk aversion, by taking into account higher orders in the perturbation method. Their paper differs from ours in that risk aversion affects macroeconomic quantities but firms still absorb shocks entirely instead of allowing for an active action to it through adjusting technology.

<sup>5</sup>By now, the literature on investment- or production-based asset pricing is vast. Recent contributions include Papanikolaou (2011), Gârleanu, Panageas, and Yu (2012), Ai, Croce, and Li (2013), Belo, Lin, and Bazdresch (2014), Croce (2014), Kung (2015), Kung and Schmid (2015), and Chen (2016) among many others; none of these works, however, study state-contingent technology.

that, explains the volatility of the price-dividend ratio of the dividend claim. With technology choice, the macroeconomy reacts to changes in risk-aversion by varying the volatility of output, investment, and consumption and, through that, links the macroeconomy to asset prices.

## 2 A macro-finance economy with state-contingent technology

Consider a representative agent who owns an all-equity representative firm, which uses productive capital to generate one real good and operates in discrete time with infinite horizon.

### 2.1 Firms

Let  $\Theta_t$  be the exogenous technological productivity level at time  $t$ . We assume that  $\log \Theta_t$  follows an AR(1) process with trend,

$$\log \Theta_{t+1} = \log Z_{t+1} + \phi (\log \Theta_t - \log Z_t) + \varepsilon_{t+1}, \quad \text{and} \quad \log Z_t = \mu t, \quad (1)$$

where  $|\phi| < 1$  and  $\varepsilon_{t+1} \sim N(0, \sigma^2)$  denotes the exogenous shock.

Departing from the standard production economy, we assume that the representative firm modifies the underlying productivity shocks. Following Cochrane (1993) and Belo (2010), at time  $t$ , a state-contingent technology  $\Omega_{t+1}$  is chosen through a CES aggregator

$$\mathbb{E}_t \left[ \frac{\Omega_{t+1}^{(1-\alpha)\nu}}{\Theta_{t+1}^{(1-\alpha)\nu}} \right] \leq 1, \quad (2)$$

where  $\mathbb{E}_t$  is the conditional expectation operator. In (2), the variable  $\alpha \in (0, 1)$  stands for the capital share in output and the curvature  $\nu$  captures the representative firm's technical ability to modify technology. When  $\nu < 1$ , increasing the volatility of  $\Omega_{t+1}$  also increases average productivity. For this reason, we assume that  $\nu > 1$ . With this assumption, as  $\nu$  increases, distorting the underlying shocks reduces average productivity. When  $\nu \rightarrow +\infty$ , it is infinitely

costly to modify the exogenous productivity. Therefore, we obtain  $\Omega_{t+1} = \Theta_{t+1}$ .

Appendix A provides intuition for the reduced-form approach in modeling technology choice. Briefly, we consider the reduced-form specification of technology choice as representing the risk-return trade-off of aggregate productivity, from choosing the mixture of different technologies. Deciding on how to allocate the aggregate capital to the various technologies and different mixtures imply different mean-variance characteristics for aggregate productivity, where higher risk leads to higher average productivity. Thus, we interpret the technology modifications set in (2) as a simple abstract form of modeling state-contingent technologies implying flexibility for optimal future productivity. More specifically, constraint (2) determines the representative firm's ability to trade off higher realizations of shocks in some states at time  $t + 1$  with lower realizations in other states. The optimal choice offsets the marginal benefit from smoothing consumption over time and states with the marginal cost of lower average productivity (or a tradeoff between static efficiency and flexibility similar to Mills and Schumann (1985)).

Output,  $Y_t$ , is given by

$$Y_t = K_t^\alpha \Omega_t^{1-\alpha}, \quad (3)$$

where  $K_t$  denotes the capital stock at the beginning of period  $t$ .

Capital accumulates according to

$$K_{t+1} = (1 - \delta)K_t + g_t, \quad (4)$$

where  $\delta$  is the depreciation rate and  $g_t$  stands for the capital formation function. We specify  $g$  as in Jermann (1998), i.e.,

$$g_t = \left[ \frac{a_1}{1 - 1/\chi} \left( \frac{I_t}{K_t} \right)^{1-1/\chi} + a_2 \right] K_t, \quad (5)$$

where  $I_t$  denotes investment at time  $t$ , the curvature  $\chi > 0$  governs capital adjustment costs, and  $a_1$  and  $a_2$  are constants. These specifications imply that capital adjustment costs are high when  $\chi$  is low and that capital adjustments are costless when  $\chi \rightarrow \infty$ . Following Boldrin,

Christiano, and Fisher (2001), we set  $a_1$  and  $a_2$  such that there is no cost to capital adjustment in the deterministic steady-state

$$a_1 = (e^\mu - 1 + \delta)^{1/\chi} \quad \text{and} \quad a_2 = \frac{1}{1 - \chi} (e^\mu - 1 + \delta).$$

## 2.2 Households

To separate the elasticity of intertemporal substitution (EIS) from risk aversion, we assume that the representative agent exhibits recursive preferences (Kreps and Porteus (1978), Epstein and Zin (1989, 1991), and Weil (1989)), whose utility at time  $t$  is represented by

$$U_t = \left\{ (1 - \beta) C_t^{1-1/\psi} + \beta \mathbb{E}_t [U_{t+1}^{1-\gamma_t}]^{\frac{1-1/\psi}{1-\gamma_t}} \right\}^{\frac{1}{1-1/\psi}}, \quad (6)$$

where  $0 < \beta < 1$  denotes the subjective time discount factor,  $C_t$  stands for aggregate consumption at time  $t$ ,  $\psi > 0$  represents the EIS, and  $\gamma_t$  denotes the state dependent relative risk aversion.

Every period the representative agent maximizes her utility (6) by choosing consumption  $C_t$  and investment  $I_t$  given the aggregate output  $Y_t = C_t + I_t$ . In addition, the agent chooses the productivity  $\Omega_{t+1}$  for every future state next period, given the conditional distribution of the exogenous productivity  $\Theta_{t+1}$  and according to the constraint (2).

The representative agent discounts consumption with her stochastic discount factor<sup>6</sup> given by

$$M_{t,t+1} = \beta \left[ \frac{C_{t+1}}{C_t} \right]^{-\frac{1}{\psi}} \left[ \frac{U_{t+1}^{1-\gamma_t}}{\mathbb{E}_t (U_{t+1}^{1-\gamma_t})} \right]^{\frac{\frac{1}{\psi} - \gamma_t}{1-\gamma_t}}. \quad (7)$$

Besides the macroeconomic quantities, we also study asset prices in the model with technology choice. Specifically, we compute the returns  $R_{f,t}$  on the risk-free asset, which pays one unit of consumption next period, and the returns  $R_{i,t}$  on real investment, which are equal to the returns on the aggregate consumption claim (Restoy and Rockinger (1994)). In addition,

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<sup>6</sup>See Melino and Yang (2003) and Dew-Becker (2014) for a stochastic discount factor with recursive preferences and time-varying risk aversion.



we study the returns on a risky stock with next period dividends,  $D_{t+1}$ , as follows

$$R_{s,t+1} = \frac{P_{t+1} + D_{t+1}}{P_t}, \quad (8)$$

where  $P_t$  denotes the price of the dividend claim at time  $t$ . Since, the properties of the dividends of the representative firm generated by the model differ from those of the dividends of the aggregate stock market in the data, we also price a claim to a dividend process. Following Kaltenbrunner and Lochstoer (2010), we assume that the growth in dividend, denoted by  $\Delta d_{t+1}$ , evolves according to

$$\Delta d_{t+1} = \mu + d_1 (\theta_t - c_t) + d_2 \epsilon_{t+1} + d_3 \epsilon_{t+1}^d, \quad (9)$$

where  $\theta_t$  and  $c_t$  denote log deviations of the exogenous productivity and consumption, respectively, from their steady states. Further,  $\epsilon_{t+1}^d$  is i.i.d.  $N(0, 1)$  and  $d_1, d_2, d_3$  are constant coefficients.

### 2.3 The equilibrium conditions

With recursive preferences, the current value Lagrangian function of the maximization problem with state-contingent technology can be written as

$$L_t = \left\{ (1 - \beta) C_t^{1-1/\psi} + \beta \mathbb{E}_t \left[ U_{t+1}^{1-\gamma} \right]^{\frac{1-1/\psi}{1-\gamma}} \right\}^{\frac{1}{1-1/\psi}} - \lambda_t^1 (C_t - K_t^\alpha \Omega_t^{1-\alpha} + I_t) - \lambda_t^2 [K_{t+1} - (1 - \delta)K_t - g_t] - \lambda_t^3 \left\{ \mathbb{E}_t \left[ \frac{\Omega_{t+1}^{(1-\alpha)\nu}}{\Theta_{t+1}^{(1-\alpha)\nu}} \right] - 1 \right\}, \quad (10)$$

where  $\lambda_t^1$ ,  $\lambda_t^2$ , and  $\lambda_t^3$  denote Lagrangian multipliers for the three constraints and where the social planner maximizes over  $C_t$ ,  $I_t$ ,  $K_{t+1}$ ,  $\Omega_{t+1}$  at each state next period.

Four sets of first-order conditions characterize equilibrium; the first two conditions are the first two constraints that appear in the Lagrangian in (10), that is, consumption equals

aggregate output minus investment and that capital accumulates according to (4).<sup>7</sup> The third condition characterizes the optimal amount of investment and the fourth is a set of conditions that determine the optimal productivity choice for every state next period.

The optimal amount of investment in period  $t$  is characterized by the marginal  $q$  condition,

$$\frac{1}{g_{I,t}} = \mathbb{E}_t \left[ M_{t,t+1} \left( \alpha \frac{Y_{t+1}}{K_{t+1}} + \frac{1 - \delta + g_{K,t+1}}{g_{I,t+1}} \right) \right], \quad (11)$$

where  $g_{I,t}$  and  $g_{K,t}$  are the partial derivatives of the capital formation function with respect to investment and capital, respectively, in period  $t$ . The left hand side of (11) shows the marginal cost of investment, which is the amount of investment required to generate a unit of productive capital. The right hand side of (11) describes the marginal benefit from an additional unit of capital, which stems from next period's marginal product of capital and the remaining marginal value of future capital stock. Thus, the firm optimally equates the marginal costs with the marginal benefits of investment. From this first-order condition, returns on an additional unit of investment are:

$$R_{i,t+1} = g_{I,t} \left( \alpha \frac{Y_{t+1}}{K_{t+1}} + \frac{1 - \delta + g_{K,t+1}}{g_{I,t+1}} \right). \quad (12)$$

In our model, the representative firm in a period  $t$  optimally chooses the productivity  $\Omega_{t+1}$  state-by-state for next period, which is given by

$$\left( \frac{\Omega_{t+1}}{\Theta_{t+1}} \right)^{(1-\alpha)\nu} = \frac{(M_{t,t+1} \Theta_{t+1}^{1-\alpha})^{\frac{\nu}{\nu-1}}}{\mathbb{E}_t \left[ (M_{t,t+1} \Theta_{t+1}^{1-\alpha})^{\frac{\nu}{\nu-1}} \right]}, \quad (13)$$

where the ratio on the left hand side is the transformation of the exogenous productivity.<sup>8</sup> Equation (13) describes the tradeoff embedded in the distribution of  $\Omega$ . On the one hand, it can be beneficial to increase productivity in states where the productivity is exogenously high and decrease it where the productivity is exogenously low. In this way, next period's average productivity is maximized since the cost of deviating from the exogenous productivity

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<sup>7</sup>The third constraint is automatically satisfied by the optimal productivity choice.

