Competitive on-the-job search*

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Abstract

The paper proposes a model of on- and off-the-job search that combines convex hiring costs and directed search. Firms permanently differ in productivity levels, their production function features constant or decreasing returns to scale, and search costs are convex in search intensity. Wages are determined in a competitive manner, as firms advertise wage contracts (expected discounted incomes) so as to balance wage costs and search costs (queue length). An important assumption is that a firm is able to sort out its coordination problems with their employees in such a way that the on-the-job search behavior of workers maximizes the match surplus. Our model has several interesting features. First, it is close in spirit to the competitive model, with a tractable and unique equilibrium, and is therefore useful for empirical testing. Second, the resulting equilibrium gives rise to an efficient allocation of resources. Third, the equilibrium is characterized by a job ladder: unemployed workers search for low-productivity, low-wage firms. Workers in low-wage firms search for firms slightly higher on the productivity/ ladder, and so forth up to the workers in the second most productive firms who only apply to the most productive firms. Finally, the model rationalizes empirical regularities of on-the-job search and labor turnover. First, job-to job mobility falls with average firm tenure and firm size. Second, wages increase with firm size, and wage growth is larger in fast-growing firms.

*Some of the results in this paper are also availiable in the note "Job-to-job movements in a simple search model", forthcomming in the *AER papers and proceedings* (2010).

1 Introduction

The objective of this paper is to provide a simple, competitively flavoured model consistent with a variety of facts described in longitudinal data sets on firms and workers flows. In particular, the model should be able to explain that 1) firms with different productivities and size coexist in the labor market, 2) On-the-job search is prevalent and worker flows between firms are large, and 3) more productive firms are larger and pay higher wages than less productive firms.

Our model contains three key elements. First, it applies the *competitive search equilibrium* concept, initially proposed by Moen (1997). Thus, firms publicly advertise wages, and workers observe the advertised wages prior to making their decision where to direct their search. Second, we assume that firms have access to a search technology with *convex costs of maintaining vacancies* (Bertola and Caballero, 1992; Bertola and Garibaldi, 2001). In contrast with the standard model with linear costs of creating vacancies and constant returns in production, this implies that the firms' size distribution is well defined. Third, we follow Moen and Rosen (2004) and allow for *efficient contracting.* The contracts are thus designed so as to resolve any agency problems between employers and employees, so that their joint income is maximized.

With these assumptions we obtain a tractable model of on-the-job search, closely related to the competitive model, in which on-the-job search is an optimal response to search frictions and heterogenous firms. The equilibrium leans towards a job ladder, where unemployed workers search for low-productivity firms offering low wages, and then gradually advances to higher paid jobs. Productive firms pay higher wages and grow faster than less productive firms, consistent with the stylized facts. We believe our model is interesting, for several reasons. First, as job-to-job flows are huge (Davis and Haltiwanger 1999) and important for economic growth (Lentz and Mortensen 2006), understanding the reasons behind on-the-job search is in itself important. In particular, it is of interest to set up a model of efficient on-the-job search, and derive its implications for wage distributions and job flows. Second, several influential papers structurally estimate models of on-the-job search based on random search (particularly the Burdett-Mortensen model), see Postel-Vinay and Robin (2002), Bagger and Lentz, Lise et. al. (2008), Lentz and Mortensen (2007) and Moscarini and Postel-Vinay (2009). We deliver an alternative framework, based on directed search, suitable for empirical analysis. Note also that our model delivers different empirical predictions than the existing models based on random search. For instance, the Burdett-Mortensen predicts a weak relationship between the wage before the job switch and the distribution of wages after the job switch. More specifically, previous wages only constitutes a truncation point for the distribution of wages after the switch. According to our model, workers employed in firms offering relatively high wages (i.e., have high productivity) search for jobs that offer strictly higher wages than do workers employed in firms offering lower wages initially.

In a robustness section of the paper we analyze the equilibrium of the model with alternative assumptions regarding the search and adjustment costs. We show that if the convexity of the cost of maintaining vacancies vanishes, on-the-job search vanishes as well, as only the most productive firms post vacancies. We also augment the model by including convex costs of *hiring* workers (not posting vacancies, see Lucas, Blanchard, and Sargent). Hiring costs differ from search cost in that they are independent of labor market conditions such as the tightness of the labor market. In the resulting equilibrium, firms of different productivities may coexist in the market, but there is no on-the-job search. Hence our assumptions regarding convex adjustment costs seem to be necessary to obtain on-the-job search with directed search. Finally we show that if search costs are inversely related to the size of the firm, a firm's growth rate is independent of size, and hence Gibrath's law is satisfied.

We also calculate numerically the equilibrium of the model with two firm types. Compared to standard search models, our model with on-the-job search delivers unexpected effects, even though it converges to traditional models as a special case (Pissarides 2000). We find that an increase in average productivity, caused by an exogenous shift in the fraction of high-type firms in the market can actually lead to an *increase* in unemployment and a reduction in entry for a subset of the parameter space. This will never happen in random search models, and is caused by composition effects between queue length across different submarkets.

There exist papers with on-the-job search where firms advertise wages, see Moen and Rosen (2004) and Shi (2008) as well as Menzio and Shi (2010) (written simultaneously with our paper). Our paper differs from the previous papers in that firms in our paper have permanently different productivities. As firms are ex ante identical (before they sink a cost and enter the market), the zero profit constraint only holds for expected profit ex ante, it does not hold in each submarket. Hence our model is not block recursive (Menzio and Shi), and the stocks of agents in each market matters. This makes existence proofs much harder. Our paper is also related to Mortensen and Wright (2001), who analyze competitive search equilibrium when workers income during unemployment differ.

The paper proceeds as flows. Section 2 presents the model and define equilibrium. Sections 3 and 4 characterizes equilibrium, while section 5 looks at extensions. Section 6 points to testable differences between our model and the Burdett-Mortensen model. Section 7 presents baseline simulation while section 8 concludes.

2 Model and equilibrium

The structure of our model is as follows

- The labor market is populated by a measure 1 of identical workers. Individuals are neutral, infinitely lived, and discount the future at rate r.

- The technology requires an entry cost equal to K. Conditional upon entry, the firm learns its productivity, which can take any value between y_1, and y_n with $y_1 < y_2 < ... < y_n$. The probability that each productivity is selected is α_i with

$\Sigma_i \alpha_i = 1$. The productivity of a firm is a fixed throughout its life. Unemployed workers have access to an income flow $y_0 < y_1$.

- Firms post vacancies and wages to maximize expected profits. Vacancy costs $c(v)$ are convex in the number of vacancies posted. Unless otherwise stated we assume that $c(0) = c'(0) = 0$. In the numerical analysis we assume that the vacancy costs are zero up to a point ν, at which they become infinite.[1]

- Firms die at rate δ. In addition, workers separates from firms at an exogenous rate s.

- Wage contracts are complete, and resolve any agency problems between employers and employees. In particular, the wage contract ensures efficient on-the-job search.

The last point deserves a comment. In this context, efficient on-the-job search implies that the worker, when searching on the job, searches for the jobs that maximizes the joint surplus of the worker and the firm. There are various wage contracts that implement this behavior. For example, the worker pays the firm its entire pdv value up front and then gets a wage equal to y_i. In other words, the worker buys the job from the firm and acts thereafter a residual claimant. As an alternative contract, the worker gets a constant wage and pays a quit fee equal to the continuation value of the firm if a new job is accepted (see Moen and Rosen (2004) for more examples). In principle the worker and the firm may also write directly into the wage contract the search behavior of the worker. In any event, the wages paid to the worker in the current job do not influence her on-the-job search behavior.[2]

The search market endogenously separates into submarkets, consisting of a set of workers and firms with vacancies searching for each-other. In each submarket, the flow of matches is determined by a constant-returns-to scale matching function $x(N, V)$, where N and V are the measure of workers and firms in that submarket, respectively. Let $\theta = V/U$, and define $p(\theta) = x(u, V)/u = x(1, \theta)$ and $q(\theta) = x(u, V)/V = x(1/\theta, 1)$. Finally. let $\eta = |q'(\theta)\theta/q|$ denote the absolute value of the elasticity of η with respect to θ. It is convenient to assume that $\eta(\theta)$ is non-decreasing in θ.

Firms advertise contracts and workers apply to one of the contracts. For any given contract σ, let $W(\sigma)$ denote the associated net present income of the worker that obtains the job. As will be clear below, $W(\sigma)$ is a rather complicated object,

[1] Convex costs of maintaining vacancies may be rationalized be decreasing returns to scale in the firm's recruitment department. Convex hiring costs can be seen as a generalization of Pissarides (2000) and Burdet Mortensen (1999), where the number of vacancies is exogenously fixed. Ananlogusly, the search costs of workers are usually assumed to be convex (Pissarides 2000). Finally, ourassumption of convex hiring costs have empirical support, see Yashiv (2000a,b).

[2] It follows from this that a worker employed in a firm of type i will never search for a job in another firm of type $j \leq i$. Such jobs cannot profitably offer a wage that exceeds the productivity in the current firm.

as it includes the expected income to the worker from on-the-job search, which again depends on wages advertised by more productive firms and the probability rates of getting these jobs.

Consider an economy where a countable set of NPV wages $W_1, ..., W_k...$ are advertised, each by a strictly positive measure of firms. We require that $W \leq y_n$. The firms that advertise a given wage and the workers that apply to those firms form a submarket. Let $\theta_1, ..., \theta_k...$ denote the associated vector of labor market tightness. The set of pairs (θ_k, W_k) is denoted by Ω.

Let M_i $i = 0, 1, ..., n$ denote the joint expected discounted income flow of a worker and a job in a firm of type i, where the gains from on-the-job search is included. Since on-the-job search is efficient, it follows that M_i is given by

$$rM_i = y_i + (s + \delta)(M_0 - M_i) + \max_k p(\theta_k)[W_k - M_i] \tag{1}$$

The first term is the flow production value created on the job. The second term captures the expected capital loss due to job separation, which happens at rate $s + \delta$, and reduces the joint income to M_0 (since the firm then earns zero on this match). The last term shows the expected joint gain from on-the-job search. Since the current wage is a pure transfer from the employer to the worker, it does not appear in the expression.

From (1) it follows that the optimal search behaviour of a worker depends on her current position, as this influences M_i. Hence our model is characterized with what we refer to as endogenous worker heterogeneity: the current income flow y_i influences the gain from on-the-job search and the search behaviour of the worker in question. We refer to a worker that currently works in a firm of type i as a type $i-$searching worker or just type i worker (note that all worker "types" are equally productive).

The indifference curve of a worker of type i shows combinations of θ and W that gives a joint income equal to M_i. We can represent this as $\theta_i = f_i(W; M)$.[3] It follows that f_i is defined implicitly by the equation

$$rM_i = y_i + (s + \delta)(M_0 - M_i) + p(f_i(W, M))[W - M_i] \tag{2}$$

where M_i is the equilibrium joint income in firm i. It follows that for $M_i < W_i$

$$f_i(W; M) = p^{-1}\left(\frac{(r + s + \delta)M_i - y_i - (s + \delta)M_0}{W - M_i}\right) \tag{3}$$

The indifference curve is defined for all W, not only the values advertised in equilibrium. Define

$$f(W; M) = \min_{i \in \{0,1,...,n\}} f_i(W; M) \tag{4}$$

The function $f(W; M)$ is thus the lower envelope of the set of functions $f_i(W; M)$. In equilibrium, $f(W; M)$ shows the relationship between the wage advertised and the

[3]Strictly speaking, f_i only depends on M_i and M_0, but we write it as a function of the vector M for convenience

labor market tightness in a submarket. Suppose that for a given W, the minimum in (4) is obtained for worker type i'. This worker type will then flow into the market up to the point where $\theta = f_{i'}(W; M)$. At this low labor market tightness, no other worker types want to enter this submarket. The labor market tightness is thus given by $f_{i'}(W; M)$, and only workers of type i' enter the market.

Then we turn to the firms. It follows that at any point in time, a firm of type j maximizes the value of search given by[4]

$$\pi_j = -c(v) + v_j q(\theta)[M_j - W_j]. \tag{5}$$

where W_j is the wages paid by the firm. The first part is the flow cost of posting vacancies, while the second part is the gain from search. The firm's maximization problem reads

$$\max_{v,W} -c(v) + vq(\theta)[M_j - W] \text{ s.t. } \theta = f(W, M)$$

Denote the associated maximum profit flow by π_j^*. The expected profit of a firm entering the market as a type j firm is thus

$$\Pi_j = \frac{\pi_j^*}{r+\delta} \tag{6}$$

Denote the vector of wages that solves $j's$ maximization problem by $W_j(M)$. Below we show that W_j is finite. Denote the optimal measure of vacancies by $v_j(M)$. The number of vacancies posted by firm j is independent of its choice of v_j.