<sup>8</sup> $\Omega_{t+1}$  is a function of the state in period  $t$  and  $\Theta_{t+1}$ .

is a function of the ratio of transformation.<sup>9</sup> We see this from the case of CRRA preferences,  $\gamma_t = 1/\psi$ , and risk neutrality,  $\gamma_t = 0$ , where the stochastic discount factor is constant and, thus, cancels out from (13). As a result, the log optimal endogenous technology is proportional to the log exogenous productivity,

$$\log \Omega_{t+1} \propto \frac{\nu}{\nu - 1} \log \Theta_{t+1}. \quad (14)$$

On the other hand, when the representative agent is risk averse it is optimal to shift productivity to high “value” states, that is, states of high marginal utility  $M$ . Given the above tradeoff in the model with endogenous technology choice, it can be optimal to amplify or reduce exogenous volatility and it can be optimal to choose a positive or negative correlation between endogenous and exogenous productivity.

### 3 The log-linearized real economy

To understand the economic mechanism behind technology choice, we derive the log-linear approximation of the macroeconomic dynamics. Asset prices are then solved using a projection method utilizing the log-linear dynamics of the state vector. Appendix B contains proofs and additional details of the log-linearization.

The proposition below summarizes the log-linear economy in equilibrium, where lower-case letters denote percentage deviations from steady-state values of detrended variables.

**Proposition 1.** *The state vector of the economy is  $(k_t, \omega_t, \theta_t, \gamma_t)$ . However, percentage deviations of a macroeconomic variable  $x_t \in \{y_t, c_t, i_t, u_t\}$  do not depend on risk aversion,  $\gamma_t$ , and, thus, is given by*

$$x_t = x_k k_t + x_\omega \omega_t + x_\theta \theta_t$$

where expressions for the coefficients and the steady states are in Appendix B. All the coefficients

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<sup>9</sup>For example, a ten percent increase in productivity when  $\theta$  is high has the same cost as a ten percent increase in productivity when  $\theta$  is low. Therefore, increasing productivity when  $\theta$  is high and decreasing it when it is low maximizes average productivity.

are independent of the technology choice parameter  $\nu$  and risk aversion. The law of motion of the percentage deviations from the steady-state values of the state vector is given by the law of motion of  $\gamma_t$ , and

$$\begin{aligned}\theta_{t+1} &= \phi\theta_t + \epsilon_{t+1}, \\ k_{t+1} &= \frac{1-\delta}{e^\mu}k_t + \left(1 - \frac{1-\delta}{e^\mu}\right)i_t, \\ \omega_{t+1} &= \phi\theta_t + \sigma_\omega(\gamma_t)\epsilon_{t+1},\end{aligned}\tag{15}$$

where  $\epsilon \sim \mathbb{N}(0, \sigma^2)$ . The technology choice is represented by the sensitivity of the endogenous productivity to exogenous shocks, which is given by

$$\sigma_\omega(\gamma_t) = \frac{(1-\alpha)\nu - \frac{1}{\psi}c_\theta - (\gamma_t - \frac{1}{\psi})u_\theta}{(1-\alpha)(\nu-1) + \frac{1}{\psi}c_\omega + (\gamma_t - \frac{1}{\psi})u_\omega}.\tag{16}$$

Finally, substituting in the laws of motion of  $\omega_t$  and  $\theta_t$  we obtain

$$x_{t+1} = x_k k_{t+1} + \tilde{x}_\theta \theta_t + \sigma_x(\gamma_t)\epsilon_{t+1},\tag{17}$$

where  $\tilde{x}_\theta = \phi(x_\omega + x_\theta)$  and  $\sigma_x(\gamma_t) = \sigma_\omega(\gamma_t)x_\omega + x_\theta$  for  $x \in \{y, c, i, u\}$ .

In Proposition 1, sensitivities with respect to  $\omega$ , i.e.,  $x_\omega$ , represent sensitivities with respect to the current or actual level of productivity, whether this is endogenous, as in the case of technology choice, or exogenous, as in the standard RBC model. Whereas  $\theta$  affects the macroeconomy because it controls the expected productivity and the magnitude of the effects depend on the persistence of the exogenous shocks. If  $\phi = 0$ , then all sensitivities with respect to  $\theta$  are zero, i.e.,  $x_\theta = 0$  for all  $x \in \{y, c, i, u\}$ .

The above proposition formalizes that risk aversion has no first-order effects. This is not surprising, since risk aversion only affects the optimal relation between risk and return in a given period and log-linearization does not consider the second moments of the macroeconomic

variables.<sup>10</sup> Further, Tallarini (2000) shows that risk aversion has only small second-order effects on the dynamics of the macroeconomy.

### 3.1 Technology choice, $\sigma_\omega(\gamma_t)$

Technology choice allows to optimally choose productivity risk over one period through  $\sigma_\omega$ , which depends on risk aversion and drives the conditional second moments of the macroeconomic variables as shown by (17). The next corollary shows limit properties of technology choice.

**Corollary 1.** *The following are limit properties of technology choice:*

$$\lim_{\nu \rightarrow \infty} \sigma_\omega(\gamma_t) = 1 \quad \text{and} \quad \lim_{\gamma_t \rightarrow \infty} \sigma_\omega(\gamma_t) = -\frac{u_\theta}{u_\omega}.$$

When  $\sigma_\omega(\gamma_t)$  equals unity, there is no transformation in productivity and we recover the standard RBC model. When  $\sigma_\omega(\gamma_t) > 1$ , it is optimal to choose amplified shocks that comove with the underlying shocks, that is, it is optimal to shift productivity from low productivity states to high productivity states. When  $0 \leq \sigma_\omega(\gamma_t) \leq 1$ , it is optimal to choose less volatile shocks that comove with the underlying shocks and when  $-1 \leq \sigma_\omega(\gamma_t) \leq 0$ , it is optimal to choose less volatile shocks that move counter to the underlying shocks. It is even possible to have  $\sigma_\omega(\gamma_t) \leq -1$ , in which case technology choice not only more than offsets the underlying shocks but also amplifies them.

The representative firm shifts productivity across states depending on the tradeoff between maximizing average productivity and transferring productivity from low value states to high value states. On the one hand, the firm maximizes productivity by shifting it to states with high exogenous productivity, where the transformation cost is lower. This mechanism is driven

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<sup>10</sup>Log-linearizing the stochastic discount factor (7) gives:

$$\ln M_{t,t+1} = -\frac{1}{\psi}(c_{t+1} - c_t) - \left(\gamma_t - \frac{1}{\psi}\right)[u_{t+1} - \mathbb{E}_t(u_{t+1})].$$

Therefore, risk aversion  $\gamma_t$  only affects the price of risk.

by the terms  $(1 - \alpha)\nu$  and  $(1 - \alpha)(\nu - 1)$  in (16). When agents have risk-neutral CRRA utility ( $\gamma = 1/\psi = 0$ ) then  $\sigma_\omega$  takes the same value,  $\nu/(\nu - 1)$ , as the exact solution in (14). For this case, a lower  $\nu$  implies lower transformation cost and, thus, more productivity is shifted to high productivity states.

On the other hand, when agents are risk averse, the firm also wants to shift productivity to high value states. With  $\gamma_t > 1/\psi$  the value of a state decreases with consumption and the continuation utility, as shown in (7). Consequently, technology choice offsets some of the fluctuations in consumption and continuation utility coming from exogenous shocks and  $\sigma_\omega(\gamma_t)$  decreases with  $c_\theta$  and  $u_\theta$ . Naturally, the more sensitive is consumption and utility to exogenous shocks (higher  $c_\theta$  and  $u_\theta$ ) the larger is the optimal shift in productivity. This can be seen in the numerator of  $\sigma_\omega(\gamma_t)$ . As productivity shifts to low value states, consumption and the continuation utility increase in those states along with their value. Depending on the sensitivity of the value of a state to  $\omega_t$ , which is determined by  $c_\omega$  and  $u_\omega$ , optimal technology choice pushes  $\sigma_\omega(\gamma_t)$  toward zero, as shown by the denominator in (16). If  $\sigma_\omega(\gamma_t) = 0$ , then all one-period productivity risk is eliminated.

Regarding the cost of productivity transformation, we emphasize that the effects of  $c_\theta$ ,  $u_\theta$ ,  $c_\omega$ , and  $u_\omega$  depend on the cost of transformation,  $\nu$ . When  $\nu$  is high, less productivity is shifted through technology choice and  $\sigma_\omega(\gamma_t)$  is close to one. In the limit ( $\nu \rightarrow \infty$ ), we recover the standard RBC case with no shift in productivity.

### 3.2 Two implications

The technology choice model generates two implications that directly link the macroeconomy with asset prices. According to the model, the cost of technology choice ( $\nu$ ) controls the volatility of the risk-free rate. To see why, consider the expected growth rate in productivity, which is given by

$$\mathbb{E}_t(\log \Omega_{t+1} - \log \Omega_t) = \mu - \phi(1 - \phi)\theta_{t-1} + [\phi - \sigma_\omega(\gamma_{t-1})]\epsilon_t.$$

When the expected growth rate in productivity and, hence, output is high (low), the value of intertemporal substitution is high (low), which is reflected in a high (low) risk-free rate. More importantly, the larger is the fluctuations in the above expected growth rate the higher is the volatility of the risk-free rate, where the extent by which it fluctuates is determined by technology choice. Specifically, a decrease in the cost of technology choice (lower  $\nu$ ) amplifies technology choice and pushes  $\sigma_\omega(\gamma_t)$  further away from one, as inferred from:

$$\frac{\partial \sigma_\omega(\gamma_t)}{\partial \nu} = \frac{(1 - \alpha)[1 - \sigma_\omega(\gamma_t)]}{(1 - \alpha)(\nu - 1) + \frac{1}{\psi}c_\omega + (\gamma_t - \frac{1}{\psi})u_\omega}.$$

As a result, the lower is  $\nu$  the higher is the risk-free rate volatility. In the calibration, we use this property of the model to pin down the value of  $\nu$ .

The second implication stems from the fact that  $c_\theta$  and  $u_\theta$  are non-zero if  $\phi > 0$ . That is, the value of a state is not only determined by the actual (endogenous) level of productivity, but also by the level of  $\theta$ , because it determines expected future endogenous productivity. For this reason, even when risk aversion is infinite the optimal technology choice does not eliminate all productivity risk but results in  $\sigma_\omega(\gamma_t)$  being equal to  $-u_\theta/u_\omega$ , as shown in Corollary (1). All one-period risk is eliminated only when  $\phi = 0$ , in which case  $u_\theta = 0$ , and risk aversion is infinite.<sup>11</sup> Otherwise, it is optimal to more than offset exogenous productivity shocks, that is  $\sigma_\omega(\gamma_t)$  is negative. This is optimal when negative productivity shocks are very costly, because it allows building up capital as a response to such negative and persistent shocks. This can be seen from the fact that investment typically reacts negatively to  $\theta$  as inferred from  $i_\theta = -c_\theta C/I$ .<sup>12</sup>

The second implication, that is whether  $\sigma_\omega(\gamma_t)$  is positive or negative, relates to how  $\gamma_t$  affects the conditional volatilities of output and investment. From Proposition 1, we know that the conditional volatility of investment is given by the absolute value of  $\sigma_i(\gamma_t) = \sigma_\omega(\gamma_t)i_\omega + i_\theta$ , where  $i_\theta$  is typically negative; and the conditional volatility of output is given by the absolute value of  $\sigma_y(\gamma_t) = \sigma_\omega(\gamma_t)y_\omega$ , since  $y_\theta = 0$ . Further, the effect of risk aversion on technology

<sup>11</sup>This corresponds to the case of utility smoothing discussed in Backus, Routledge, and Zin (2013).