Let the vector $N = (N_0, N_1, ...N_j...)$ denote the measure of workers in type j firms. Let the vector $\widetilde{\tau}_j = (\tau_{j1}, \tau_{j2}, ...)$ denote the distribution of vacancies posted by firms of type j over the different submarkets. Similarly, let $\widetilde{\kappa}_j = (\kappa_{j1}, \kappa_{j2}, ...)$ denote the distribution of searching type j workers over the different submarkets. Finally, let k denote the total number of firms. In steady state, inflow of workers into type j firms has to be equal to outflow, hence

$$\sum_k \alpha k v_j \widetilde{\tau}_{jk} q(\theta_k) = [s + \delta + \sum_k p_{jk}(\theta_k)\widetilde{\kappa}_{jk}]N_j \tag{7}$$

for $j > 0$. For unemployed workers, the corresponding inflow-outflow equation reads

$$(s+\delta)(1-N_0) = \sum_k p_{0k}(\theta_k)\widetilde{\kappa}_{0k}]N_0 \tag{8}$$

The labor market tightness θ_k in market k is given by

$$\theta_k = k\frac{\sum_j \alpha_j \widetilde{\tau}_{jk} v_j}{\sum_j \widetilde{\kappa}_{jk} N_j} \tag{9}$$

We are now in a position to define the general equilibrium.

[4] At any point in time, the firm decides on the number of vacancies to be posted and the wages attached to them. This only influences profits through future hirings, and is independent of the stock of existing workers.

Definition 1 *General equilibrium is defined as a vector of asset values* M^**,wages* W^**, employment stocks* N^**, labor market tightness* θ^**, distributions of searching workers* $\widetilde{\kappa}^*$*, and distributions of vacancies* $\widetilde{\tau}^*$*, and a number* k *such that*

1. *Profit maximization: i)* $W^* = \cup_{j=1}^{n} W_j(M^*)$ *ii)* $v_j = v_j(M^*)$*iii)If* $\tau_{jk} > 0$*, then* $W_k^* \in W_j(M^*)$
2. *Optimal worker search:* $rM_i^* \geq y_i + (s+\delta)(M_0 - M_i^*) + p(\theta_k)[W_k - M_i^*]$*, with equality if* $\kappa_{ik} > 0$
3. *Optimal entry: The expected profit of entering the market is equal to the entry cost* K*, i.e.,*
$$E\Pi_j = K$$
4. *Aggregate consistency: The flow equations (7), (8) and (9) are satisfied.*

In addition we make the following *equilibrium refinement*: if more than one allocation satisfies the equilibrium conditions, the market picks the equilibrium where aggregate output is highest. This can be rationalized by assuming that a market maker sets up the markets (as in Moen 1997).

3 Characterizing equilibrium

Before we prove existence of equilibrium, we will derive some properties of the equilibrium (assuming that it exists). First we will derive properties for $f(W, M)$.

Consider an arbitrary Ω, and let M denote the corresponding vector of asset values defined by (1). Let W^s denote the highest wage in Ω. By construction, M exists and is unique. Furthermore, in the appendix we show that $y_0/r \leq M_0 < M_1, ... < M_n$ and $(r+s+\delta)M_i - y_i$ is decreasing in i. If we restrict our attention to the case were $W^s < y_n$, it follows that $M_n = \frac{y_n + sU}{r+s}$.

If $M_j \geq W^s$, workers employed in firms of type j or higher do not search. In this case $(r+s+\delta)M_j = y_j + (s+\delta)M_0$, and it follows from (2) that$f_i(W, M) = 0$ for $W > M_j$. It follows that $f(W, M)$ is discontinuous at $W = M_j$. If $M_i < W^s$, type i-searching workers will search, hence $f_i(W, M) = f(W, M)$ for some W (at least the wages they actually search for). In this case $(r+s+\delta)M_j > y_j + (s+\delta)M_0$, and it follows from (3) that $f_i(W; M) > 0$ for all W.

Lemma 1 *For any vector* Ω *and associated vector* M *defined by (1), the following holds*

a) Single crossing: For any i, j*,* $i < j$*,* $M_j < W^s$*, the equation* $f_i(W; M) = f_j(W; M)$ *has exactly one solution, and at this point*

$$|\frac{df_i(W; M)}{dW}| < |\frac{df_j(W; M)}{dW}|$$

b) Suppose $M_{n'} < W^s < M_{n'+1}$. Then there exists an increasing vector of wages $(W^0, W^1, W^2, ..., W^{n'})$ (not necessarily in Ω), with $W^0 = y_0$, with the following properties: If $W \in (W^i, W^{i+1})$, $i < n'$, then $f(W; M) = f_i(W; M)$. If $W^{n'} < W \leq M_{n'}$ then then $f(W; M) = f_{n'}(W; M)$. If $W > M_{n'}$ then $f(W; M) = 0$.

c) $f(\theta)$ is discontinuous at $M_{n'+1}$, as $\lim_{W \to M^-_{n'+1}} f(W; M) > 0$ while $\lim_{W \to M^+_{n'+1}} f(W, M) = 0$. For all other values of W, f is continuous, and strictly decreasing in W for all $W < M_{n'+1}$. f is continuously differentiable except at the points $(W^1, W^2, ..., W^{n'}, M_{n+1})$. At this points, $\lim_{W \to W^{i-}} df(W; M)/dW > \lim_{W \to W^{i+}} df(W; M)/dW$.

From c) it follows that the function f is not globally convex.

In order to characterize the equilibrium of the market, the following result is useful (recall that $\eta = -q'(\theta)\theta/q$):

Lemma 2 *a) Let $W > M_0$ be an NPV wage that is advertised in equilibrium. In the corresponding submarket, there is exactly one type of firms, say j, and one type of workers, say i (working in a type i firm).*

b) Suppose there exist a submarket where workers employed in firms of type i search for jobs in firms of type j. The wage W_{ij} in this submarket is uniquely given by

$$\frac{\eta}{1-\eta} = \frac{W_{ij} - M_i^*}{M_j^* - W_{ij}} \tag{10}$$

The lemma simplifies characterization of equilibrium. Each worker-firm combination leads to at most one operating submarket, and each submarket can be attributed to exactly one worker-firm combination. Let submarket ij denote a market in which workers currently employed in firms of type i and firms of type j search for each-other. Hence we can describe the vector of distributions $\widetilde{\kappa}$ as an $n \times n$ matrix κ, where where κ_{ij} gives the fraction of workers employed in firms of type i that search in the ij submarket. Note that $\kappa_{ij} = 0$ for all $j < i$. Similarly, we can write the vector of distributions $\widetilde{\tau}$ as an $n \times n$ matrix τ, where τ_{ij} denote the fraction of firms of type j searching in the ij submarket. Then $\tau_{ij} = 0$ if $i \geq j$. The first order condition for v_{ij} writes (where θ_{ij} is the labor market tightness in the market)

$$c'(v) = (M_j - W_{ij})q(\theta_{ij}) \tag{11}$$

By slightly rearranging the first order conditions we obtain from 1

$$rM_i = y_i + (s+\delta)(M_0 - M_i) + \max_j p(\theta_{ij})\eta[M_j - M_i] \tag{12}$$

$$W_{ij} = M_i + \eta(M_j - M_i) \qquad \text{for all } i, j|\kappa_{ij} > 0 \tag{13}$$

$$c'(v_j) = (1-\eta)(M_j - M_i)q(\theta_{ij}) \quad \text{for all } i, j|\kappa_{ij} > 0 \tag{14}$$

The first conditions defines joint income and ensures efficient on-the-job search. The second equation defines the traditional efficient rent sharing in competitive search equilibrium, the Hosios condition. The third equation equates the marginal cost of vacancy posting to its expected benefit. Since the value of search is the same in all submarkets a firm operates, v_j is independent of i. Note that the wage contract posted by the firm is also constant throughout the life of the firm and does not feature any transitional dynamics.

Remark 1 *Note that as long as $y_i > y_0$, firms of type i are active in equilibrium. Since workers search equally well on and off jobs, the joint income of a worker and a firm of type i, $M_{i,}$ is then strictly greater than M_0. The firm will thus offer a wage $W = M_0 + (1-\beta)(M_i - M_0)$ and attract some workers.*

Remark 2 *The net present value of profits Π_j is then given by (6). With quadratic costs, the NPV profit reads*

$$\Pi_j = \frac{[(1-\beta)(M_j - M_i)q(\theta_{ij})] - c(v_{ij})}{r+s+\delta}$$

Proposition 1 *The equilibrium exists*

Our next proposition states that the equilibrium allocation is efficient

Proposition 2 *The equilibrium is efficient*

Our next lemma characterizes wage distributions and search behavior of workers and firms

Proposition 3 *Maximum separation:*

a) Let $i < j$. Then workers in a firm of type j always search for jobs with strictly higher wages than workers employed in firms of type $l < j$. Firms of type j always offer a strictly higher wage than firms of type i.

b) Let I_k denote the set of worker types searching for firms of type k. Consider I_k and I_l, $k > l$. Then all elements in I_k are greater than or equal to all elements in I_l. Hence I_k and I_l have at most one common element.

It follows that the market, to the largest extent possible, separates workers and firms so that the low-type workers search for the low-type firms. Note the similarity with the non-assortative matching results in the search literature (Shimer and Smith (2001), Eeckout and Kirkcher (2008). If the production technology is linear in the productivities of the worker and the firm, it is optimal that the high-type firms match with the low-type workers and vice versa. Similarly, in our model it is optimal that the workers in a firm with a high current productivity search for vacancies with high productivity, and vice versa.

From an efficiency point of view, the result can be understood as follows:, recall that if vacancies are filled quickly that requires long worker queues, and the flip-side of the coin is that workers find jobs slowly. It is therefore optimal that the most "patient" workers, i.e., the workers with the highest current wage, search for the most "impatient" firms, the firms with the highest productivity. It is also trivial to extend the efficiency result above to the n-firm case.

Note that the growth rate of a firm of a given type depends on the wage that it offers. Thus, firms of different productivities may offer different wages and attract workers at different speeds, as an efficient response to search frictions. Furthermore, the size of a firm in a given market converges to a steady state level. Thus, firms do not grow indefinitely.

It is particularly interesting to analyze the search behaviour of employed workers when the number of firm types grow large. To this end, let $G(y)$ denote the cumulative distribution function of a continuous distribution on $[y_0, y^{\max}]$. We require that for all y, the density $g(y) > \varepsilon$ for some $\varepsilon > 0$. We define a sequence of equilibria in the following way. For $t = 1$, $n = 1$, $y_1 = \frac{y^{\max} - y_0}{2}$, and $\alpha_1 = 1$. To obtain $t = 2$, we divide the interval above and below y_1 in two new intervals. Hence for $t = 2$, $n = 3$. Then we divide each of these intervals in two to obtain $t = 3$ etc. Thus, for an arbitrary t it follows that $n = 2(t + 1) - 1$, $y_i = (y^{\max} - y_0)\frac{i}{n+1}$. It follows that if y is in the support of the distribution for $t = t'$, it will also be in the support for all $t > t'$.

Suppose y is in the support of the distribution for all $t \geq t'$. Let $I_{y,t}$ denote the set worker "types" that apply for $t > t'$, that is, the productivity of the employers of the workers applying to a firm with productivity y, and let $\Delta^w_{y,t} y$ denote the difference between the highest and lowest productivity among the employers. $I_{y,t}$. Similarly, for a worker working in a firm of productivity y, let $J_{y,t}$ denote the set of firms the worker apply to, and let $\Delta^w_{y,t} y$ denote the difference between the most and least productive of these firms.

a firm attracts both employed and unemployed in the support of the productivity distribution The unemployment rate is strictly positive and bounded away from zero and from one for all t. Hence, unemployed workers must search for a strictly positive measure of firms, bounded away from zero. Furthermore, due to maximum separation, unemployed workers will search for firms with lower productivity than employed workers, and the sets I_0 and I_k, $k > 0$ have at most one element in common.

Proposition 4 *a) Suppose y^i is in the support of the distribution of productivities for some t'. Then $\lim_{t\to\infty} \Delta^w_t y' = \lim_{t\to\infty} \Delta^f_t y' = 0$.*

b) As $t \to 0$, the fraction of firms that searches for both employed and unemployed workers converges to zero.