<sup>12</sup>We cannot rule out that  $c_\theta$  is negative but for all reasonable parameters used in the calibration, we see that  $c_\theta$  is positive.

choice is given by

$$\frac{\partial \sigma_\omega(\gamma_t)}{\partial \gamma_t} = - \frac{u_\theta + \sigma_\omega(\gamma_t)u_\omega}{(1 - \alpha)(\nu - 1) + \frac{1}{\psi}c_\omega + (\gamma_t - \frac{1}{\psi})u_\omega}.$$

The above expression is typically negative and largely independent of  $\sigma_\omega(\gamma_t)$  because  $u_\omega$  is much smaller than  $u_\theta$ . As a result of the above properties, when  $\sigma_\omega(\gamma_t)$  is positive an increase in  $\gamma_t$  leads to a decrease in the conditional volatility of output. In this case, the conditional volatility of investment may decrease or increase depending on the relative magnitudes of  $i_\omega$  and  $i_\theta$ .

When  $\sigma_\omega(\gamma_t)$  is negative, an increase in risk aversion leads to an increase in both output and investment volatility. In our calibrated model,  $\sigma_\omega(\gamma_t)$  is on average negative which generates a testable prediction, namely that it makes the conditional volatilities of investment and output counter-cyclical, for the relation between the macroeconomy and asset prices.

## 4 Solution method and asset prices

We solve the model numerically first by log-linearizing the economy. The risk-free rate is then obtained in closed form. The price-to-dividend ratios of the consumption claim and the dividend claim are solved numerically using a projection method. The relevant Euler condition for the dividend claim is given by

$$\frac{P_t}{D_t} = \mathbb{E}_t \left[ M_{t,t+1} e^{\Delta d_{t+1}} \left( 1 + \frac{P_{t+1}}{D_{t+1}} \right) \right], \quad (18)$$

and similarly for the consumption claim.

Starting with cash-flows, the following proposition presents the log consumption growth for the log-linearized approximation of the model's equilibrium.

**Proposition 2.** *Given the log-linear approximation of the equilibrium in Proposition 1, the log consumption growth is conditionally normal,  $\Delta c_{t+1} = \mu_t + \sigma_c(\gamma_t) \epsilon_{t+1}$ . Its conditional mean is given by*

$$\mu_t = \mu + \mu_k k_t + \mu_\theta \theta_{t-1} + \sigma_\mu(\gamma_t) \epsilon_t, \quad (19)$$



where  $\mu_k = \delta c_k(i_k - 1)$ ,  $\mu_\theta = \delta c_k i_\theta - c_\theta(1 - \phi)$ , and  $\sigma_m(\gamma_t) = \delta c_k \sigma_i(\gamma_t) + c_\theta - \sigma_c(\gamma_t)$ . The coefficients  $\sigma_c(\gamma_t)$  and  $\sigma_i(\gamma_t)$  are defined in (17).

The stochastic discount factor in (7) is also log-normally distributed in the log-linear approximation.

**Proposition 3.** *Given the log-linear approximation of equilibrium in Proposition 1, the log stochastic discount factor is conditionally normal:*

$$\log M_{t,t+1} = \log \hat{\beta} - \frac{1}{\psi} \mu_t - \sigma_m(\gamma_t) \epsilon_{t+1}, \quad (20)$$

where

$$\log \hat{\beta}(\gamma_t) = \log \beta + \frac{1}{2}(1 - \gamma_t) \left( \gamma_t - \frac{1}{\psi} \right) \sigma_u(\gamma_t)^2 \sigma^2, \quad (20a)$$

$$\sigma_m(\gamma_t) = \frac{1}{\psi} \sigma_c(\gamma_t) + \left( \gamma_t - \frac{1}{\psi} \right) \sigma_u(\gamma_t), \quad (20b)$$

where  $\mu_t$  is given in Proposition 2,  $\sigma_u(\gamma_t)$  is defined in (17), and  $\sigma$  denotes the standard deviation of the exogenous shock  $\epsilon$  defined in (1).

The price of risk is given by the absolute value of  $\sigma_m(\gamma_t)\sigma$ . The introduction of technology choice changes the sensitivities to the exogenous shock, the  $\sigma_x$ 's, which in turn affects the price of risk.<sup>13</sup> To see the effect of technology choice on the price of risk, we express  $\sigma_m(\gamma_t)$  in terms of  $\nu$ , that is, by substituting in the expressions of  $\sigma_c(\gamma_t)$  and  $\sigma_u(\gamma_t)$  as derived in Proposition 1:

$$\sigma_m(\gamma_t) = \frac{(\nu - 1)A_\theta(\gamma_t) + \nu A_\omega(\gamma_t)}{\nu - 1 + A_\omega(\gamma_t)/(1 - \alpha)}, \quad A_x(\gamma_t) = \frac{1}{\psi} c_x + \left( \gamma_t - \frac{1}{\psi} \right) u_x, \quad x \in \{\theta, \omega\}. \quad (21)$$

Taking the first derivative with respect to  $\nu$  gives

$$\frac{\partial \sigma_m(\gamma_t)}{\partial \nu} \propto \frac{A_\theta(\gamma_t) + A_\omega(\gamma_t)}{1 - \alpha} - 1, \quad (22)$$

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<sup>13</sup>In the absence of technology choice, both  $\sigma_u$  and  $\sigma_c$  are independent of  $\gamma_t$ , as shown by Proposition 1.

which for reasonable parameters is positive. This implies that the more flexible is technology choice (the lower is  $\nu$ ), the lower is the price of risk. In fact, at the lowest possible value ( $\nu = 1$ ), we obtain  $\sigma_m(\gamma_t) = 1 - \alpha$  in which case the price of risk becomes very low and risk aversion does not affect it.

Despite the effect of technology choice on the price of risk, the technology choice model is on the same footing as the standard RBC model in fitting the price of risk. The reason is as follows: The unconditional price of risk at the steady state, is roughly determined by  $\sigma_c$  and  $\sigma_u$ . The coefficient  $\sigma_c$  is pinned down by the unconditional volatility of consumption growth and the technology choice model can fit it. The coefficient  $\sigma_u$  is determined by the long-run volatility of  $\mu_t$ .<sup>14</sup> From Proposition 2 it follows that technology choice affects only  $\sigma_\mu$ , but this coefficient has a negligible effect on the long-run volatility of  $\mu_t$ . As a result, both the technology choice and the standard RBC model can match the price of risk.

Moving on to asset prices, our focus is on the one-period interest rate or risk-free rate and the price of the stock, which is a claim to the dividend stream defined in (9). The following proposition provides the expression for the risk-free rate and an approximate expression of the log price-dividend ratio of the dividend claim.

**Proposition 4.** *Given the stochastic discount factor in Proposition 3, the continuously compounded one-period risk-free rate is*

$$r_{f,t} = -\log \hat{\beta}(\gamma_t) + \frac{1}{\psi} \mu_t - \frac{1}{2} \sigma_m(\gamma_t)^2 \sigma^2. \quad (23)$$

*The log-linear approximation of the stock price-dividend ratio, assuming an AR(1) process for*

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<sup>14</sup>The log-linear approximation of the utility is given as follows:

$$u_t = c_t + \sum_{\tau=1}^{\infty} \hat{\beta}^\tau (\mu_{t+\tau} - \mu),$$

where  $\hat{\beta} = \beta e^{\mu(1-1/\psi)}$  and  $\mu_{t+\tau} = \mathbb{E}_t(c_{t+\tau+1} - c_t)$ .

$\gamma_t$ , is given by

$$p_t - d_t \approx \overline{p - d} + \mathbb{E}_t \sum_{\tau=0}^{\infty} \hat{J}^\tau \xi_{t+\tau}, \quad (24)$$

where  $\xi_t$  is a zero mean variable given by

$$\xi_t = d_1(\theta_t - c_t) - \frac{1}{\psi}(\mu_t - \mu) + \left[ \xi_1(\gamma)(\gamma_t - \gamma) + \frac{1}{2}\xi_2(\gamma)(\gamma_t - \gamma)^2 \right] \sigma^2. \quad (25)$$

All expressions above are defined in Appendix C.

The level of the risk-free rate is principally determined by the subjective discount factor  $\beta$  and the EIS  $\psi$ . For standard parameters, the risk-free rate is driven mainly by fluctuations in expected consumption growth  $\mu_t$ . That is, fluctuations in risk aversion are typically not quantitatively important for the volatility of the risk-free rate. What is important, as already discussed, is technology choice. Qualitatively, for reasonable parameters  $\sigma_\mu(\gamma_t)$  decreases with  $\nu$ , which implies that a more flexible technology choice makes  $\mu_t$  more volatile. Quantitatively, we expect that with technology choice one can match the volatility of the risk-free rate in the data.

A fundamental asset pricing quantity is the stock return volatility or the volatility of the price-dividend ratio of the stock market. Proposition 4 provides an approximate analytical expression of how the log price-dividend ratio of the dividend claim varies with the state of the economy. From the proposition, we see that three factors drive the  $p - d$  ratio: (i) the expected consumption growth  $\mu_t$ , which drives the risk-free rate, (ii) expected cash-flow growth driven by  $(\theta_t - c_t)$ , and (iii) risk aversion. The first factor, namely  $\mu_t$ , is important only when intertemporal substitution is quite inelastic ( $\psi$  being very low). Further, only the persistent fluctuations of  $\mu_t$  coming from  $k_t$  and  $\theta_{t-1}$ , in equation 19, matter since the fluctuations coming from  $\epsilon_t$  are short lived. For this reason, technology choice has negligible effect on the volatility of the  $p - d$  ratio.

The dividend growth driven by  $(\theta_t - c_t)$  is potentially important for the fluctuations of

the log price-dividend ratio if  $d_1$  is large enough. Effectively, a positive shock to exogenous productivity that leads to a significant and persistent shock to dividend growth also leads to a substantial increase in the stock price relative to dividends.

Finally, the introduction of time-variation in risk aversion can augment the stock market volatility, principally through the linear term  $\xi_1(\gamma)(\gamma_t - \gamma)$ . In consumption based asset pricing models, e.g. Campbell and Cochrane (1999), an increase in risk aversion typically leads to a decrease in prices because it increases the price of risk and, consequently, the expected returns as well. In our model, this requires  $\xi_1(\gamma)$  to be negative. However, many factors and parameters determine the sign and magnitude of  $\xi_1(\gamma)$ . For example, an EIS lower than 1 and low dividend risk (low  $d_2$ ) may lead to a positive  $\xi_1(\gamma)$ . Yet, in our preferred calibration it is negative and large in absolute value. Thus, the time-varying risk aversion introduces substantial fluctuations in the stock market.

## 5 The calibrated economy with technology choice and time-varying risk aversion

In this section, we present models with and without technology choice and with and without time-varying risk aversion. After presenting the calibration of the models, our discussion is centered on the unique feature of the model with technology choice and time-varying risk aversion. Namely, the cyclical evolution of the conditional volatility of investment and its ability to predict asset pricing moments. Without technology choice or without time-varying risk aversion, the conditional volatility of macroeconomic quantities is constant.