Thus, as the number of firms grows, the search pattern of the workers get close to a pure job ladder. Unemployed workers randomize over a set of firms with low productivity. Workers employed in firms with productivity y applies to firms with productivity on a small interval around y' for some $y' > y$.

4 Robustness

In this section we explore the equilibrium of our model under slightly different assumption. First we analyze equilibrium when the vacancy cost function is almost linear. We show that in this case there is no on-the-job search, only the most productive firms hire workers and they hire directly from the unemployment pool. This violates the stylized facts, referred to in the introduction, that on-the-job search is prevalent and that firms with different productivities post vacancies.

Then we explore if the stylized facts can be explained with linear vacancy costs if we include another friction, convex adjustment cost of labor as laid out in Sargent (1987). Adjustment costs are organizational costs and training costs that firms incur when hiring new workers, and in contrast with search costs they are associated with look at the effects of hiring costs on the equilibrium of the model. We show that although convex adjustment costs imply that not only the most efficient firms recruit in equilibrium, there will still be no on-the-job search, and all firms pay equal wages. Again we conclude that this model is inconsistent with the stylized facts discussed in equilibrium.

Finally, our model as laid out in the previous sections imply that firms' growth rate declines over time, and that the firm size eventually converges to a finite size. This contradicts Gibraths law. In the third subsection we show that the equilibrium of the model satisfies Gibraths law if search costs are inversely related to firm size.

4.1 Almost linear vacancy costs

Here we characterize the equilibrium of the model when the degree of convexity of $c(v)$ vanishes. To this end, we write the vacancy cost function as

$$c(v) = c_0 v + cv^2/2$$

where c_0 and c are constants.[5]

Consider a firm of the highest type. Suppose this firm searches for unemployed workers. The number of vacancies posted and the profit of the firm is then given by

$$\begin{aligned} v_{n0} &= \frac{(1-\eta)q_{n0}(M_n - M_0) - c_0}{c} \\ \pi_{n0} &= \frac{[(1-\eta)q_{n0}(M_n - M_0) - c_0]^2}{2c} \end{aligned}$$

It follows that $q_{n0}(M_n - M_0) - c_0$ converges to zero as c_0 converges to zero, otherwise profits Π_0, and hence the expected profit of entering the market would go to infinity. However, as $q_{n0}(M_n - M_0) - c_0$ goes to zero, it is trivial to show that if $q_{j0}(M_j - M_0) - c_0$ becomes negative for sufficiently small values of c for all $j < n$ (see appendix for details). Hence no other firm type hires unemployed workers. However, in steady

[5] Note that $c'(0) = c_0 > 0$. At this point we deviate from our previous analysis, where we assumed that $c'(0) = 0$.

state it then follows that $N_1 = N_2 = ... = N_{n-1} = 0$ (this also follows directly from (3). We have thus shown the following result:

Proposition 5 *For sufficiently low values of c, only type n firms post vacancies. Hence there is no on-the-job search*

Hence, in order to obtain on-the-job search, search costs must be convex.

4.2 Hiring costs

Sargent (1987) argues that sluggish employment adjustment may be due to convex hiring costs. Hiring cost differs from search costs in that they do not depend on aggregate variables like the labor market tightness, they are entirely related to adjustment costs internal to the firms. Let us analyze whether conveyed hiring costs together with linear vacancy costs can explain the stylized facts.

We assume that the adjustment costs depend on gross hiring $h = qv$, and that the costs can be written as $\gamma(h)$, where $h = qv$. Furthermore, we assume that $\gamma(0) = \gamma'(0) = 0$ that $\gamma'()$ and $\gamma''()$ are strictly positive for $qv > 0$, and that $\lim_{h\to\infty} \gamma'(h) = \infty$.

Consider a firm of type j that searches for workers and offers a wage W_j. The profit flow from hiring can then be written as

$$\pi_j = -c_0 v - \gamma(h) + qv(M_j - W_j)$$

It is instructive to divide the firms' maximization problem into two steps:

- For a given h, minimize $c_0 v + qvW_j$ subject to $h = q(f(W_j))v$, and denote the minimum by $C(h)$
- For a given $C(h)$, find the optimal h, i.e., the h that solves $hM_j - C(h) - \gamma(h)$

The Lagrangian associated with the first problem writes

$$L = c_0 v + qvW - \lambda[q(f(W; M))v - h]$$

Let $q_w = \frac{d}{dW} q(f(W)$, and let $\eta_w = q_W W/q$. It follows that the first order condition of the first problem reads

$$W^* = c_0 \frac{\eta_w}{q} \qquad \lambda = \frac{c_0}{q^*} + W^* \qquad h = vq^* \tag{15}$$

where $q^* = q(f(W^*))$. Note that *the optimal wage W^* is independent of both h and M_j, and*

$$C(h) = (W^* + \frac{c}{q^*})h$$

The unit cost is thus the wage rate plus the expected cost c/q^* of finding a worker. If the firm will double its hiring rate, it does so by doubling the vacancy rate, leaving the wage constant. This leads to a doubling of the wage bill (since twice as many are hired) and a doubling of the vacancy cost, and thus also a doubling of $C(h)$.

The first order condition for the optimal hiring rate is thus given by

$$\gamma'(h) = M_j - W^* - \frac{c}{q^*} \tag{16}$$

which uniquely determines h.

Proposition 6 *Suppose the vacancy costs are linear while hiring costs are convex in the hiring rate $h = vq$. Then the following is true*

a) The optimal wage is independent of the productivity of the firm.

b) Firms with different productivities may coexist and post vacancies in equilibrium

c)) There is no on-the-job search

The first point, that the hiring wage is independent of firm type, follows directly from (15), as the wage only depends on c_0 and properties of the matching function. Note that a productive firm with a high M_j will hire many workers (from 16), however this induces the firm to post many vacancies, not to set the wages high.

The convexity of the hiring function sets a limit on how big the most productive firms will grow. From (16) it follows that $M_n > W^* - c/q^*$, and this open up for the possibility that $M_i > W^* - c/q^*$ also for $i < n$ and hence that these firms may also hire (see the appendix for a formal proof).

Efficient on-the-job search implies that workers will only search for jobs if $W > M_j$. At the same time firms will only attract workers in the first place if $M_j > C$. Hence it will always be less costly to search for unemployed workers, and on-the-job search will not take place.

To conclude, it follows that convex hiring costs can not explain the stylized facts that on-the-job search is prevalent or that high-type firms pay higher wages than low-type firms.

Thus, convex hiring cost may explain why firms of different productivities coexist in equilibrium. However, convex can neither explain on-the-job search nor the observed relationship between wages and firm productivity.

4.3 Search costs decreasing in firm size

The model presented above implies a constant gross hiring rate of workers. As the firms grow, so does the number of separations, and as a result there exists a steady state level of employment. Thus, the growth rate of any given firm is decreasing with firm age and size, and is hence violating Gibraths law.

The dynamic properties of the firms depend crucially on the functional form of the cost of hiring vacancies. To see this, suppose the cost of posting vacancies can be written as

$$c = Nc(\frac{v}{N})$$

The cost function can be rationalized as follows: Suppose $c(v_i)$ is the individual effort cost of assisting in the hiring process by exerting v_i units of effort. Since $c()$ is convex, it is optimal to distribute effort equally over the work force so that each worker contributes $v_i = v/N$ units, where v is the total effort level of the firm, i.e., the number of vacancies. Total effort cost is then $Nc(\frac{v}{N})$. In all other aspects the model proceeds exactly as before. In the appendix we show that the behaviour of the firm is exactly as before, the only difference is that M_j now writes

$$M_j = \frac{y_j + \max_k p_{jk}(W_{jk} - M_j) + (s+\delta)M_0 + [\widehat{v}_j^* q_{ij}^*(1-\eta)(M_j - M_i) - c'(\widehat{v})]}{r+s+\delta} \qquad (17)$$

where $\widehat{v}$ is the ration of vacancies to workers. In particular, The expression is the same as our previous expression for M_j, except for the last term in the nominator which captures the improved hiring opportunities in the future by hiring more workers today.

All equilibrium conditions are preserved. In particular, the wage and vacancy equation in submarkets ij where firms of type j are active reads

$$W_{ij}^* = \eta(M_j - M_i) \qquad (18)$$

$$\widehat{v}_j^* = (1-\eta)q_{ij}^*(M_j - M_i)c \qquad (19)$$

and $q^* = q(\theta(W_{ij}^*, \overline{M}_i))$. Since M_j is time-independent for all j, it follows that $\widehat{v}_j$, the ratio of vacancies to workers defined by (19) is independent of N. It follows that the growth rate of a firm j searching in submarket ij contingent upon survival is

$$\frac{\dot{N}}{N} = vq_{ij}^* - p - s$$

where p is the rate at which workers leave the firm after successful on-the-job search.

5 Empirical implications

In this section we will briefly discuss testable differences in predictions between our model and some other important models of on-the-job search. To this end, let $D_w(w|w^o)$ denote the distribution of wages obtained after successful on-the-job search of a worker with a wage w^o prior to the job switch. Analogously, let $D_p(p|p_0)$ denote the distribution of productivities in the new firms contingent on the productivity of the employer prior to the job change. Finally, let $D_f(w|w^n)$ denote the distribution of wages *prior* to the job switch for a worker that obtains a wage w^n after successful on-the-job search.

The Burdett-Mortensen (BM) model. In the BM model, a firm's output is proportional to the labor force,and firms post wages prior to being matched with workers. Firms and workers match randomly, hence the distribution of wages D_w after successful on-the-job search is equal to the wage distribution over vacancies truncated at previous wage w^o ($w \geq w_o$). Hence, if the wage distribution over vacancies is denoted by $F^w(w)$, it follows that

$$D_w(w|w^o) = \frac{F^w(w) - F^w(w^o)}{1 - F^w(w^o)}$$

The support of the distribution D is $[w^o, w^s]$, where w^s is the supremum of the support of advertised wages. Second, consider two workers with different initial wages w_l^o and w_h^o, $w_l^o < w_h^o$. Consider the distribution of wages after successful on-the-job search. Let $D^{w \geq w_j}(w|w_i^o)$ denote the distribution function of new wages w, contingent on $w \geq w_j$, as a function of the old wage w_i^o. Then for any $w_j \geq w_h^o$,

$$\begin{aligned} D_w^{w \geq w_j}(w|w_i^o) &= \frac{D_w(w|w_i^o) - D_w(w_j|w_i^o)}{1 - D_w^{w \geq w_j}(w_j|w_i^o)} \\ &= \frac{F^w(w) - F^w(w_j)}{1 - F^w(w_j)} \end{aligned}$$

independently of w_i^o. Hence $D_w^{w \geq w_j}(w|w_l^o) = D_w^{w \geq w_j}(w|w_h^o)$ for any $w_j > w_o^h$.

Similarly, the distribution of prior wages $D_f(w|w^n)$ is equal to the distribution of wages over employees (including unemployment benefit) truncated at $w \leq w^n$. Consider two wages w_l^n and w_h^n, $w_l^n \leq w_h^n$, and let $D_f^{w \leq w_j}(w|w^n)$ denote the distribution of the prior wage w prior to the job change. It follows that as long as $w_j \leq w_l^n$, $D_f^{w \leq w_j}(w|w_l^n) = D_f^{w \leq w_j}(w|w_h^n)$.

The Postel-Vinay and Robin (PR) wage setting procedure. Postel-Vinay and Robin (2002) assume that after successful on-the-job search, the incumbent firm and the new firm compete for the worker in a Bertrand fashion. Furthermore, firms compete in NPV wages, hence a worker takes into account that expected future wages (after encountering another job offer) will be higher the higher is the productivity of the employer. The latter is referred to as the option value of the job.

If we consider productivity instead of wages, the results for the BM model carries over to this model. If we let D_p denote the distribution of productivities of the new firm after successful on-the-job search, it follows that D_p is equal to the productivity distribution of the vacancies truncated at the productivity level of the previous employer. Hence, if the productivity distribution over vacancies is given by $F^p(w)$, it follows that

$$D_p(w|w^o) = \frac{F^p(w) - F^p(w^o)}{1 - F^p(w^o)}$$

Consider then the distribution of wages $D_w(w|w_o)$. There is no one-to-one correspondence between wages and productivity in a given job. However, wages and productivity are positively correlated. Hence there will be a positive correspondence between wages in a previous job and wages in the new job, even though the distribution of underlying productivities does not show such a correspondence.

Due to the fact that the option value is increasing in productivity, the model predicts that, contingent on prior wage (or productivity of the employer) there is a *negative* relationship between wages in the new job and the productivity of the new employer.

Consider then the distribution of wages in a given firm. On average, a high-productivity firm will pay higher NPV wages when attracting workers, since they will be willing to bid higher and be able to attract workers previously employed in more productive firms. On the other hand, since the option value of staying in a high-productivity firm is higher than the option value of staying in a low-productivty firm, hence contingent on the productivity of the previous employer, the more productive workers pay less.

Competitive on-the-job search. Our model is not a model of wages, but rather of NPV wages. However, assume that the wage that the worker obtains in a firm is constant, and that the workers' search behaviour is contracted upon directly. As the value of job search is lower the higher is the worker in the hierarchy, it follows easily that $w = w(W)$, $w\prime(W) > 0$.

With this assumption, it follows that the distribution $D_w(w|w^o)$ then has a spike, at a discrete distance above w^o. Furthermore, the support of D_w is an interval $[w^l, w^h]$, where $w^o < w^l < w^h < w^s$. As the number of firm types goes to infinity, $w^h - w^l$ converges to zero. Furthermore, it follows from our earlier results that $D(w|w_h^o)$ is strictly above $D(w|w_l^o)$, the infimum of the support of the former is strictly greater than the supremum of the support of the latter.

Another prediction from competitive on-the-job search is that more productive firms pay higher wages than less productive firms, even if they attract workers from firms with the same productivity.

Thus, the BM model and the competitive search model has different predictions regarding the relationship between wages before and after a job change. Regarding wage schedules, the CS model and the PR-wage setting procedure give rise to differences that are more subtle, and where it may be necessary to solve the models numerically to spot the differences. However, the models have very different predictions regarding the relationship between wages and productivity. The competitive search model predicts that if two firms with different productivities attract workers with equally productive employers, the high-productivity firm pays the higher wage. The PR model predicts the opposite.

6 Equilibrium with two types of firms

Consider the special case with two types of firms. We also assume that the matching function is Cobb-Douglas, $x(u, v) = Au^\beta v^{1-\beta}$. In this case we can get some more structure and therefore also some more results regarding the opening and closing of submarkets.

Our first observation is that the $_{12}$ market is always open. Suppose not. Then a high-type firm that opens vacancies with a wage slightly above y_1 would attract applications for all workers employed in type 1 firms. The firm would thus obtain an infinitely high arrival rate q of job offers, and would make infinitely high profit. A deviation from equilibrium would thus surely be profitable.

The next question is whether the $0, 2$ market will open up (stairways to heaven). If not, we say that we have a pure job ladder. Whether or not we have a pure job ladder depends on parameter values. However, with very mild restrictions on $c(v)$ (that $\lim_{v\to\infty} c'(v)/v = \infty$) we can show the following proposition:

Proposition 7 *a) Suppose K is high, so that few firms enter the market. Then high-type firms search both for unemployed and employed workers.*

b) Suppose there exists a pure job ladder for some values of K. Provided that the number of vacancies is not too flexible (c'' is sufficiently large around the equilibrium point), then there exists a K^* such that there is a pure job ladder for $K < K^*$ while both the $1, 2$ and the $0, 2$ market open up if $K > K^*$.

We need the qualifier in order to ensure that the measure of vacancies in the economy goes to zero when the measure of firms goes to zero. The proposition thus states that the pure job ladder equilibrium only prevails if the frictions in the market are sufficiently low. Thus, contrary to what one may expect, a pure job ladder (if it emerges at all) emerges when there are many jobs relative to workers and hence the unemployment rate is low.

Our second question regards the relationship between the share of high-type firms in the equilibrium. Let a balanced increase in α_2 denote an increase in α_2 where other variables (for instance the entry cost K) is adjusted so that the number of firms k is kept constant. We are able to show the following result:

Proposition 8 *a) For high values of α_2, both the $0, 2$ and the $1, 2$ submarkets are active. For low values of α_2, only the 12 market is active.*

b) Consider balanced changes in α_2. Suppose $c''(e)$ is large. Then there exists a unique $\alpha = \alpha^*$ such that the $_{02}$ market is open if and only if $\alpha_2 > \alpha^*$.

Two remarks regarding b) is warranted. The first regards the fact that we are only considering balanced changes. The reason is the following. Suppose that $\alpha_2 = \alpha^*$, so that there is a pure job ladder and the 2-firms are just indifferent by entering the $_{12}$ and the $_{02}$ market. Consider an increase in α_2. This has two effects on equilibrium. First it becomes more crowded in the $_{12}$ market, and this favours the $_{02}$ market. However, if we let k vary, it follows that k will increase, and as seen in proposition (7) this favours the pure job ladder equilibrium. In general we are not able to show which force is the stronger, and hence cannot guarantee that there is a unique switching point. However, with balanced changes we can.

The second comment regards the requirements on $c''(e)$. When α_2 increases, the direct effect is that p_{01} decreases, as there are fewer low-type firms. Our proof of uniqueness of the switching point depends on p_{01} being decreasing in α_2. However, low-type firms may post more vacancies, and in principle this may imply that p_{01} increases in α_2. As we have not been able to rule this out, we instead put restrictions on $c''(e)$ so that the flexibility of e is not too big.

6.1 Basic Calibration and Comparative Static

Table 1 and 2 report the basic parameter values for our calibration. The calibration is based on quarterly statistics and the pure interest rate is 1 percent. The productivity level in low type firms is set to a baseline reference value of $y_1 = 1$, while the premium for the high type is 15 percent. The flow value of unemployment z is 0.55, a value far the replacement rate observed in real life labour markets. The matching function is Cobb Douglas with an elasticity β equal to 0.5. The parameter of the search cost is 0.15, while the entry cost k is 5, a value roughly equal to five times times the output produced by a low productivity job. The sum of the separation s and the firm death rate is 0.06. The proportion of low productivity firms is 0.155. In the specification of the model presented in this section, we assume that the convexity of the vacancy is extreme so that each firm can post at most a maximum number of vacancies $\overline{v} = 0.15$. The rest of the parameters are reported in 1

The baseline equilibrium features an unemployment rate equals to 7.7 percent and a job finding probability equal to 0.7, in line with the basic quarterly statistics in advanced economies. Unemployment flows are 5.5 percent, consistent with the quarterly job creation rate in the US manufacturing sector compiled by Davis and Haltiwanger. Job to job mobility is slightly below 5 percent. In Table 1 most of the unemployed workers search for low productivity firms, as indicated by $k_{01} = 0.99$. Similarly, high productivity firms search mainly among the employed sector, as indicated by the fraction of firms hiring from the employment pool ($\tau = 0.99$) The equilibrium allocation is described in the central part of Table 1. The job finding rate for unemployed workers p_{01} is the largest among the various job finding rates, but the bulk of workers in the labor market is employed in type 2 firms. Indeed, type 2 firms absorb 68 percent of the total workforce. As a result, the submarket 02, albeit significant, represents a fringe of the entire economy.

The idea of the baseline simulation from Table 1 to Table 2 is to show that a small decrease in the share of high productivity firms α lead the economy to move toward a pure job ladder equilibrium. Indeed, the only parameter that changes between Tables and 1 and 2 is α. Recall that in the baseline specification of Table 1 the equilibrium value of τ is very close to one and as a result the submarket 02 is very small. A small decrease α, similarly to that experienced from Table 1 to Table i2 leads to an equilibrium value of $\tau > 1$, a value that is not consistent with all three submarkets being operative. In other words, as α falls with respect to the value assigned in Table 1, the economy moves to a *pure job ladder equilibrium*. In moving from Table 1 to Table 2 α falls from 0.155 to 0.154, suggesting that α^* in our numerical example is inside this small interval. The economy described in Table 2 does look very similar to that described in Table 1, even though two only submarkets are operative. Note also that the equilibrium value of unemployment M_0^* does slightly fall as α falls. This is not surprising since in a pure job ladder equilibrium the share of high productivity firms is higher. Workers start out in low productivity firms and eventually graduate to high type jobs through on the job search. Eventually, firm and match specific shocks at rate δ and s induce another round of job ladder. The bottom part of the Table 1 features also an important relationship between firm size and firm wages, where the latter are measured in terms of PDV wages. Clearly, high type firms are larger in size and pay higher wage.

6.2 Comparative Static

The main object of the simulations carried out in this section is to show the mechanics of the model for different values of α in the baseline simulation provided above. As $\overline{y} = (1-\alpha)y_1 + \alpha y_2$, an increase in α is akin to an increase in average productivity. The basic charts of the simulations are provided in Figure 1 and 2. First note that when $\alpha = 0$ or 1, the model collapses to the traditional matching model without on-the-job search (Pissarides 2000). As expected, the transition rate from unemployment to employment is higher and unemployment lower when $\alpha = 1$ than when $\alpha = 0$. (In Figure 1 unemployment falls from 0.0968 to 0.083 as α increases from 0 to 1). We refer to this as a pure *productivity effect*, and it is caused by higher entry of firms and a higher f when output per firm is high.

For interior values, an increase in α comes along with important *composition effects*. While the value functions increases smoothly as the economy becomes more productivity (top left panel in Figure 2), the increase in the job finding rate p_{01} in the pure job ladder is humped-shaped. For a fixed number of firms, an increase in α reduces the number of jobs available to the unemployed (which are hired in firm of type 1), and increase the jobs available to the employed (which are hired in firm of type 2). This composition effect tend to reduce the job finding rate p_{01}. The productivity effect increases the number of firms, and hence work in the opposite direction, but in the pure job ladder equilibrium it only dominates the composition effect for very low values of α. Note also that job-to-job movements, by definition equal to zero at the extremes, tends naturally to grow as the economy operates into a pure job ladder equilibrium.

For higher values of α, the mixed job ladder equilibrium emerges, with a different type of composition effects. In particular, the $_{02}$ submarket is characterized by lower job-finding rates. A higher α on some intervals imply larger variations in the queue lengths among unemployed workers, and this tends to increase unemployment. For relatively low levels of α this effect dominates the productivity effects. Eventually, as the share of high productivity firms increases toward 1, the pure productivity effects emerges and unemployment falls.

Finally, the non monotonic behavior of entry deserves few comments. When α is low, the value of a high-type firm (given by 6) is extremely high since they grow so quickly. This explains the hump-shaped form of f, the number of firms in the economy.

7 Conclusion

We have developed a competitive flavored matching model where on-the-job search is an optimal response to productivity differences between firms and costly search. In the resulting equilibrium, workers hired in firms with different productivities, and

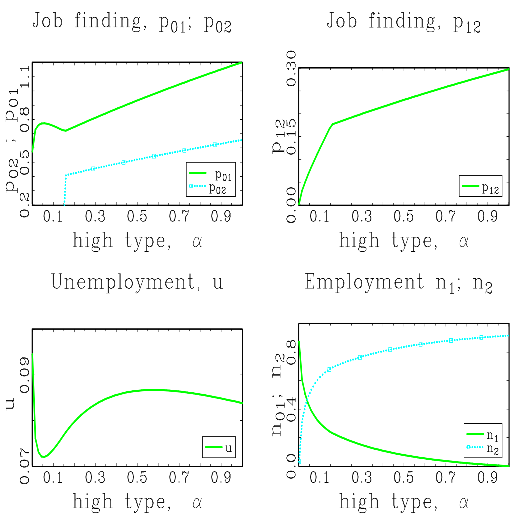

Figure 1: Increase in Average Productivity, Stocks and Job Finding Rates

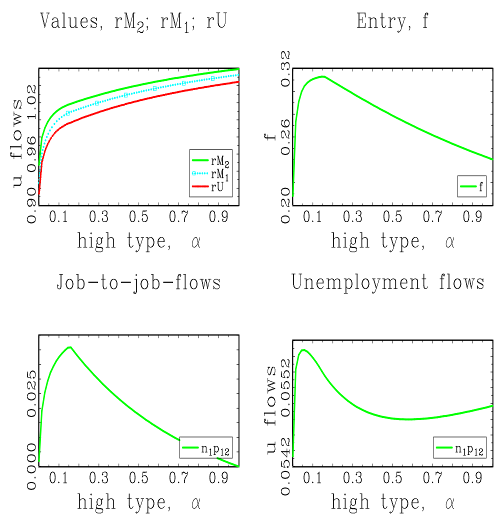

Figure 2: Increase in Average Productivity, Value Functions, Flows and Entry

firms with different productivities search in separated job markets, in such a way unemployed workers search for low-type firms, while employed workers search for jobs in firms that are more productive. The equilibrium thus features a job ladder, where workers gradually moves to jobs with higher wages. The model can thus explain the following stylized facts about the labor market: 1) firms with different productivities coexist in the labor market, 2) On-the-job search is prevalent and worker flows between firms are large, and 3) more productive firms are larger and pay higher wages than less productive firms. The equilibrium is efficient.