### 5.1 Calibration

Table 1 shows values for the parameters that are common across models. Specifically, the productivity mean growth rate ( $\mu$ ) is 0.4%, persistence of productivity shocks ( $\phi$ ) is 0.9999, the capital share ( $\alpha$ ) is 0.36, and the quarterly depreciation rate ( $\delta$ ) is 2.1%. When relative

Table 1: Common model parameters - quarterly frequency

Description	Parameter	Value
Exogenous productivity mean growth rate	$\mu$	0.4%
Persistence of exogenous productivity shocks	$\phi$	0.9999
Output capital share	$\alpha$	0.36
Capital depreciation rate	$\delta$	2.1%
(Mean) coefficient of relative risk aversion	$\gamma$	5.0
Elasticity of intertemporal substitution	$\psi$	1.50
Market dividend growth exposure to $\theta_t - c_t$	$d_1$	0.055
Market dividend growth exposure to exogenous shocks	$d_2$	0.70
Market dividend growth idiosyncratic volatility	$d_3$	5.7%

risk aversion ( $\gamma$ ) is constant we set it to 5 and the EIS is 1.5. All these parameter values are taken from Kaltenbrunner and Lochstoer (2010). When risk aversion varies, we set its mean to 5. Table 1 also shows the exposure of the exogenous market dividends to growth ( $d_1$ ), exogenous shocks ( $d_2$ ), and idiosyncratic volatility ( $d_3$ ), respectively, that are calibrated to the historical moments of aggregate stock market dividends as well as the contemporaneous correlation with consumption growth. These parameter values slightly differ from the ones used in Kaltenbrunner and Lochstoer (2010) as our data covers a longer period.

Table 2 shows values for the parameters that vary across models. TCV is our model with technology choice and time-varying risk aversion. NTCV stands for no technology choice with time-varying risk aversion. TCC denotes an economy with technology choice and constant risk aversion. NTCC is the no technology choice and constant risk aversion benchmark corresponding to model LRR II in Kaltenbrunner and Lochstoer (2010), albeit with slightly different parameters.

For each model, we determine the remaining parameter values by matching moments of the data. Specifically, we use the subjective discount factor ( $\beta$ ) to match the mean of the risk-free rate, the capital adjustment cost parameter ( $\chi$ ) to match the ratio of the volatility of consumption growth to the volatility of output growth, the technology choice parameter ( $\nu$ ) to match the volatility of the risk-free rate, and the volatility of exogenous productivity shocks

Table 2: Calibrated model parameters

TCV is the model with technology choice and time-varying risk aversion, NTCV is the model without technology choice but with time-varying risk aversion, TCC is an economy with technology choice and constant risk aversion, and NTCC is the no technology choice and constant risk aversion benchmark.

Description	Parameter	Values			
		TCV	NTCV	TCC	NTCC
Subjective discount factor	$\beta$	0.9991	0.9981	0.9991	0.9981
Capital adjustment cost parameter	$\chi$	12.4	1000	12.5	1500
Technology choice parameter	$\nu$	6.5	$\infty$	6.0	$\infty$
Volatility of exogenous productivity shocks	$\sigma$	4.17%	4.46%	4.21%	4.46%
Linear coefficient of risk aversion	$\eta_1$	3.15	2.45		
Quadratic coefficient of risk aversion	$\eta_2$	0.482	0.252		
		Averages across simulations			
Mean of relative risk aversion		5.18	5.11		
Standard deviation of relative risk aversion		0.97	0.81		
Minimum of relative risk aversion		3.34	3.55		
Maximum of relative risk aversion		7.30	6.84		

( $\sigma$ ) to match the volatility of consumption growth. Risk aversion evolves according to

$$\gamma_t = \gamma - \lambda(\theta_t) \theta_t, \quad \lambda(\theta_t) = \eta_1 - \eta_2 \theta_t, \quad (26)$$

where we set the linear coefficient  $\eta_1$  to match the volatility of stock market returns and the quadratic coefficient  $\eta_2$  to bound  $\gamma_t$  away from zero.

## 5.2 Performance of the models vis-à-vis the data

We evaluate the performance of the models with respect to quarterly data in Table 3. Columns 2 and 3 show the mean estimate of standard macro-finance variables from the data along with their standard errors (*s.e.*), which are Newey and West (1987) corrected with 24 lags. For each model, we report corresponding averages obtained from 1000 simulated paths with 300 quarters, where we use a burn-in of 100 quarters. The parentheses next to the model statistics show the *t*-statistics ( $t - st$ ) of the hypotheses that the data estimates are generated from the model averages.

From Table 3, we see that we cannot reject the hypotheses that the consumption growth, the volatility of consumption growth, the first autocorrelation of consumption growth, and the

Table 3: Calibrated models versus data

$\Delta x$  denotes the first-difference of the natural logarithm of a variable  $X$ .  $y$  denotes (the natural logarithm of) total output;  $c$  denotes total consumption;  $i$  denotes total investment. For a variable  $x$ ,  $\sigma(x)$  denotes its volatility;  $ac_1(x)$  is its first-order autocorrelation and  $\rho(x, z)$  is its correlation with variable  $z$ . The data are described in Appendix D. The parentheses next to the data estimates show the standard errors (*s.e.*), which are Newey and West (1987) corrected with 24 lags. TCV is the model with technology choice and time-varying risk aversion, NTCV is the model without technology choice but with time-varying risk aversion, TCC is an economy with technology choice and constant risk aversion, and NTCC is the no technology choice and constant risk aversion benchmark. The model statistics are averages of 1000 simulated paths of 300 quarters with a burn-in of 100 quarters. The parentheses next to the model statistics show the  $t$ -statistics ( $t - st$ ) of the hypotheses that the data estimates are generated from model averages. Macroeconomic and dividend data are annual. The corresponding data from the models are time aggregated. All other statistics are quarterly.

	Data		TCV		NTCV		TCC		NTCC	
	<i>est.</i>	<i>s.e.</i>	<i>avg.</i>	<i>t - st</i>	<i>avg.</i>	<i>t - st</i>	<i>avg.</i>	<i>t - st</i>	<i>avg.</i>	<i>t - st</i>
$\mu(\Delta c)$	1.74	(0.38)	1.58	(0.41)	1.58	(0.42)	1.58	(0.41)	1.58	(0.42)
$\sigma(\Delta c)$	2.70	(0.54)	2.70	(0.00)	2.70	(0.00)	2.70	(0.01)	2.70	(0.00)
$ac_1(\Delta c)$	0.48	(0.07)	0.44	(0.58)	0.53	(0.68)	0.46	(0.34)	0.53	(0.69)
$\sigma(\Delta c)/\sigma(\Delta y)$	0.55	(0.06)	0.55	(0.04)	0.55	(0.15)	0.55	(0.05)	0.55	(0.14)
$\sigma(\Delta i)/\sigma(\Delta y)$	2.71	(0.14)	1.86	(6.10)	1.94	(5.53)	1.87	(6.04)	1.94	(5.53)
$\mu(R_f)$	0.14	(0.15)	0.14	(0.02)	0.21	(0.44)	0.12	(0.16)	0.22	(0.49)
$\sigma(R_f)$	0.84	(0.10)	0.83	(0.04)	0.27	(5.60)	0.85	(0.09)	0.24	(5.82)
$\mu(R_i - R_f)$			0.19		0.51		0.35		0.37	
$\sigma(R_i)$			1.32		3.07		1.66		2.21	
$SR_i$			0.21		0.17		0.24		0.17	
$\sigma(\Delta d)$	11.10	(2.12)	11.03	(0.03)	11.21	(0.05)	11.06	(0.02)	11.21	(0.05)
$ac_1(\Delta d)$	0.18	(0.14)	0.27	(0.59)	0.27	(0.64)	0.27	(0.60)	0.27	(0.64)
$\rho(\Delta c, \Delta d)$	0.52	(0.15)	0.49	(0.21)	0.53	(0.11)	0.49	(0.16)	0.53	(0.11)
$\mu(p - d)$	4.79	(0.10)	4.79	(0.01)	5.13	(3.27)	5.03	(2.31)	5.32	(5.11)
$\sigma(p - d)$	0.44	(0.05)	0.34	(1.96)	0.29	(2.89)	0.10	(6.21)	0.10	(6.34)
$\rho(p - d, r_f)$	0.03	(0.17)	0.31	(1.64)	0.85	(4.80)	0.40	(2.19)	1.00	(5.70)
$\mu(R_m - R_f)$	2.04	(0.39)	1.97	(0.19)	1.53	(1.30)	1.24	(2.03)	1.01	(2.61)
$\sigma(R_m)$	11.16	(2.21)	11.19	(0.01)	11.18	(0.01)	7.82	(1.51)	8.32	(1.29)
$SR_m$	0.18	(0.05)	0.18	(0.02)	0.14	(0.91)	0.16	(0.47)	0.12	(1.23)

ratio of the volatility of consumption growth to the volatility of output growth are generated by the four models. To the contrary, we reject the hypotheses that the ratio of the volatility of investment growth to the volatility of output growth are generated by the four models. All the models replicate the level of the risk-free rate but only for the two models with technology choice, TCV and TCC, we cannot reject the null hypothesis that volatility of the risk-free rate is generated by either TCV or TCC.

Table 3 also shows the expected excess return on investments, the volatility of investment returns, and the Sharpe ratio of investment returns for the models. We see that the model with technology choice and time-varying risk aversion has the lowest quarterly return but also has the lowest volatility of investment returns. Hence, its investment based Sharpe ratio is not only comparable to the ones of the other models but is second only to the one of the model with technology choice with constant risk aversion. Further, we see that there is no difference between the calibrated aggregate stock market dividends across the models.

Turning to the statistics on the log price-dividend ratio, we see that our model perfectly matches the mean estimate of the log price-dividend ratio of the data without targeting it in the calibration. In addition, our model is the only one where we cannot reject the null hypothesis that the data is generated by the model. The model with technology choice and time-varying risk aversion produces the largest volatility for the log price-dividend ratio. Nevertheless, in this case the null hypotheses is rejected for all models.<sup>15</sup> In the data, the correlation between the log price-dividend ratio and the risk-free rate is basically zero. It is in general very difficult for asset pricing models to produce such a low correlation. For example the model of Kaltenbrunner and Lochstoer (2010) without technology choice and constant risk aversion in our calibration produces a correlation between these two quantities of 1. In our model, the correlation between the log price-dividend ratio and the risk-free rate turns out to be 0.31 and only for this model, we cannot reject the null hypothesis.

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<sup>15</sup>Although the null is rejected for the TCV model its volatility of the log price-dividend ratio is 0.34 while the NTCC model produces a volatility of 0.10. In addition, Boudoukh, Michaely, Richardson, and Roberts (2007) show that in their sample the volatility of the log price-dividend ratio declines from 0.41 to roughly 0.30 with share repurchases, which is closer to what our TCV model generates.



Why is the correlation between the log price-dividend ratio and the risk-free rate low in our model with technology choice and time-varying risk aversion? It is because in our calibration technology choice and risk aversion move counter to an exogenous shock thereby delaying any impact of the shock on investment and consumption. This produces predictability in consumption growth, which generates fluctuations in the risk-free rate but since the predictability is short lived it does not drive the price of a claim on dividends or consumption. Thus, technology choice increases the volatility of the risk-free rate and reduces the correlation of the risk-free rate with the price-dividend ratio.

Next, we discuss the performance of the models on pricing the exogenous aggregate dividend stream. In the data, the quarterly excess return on the aggregate stock market is 2.04. In the models, this return ranges from 1.01 to 1.97. While the range of the excess returns across the models is large, this difference is not due to technology choice. As pointed out in Kaltenbrunner and Lochstoer (2010) the equity premium in the production based asset pricing model with long-run risk is quite sensitive to the parameters. Specifically, using parameter values for the benchmark economy that are more similar to ones in the technology choice model with time-varying risk aversion leads to an excess stock market return of 1.97.

However, with such parameters the model without technology choice and with constant risk aversion performs less well than shown in Table 3 on the targeted moments of aggregate consumption and the risk-free rate. Hence, overall there is no economically significant difference between the four models on their ability to replicate stock market moments. This point is supported by the models' stock market volatility and Sharpe ratio since we cannot reject the null hypothesis for both variables in all four cases. In addition, since stock market volatilities in all models are close enough to the data it is not surprising that there is little variation in risk aversion in the TCV model, which ranges from 3.34 to 7.30. Nevertheless, the time-varying risk aversion does increase the volatility from roughly 8% per quarter to 11.19% (TCV) or 11.18% (NTCV) per quarter, which perfectly matches the data.