Interestingly, we find that if the costs of maintaining vacancies are close to being linear, only the most productive firms open vacancies, and hence there will be no on-the-job search. If we include hiring costs (convex in the flow qv of new hires), more than one firm type may open vacancies. However, in this case wages will be independent of productivity and there will be no on-the-job search. Hence convex hiring costs seem to be necessary in order to explain the stylized facts of on-the-job search.

8 APPENDIX:

8.1 Proof of lemma 1

Proof: a) An unemployed worker obtains y_0/r, hence $y_0/r \leq M_0$. A type n worker cannot gain from on-the-job search since $W \leq y_n$, and hence $M_n = y_n/(r+s+\delta)$. A worker and a firm of type j (hereafter referred to as a worker of type j) can always obtain a strictly higher joint income than a worker of type $i < j$ by following exactly the same search strategy as the type i worker, hence $M_j > M_i$. Analogously, a worker of type i can always mimic the search strategy of a worker of type $j > j$. Let the associated joint income be denoted M_i'. From (1),

$$(r+s+\delta)(M_j - M_i') = y_j - y_i$$

or

$$(r+s+\delta)M_j - y_j = (r+s+\delta)M_i' - y_i$$

Since $M_i' \leq M_i$ it follows that $(r+s+\delta)M_j - y_j \leq (r+s+\delta)M_i - y_i$.

b) We want to show that the indifference curve has the following single crossing property: Suppose $i < j \leq n'$. Then there exists a wage W' such that $f_i(W', M) = f_j(W', M)$, $f_i(W, M) < f_j(W, M)$ for all $W < W'$, and $f_i(W, M) > f_j(W, M)$ for all $W > W'$.

Note that $\lim_{W \to M_j^+} f_j = \infty$ while $f_i(M_j, M) < \infty$. Thus, for W close to but above M_j we have that $f_i(W,M) < f_j(W,M)$. The ratio of $p(f_i)$ to $p(f_j)$ reads

$$\frac{p(f_i)}{p(f_j)} = \frac{(r+s+\delta)M_i - y_i - (s+\delta)M_0}{(r+s+\delta)M_j - y_j - (s+\delta)M_0}\frac{W-M_j}{W-M_i}$$
$$\lim_{W\to\infty}\frac{p(f_i)}{p(f_j)} = \frac{(r+s+\delta)M_i - y_i - (s+\delta)M_0}{(r+s+\delta)M_j - y_j - (s+\delta)M_0} < 1$$

(from a in this lemma). Thus, for sufficiently large values of W, $p(f_i(W;M)) < p(f_j(W,M))$, and hence $f_i(W;M) < f_j(W,M)$. Since f_i and f_j are continuous it follows that there exists a value W' such that $f_i(W';M) = f_j(W',M)$.

c) Suppose now that $i < j < k$. Let W^{ik} be the unique solution to $f_i(W;M) = f_k(W;M)$. Suppose $W^{ik} < W^{ij}$. From the single-crossing property just arrived it follows that $f_i < f_j$ for all $W < W^{ij}$ and that $f_k < f_j$ for all $W > W^{ik}$. Since, by assumption,$W^{ik} < W^{ij}$, it follows that hence $f_j(W;M) > f(W,M)$. Since all workers types at or below except the highest search this is a contradiction.

For wages below M_1, only type zero workers will apply, hence $f(W,M) = f_0(W;M)$. At $W^1 = W^{01}$, $f_0(W^1;M) = f_1(W;M)$. Let $W^2 = W^{12}$. Then $f(W) = f_2(W;M)$ for $W \in [W^1, W^2]$ (which may have zero measure) and so forth. Furthermore, by definition type n' gains from search, and hence searches for a job with a wage $W \leq W^s$. Hence there must be an interval $[W, W^s]$ at which $f_{n'} = f_n$

d) For all $i \leq n'$, each of the functions $f_i(W;M)$ is continuously differentiable for $W > M_i$. It follows that $f(W;M) = \min_i f_i(W;M)$ is continuos and piecewise differentiable for all $W < M_{n'+1}$. However, as the nominator in (3) is zero for $i = n'+1$, it follows that $f_{n'+1}$ is zero for any $W > M_{n'+1}$. Hence f is discontinuous at $M_{n'+1}$,as it jumps down to zero at this point.

8.2 Proof of lemma 2

Suppose a submarket attracts $i-$workers and j-workers, $i < j$. Denote the npv wage in the submarket by W'. Then $f_i(W',M) = f_j(W',M) = f(W',M)$. From lemma (1) this can only be the case if $j = i+1$, that is, if $W' = W^i$.

From lemma 1 it follows that

$$\begin{aligned}
\lim_{W\to W^{i-}}\frac{\partial q(\theta(W^i))}{\partial W} &= \lim_{W\to W^{i-}} q'(\theta(W_i))\frac{\partial\theta(W^i)}{\partial W} \\
&< \lim_{W\to W^{i-}} q'(\theta(W_i))\frac{\partial\theta(W^i)}{\partial W} \\
&= \lim_{W\to W^{i-}}\frac{\partial q(\theta(W^i))}{\partial W}
\end{aligned}$$

It follows that $W = W^i$ cannot be a solution to any firm's maximization problem and hence cannot be an equilibrium wage. Hence a submarket cannot contain two different worker types.

Suppose then that two firm types i and j, $i < j$ offer the wage W'. The optimal wage for firm j solves

$$\max_{v,W} -c(v) + vq(f(W))[M_j - W]$$

with first order condition for W given by

$$\frac{q'(f(W))f'(W)}{q} = \frac{1}{M_j - W} \tag{20}$$

The left-hand side is independent of j, while the right hand side is increasing in j. It follows that the first order conditions cannot be satisfied for two different firm types simultaneously.

In order to derive (10), first note that

$$\frac{dp^{-1}(\theta)}{d\theta} = \frac{1}{p'(\theta)} = \frac{1}{q + q'(\theta)}$$

(since $p(\theta) = \theta q(\theta)$). From (3) it thus follows that

$$f'(W) = -\frac{1}{q + q'(\theta)} \frac{\theta q(\theta)}{W - M_i} \tag{21}$$

which inserted into (**??**) gives

$$-\frac{q'(\theta)}{q + q'(\theta)} \frac{\theta q(\theta)}{W - M_i} = \frac{q}{M_j - W}$$

Inserting $\eta = -q'(\theta)\theta/q(\theta)$ and reorganizing slightly gives

$$\frac{\eta}{1 - \eta} = \frac{W - M_i}{M_j - W}$$

By assumption, the left-hand side is decreasing and the right-hand side is increasing in W. Thus, for given M_i and M_j the equation uniquely defines W. Trivially, the first order condition for vacancies is given by (11)

8.3 Proof of existence

The strategy for the proof is to construct a mapping for which the equilibrium of the model is a fixed point, and then apply Brouwer's fixed point theorem. We assume that the matching function is Cobb-Douglas, $x(u, v) = Au^{\beta}v^{1-\beta}$ (generalizing the proof is simple).

To this end, let $\overline{\kappa}$ denote a matrix describing submarket choices of workers κ_{ij}, $\kappa_{ij} = 0$ if $i \geq j$, and $\sum_{j=i+1}^{n} \kappa_{ij} = 1$ for all i. Similarly, let $\overline{\theta}$ denote a matrix of labor market tightnesses θ_{ij}, $\theta_{ij} = 0$ if $i \geq j$. We require that $0 \leq \theta_{ij} \leq \theta^{\max}$ for all $i < j$, where $\theta^{\max}$ will be defined below. Finally, let the real number k denote the measure of firms in the economy. We require that $k \leq k^{\max}$. It follows that the set $D^n \in R^{2(n+1)^2+1}$ of allowed vectors $(\overline{\kappa}, \overline{\theta}, k)$ is closed and convex.

We want to construct a continuos mapping $\Gamma : D^n \to D^n$, and proceed as follows: Let $\overline{p}$ denote the matrix of transition probabilities $p_{ij} = A\theta_{ij}^{1-\beta}$. Analogous with (**??**), define

$$(r + s + \delta)M_{ij} = y_i + (s + \delta)M_0 + p_{ij}\beta(M_j - M_{ij}) \tag{22}$$

(where M_0 is replaced with M_0). Let $\overline{M}$ denote the matrix of values M_{ij}. Given the matrix $\overline{p}(\overline{\theta})$, the matrix $\overline{M}$ is uniquely defined as a continuos function of $\overline{\theta}$, $\overline{M}(\overline{\theta})$. To see this, first note that M_n is independent of $\overline{\theta}$. Suppose M_{ij} and $M_i = \max_j M_{ij}$ are uniquely defined as continuos functions of $\overline{\theta}$ for all $i, j > i$, for all $i > k$. It then follows from equation (22) and the definition of M_i that this also holds for M_{kj}, $j > k$, and M_k. Thus it holds for all i, j such that $j > i$.

The gross income flow of a firm of type j of posting a vacancy in submarket i is given by $\rho_{ij} = q(\theta_{ij})(1-\beta)(M_j - M_{ij})$. Define $\rho_j = \max_i \rho_{ij}$. Now define θ_{ij}^a implicitly by the function

$$q(\theta_{ij}^a)(1-\beta)(M_j - M_{ij}) = \rho_j$$

The equation thus shows the values of θ_{ij} such that the firm of type j is indifferent between searching in submarket i and in the best submarket given $\overline{\theta}$. Finally, let v_j^a be defined by the equation $c'(v_j^a) = \rho_j$ (the optimal number of vacancies given ρ_j). It follows that both $\overline{\theta}^a$ and v_i^a are continuos functions of $\overline{\theta}$.

Given the initial vector $\overline{\theta}$, equation (7) uniquely defines $N_0, N_1, ...N_n$ as continuous functions of $\overline{\theta}$. In each submarket, aggregate consistency requires that

$$N_i \kappa_{ij} \theta_{ij} = k\alpha_i \tau_{ij} v_{ij} \tag{23}$$

Sum over i. This gives

$$\sum_{i<j} N_i \kappa_{ij} \theta_{ij} = \sum_{i<j} k\alpha_i \tau_{ij} v_{ij}$$

We now insert $v_{ij} = v_j^a$ into this equation. Define the constant ζ_j by the expression

$$\sum_{i<j} N_i \kappa_{ij} \theta_{ij} = \zeta_j k \alpha_i v_j^a \tag{24}$$

Finally, define

$$\widehat{\theta}_{ij} = \zeta_j(\widehat{\theta})\theta_{ij}^a(\widehat{\theta})$$

This is our updating rule for rule for θ unless the upper bound $\theta^{\max}$ binds, in which case $\widehat{\theta}_{ij} = \theta^{\max}$.

Consider the searching workers. Suppose M_i is obtained for $j \in J_i$. For all $j \notin J_i$, define

$$\widehat{\kappa}_{ij} = \frac{M_{ij}}{M_i} \kappa_{ij}$$

Note that $\widehat{\kappa}_{ij}$ is continuous in $\overline{\theta}$ and κ_{ij}. Define the constant η_i by the expression

$$\eta_i \sum_{j \in J_i} \kappa_{ij} + \sum_{j \notin J_i} \widehat{\kappa}_{ij} = 1$$

For all $j \in J_i$, the updating rule reads

$$\widehat{\kappa}_{ij} = \eta_i \kappa_{ij}$$

Finally, the expected profit of a firm of type i entering the market and searching for a firm $j \in J_i$ reads

$$\Pi_i = \frac{1}{r+\delta} \left\{ v_i^a q(\theta_{ij})(M_{ij} - M_i)(1-\beta) - c(v_i^a) \right\}$$

The expected profit of entering, given the initial parameter values, is

$$E\Pi = \sum_i \alpha_i \Pi_i$$

The updating rule for k reads

$$\widehat{k} = k \frac{E\Pi}{K}$$

unless the upper bound $k^{\max}$ binds, in which case $\widehat{k} = k^{\max}$.

We have thus constructed a mapping $\Gamma : D^n \to D^n$, which by construction is continuos. It follows from Brouwers fixed point theorem that the mapping has a fixed point.

Our next step is to show that a fixed point of Γ is an equilibrium of our model. Denote the fixed point by D^*. First, given the asset value matrix M_{ij}, the firm sets the optimal sharing rule by construction. Furthermore, by the very definition of $\widehat{\theta}$ it follows that the all firm are indifferent by entering any submarket $_{ij}$. Thus, the firms' search behavior is optimal.

Second, from the updating rule for κ it follows that if $\kappa_{ij}^* > 0$, then it is optimal workers in firm j to search for a position in firm j.

Third, we have to show that the model is internally consistent, and satisfies (23). At the fixed point, $\zeta_j = 1$ for all j. Hence (24) is satisfied. However, this means that the weights τ_{ij} give us enough degrees of freedom to satisfy (23).

By construction, the labor market tightness θ_{ij}^* is defined even in submarkets where $\kappa_{ij} = 0$, i.e., even in empty submarkets. We have thus ruled out the situations where no agents enter a submarket which potentially may be active because no-one else enter the market.

Finally, we characterize the bounds. Consider the equilibrium with $\alpha_n = 1$ (only firms of the highest productivity). Then the model collapses to the standard search model, and it is trivial to show that this equilibrium exists. Define k^1 and θ^1 as the equilibrium values of k and θ in this equilibrium, respectively. Define $k^{\max} = k^1/\alpha_n$.

With this number of firms, we know that $M^n \leq M'$, where the latter is the joint income of a worker and a firm (type n) in equilibrium when $\alpha_n = 1$. It follows that the expected income of entering when $\alpha_n < 1$ is lower than K. Hence $k^* < k^{\max}$. Finally, let W'_{01} and θ'_{01} denote the solution to the maximization problem of a firm of type 1 in the $\alpha_n = 1$ equilibrium. By construction, $\theta^*_{ij} < \theta'_{01} = \theta^{\max}$. (Note that we could have worked with queue length $1/\theta$ instead, in which case defining an upper bound for θ would be easier).

8.4 Proof of efficiency

The welfare function is defined as

$$W = \int_0^\infty [\sum_{j=0}^n N_j y_j - \sum_{j=1}^n \alpha_j k c(v_j) - aK]e^{-rt}dt$$

Where v_j is the number of vacancies of a firm of type j. The law of motions are

$$\begin{aligned} \dot{N}_j &= \sum_{i=0}^{j-1} x(\kappa_{ij}N_i, \alpha_j k \tau_{ij} v_j) - \sum_{k=j+1}^{n} x(\kappa_{jk}N_j, \alpha_i k \tau_{kj} v_j) - (s+\delta)N_j \\ \dot{k} &= a - \delta k \end{aligned}$$

The initial conditions take care of the requirement that $\sum_i N_i = 1$. The controls are a, κ_{ij}, τ_{ij} and v_j. All κ_{ij}, τ_{ij} have to be between zero and 1, this will be discussed later. The current-value Hamiltonian reads

$$\begin{aligned} H &= \sum_{j=0}^n N_j y_j - \sum_{j=1}^n \alpha_j k c(v_j) - aK \\ &+ \sum_{j=0}^n \lambda_j [\sum_{i=0}^{j-1} x(\kappa_{ij}N_i, \alpha_j k \tau_{ij} v_j) - \sum_{k=j+1}^n x(\kappa_{jk}N_j, \alpha_k k \tau_{jk} v_j) - (s+\delta)N_j] \\ &+ A[a - \delta f] \end{aligned}$$

The controls are chosen so as to maximize H. Note that $x_v = (1-\eta)q(\theta)$, where $\eta = -q'(\theta)\theta/q$. [6]The first order conditions for the other controls read

$$A = K \tag{25}$$

$$p_{ij}(\lambda_j - \lambda_i) = \max_k p_{ik}(\lambda_i - \lambda_k) \text{ if } \kappa_{ij} > 0 \tag{26}$$

$$q_{ij}(\lambda_j - \lambda_i) = \max_k q_{kj}(\lambda_j - \lambda_k) \text{ if } \tau_{ij} > 0. \tag{27}$$

[6]To see this, note that $q(\theta) = x(\frac{1}{\theta}, 1)$. From the Euler equation it follows that

$$x_u(\frac{1}{\theta}, 1)\frac{1}{\theta} + x_v = x(\frac{1}{\theta}, 1) = q$$

which gives the experssion in the text.

First order conditions with respect to v_j gives We thus get the following first order conditions for vacancy creation:

$$c'(v_j) = (1-\beta)q(\theta_{ij})[\lambda_j - \lambda_i] \tag{28}$$

for all ij for which $\kappa_{ij>0}$ (note that the right-hand side is the same for all active submarkets). Finally, the value functions for the adjoint variables are given by (in steady state)

$$\begin{aligned}
(r+s+\delta)\lambda_j &= y_j + \beta \max_{k>j} p_{jk}(\lambda_k - \lambda_j) + (s+\delta)\lambda_u \qquad (29) \\
(r+\delta)A &= \sum_{j=1}^{n} \alpha_j[(1-\beta)v_j \max_k q_{jk}(\lambda_k - \lambda_j) - c(v_j)]
\end{aligned}$$

It follows that the first order conditions of the planner is exactly equal to the market solution. More than that, the maximization problem for the controls is exactly equal to the maximization problem of the firm. Thus, the planner's solution and the decentralized solution is the same.

8.5 Proof of proposition 3

a) Let $j > i$, and suppose a firm of type j advertise a wage W^l with job finding rate q^l, while the firm of type i advertise a wage W^h with job finding rate q^h. Profit maximization then implies that

$$\begin{aligned}
q^h(M_i - W^h) &\geq q^l(M_i - W^l) \\
q^l(M_j - W^l) &\geq q^h(M_j - W^h)
\end{aligned}$$

Combining the two gives

$$q^h(M_i - W^h) - q^h(M_j - W^h) \geq q^l(M_i - W^l) - q^l(M_j - W^h)$$

or

$$(q^h - q^l)(M_i - M_j) \geq 0$$

Since $M_i < M_j$, this is a contradiction.

From lemma 1 we know that $f_i(W) = f(W)$ at the interval $[W^{i-1}, W^i]$. Furthermore, from the proof of lemma 2 we know that W^i cannot be an equilibrium point. It follows that a worker of type j always searches for higher wages than a worker of type $i < j$.

b) Suppose to the contrary that I_l has an element, say i, that is strictly greater than one element in I_k, say j. From a) it follows that worker i searches for strictly higher wages than worker j. Hence firm l advertises a wage that is strictly higher than a wage advertising by firm k, $l < k$. We know from a) that this is a contradiction.

8.5.1 Proof of 4

a) Suppose y is in the support of G for $t \geq t'$ for some t'. From the maximum separation result it follows that all the workers in $I_{y,t,}$ except possibly at the end points search solely for firm y. We want to show that $\lim_{t\to 0} \Delta^w_{y,t} y > 0$. Suppose not. Then there exists a $\Delta' > 0$ such that for any t^* there exists a $t > t^*$ such that $\Delta^w_{y,t} y \geq \Delta'$. The probability mass of the end points of $\Delta^w_{y,t} y$ converges to zero as $t \to 0$. Hence there exists a $\Delta > 0$ such that for any t' there exists a $t > t'$ such that the measure of workers searching for firm y is greater than or equal to Δ. Each firm posts a finite number of vacancies. The probability that a firm obtains a probability y converges to zero as $t \to \infty$. Hence, for any $\varepsilon > 0$ and any t' there exists a $t > t'$ such that the fraction of the aggregate measure of vacancies posted by type y-firms to the measure of applications sent to these firms is below ε. Hence the probability rate of getting a job for the applicants is arbitrarily low. Since wages advertized never exceeds productivity y, it follows that the gain from search for these workers will be arbitrarily low, and this is inconsistent with optimal search. The proof of the claim that $\lim_{t\to\infty} \Delta^f_t y' = 0$ is analogous.

b) Suppose a firm with productivity y_i attracts both unemployed and employed workers. Then we know that firms of type $i-1$ only attract unemployed workers, while workers of type $i+1$ only attract employed workers. Thus, for any t, at most one firm type attracts both unemployed and employed workers. As t goes to infinity, the fraction of firms that obtain any given productivity goes to zero. Hence the fraction that attracts both unemployed and employed workers goes to zero as t grows to infinity.

8.5.2 Proof of proposition 5

Since $c_0 > 0$, it follows that $p_{ij} < \infty$ for all i, j. In particular, an upper bound for θ, $\overline{\theta}$, is given by the equation $q(\overline{\theta}) = (y^{\max} - z)/(r + s + \delta)$. An upper bar on p is thus $\overline{p} = p(\overline{\theta})$. From (12) it follows that there exist a lower bound ΔM on $M_n - M_{n-1}$. Suppose firm j enters submarket 0. Since $M_j < M_n$, it follows that $q_{n,0} > q_{j,0}$. Hence

$$
\begin{aligned}
& \lim_{c\to 0}(1-\eta)q_{i0}(M_i - M_0) - c_0 \\
< & \lim_{c\to 0}(1-\eta)q_{n0}(M_i - M_0) - c_0 \\
< & \lim_{c\to 0}(1-\eta)q_{n0}(M_n - M_0) - c_0 - (1-\eta)q_{n0}\Delta M \\
< & \ 0
\end{aligned}
$$

Hence the firm will obtain negative profit by posting vacancies in the market, and hence will not enter. This completes the proof.

8.5.3 Proof of proposition 6

a) and c) are proved in the text. We have only left to show b). Suppose only the highest type of firms hire workers. The equilibrium is then independent of the productivity of the less productive firms. Let $M_n^* > W + \frac{c}{q}$ denote the equilibrium value of M_n.

Consider a firm of type y_{n-1}. If this firm deviates and search for and hire a worker, it will obtain a joint income given by

$$M_{n-1} = M_n^* - \frac{y_n - y_{n-1}}{r + s + \delta}$$

It follows that if y_{n-1} is sufficiently close to y_n, $M_{n-1} > 0$, and the firm will find it profitable to start hiring workers. The result thus follows.

8.5.4 Proof of proposition ??

Let $\widehat{v} = v/N$. A firm maximizes

$$\begin{aligned} & \int_0^\infty e^{-(r+\delta)} N[y - \widehat{v}q(f(W))W + sU + pW' - c(\widehat{v})]dt \\ & \text{S.T.} \\ \dot{N} &= N[\widehat{v}q(f(W)) - s - p] \end{aligned}$$

The current-value Hamiltonian thus reads

$$H = N[y - \widehat{v}q(f(W))W + sU + pW' - c(\widehat{v})] + \lambda N[\widehat{v}q(f(W)) - s - p]$$

First order conditions read

$$\begin{aligned} (r+\delta)\lambda &= y + sU + p(W' - \lambda) + \widehat{v}q(f(W))(\lambda - W) - c(\widehat{v}) \\ \frac{d}{dW}(\lambda - W) &= q \\ q(\lambda - W) &= c'(v) \end{aligned}$$

The first order conditions are exactly equal to the first order conditions in the standard model with λ substituted in for M. The result thus follows.

8.6 Proof of proposition 7

Consider a situation where K is arbitrarily high. For firms to be willing to enter, it follows that Π_2 must be arbitrarily high. However, this can only be the case if the arrival rate of jobs in the market the firm searches for is arbitrarily high, i.e., k must be arbitrarily small.

It follows that p_{01} is arbitrarily low, and thus also that N_1 is arbitrarily low. Suppose all high-type firms were searching for employed 29workers. Since the ratio of high-to low type firms is bounded, the labor market tightness in this market would be bounded. Hence the firms could not obtain an arbitrarily high profit, and we cannot be in equilibrium. In the unemployed submarkets, the labor market tightness by contrast is arbitrarily small and the firms get an arbitrarily high profit.

Suppose then that for some K', there exists a pure job ladder. We want to show that then there is a pure job ladder for all $K < K'$. First we keep the number of vacancies per firm constant. By a revealed preference type of argument one cans show that k is decreasing in K.

We first want to show that $el_k p_{01} > el_k p_{02}$. First note that

$$\begin{aligned} el_k p_{01} &= el_k(\frac{\alpha_1 k}{u})^{1-\beta} = (1-\beta)(1-el_k u) \\ el_k p_{12} &= el_k(\frac{\alpha_2 k}{N_1})^{1-\beta} = (1-\beta)(1-el_k N_1) \end{aligned}$$

Now

$$N_1 = \frac{p_{01}u}{s+\delta+p_{12}} \tag{30}$$

$$u = \frac{s+\delta}{s+\delta+p_{01}} \tag{31}$$

For given stocks u and N_1, $el_k p_{01} = el_k p_{12}$. From (30) and (31) it then follows that $el_k N_1 > el_k u$ (since $p_{01}u = (s+\delta)(1-u)$ is increasing in k). It thus follows that $el_k p_{01} > el_k p_{12}$.