We have also calibrated four economies with low EIS. The calibrated parameter values and the performance of those models vis-à-vis the data are summarized in Table 7 and 8 in the

Appendix E. Overall the low EIS cases do perform about equally well with respect to the empirical moments shown in Table 3 although they require quite high levels of risk aversion and subjective discount factor.

From the predictive regressions in Table 4, we see that only the models with time-varying risk aversion generate a realistic level for the standardized regression coefficient of the log price-dividend ratio. Specifically, in the data the absolute level of the coefficient increases from 0.13 at one quarter to 0.53 at 28 quarters with  $t$ -statistics ranging from 3.03 to 3.75. The TCV and NTCV models generate about half of the predictability in the data and this for all horizons. Further, we cannot reject the null hypothesis that the data are generated by either the TCV or NTCV model. In both models with constant risk aversion the regression coefficients are about one third of the ones in the models with time-varying risk aversion and the null is rejected for every single regression coefficient. From Table 9 in the Appendix E we learn that these results reproduce also when we use a low EIS.

In summary, the model without technology choice has difficulty simultaneously matching the targeted moments discussed above along with the volatility of the risk-free rate, the moments of the log price-dividend ratio, and the correlation between the risk-free rate and the log price-dividend ratio. In particular, only the model with technology choice and time-varying risk aversion reproduces the correlation between the risk-free rate and the log price-dividend ratio. In addition, the model without time-varying risk aversion has difficulty in producing predictability in excess returns by the log price-dividend ratio.

### 5.3 Testing the model

After establishing that the model with technology choice and time-varying risk aversion reproduces standard macroeconomic and asset pricing moments at least as well as a benchmark model à la Kaltenbrunner and Lochstoer (2010), we now turn to empirical evidence on the unique features of our model.

Here, we explore the ability of the model with technology choice and time-varying risk

Table 4: Excess return predictability by  $p - d$

The table shows the standardized regression coefficient on the log price-dividend ratio from a standard predictive regression using the stock market excess return for horizons from one quarter to 28 quarters. The data are described in Appendix D. The  $t$ -statistics ( $t - st$ ) are Newey and West (1987) corrected with 24 lags. TCV is the model with technology choice and time-varying risk aversion, NTCV is the model without technology choice but with time-varying risk aversion, TCC is an economy with technology choice and constant risk aversion, and NTCC is the no technology choice and constant risk aversion benchmark. The model statistics are averages of 1000 simulated paths of 300 quarters with a burn-in of 100 quarters. The parentheses below the regression coefficients for the models show the  $t$ -statistics ( $t - st$ ) of the hypotheses that the data estimates are generated from model averages. The data have quarterly frequency starting in 1947.

	$\rho(p - d_t, \sum_{s=1}^{\tau} [R_{m,t+s} - R_{f,t+s-1}])$									
	$\tau = 1$	$\tau = 2$	$\tau = 4$	$\tau = 8$	$\tau = 12$	$\tau = 16$	$\tau = 20$	$\tau = 24$	$\tau = 28$	
Data	-0.13	-0.19	-0.26	-0.35	-0.39	-0.42	-0.47	-0.50	-0.53	
$t - st$	(3.03)	(3.33)	(3.34)	(3.23)	(2.97)	(2.81)	(3.20)	(3.52)	(3.75)	
TCV	-0.07	-0.10	-0.14	-0.19	-0.23	-0.27	-0.30	-0.32	-0.34	
$t - st$	(1.38)	(1.55)	(1.53)	(1.40)	(1.19)	(1.03)	(1.17)	(1.29)	(1.34)	
NTCV	-0.07	-0.10	-0.14	-0.20	-0.24	-0.27	-0.30	-0.32	-0.34	
$t - st$	(1.35)	(1.52)	(1.50)	(1.37)	(1.17)	(1.00)	(1.15)	(1.26)	(1.31)	
TCC	-0.02	-0.03	-0.05	-0.06	-0.08	-0.09	-0.10	-0.11	-0.11	
$t - st$	(2.48)	(2.74)	(2.75)	(2.63)	(2.38)	(2.22)	(2.53)	(2.76)	(2.93)	
NTCC	-0.02	-0.03	-0.05	-0.06	-0.08	-0.09	-0.10	-0.11	-0.11	
$t - st$	(2.47)	(2.73)	(2.73)	(2.62)	(2.38)	(2.22)	(2.52)	(2.76)	(2.93)	

Table 5: Conditional volatilities

For a variable  $x$ ,  $\sigma(\sigma_x)/\mu(\sigma_x)$  denotes its volatility of volatility normalized by the mean of volatility;  $ac_1(\sigma_x)$  is its first-order autocorrelation and  $\rho(\sigma_x, z)$  is its correlation with variable  $z$ . The data are described in Appendix D. The parentheses next to the data estimates show the standard errors (*s.e.*), which are Newey and West (1987) corrected with 24 lags. The conditional volatility series of output, consumption, and investment in the data are obtained by fitting an ARMA(1,1)-EGARCH(1,1) to each growth rate series. In the constrained estimation for the process for volatility (EGARCH), we set the AR(1) coefficient equal to 0.9999 to mimic the persistence of the exogenous shocks in the model economies. Both the high and low EIS economies correspond to the TCV model, which is the model with technology choice and time-varying risk aversion. The model statistics are averages of 1000 simulated paths of 300 quarters with a burn-in of 100 quarters. The parentheses below the regression coefficients for the models show the  $t$ -statistics ( $t-st$ ) of the hypotheses that the data estimates are generated from model averages. The data have quarterly frequency starting in 1947.

	Data				High EIS			Low EIS		
	constrained		unconstrained		<i>avg.</i>	$t-st$	$t-st$	<i>avg.</i>	$t-st$	$t-st$
	<i>est.</i>	<i>s.e.</i>	<i>est.</i>	<i>s.e.</i>		(con.)	(unc.)		(con.)	(unc.)
$\sigma(\sigma_y)/\mu(\sigma_y)$	0.32	(0.04)	0.32	(0.02)	0.39	(2.03)	(3.60)	0.47	(4.09)	(7.05)
$\sigma(\sigma_c)/\mu(\sigma_c)$	0.43	(0.06)	0.35	(0.04)	0.42	(0.15)	(1.67)	0.17	(4.53)	(4.41)
$\sigma(\sigma_i)/\mu(\sigma_i)$	0.37	(0.06)	0.30	(0.05)	0.35	(0.39)	(0.91)	0.59	(3.81)	(5.43)
$ac(\sigma_y)$	0.98	(0.01)	0.89	(0.02)	0.97	(1.23)	(3.19)	0.94	(4.15)	(2.00)
$ac(\sigma_c)$	0.97	(0.02)	0.93	(0.02)	0.97	(0.33)	(1.64)	0.95	(1.39)	(0.87)
$ac(\sigma_i)$	0.98	(0.02)	0.98	(0.02)	0.97	(0.82)	(0.34)	0.92	(4.10)	(3.33)
$\rho(p-d, \sigma_y)$	-0.75	(0.07)	-0.59	(0.07)	-0.46	(4.16)	(1.70)	0.88	(23.31)	(19.94)
$\rho(p-d, \sigma_c)$	-0.82	(0.04)	-0.76	(0.05)	0.31	(27.20)	(20.35)	0.94	(42.47)	(32.37)
$\rho(p-d, \sigma_i)$	-0.65	(0.08)	-0.67	(0.08)	-0.65	(0.06)	(0.22)	0.61	(15.83)	(16.68)

aversion to replicate the conditional volatility of output, consumption, and investment.<sup>16</sup> We do this for the high and low EIS cases.

From Table 5, we see that technology choice model with time-varying risk aversion and high EIS does a fairly good job in replicating the fluctuations in the conditional volatilities of output, consumption, and investment. Specifically, in the data these standardized volatilities are 0.32, 0.43, and 0.37 in the constrained estimation while in our model with high EIS these are 0.39, 0.42, and 0.35, respectively. The conditional volatilities behave similarly in the unconstrained estimation and we cannot reject the null hypothesis that the constrained and unconstrained data are generated by the technology choice model with time-varying risk aversion and high EIS. For the model with low EIS all null hypotheses are rejected. In addition, the first-order autocorrelation of these three macroeconomic volatilities are very close to the data. Only for the

<sup>16</sup>In the other models, the conditional volatilities are constant since we obtain time-varying conditional volatilities only if  $\sigma_\omega$  is time-varying.

unconstrained autocorrelation of the volatility of output do we reject the null. For the model with low EIS only the null hypotheses for the autocorrelation of the volatility consumption are not rejected.

We emphasize the correlations between the log price-dividend ratio and the conditional volatilities of output, consumption, and investment in Table 5. While these results are mixed, they have important implication for a relation between current macroeconomic quantities and expected excess returns. First, we see that in the data the correlation between the log price-dividend ratio and the volatility of investment is either -0.65 or -0.67, depending on whether we constrain the estimation or not. Second, in the technology choice model with time-varying risk aversion and high EIS the average is -0.65. For the economy with low EIS the average is 0.61. From this, we expect that in predictive regressions with the conditional volatility of investment the high EIS model produces a sign that is in line with the data while the model with low EIS produces the wrong sign for the regression coefficient. Further, although the model with high EIS does fairly well on the correlation between the log price-dividend ratio and the volatility of output, its average is a bit lower than in the data. Consequently, the consumption volatility is not only lower than what we see in the data, but also slightly positive instead of negative.

Finally, we discuss the predictive regression analysis in Table 6. We start by establishing that the conditional volatility of investment predicts excess stock market returns at least as well as the log price-dividend ratio. The standardized regression coefficients at both the short horizon of one quarter and the long horizon of 24 quarters are basically identical to the ones using the log price-dividend ratio and this is independent of how we estimate the process for volatility. The  $t$ -statistics ( $t - st$ ), which are Newey and West (1987) corrected with 24 lags, range from 2.5 to 4. Turning to our technology choice model with time-varying risk aversion and high EIS, we see that the model produces about half of the predictability at all horizons, which is consistent with the evidence in Table 4. Here, however, the slope of the standardized regression coefficients over the regression horizon is not steep enough and, thus, starting from 12 quarters on we do reject the null hypotheses. Even then Table 6 strongly supports our model and rules out a specification with low EIS.

Table 6: Excess return predictability by conditional investment volatilities

The table shows the standardized regression coefficient on the conditional volatility of investment from a standard predictive regression using the stock market excess return for horizons from one quarter to 28 quarters. The data are described in Appendix D. The  $t$ -statistics ( $t - st$ ) are Newey and West (1987) corrected with 24 lags. The conditional volatility series of output, consumption, and investment in the data are obtained by fitting an ARMA(1,1)-EGARCH(1,1) to each growth rate series. In the constrained estimation for the process for volatility (EGARCH), we set the AR(1) coefficient equal to 0.9999 to mimic the persistence of the exogenous shocks in the model economies. Both the high and low EIS economies correspond to the TCV model, which is the model with technology choice and time-varying risk aversion. The model statistics are averages of 1000 simulated paths of 300 quarters with a burn-in of 100 quarters. The parentheses below the regression coefficients for the models show the  $t$ -statistics ( $t - st$ ) of the hypotheses that the data estimates are generated from model averages. The data have quarterly frequency starting in 1947.