Note that $el_k(\lambda_2 - \lambda_0) < el_k(\lambda_1 - \lambda_0)$ (since λ_1 is increasing in k). From (26) it follows that $el_k p_{02} > el_k p01$. From (29) it follows that we can write

$$\begin{aligned} \lambda_2 - \lambda_0 &= \frac{y_2 - y_0}{r+s+(1-\beta)p_{02}} \\ \lambda_2 - \lambda_1 &= \frac{y_2 - y_1}{r+s+(1-\beta)p_{12}} \end{aligned}$$

Taking elasticities give

$$\begin{aligned} el_k(\lambda_2 - \lambda_0) &= el_p \frac{y_2 - y_0}{r+s+(1-\beta)p_{02}} el_k p_{02} \\ el_k(\lambda_2 - \lambda_1) &= el_p \frac{y_2 - y_1}{r+s+(1-\beta)p_{12}} el_k p_{12} \end{aligned}$$

Now $el_x \frac{1}{a+x} = -\frac{x}{a+x}$ (for any constant a) which is decreasing in k. Furthermore, we have seen that $el_k p_{02} > el_k p_{12}$. It thus follows that

$$el_k(\lambda_2 - \lambda_1) > el_k(\lambda_2 - \lambda_0)$$

Finally, from (27) it follows that the result holds if

$$el_k q_{02} + el_k(\lambda_2 - \lambda_0) < el_k q_{12} + el_k(\lambda_1 - \lambda_0)$$

Since $el_k p_{02} > el_k p_{12}$ it follows that $el_k p_{02} < el_k p_{12}$, and we have just shown that $el_k(\lambda_2 - \lambda_1) > el_k(\lambda_2 - \lambda_0)$. The result thus follows.

8.7 Proof of claim related to α^*

a) Sketch of proof: For any number $\varepsilon > 0$. Consider a high-type firm that sets $w = y_1 + \varepsilon$. As $\alpha_0 \to 1$, the arrival rate of workers to this firms goes to infinity, independently of which wage $w \in (y_1, y_2)$ the other high-type firms choose. Thus profits go to infinity. If a high-type firm searches for unemployed workers, the arrival rate of workers to the firm will be bounded, and hence also profit. The claim thus follows. By a similar argument, it also follows that at least some high-type firms searches for employed workers as long as $\alpha > 0$.

b) We want to show the following claim: For a given number of firms k, there exists a unique α^* with the following property: If $\alpha > \alpha^*$ there exists a pure job ladder. If $\alpha < \alpha^*$ some high-type firms search for unemployed workers. We start by assuming that the number of vacancies per firm is constant.

Consider first the case where $\alpha \to 1$. Note that λ_{12} is limited above. We want to show that $\lim_{\alpha\to 1} q_{12} = \infty$. Suppose not, and suppose instead that q is bounded by $\overline{q}$. Since λ_{12} is limited above by $\overline{\lambda} = y_2/(r + s + \delta)$ it follows that v is limited above by $\frac{\overline{\lambda}\overline{q}}{c}$.

Let $\overline{N_1}$ denote the value of N_1 in the limit as $\alpha \to 1$. Clearly $\overline{N_1} > 0$ and $rM_0 > z$. It follows that

$$\begin{aligned}
\lim_{\alpha\to 1} q_{12} &= \lim_{\alpha\to 1} A[\frac{(1-\alpha)kv}{N_1}]^{-\beta} \\
&\leq \lim_{\alpha\to 1} A[\frac{(1-\alpha)k\overline{v}}{N_1}]^{-\beta} \\
&= \infty
\end{aligned}$$

Hence q cannot be limited above. But then it follows that the profit of searching for employed workers goes to infinity as α goes to zero.

Consider then the profitability of a high-type firm searching for unemployed workers. Since λ_{02} is bounded above by $\overline{\lambda} = y_2/(r + s + \delta)$, the profit can only goes to infinity if q_{12} does. Suppose it does. Then workers applying to this job has a job finding rate of $p = 0$ and thus receives $rM_0 = z$. However, the workers would then prefer to search for the low-type firm and we cannot be in equilibrium. It follows that it is more profitable to search for employed than for unemployed workers if α is sufficiently close to 1.

Suppose then $a \to 0$. It follows that $N_1 \to 0$. We want to show that the proportion of high-type firms searching for employed workers goes to 0. Suppose not, and suppose the share is bounded below by $\tau^{\min} > 0$. Suppose that in the limit, $v_{12} > 0$. It follows that

$$\begin{aligned}
\lim_{\alpha\to 0} q_{12} &= \lim_{\alpha\to 1} A[\frac{(1-\alpha)k\tau v_{12}}{N_1}]^{-\beta} \\
&\leq \lim_{\alpha\to 1} A[\frac{(1-\alpha)k\tau^{\min} v_{12}}{N_1}]^{-\beta} = 0
\end{aligned}$$

Note also that $v_{12} = 0$ if and only if $q_{12}\lambda_{12} = 0$. Thus both if $v_{12} = 0$ in the limit and when it is not the assumption that $\tau^{\min} > 0$ is inconsistent with (27).

Finally we want to show that for any $\alpha > 0$, $\tau > 0$. Suppose not. Then there exists an $\alpha > 0$ such that $\tau = 0$. If (27) is satisfied we must have that $v_{12} < \infty$. But then it follows that $q_{12} = \infty$, hence (27) cannot be satisfied. Again we have derived a contradiction.

Finally we want to show that there exists a unique α^* as described above. That there exists a α^* such that (27) is satisfied with equality for $\tau = 1$ follows from continuity and the results just laid out. What is left is to show that this α^* is unique. To this end it is sufficient to show that if (27) is satisfied with equality for $\tau = 1$, then an decrease in α implies that the right-hand side of (27) is strictly greater than the left-hand side for $\tau = 1$.

In what follows we will work with α_2 rather than α, the fraction of high-type firms. We want to show that an increase in α_2 for a given k, and given that $\tau = 1$ implies that searching for unemployed workers become strictly more profitable than searching for employed workers. (from 27)

$$q_{12}(\lambda_2 - \lambda_1) < q_{02}(\lambda_2 - \lambda_0) \tag{32}$$

for α_2' marginally greater than α_2^*.

First note that an increase in α increases λ_1. Suppose λ_0 decreases. From p_{01} (29) it follows that decreases. From (29) it also follows that

$$\lambda_2 - \lambda_0 = \frac{y_2 - r\lambda_0}{r + s + \delta}$$

which thus increases. From (26) and the fact that p_{01} decreases, it follows that p_{02} decreases. From the matching function it follows that that q_{02} increases. Thus the right-hand side of 32 increases. An increase in α_2 increases p_2, and from (**??**) it follows that $(\lambda_2 - \lambda_1)$ decreases and q_{12} decreases. Thus the left-hand side of 32 decreases. Hence we are done in this case.

Suppose then that λ_0 is increasing in α_2 (which indeed seems likely). In what follows we re-scale the model by setting $z = 0$. Clearly this can be done without loss of generality, as the maximization problem is unchanged if all flows z, y_1, y_2 are reduced equally much. It follows that we can write

$$\lambda_0 = \frac{p_{01}}{r + p_{01}}\lambda_1$$

Thus, from (29)

$$\lambda_0(1 - \frac{s + \delta}{r + s + \delta})\frac{r + p_{01}}{p_{01}} = \frac{y_1 + p_{12}(\lambda_2 - \lambda_1)}{r + s + \delta}$$

Taking elasticities wrt α_2 gives

$$el\lambda_0 + X < elp_{12} + el(\lambda_2 - \lambda_1)$$

where $X = el\frac{r+p_{01}}{p_{01}} > 0$. An increase in α follows that

$$el\lambda_0 < elp_{12} + el(\lambda_2 - \lambda_1)$$

From (29) it follows that $r\lambda_0 = p_{02}(\lambda_2 - \lambda_0)$. Taking elasticities and using the above equation give

$$elp_{02} + el(\lambda_2 - \lambda_0) < elp_{12} + el(\lambda_2 - \lambda_1)$$

or

$$elp_{02} < elp_{12} + el(\lambda_2 - \lambda_1) - el(\lambda_2 - \lambda_0) \tag{33}$$

As $\frac{\delta\lambda_1}{\delta\alpha_2} > \frac{\delta\lambda_0}{\delta\alpha_2}$ and $\lambda_2 - \lambda_1 < \lambda_2 - \lambda_0$ it follows that $0 > el(\lambda_2 - \lambda_0) > el(\lambda_2 - \lambda_1)$. From (33) it thus follows that $elp_{02} < elp_{12}$ and thus that $elq_{02} > elq_{12}$. Together this implies that (32) is satisfied.

8.8 Proof of lemma ??.

Consider a firm of type j that searches for workers employed at level i having an NPV wage of $\widetilde{M}_i$. Furthermore, assume that the workers in that firm searches for jobs in firms of type k which pay W_{jk} and which they obtain at rate p_{jk} (still we supress the dependence of k in the expressions below). Since the agents are risk neutral and use the same discount factor, the timing of the payment to the worker is irrelevant, and for notational convenicence we assume that the worker is paid the entire NPV wage W_{ij} upfront. The net present value of profit of a firm with initial labor stock N_0 can be written as (we index state variables by j and choice variables and the adjungated variable by ij)

$$\begin{aligned}
\Pi_{ij} &= \int_0^\infty [N_j[y_j + sM_0 + p_{jk}W_{jk}] - \frac{\widetilde{v}_{ij}N_j}{2} - N_j\widetilde{v}_{ij}W_{ij}q(W_{ij}))]e^{-(r+\delta)t}dt \\
s.t. N_j(0) &= N_0 \\
\dot{N}_j &= \widetilde{v}_{ij}q(W_{ij})N_j - (s + p_{jk})N_j
\end{aligned}$$

where $\widetilde{v}_{ij} = v_{ij}/N_j$ and where $q(W_{ij}) \equiv q(p_i(W_{ij}, M_i)$, The Hamiltonian reads

$$H = N_j[y_j + sM_0 + p_{jk}W_{jk} - N_j\widetilde{v}_{ij}W_{ij}q(p(W_{ij})))] + \lambda_{ij}[\widetilde{v}_{ij}q(W_{ij})N_j - (\delta + p_{jk})N_j].$$

First order conditions for W reads (after some manipulation)

$$\begin{aligned}
W_{ij} &= (\lambda_{ij} - M_i)\beta \\
\widetilde{v}_{ij} &= (\lambda_{ij} - M_i)(1 - \beta)q \\
(r + \delta)\lambda_{ij} &= y_j + sM_0 + p_{jk}(W_{jk} - \lambda_{ij}) + [(\lambda - W)\widetilde{v}^*q(W^*) - \frac{\widetilde{v}_{ij}}{2c}]
\end{aligned}$$

Which gives us the conditions in the lemma (with $\widetilde{M}_{ij}$ substituted in for λ_{ij}). (Have to say something about the max, that is postponed).

8.9 Proof of proposition ??

In order to show that the problem is well defined it is sufficient to show that $\widetilde{M}_{ij}$ is bounded for all $\widetilde{v_{ij}}$ and all W_{ij}. We will show that this is always the case for sufficiently high search costs, i.e., for sufficiently low values of c. By assumption, $(r+s)M_i > y_i + s\widetilde{M}_0$, hence $q_{ij}(f(W_{ij}, M))$ is finite for any finite W_{ij}. Define

$$\overline{W}_j = \frac{y_j + \max_{k>j} p_{jk} W_{jk} + (s+\delta)\widetilde{M}_0}{r+s+\delta}$$

and define $\overline{q}_j = q_{ij}(\overline{W}_j)$. Note that $\overline{W}_j$ strictly exceeds the NPV of the income a worker generates, hence by paying $\overline{W}_j$ to all the workers the firm surely obtains a negative profit. Rewrite (17) to

$$\begin{aligned}\widetilde{M}_{ij} &= \frac{y_j + \max_k p_{jk}(W_{jk} - \widetilde{M}_{ij}) + (s+\delta)\widetilde{M}_0 - \widetilde{v}^*_{ij} q^*_{ij}(1-\beta)\widetilde{M}_i - \widetilde{v}^{*2}_{ij}/2c]}{r+s+\delta - \widetilde{v}^*_{ij} q^*_{ij}(1-\beta)} \\ &< \frac{y_j + \max_k p_{jk}(W_{jk} - \widetilde{M}_{ij}) + (s+\delta)\widetilde{M}_0 - \widetilde{v}^*_{ij}\overline{q}_j(1-\beta)\widetilde{M}_i) - \widetilde{v}^{*2}_{ij}/2c}{r+s+\delta - \widetilde{v}^*_{ij}\overline{q}(1-\beta)}\end{aligned}$$

In the second expression we have substituted in $\overline{q}_j > q_{ij}$. For sufficiently high values of $\widetilde{v}_{ij}$, the denominator is negative. Define $\widetilde{v}^0$ as the value of $\widetilde{v}_{ij}$ that makes the denominator equal to zero. It follows that

$$\widetilde{v}^0 = \frac{r+s+\delta}{(1-\beta)\overline{q}}$$

It is sufficient to show that the nominator is negative for $v = \widetilde{v}^0$, i.e., that

$$y_j + \max_k p_{jk}(W_{jk} - \widetilde{M}_{ij}) + (s+\delta)\widetilde{M}_0 - \widetilde{v}^0\overline{q}(1-\beta)\widetilde{M}_i) - (\widetilde{v}^0)^2/2c < 0$$

which is trivially satisfied for sufficiently low values of c.