	$\sigma_i: \rho(\sigma_{i,t}, \sum_{s=1}^{\tau} [R_{m,t+s} - R_{f,t+s-1}])$								
	$\tau = 1$	$\tau = 2$	$\tau = 4$	$\tau = 8$	$\tau = 12$	$\tau = 16$	$\tau = 20$	$\tau = 24$	$\tau = 28$
Data (con.)	0.11	0.15	0.21	0.31	0.39	0.43	0.45	0.49	0.48
$t - st$	(2.53)	(2.47)	(2.53)	(2.71)	(2.94)	(3.01)	(3.30)	(3.57)	(3.71)
Data (unc.)	0.12	0.15	0.21	0.32	0.41	0.45	0.48	0.52	0.52
$t - st$	(2.61)	(2.51)	(2.57)	(2.76)	(3.04)	(3.14)	(3.50)	(3.86)	(4.06)
High EIS	0.04	0.06	0.09	0.12	0.15	0.17	0.19	0.21	0.22
$t - st$ (con.)	(1.55)	(1.44)	(1.48)	(1.65)	(1.82)	(1.82)	(1.91)	(2.07)	(2.00)
$t - st$ (unc.)	(1.62)	(1.47)	(1.52)	(1.71)	(1.94)	(1.96)	(2.11)	(2.34)	(2.32)
Low EIS	-0.05	-0.07	-0.10	-0.14	-0.17	-0.19	-0.20	-0.22	-0.23
$t - st$ (con.)	(3.74)	(3.72)	(3.80)	(3.96)	(4.21)	(4.33)	(4.80)	(5.16)	(5.47)
$t - st$ (unc.)	(3.83)	(3.77)	(3.83)	(3.98)	(4.30)	(4.44)	(5.00)	(5.45)	(5.85)

## 6 Conclusions

In this paper, we embark on an abstract exploration of technology choice or state-contingent technology in a production-based economy. Our point of departure is that it is plausible to assume that production technology is state dependent. Following the literature on consumption based asset pricing, we also assume that risk aversion is state dependent.

Although technology choice directly depends on risk aversion it remains that there are no first-order effects of risk aversion on macroeconomic quantities. Instead, in our model with technology choice and risk aversion, we see that if risk aversion is time-varying, then the conditional volatility of investment evolves with risk aversion. We also see that the parameter that governs technology choice, and through that the cost of productivity transformation, also governs the volatility of the risk-free rate.

In our preferred calibration, technology choice and risk aversion move counter to an exogenous shock. Thereby, they delay the reaction of investment and consumption to the shock. Therefore, we see predictability in consumption growth, which generates fluctuations in the risk-free rate. Since technology choice is one-period ahead the generated predictability is short lived. It, thus, does not affect long lived securities such as the claim to aggregate dividends. In the model, we see that the volatility of the risk-free rate increases but the volatility of the log price-dividend ratio does not. This mechanism, hence, reduces the correlation of the risk-free rate with the log price-dividend ratio. Since in the data there almost is no correlation between the risk-free rate and the log price-dividend ratio and since without the mechanism in our model it is difficult to significantly reduce the correlation below 1, we think that this is a useful way to think about the impact of technology choice.

To further strengthen our point that asset prices and the macroeconomy are linked through variations in risk aversion, we regress excess stock market returns on the conditional volatility of investment growth and show that the model reproduces about half of the predictability in the data. This novel empirical evidence reproduces only in a model with technology choice and time-varying risk aversion.

We close by reiterating that it would be desirable to provide micro-foundations for the stylized production technology employed in the model. We leave this ambitious task for future research.



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# A Online appendix (Not for publication)

## A Technology choice

The following section provides some economic intuition for the reduced-form formulation of technology choice, which we borrow from Cochrane (1993), in our economy. Suppose that the central planner can choose to invest in a complete set of different technologies as in Jermann (2010). With a complete set we mean that there are as many independent technologies, indexed by  $i = [1, \dots, I]$ , as there are states of nature denoted by  $s = [1, \dots, S]$ . The productivity of a technology  $i$  is denoted by  $\Theta_i(s)$  for state  $s$ . Without loss of generality, let also  $\Theta_1(s)$  be the productivity next period for the exogenous benchmark technology which is log-normally distributed,

$$\log \Theta_1 = \mu + \epsilon, \tag{A1}$$

where  $\epsilon \sim N(0, \sigma^2)$ . Define

$$\vartheta_i(s) = \frac{\Theta_i(s)}{\Theta_1(s)}, \forall i = 1, \dots, I,$$

where by definition  $\vartheta_1(s) = 1$ .

Each technology produces the same good and the production of a technology  $i$  is given by

$$Y_i(s) = K_i^\alpha \Theta_i(s)^{1-\alpha},$$

where  $K_i$  is the capital invested in technology  $i$  at the beginning of the current period. The central planner has a total of  $K$  capital to allocate over the set of technologies. Let  $w_i$  be the fraction invested in technology  $i$ , i.e.,

$$w_i = \frac{K_i}{K}.$$

Then, total production can be expressed as follows:

$$Y = K^\alpha \Theta_1^{1-\alpha} \sum_{i=1}^I w_i^\alpha \vartheta_i(s)^{1-\alpha}.$$

Let us now define

$$T(\mathbf{w}, s) = \sum_{i=1}^I w_i^\alpha \vartheta_i(s)^{1-\alpha} \quad \text{and} \quad \Omega(s) = \Theta_1(s) T(\mathbf{w}, s)^{1/1-\alpha}.$$

Then, aggregate output can be rewritten as

$$Y = K^\alpha \Omega^{1-\alpha},$$

where  $\Omega$  becomes the endogenously chosen productivity or technology next period through the choice of the portfolio of technologies  $\mathbf{w} = [w_1, \dots, w_I]$ . Since, the production technology market is complete, instead of choosing  $\mathbf{w}$  the social planner can directly choose  $\Omega$  (or  $T$ ) in all future states given, of course, the joint productivity distribution of the technologies. Instead of specifying, however, the joint productivity distribution of the available technologies, we adopt the reduced-form assumption by which we can choose  $T$  given the constraint

$$\mathbb{E}[T^\nu] \leq 1, \tag{A2}$$

for some constant  $\nu$ . This implies that the endogenously chosen productivity  $\Omega$  can have any conditional distribution as long as (A2) holds. Since we log-linearize the economy, the endogenous productivity next period  $\Omega$  can be expressed as

$$\log \Omega = \log X + \sigma_\omega \epsilon + \sigma_u u, \tag{A3}$$

where  $u \sim N(0, 1)$  is an innovation to productivity orthogonal to  $\epsilon$ . The central planner can therefore choose,  $\sigma_\omega$ ,  $\sigma_u$  and  $X$  according to a certain objective and subject to the constraint (A3). Choosing  $\sigma_\omega = 1$ ,  $\sigma_u = 0$  and  $\log X = \mu$  ensures that  $\Omega = \Theta$ .

To understand the role of the parameter  $\nu$ , we can derive the optimal choice for  $\sigma_\omega$ ,  $\sigma_u$ , and  $\log X$  from maximizing average production next period, which is given by

$$\mathbb{E}[\Omega^{1-\alpha}] = X^{1-\alpha} \exp \left[ \frac{1}{2} (1-\alpha)^2 (\sigma^2 \sigma_\omega^2 + \sigma_u^2) \right].$$

Then, we can investigate the cost to average production from deviating from such a choice. Note, first, that the productivity choice constraint (A2) implies that

$$X^{1-\alpha} \leq \exp \left\{ (1-\alpha)\mu + \frac{1}{\nu} - \frac{1}{2} \nu (1-\alpha)^2 [(\sigma_\omega - 1)^2 + \sigma_u^2] \right\}.$$

Assuming, therefore, that the above constraint is binding at the optimum, we have that the average productivity next period is given by

$$\mathbb{E}[\Omega^{1-\alpha}] = \exp \left\{ (1-\alpha)\mu + \frac{1}{\nu} + \frac{1}{2} (1-\alpha)^2 [\sigma_\omega^2 \sigma^2 - \nu (\sigma_\omega - 1)^2 \sigma^2 + (1-\nu)\sigma_u^2] \right\}.$$

Maximizing next period's average production would then mean that

$$\max_{\sigma_\omega, \sigma_u} \quad \sigma_\omega^2 \sigma^2 - \nu(\sigma_\omega - 1)^2 \sigma^2 + (1 - \nu)\sigma_u^2.$$

Given this maximization problem, if  $\nu$  was less than one then increasing  $\sigma_u$  as much as possible would be the optimal decision. To avoid such examples, we restrict to cases where  $\nu > 1$  and, therefore, the optimal solution is  $\sigma_u^* = 0$ . The optimal exposure to the exogenous productivity of the benchmark technology becomes then

$$\sigma_\omega^* = \frac{\nu}{\nu - 1},$$

which ensures the maximum average production next period. If any other exposure  $\sigma_\omega = \sigma_\omega^* - \Delta$  is chosen, then the cost to the average production is proportional to  $(\nu - 1)\Delta^2$ . Therefore, the larger the parameter  $\nu$  is, the larger is the cost to average production from a deviation  $\Delta$  from the growth optimal choice. When  $\nu \rightarrow \infty$ , then it becomes infinitely costly to deviate from the exogenous benchmark productivity and  $\sigma_\omega^* \rightarrow 1$ .

## B Loglinearization

### B.1 Equilibrium conditions

With a slight abuse of notation, all variables below are normalized by the time trend, except  $\gamma_t$  and  $M_{t,t+1}$ . The equilibrium conditions for recursive preferences with technology choice, in addition to the law of motion of  $\gamma_t$ , are summarized as follows:

$$\log \Theta_{t+1} = \phi \log \Theta_t + \epsilon_{t+1}, \tag{B4}$$

$$1 = \mathbb{E}_t \left[ \frac{\Omega_{t+1}^{(1-\alpha)\nu}}{\Theta_{t+1}^{(1-\alpha)\nu}} \right], \tag{B5}$$

$$M_{t,t+1} = \beta e^{-\mu/\psi} \left[ \frac{C_{t+1}}{C_t} \right]^{-\frac{1}{\psi}} \left[ \frac{U_{t+1}^{1-\gamma_t}}{\mathbb{E}_t (U_{t+1}^{1-\gamma_t})} \right]^{\frac{\frac{1}{\psi} - \gamma_t}{1-\gamma_t}}, \tag{B6}$$

$$Y_t = K_t^\alpha \Omega_t^{1-\alpha}, \tag{B7}$$

$$Y_t = C_t + I_t, \tag{B8}$$

$$K_{t+1} = (1 - \delta)e^{-\mu} K_t + \left[ \frac{a_1}{1 - 1/\chi} \left( \frac{I_t}{K_t} \right)^{1-1/\chi} + a_2 \right] K_t e^{-\mu}, \tag{B9}$$

$$\left(\frac{I_t}{K_t}\right)^{1/\chi} = \mathbb{E}_t \left\{ M_{t,t+1} \left[ \alpha a_1 \frac{Y_{t+1}}{K_{t+1}} + (1 - \delta + a_2) \left(\frac{I_{t+1}}{K_{t+1}}\right)^{1/\chi} + \frac{a_1}{\chi - 1} \frac{I_{t+1}}{K_{t+1}} \right] \right\}, \quad (\text{B10})$$

$$\Omega_{t+1}^{(1-\alpha)\nu} = \frac{(M_{t,t+1} \Theta_{t+1}^{1-\alpha})^{\frac{\nu}{\nu-1}}}{\mathbb{E}_t \left[ (M_{t,t+1} \Theta_{t+1}^{1-\alpha})^{\frac{\nu}{\nu-1}} \right]} \Theta_{t+1}^{(1-\alpha)\nu}, \quad (\text{B11})$$

$$U_t = \left\{ (1 - \beta) C_t^{1-1/\psi} + \beta e^{\mu(1-\gamma_t)/\theta} \mathbb{E}_t \left[ U_{t+1}^{1-\gamma_t} \right]^{\frac{1-1/\psi}{1-\gamma_t}} \right\}^{\frac{1}{1-1/\psi}}. \quad (\text{B12})$$

Condition (B5) is redundant, since it is implied by condition (B11). Therefore, we have 8 first-order conditions that determine the dynamics of the 8 variables  $\Theta$ ,  $\Omega$ ,  $Y$ ,  $C$ ,  $I$ ,  $K$ ,  $U$  and  $M$ .

The key variables in the deterministic steady-state of the economy are described by

$$\begin{aligned} \Theta &= \Omega = 1, \\ Y &= K^\alpha, \\ K &= \left[ \frac{e^{\mu/\psi} - \beta(1 - \delta)}{\alpha\beta} \right]^{\frac{1}{\alpha-1}}, \\ C &= Y - I, \\ I &= (e^\mu - 1 + \delta)K, \\ U &= C \left[ \frac{1 - \beta}{1 - \beta e^{\mu(1-1/\psi)}} \right]^{\frac{1}{1-1/\psi}}, \\ M &= \beta e^{-\mu/\psi}. \end{aligned}$$

Therefore, the deterministic steady-state is independent of risk aversion parameter  $\gamma$  and the technology choice curvature  $\nu$ .

## B.2 Loglinearization: Recursive preferences with technology choice

By convention, the percentage deviation of variable  $X_t$  from its detrended steady-state value ( $X$ ) is defined as  $x_t = \log X_t - \log X$ . For example, the exogenous technology shock process can be rewritten as  $\theta_t = \phi \theta_{t-1} + \epsilon_t$  where  $\epsilon \sim \mathbb{N}(0, \sigma^2)$ . The loglinearized model depends on the three state variables  $\theta_t$ ,  $k_t$ , and  $\omega_t$ , which measure the percentage deviation from the steady-state values of the detrended variables  $\Theta_t$ ,  $K_t$ , and  $\Omega_t$ , and the risk aversion  $\gamma_t$ .

The percentage deviations of output, consumption, investment, and utility can be summarized as follows

$$x_t = x_k k_t + x_\omega \omega_t + x_\theta \theta_t + x_\gamma (\gamma_t - \gamma) \quad (\text{B13})$$

where  $x \in \{y, c, i, u\}$  and  $\gamma$  is the steady-state value of the risk aversion parameter. The



coefficients  $y_k, y_\omega, y_\theta, y_\gamma, c_k, c_\omega, c_\theta, c_\gamma, i_k, i_\omega, i_\theta, i_\gamma, u_k, u_\omega, u_\theta$ , and  $u_\gamma$  are coefficients to be determined.

We first show that  $x_\gamma$  is zero for all variables  $x \in \{y, c, i, u\}$ . Note that  $\gamma_t$  appears only in (B10), through (B6), and in (B12). We re-write (B12) as follows

$$U_t^{1-1/\psi} = (1 - \beta) C_t^{1-1/\psi} + \beta e^{\mu(1-1/\psi)} R_{u,t}^{1-1/\psi} \quad (\text{B14})$$

where

$$R_{u,t} = \mathbb{E}_t [U_{t+1}^{1-\gamma_t}]^{\frac{1}{1-\gamma_t}}. \quad (\text{B15})$$

Log-linearizing (B14) and (B15) yields

$$u_t = (1 - \beta e^{\mu(1-1/\psi)}) c_t + \beta e^{\mu(1-1/\psi)} r_{u,t}, \quad (\text{B16})$$

$$r_{u,t} = E_t(u_{t+1}). \quad (\text{B17})$$

The above implies that  $c_\gamma$  and  $u_\gamma$  are zero. Using  $R_u$  to log-linearize (B6), we obtain

$$m_{t+1} = -\frac{1}{\psi}(c_{t+1} - c_t) + \left(\frac{1}{\psi} - \gamma_t\right) [u_{t+1} - \mathbb{E}_t(u_{t+1})], \quad (\text{B18})$$

which implies that the first conditional moment of  $m_{t+1}$  is independent of  $\gamma_t$ . Finally, log-linearizing (B10), in which only the conditional expectation of  $m_{t+1}$  appears, implies that  $i_\gamma, \kappa_\gamma$ , and  $y_\gamma$  are zero. The exogenous productivity is by assumption independent of  $\gamma_t$  and the only variable that depends on risk aversion is the endogenous productivity  $\omega_t$ .

Log-linearizing condition (B5) implies that the conditional expectation of  $\omega_{t+1}$  is the same as that of  $\theta_{t+1}$ . Thus, from (B4) we obtain that

$$\omega_{t+1} = \phi\theta_t + \sigma_\omega(\gamma_t)\epsilon_{t+1}. \quad (\text{B19})$$

Matching the coefficients of  $\epsilon_{t+1}$  in (B11), for which we use (B4) and (B6), and solving for  $\sigma_\omega(\gamma_t)$  we obtain the optimal technology choice,

$$\sigma_\omega(\gamma_t) = \frac{(1 - \alpha)\nu - \frac{1}{\psi}c_\theta + (\frac{1}{\psi} - \gamma_t)u_\theta}{(1 - \alpha)(\nu - 1) + \frac{1}{\psi}c_\omega + (\gamma_t - \frac{1}{\psi})u_\omega}. \quad (\text{B20})$$

Log-linearizing the equilibrium conditions yields the remaining coefficients. For example,  $c_k$

is the positive root from the following quadratic equation

$$0 = B \left[ \frac{\alpha(C+I)k_2}{I} + k_1 \right] - \frac{\alpha(C+I) - I}{\chi I} - \left( \frac{Bk_2C}{I} - \frac{1}{\psi} - \frac{C}{\chi I} \right) c_k, \quad (\text{B21})$$

where

$$B = \frac{\alpha K^{\alpha-1}(\alpha-1)}{\alpha K^{\alpha-1} + 1 - \delta} - \frac{c_k}{\psi} + \frac{1}{\chi(\alpha K^{\alpha-1} + 1 - \delta)} \left[ \frac{\alpha(C+I)}{I} - \frac{C}{I} c_k - 1 \right], \quad (\text{B22})$$

$$k_1 = \frac{1 - \delta}{e^\mu}, \quad (\text{B23})$$

$$k_2 = \frac{e^\mu - 1 + \delta}{e^\mu}. \quad (\text{B24})$$

The other coefficients are given by:

$$c_\omega = \frac{\frac{(\alpha-1)(C+I)}{\chi I} + B \frac{k_2}{I} (1-\alpha)(C+I)}{Bk_2 \frac{C}{I} - \frac{1}{\psi} - \frac{C}{\chi I}}, \quad (\text{B25})$$

$$c_\theta = \frac{\phi \left\{ \frac{\alpha K^{\alpha-1}(1-\alpha)}{\alpha K^{\alpha-1} + 1 - \delta} - \frac{c_\omega}{\psi} + \frac{1}{\chi(\alpha K^{\alpha-1} + 1 - \delta)} \left[ \frac{(1-\alpha)(C+I)}{I} - \frac{C}{I} c_\omega \right] \right\}}{\phi \left( \frac{1}{\psi} + \frac{C}{(\alpha K^{\alpha-1} + 1 - \delta)\chi I} \right) + \frac{Bk_2C}{I} - \frac{1}{\psi} - \frac{C}{\chi I}}, \quad (\text{B26})$$

$$i_k = \frac{\alpha(C+I)}{I} - \frac{C}{I} c_k, \quad (\text{B27})$$

$$i_\omega = \frac{(1-\alpha)(C+I)}{I} - \frac{C}{I} c_\omega, \quad (\text{B28})$$

$$i_\theta = -\frac{C}{I} c_\theta, \quad (\text{B29})$$

$$u_k = \frac{u_1 c_k}{1 - u_2 k_1 - u_2 k_2 i_k}, \quad (\text{B30})$$

$$u_\omega = u_1 c_\omega + u_2 k_2 u_k i_\omega, \quad (\text{B31})$$

$$u_\theta = \frac{u_1 c_\theta + u_2 k_2 u_k i_\theta + \phi u_2 u_\omega}{1 - \phi u_2}, \quad (\text{B32})$$

where

$$u_1 = 1 - \beta e^{\mu(1-\frac{1}{\psi})}, \quad (\text{B33})$$

$$u_2 = \beta e^{\mu(1-\frac{1}{\psi})}. \quad (\text{B34})$$

As for output, the coefficients are given by  $y_k = \alpha$ ,  $y_\omega = (1 - \alpha)$ , and  $y_\theta = 0$ .

From the above equations, we see that coefficients  $u_k, u_\omega, u_\theta, c_k, c_\omega, c_\theta, i_k, i_\omega, i_\theta$  are dependent on EIS ( $\psi$ ) but independent of the risk aversion ( $\gamma_t$ ) and technology choice curvature ( $\nu$ ). Moreover, from equation (B20),  $\sigma_\omega(\gamma_t)$  depends on risk aversion and technology choice curvature ( $\nu$ ). Thus, in a standard RBC economy without technology choice, macroeconomic quantities

are not risk aversion sensitive. Introducing technology choice makes macroeconomic quantities sensitive to the risk aversion. Proposition 1 concludes the above subsection.

## C Stock prices and the risk-free rate

The stochastic discount factor,  $M$ , which is given in (B6), is log-normally distributed. As shown in Proposition 3 it can be expressed in the following form:

$$\log M_{t,t+1} = \log \hat{\beta}(\gamma_t) - \frac{1}{\psi} \mu_t - \sigma_m(\gamma_t) \epsilon_{t+1}.$$

The risk-free rate is determined via

$$r_{f,t} = -\log \mathbb{E}_t(M_{t,t+1}), \quad (\text{C35})$$

which yields the expression provided in Proposition 4.

The Euler equation of the stock is given as follows

$$e^{p_t - d_t} = \mathbb{E}_t [J_{t,t+1} (e^{p_{t+1} - d_{t+1}} + 1)], \quad (\text{C36})$$

where  $J_{t,t+1} = M_{t,t+1} D_{t+1} / D_t$  and, thus,

$$\ln J_{t,t+1} = \ln \hat{\beta}(\gamma_t) - \frac{1}{\psi} \mu_t + \sigma_m(\gamma_t) \epsilon_{t+1} + \mu + d_1(\theta_t - c_t) + d_2 \epsilon_{t+1} + d_3 \epsilon_{t+1}^d. \quad (\text{C37})$$

The log of the price-dividend ratio is approximated to be linear in the (demeaned) state vector  $z_t$ , which includes deviations of risk aversion  $\gamma_t$  from its steady state. Therefore,

$$p_t - d_t \approx \overline{p - d} + b z_t, \quad (\text{C38})$$

where  $\overline{p - d}$  is the average log price-dividend ratio. To derive approximate dynamics we assume that risk aversion follows an AR(1) process around a steady state  $\gamma$ , driven by  $\epsilon_{t+1}$  and/or an idiosyncratic shock  $\epsilon_{t+1}^\gamma$ . Consequently,

$$z_{t+1} = Z z_t + \Sigma_z(\gamma_t) \epsilon_{t+1} + \Sigma_\gamma \epsilon_{t+1}^\gamma. \quad (\text{C39})$$

When  $z_t = 0$ , then  $p_{t+1} - d_{t+1} = \overline{p - d} + b \Sigma_z(\gamma) \epsilon_{t+1} + b \Sigma_\gamma \epsilon_{t+1}^\gamma$ . Solving the Euler equation when the state is  $z_t = 0$ , we obtain the following:

$$e^{\overline{p - d}} = \hat{J} e^{\overline{p - d}} + J, \quad (\text{C40})$$

where

$$\log J = \log \hat{\beta}(\gamma) + \left(1 - \frac{1}{\psi}\right) \mu + \frac{1}{2} \{d_3^2 + \sigma^2 [d_2 - \sigma_m(\gamma)]^2\}, \quad (\text{C41})$$

$$\log \hat{J} = \log J + [d_2 - \sigma_m(\gamma)] b\Sigma_z(\gamma) + \frac{1}{2}(b\Sigma_\gamma)^2 + \frac{1}{2} [b\Sigma_z(\gamma)]^2, \quad (\text{C42})$$

and, therefore,  $\overline{p-d} = \log \left( J/(1-\hat{J}) \right)$ . Solving the Euler equation for a general state and applying a first-order approximation, we obtain the following:

$$(p_t - d_t) - \overline{p-d} \approx \hat{J} \mathbb{E}_t [(p_{t+1} - d_{t+1}) - \overline{p-d}] + \xi_t, \quad (\text{C43})$$

where

$$\xi_t = d_1(\theta_t - c_t) - \frac{1}{\psi}(\mu_t - \mu) + \left[ \xi_1(\gamma)(\gamma_t - \gamma) + \frac{1}{2}\xi_2(\gamma)(\gamma_t - \gamma)^2 \right] \sigma^2, \quad (\text{C44})$$

$$\xi_1(\gamma) = \left(1 - \frac{1}{\psi}\right) \left[ \frac{1}{2}\sigma_u^2 + \sigma'_u \sigma_u \left(\gamma - \frac{1}{\psi}\right) \right] - d_2\sigma'_m + \frac{1}{\psi} [\sigma'_c \sigma_m + \sigma_c \sigma'_m] \quad (\text{C45})$$

$$+ \hat{J} \left[ \frac{\partial(b\Sigma_z)}{\partial\gamma} (d_2 - \sigma_m + b\Sigma_z) - b\Sigma_z \sigma'_m \right], \quad (\text{C46})$$

$$\xi_2(\gamma) = \left[ \sigma'_m + \frac{\partial(b\Sigma_z)}{\partial\gamma} \right]^2. \quad (\text{C47})$$

In the above expressions,  $\sigma_x$  refers to  $\sigma_x(\gamma)$  and  $\sigma'_x$  refers to the first derivative of  $\sigma_x(\gamma)$  with respect to  $\gamma$ , for some variable  $x$ . Solving forward the above equation, we obtain the following expression:

$$p_t - d_t \approx \overline{p-d} + \sum_{\tau=0}^{\infty} \hat{J}^\tau \mathbb{E}_t \xi_{t+\tau}. \quad (\text{C48})$$

We can provide similar expressions for the consumption claim.

## D Data

We collect macroeconomic variables from the NIPA tables over the period 1929 to 2017. Output series are taken to be the total output reported, the consumption series is the consumption of non-durables and services, and the investment series is the non-residential fixed investments. All macroeconomic variables are deflated by inflation computed from the CPI index of the Bureau of Labor Statistics and normalized by the civilian noninstitutional population with age over 16, from the Current Population Survey (Serial ID LNU00000000Q).

For the calibration we use the annual data. For predictive regression based on macroeco-

conomic variables, we use quarterly data starting in 1947.

We use quarterly CRSP value-weighted returns as the market return and the Fama 3-month T-bill rate as the risk-free rate from WRDS from 1927 to 2017. Real returns equal nominal returns deflated by inflation. The price-dividend ratio is inferred from the CRSP value-weighted returns with and without dividends.

## **E Low EIS models**

Table 7: Calibrated model parameters - low EIS

TCV is the model with technology choice and time-varying risk aversion, NTCV is the model without technology choice but with time-varying risk aversion, TCC is an economy with technology choice and constant risk aversion, and NTCC is the no technology choice and constant risk aversion benchmark. For all economies with low EIS, we set the time discount factor to just below 1 and use the EIS to match the mean of the risk-free rate.

Description	Parameter	Values			
		TCV	NTCV	TCC	NTCC
Subjective discount factor	$\beta$	0.9999	0.9999	0.9999	0.9999
(Mean) coefficient of relative risk aversion	$\gamma$	50	50	50	40
Elasticity of intertemporal substitution	$\psi$	0.60	0.60	0.53	0.48
Capital adjustment cost parameter	$\chi$	6.4	6.9	6.9	7.0
Technology choice parameter	$\nu$	11.1	$\infty$	12.7	$\infty$
Volatility of exogenous productivity shocks	$\sigma$	4.87%	4.73%	4.95%	4.73%
CRRA function $\lambda$ linear coefficient	$\eta_1$	120	120	-	-
CRRA function $\lambda$ quadratic coefficient	$\eta_2$	70.0	70.1	-	-
		Averages across simulations			
Mean CRRA		52.8	52.7		
Standard deviation of CRRA		21.4	20.7		
Minimum CRRA		12.1	12.9		
Maximum CRRA		115.7	113.5		

Table 8: Calibrated models: Low EIS

$\Delta x$  denotes the first-difference of the natural logarithm of a variable  $X$ .  $y$  denotes (the natural logarithm of) total output;  $c$  denotes total consumption;  $i$  denotes total investment. For a variable  $x$ ,  $\sigma(x)$  denotes its volatility;  $ac_1(x)$  is its first-order autocorrelation and  $\rho(x, z)$  is its correlation with variable  $z$ . The data are described in Appendix D. The parentheses next to the data estimates show the standard errors (*s.e.*), which are Newey and West (1987) corrected with 24 lags. TCV is the model with technology choice and time-varying risk aversion, NTCV is the model without technology choice but with time-varying risk aversion, TCC is an economy with technology choice and constant risk aversion, and NTCC is the no technology choice and constant risk aversion benchmark. The model statistics are averages of 1000 simulated paths of 300 quarters with a burn-in of 100 quarters. The parentheses next to the model statistics show the  $t$ -statistics ( $t - st$ ) of the hypotheses that the data estimates are generated from model averages. Macroeconomic and dividend data are annual. The corresponding data from the models are time aggregated. All other statistics are quarterly.

	Data		TCV		NTCV		TCC		NTCC	
	<i>est.</i>	<i>s.e.</i>	<i>avg.</i>	<i>t - st</i>	<i>avg.</i>	<i>t - st</i>	<i>avg.</i>	<i>t - st</i>	<i>avg.</i>	<i>t - st</i>
$\mu(\Delta c)$	1.74	(0.38)	1.60	(0.36)	1.60	(0.36)	1.60	(0.36)	1.60	(0.36)
$\sigma(\Delta c)$	2.70	(0.54)	2.70	(0.00)	2.70	(0.01)	2.70	(0.00)	2.70	(0.00)
$ac_1(\Delta c)$	0.48	(0.07)	0.29	(2.62)	0.27	(2.91)	0.30	(2.59)	0.27	(3.00)
$\sigma(\Delta c)/\sigma(\Delta y)$	0.55	(0.06)	0.55	(0.09)	0.55	(0.00)	0.55	(0.00)	0.55	(0.01)
$\sigma(\Delta i)/\sigma(\Delta y)$	2.71	(0.14)	1.97	(5.34)	1.97	(5.30)	2.03	(4.92)	2.06	(4.70)
$\mu(R_f)$	0.14	(0.15)	0.35	(1.34)	0.16	(0.14)	0.27	(0.82)	0.16	(0.10)
$\sigma(R_f)$	0.84	(0.10)	0.86	(0.18)	0.27	(5.57)	0.83	(0.04)	0.21	(6.19)
$\mu(R_i - R_f)$			0.09		-0.26		0.45		0.65	
$\sigma(R_i)$			1.24		1.17		1.95		2.67	
$SR_i$			0.09		-0.24		0.26		0.24	
$\sigma(\Delta d)$	11.10	(2.12)	11.47	(0.18)	11.34	(0.12)	11.54	(0.21)	11.35	(0.12)
$ac_1(\Delta d)$	0.18	(0.14)	0.27	(0.63)	0.27	(0.61)	0.27	(0.65)	0.27	(0.62)
$\rho(\Delta c, \Delta d)$	0.52	(0.15)	0.55	(0.20)	0.53	(0.11)	0.56	(0.25)	0.53	(0.11)
$\mu(p - d)$	4.79	(0.10)	4.34	(4.27)	4.43	(3.44)	4.67	(1.16)	4.75	(0.38)
$\sigma(p - d)$	0.44	(0.05)	0.22	(4.18)	0.20	(4.55)	0.08	(6.66)	0.08	(6.63)
$\rho(p - d, r_f)$	0.03	(0.17)	0.32	(1.68)	0.86	(4.86)	0.32	(1.71)	0.45	(2.45)
$\mu(R_m - R_f)$	2.04	(0.39)	2.01	(0.12)	2.06	(0.05)	1.39	(1.64)	1.46	(1.48)
$\sigma(R_m)$	11.16	(2.21)	11.10	(0.03)	11.08	(0.04)	8.01	(1.43)	8.34	(1.28)
$SR_m$	0.18	(0.05)	0.18	(0.06)	0.19	(0.05)	0.18	(0.15)	0.18	(0.16)

Table 9: Excess return predictability by  $p - d$ : Low EIS

The table shows the standardized regression coefficient on the log price-dividend ratio from a standard predictive regression using the stock market excess return for horizons from one quarter to 28 quarters. The data are described in Appendix D. The  $t$ -statistics ( $t - st$ ) are Newey and West (1987) corrected with 24 lags. TCV is the model with technology choice and time-varying risk aversion, NTCV is the model without technology choice but with time-varying risk aversion, TCC is an economy with technology choice and constant risk aversion, and NTCC is the no technology choice and constant risk aversion benchmark. The model statistics are averages of 1000 simulated paths of 300 quarters with a burn-in of 100 quarters. The parentheses below the regression coefficients for the models show the  $t$ -statistics ( $t - st$ ) of the hypotheses that the data estimates are generated from model averages. The data have quarterly frequency starting in 1947.

	$\rho(p - d_t, \sum_{s=1}^{\tau} [R_{m,t+s} - R_{f,t+s-1}])$									
	$\tau = 1$	$\tau = 2$	$\tau = 4$	$\tau = 8$	$\tau = 12$	$\tau = 16$	$\tau = 20$	$\tau = 24$	$\tau = 28$	
Data	-0.13	-0.19	-0.26	-0.35	-0.39	-0.42	-0.47	-0.50	-0.53	
$t - st$	(3.03)	(3.33)	(3.34)	(3.23)	(2.97)	(2.81)	(3.20)	(3.52)	(3.75)	
TCV	-0.09	-0.13	-0.17	-0.23	-0.28	-0.31	-0.33	-0.35	-0.36	
$t - st$	(0.97)	(1.12)	(1.11)	(1.03)	(0.89)	(0.78)	(0.95)	(1.10)	(1.20)	
NTCV	-0.09	-0.13	-0.18	-0.25	-0.29	-0.32	-0.34	-0.36	-0.37	
$t - st$	(0.87)	(1.01)	(1.00)	(0.93)	(0.80)	(0.70)	(0.86)	(1.00)	(1.10)	
TCC	-0.01	-0.02	-0.02	-0.03	-0.04	-0.04	-0.05	-0.05	-0.05	
$t - st$	(2.74)	(3.02)	(3.04)	(2.93)	(2.68)	(2.53)	(2.88)	(3.17)	(3.37)	
NTCC	-0.01	-0.02	-0.03	-0.04	-0.05	-0.05	-0.06	-0.06	-0.06	
$t - st$	(2.67)	(2.95)	(2.97)	(2.86)	(2.62)	(2.46)	(2.81)	(3.08)	(3.28)	