Then we turn to uniqueness. Note that if W_{ij} and v_{ij} satisfies (18) and (**??**), then this is a local maximum for $\widetilde{M}_{ij}$. Suppose W'_{ij} and v'_{ij} constitute a local but not a global maximum for joint income, and let $\widetilde{M}'_{ij}$ denote the corresponding joint income. Then it follows that for $\widetilde{M}'_{ij}$, the first order conditions (18) and (**??**) have at least two solutions. However, given $\widetilde{M}'_{ij}$ the firm's maximization problem is exactly as in the previous section

8.10 Computation of the General Equilibrium

8.11

To solve the model one needs to set following 10 parameters: r, s, δ, y_1 and y_2, z, β c, k, α. In addition, the matching function we use is cobb douglas with share parameter β and with constant A.

The procedure to compute the equilibrium is as follows. First, the procedure tries to solve for the model with three submarkets. If this fails the procedure switches to the pure job ladder equilibrium. The solution is basically computed in four steps. The first steps (step i) solves for the asset equations in the general model, the second steps (step ii) computes τ, the proportion of good firms that hire directly from the unemployment pool and the final steps solves for the stock. Step three (step iii) is reached only if the proportion of firms that hires directly from the unemployed is less than one. In case this proportion τ is greater than one, the procedure goes to the step four (step iv) and solves for the pure job ladder equilibrium.

8.11.1 Step i): Solving for the Asset values in the general model

The procedure starts from assigning an arbitrary initial guess value of $M_1 = M_1^{'}$ and $rM_0 = rM_0^{'}$. Given the initial guess, one can compute recursively $M_2^{'}, p_{01}^{'}, p_{02}^{'}, p_{12}^{'}$

$$
\begin{aligned}
M_2^{'} &= \frac{y_2 + (r+\delta)M_0^{'}}{r+\delta+s}; \qquad \text{using } M_2 = \frac{y_2 + (r+\delta)M_0}{r+\delta+s} \\
p_{12}^{'} &= \frac{(r+\delta+s)M_1^{'} - y_1 + rM_0^{'}}{\beta(M_2^{\prime} - M_1^{\prime})} \qquad \text{using } M_1 = \frac{y_1 + (s+\delta)M_0 + p_{12}\beta(M_2 - M_1)}{r+\delta+s} \\
p_{01}^{'} &= \frac{rM_0^{\prime} - z}{\beta(M_1^{\prime} - M_0^{\prime})} \qquad \text{using } rM_0 = z + \beta p_{01}(M_1 - M_0) \\
p_{02}^{'} &= \frac{rM_0^{\prime} - z}{\beta(M_2^{\prime} - M_0^{\prime})} \qquad \text{using } rM_0 = z + \beta p_{02}(M_2 - M_0)
\end{aligned}
$$

Given these values we define the function $d(M_1^{'}, M_0^{'})$ as the difference in profits across high type firms so that

$$
\begin{aligned}
d(\Pi) &= \Pi_{12}() - \Pi_{02}() \\
d &= \frac{[(M_2^{\prime} - M_0^{\prime})(1-\beta)p_{12}^{\prime}]^2}{2} - \frac{[(M_2^{\prime} - M_1^{\prime})(1-\beta)p_{02}^{\prime - \frac{\beta}{1-\beta}}]^2}{2}
\end{aligned}
$$

For given value of $M_0^{\prime}$, the procedure updates the value of $M_1^{'}$ so that

$$M_1^{''} = M_1^{'} - \lambda d(\Pi)$$

where $\lambda > 0$ is an adjustment parameter. In other, words we reduce the value $M_1^{''}$ as long as $d()$ is positive. Given $M_1^{''}$ and holding fixed $M_0^{'}$ update $M_2^{''}, p_{01}^{\prime\prime}, p_{02}^{\prime\prime}, p_{12}^{\prime\prime}$ using $M_2^{''}$ and proceed further until

$$d(\Pi) \simeq 0$$

Given M'' expected profits at entry are

$$dE\Pi = \alpha\Pi_{01} + \Pi_{12} - k$$

and update the value of M'_0 so that

$$M''_0 = M'_0 + \lambda_1 dE\Pi$$

Given M''_0, update the asset values and redo the procedure for finding $d(\Pi) \simeq 0$, and calculating M'''_0. The equilibrium in the first step is obtained for a couple M_1^* and M_0^* so that

$$\begin{aligned} d(\Pi) &\simeq 0 \\ dE\Pi(\Pi) &\simeq 0 \end{aligned}$$

8.11.2 Step ii): Obtaining the fraction of firms τ that hire directly from the employed

The first step of the model has solved for M_1, rM_0 $M, p_{01}, p_{12}, p_{02}$. The rest of the equations are obtained from

$$\begin{aligned} (p_{01})^{\frac{1}{1-\beta}} &= \frac{(1-\alpha)kv_1(0)}{k_{01}n_0} \\ (p_{12})^{\frac{1}{1-\beta}} &= \frac{\tau\alpha kv_2(1)}{n_1}^{1-\beta} \\ (p_{02})^{\frac{1}{1-\beta}} &= \frac{(1-\tau)\alpha kv_2(0)}{(1-k_{01})n_0} \end{aligned}$$

and the flows conditions

$$\begin{aligned} p_{02}k_{02}n_0 + p_{12}k_{01}n_1 &= (\delta + s)n_2 \\ p_{01}k_{01}n_0 &= (\delta + s + p_{12})n_1 \\ n_0 + n_1 + n_2 &= 1 \\ k_{01} + k_{02} &= 1 \end{aligned}$$

Since $\frac{n_1}{k_{01}n_0} = \frac{p_{01}}{\delta+s+p_{12}}$ dividing the equation for $\theta_{01} = (p_{01})^{\frac{1}{1-\beta}}$ and $\theta_{12} = (p_{12})^{\frac{1}{1-\beta}}$ one obtains immediately and expression for τ as

$$\tau^* = \frac{\theta_{12}}{\theta_{01}} \frac{\alpha v_1(0)}{(1-\alpha)v_2(1)} \frac{p_{01}}{\delta + s + p_{12}}$$

where $v_i = c(M_i - M_0)_i q(p_i)$ $i = 1, 2$. If $\tau^* < 1$ the equilibrium with all submarket is consistent and steps iii can be completed. Conversely, if $\tau^* > 1$ the routine solves for the pure job ladder equilibrium.

8.12 Step iii): Obtaining stocks in the general model

Assume $k = k'$ and $k_{01} = k'_{01}$ and obtain recursively

$$\begin{aligned} n'_0 &= \frac{\delta + s}{\delta + s + p_{01}k'_{01} + p_{12}(1 - k'_{01})} \\ n'_1 &= \frac{p_{01}k_{01}n_0}{\delta + s + p_{12}} \\ n'_2 &= 1 - n'_0 - n'_1 \end{aligned}$$

Given these values obtain the function dk as

$$dk = (1 - k_{01})\theta(p_{02})n_0 - (1 - \tau)(1 - \alpha)k'v_2(0)$$

and update

$$k'' = k' + \lambda dk$$

Continue the procedure as long as k'' is such that

$$dk \simeq 0$$

With the completion of step iii the general equilibrium is fully solved.

Given k'' obtain the function dk

$$dk = k - \frac{n_1\theta_{12}(p_{12})}{\tau * (1 - \alpha) * v_2(1)}$$

and update the value of k' so that

$$k'' = k' - \lambda_1 dk$$

Given k'', update the stocks and redo the procedure for finding $d(k) \simeq 0$, and calculating k'''. The equilibrium in the first step is obtained for a couple k^* and k^* so that

$$\begin{aligned} d(k) &\simeq 0 \\ dE\Pi(k) &\simeq 0 \end{aligned}$$

8.12.1 Step iv. Solve for the pure job ladder equilibrium

The step iv is reached only if the routine finds a value of $\tau > 1$ in step ii. The procedure starts from an arbitrary initial guess value of $M_1 = M'_1$ and $rM_0 = rM'_0$. Given the initial guess, it computes recursively $M'_2, p'_{01}, p'_{02}, p'_{12}$

$$
\begin{aligned}
M_2' &= \frac{y_2 + (r+\delta)M_0'}{r+\delta+s}; \qquad \text{using } M_2 = \frac{y_2+(r+\delta)M_0}{r+\delta+s} \\
p_{12}' &= \frac{(r+\delta+s)M_1' - y_1 + rM_0'}{\beta(M_2'-M_1')} \qquad \text{using } M_1 = \frac{y_1+(s+\delta)M_0 + p_{12}\beta(M_2-M_1)}{r+\delta+s} \\
p_{01}' &= \frac{rM_0' - z}{\beta(M_1'-M_0')} \qquad \text{using } rM_0 = z + \beta p_{01}(M_1 - M_0) \\
n_0' &= \frac{\delta+s}{\delta+s+p_{01}'} \\
n_1' &= \frac{p_{01}'(\delta+s)}{(\delta+s+p_{01}')(\delta+s+p_{12}')} \\
n_2' &= 1 - n_1' - u_1' \\
k' &= \frac{n_1'\theta_2(p_{12}')}{(1-\alpha)v_2(1)}
\end{aligned}
$$

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Table 1: Baseline Calibration with three submarkets

Parameter	Notation	Value
Pure Discount Rate	r	0.010
Separation Rate	s	0.040
Firm Bankruptcy Rate	δ	0.020
Bargaining Share	β	0.500
entry cost	k	5.000
low type proportion	α	0.1550
high type productivity	y_1	1.000
low type productivity	y_2	1.150
unemployed income	z	0.550
search cost parameter	c	0.150
Maximum vacancies per firm	$v*$	0.150
matching function parameter	A	1.000
matching function elasticity	β	0.500
Equilibrium Values		
Joint Income 1	rM_1	1.0089
Joint Income 2	rM_2	1.0184
unemployment flow value	rU	0.9965
unempl. job finding rate in low type	p_{01}	0.7177
on the job finding rate	p_{12}	0.1762
unempl. job finding rate directly to high type	p_{02}	0.4072
Equilibrium Quantities		
Unemployment	n_0	0.0772
Employment in Low productivity type	n_1	0.2343
Employment in High productivity type	n_2	0.6885
Proportion of unemployed in submkt 01	k_{01}	0.9990
Number of Firms	f	0.3133
Proportion of high type firms in submarket 12	τ	0.9982
Worker Flows		
Unemployment Flows	$n_0 * (p_{01} + p_{02})$	0.0554
Job to Job Flows	$n_1 * p_{12}$	0.0413
Firm Size, PDV Wages and Profits		
Profits in submarket 01	Π_{01}	3.5846
Profits in submarket 02	Π_{02}	12.7165
Profits in submarket 12	Π_{12}	12.7165
Firm Size in submarket 01	N_{01}	0.8851
Firm Size in submarket 02	N_{02}	6.1402
Firm Size in submarket 12	N_{12}	14.1918
Wages in submarket 01	rW_{01}	1.0027
Wages in submarket 02	rW_{02}	1.0074
Wages in submarket 12	rW_{12}	1.0107
Source: Authors' calculation		

Table 2: Baseline Calibration with two submarkets

Parameter	Notation	Value
Pure Discount Rate	r	0.010
Separation Rate	s	0.040
Firm Bankruptcy Rate	δ	0.020
Bargaining Share	β	0.500
entry cost	k	5.000
low type proportion	α	0.1540
high type productivity	y_1	1.000
low type productivity	y_2	1.150
unemployed income	z	0.550
search cost parameter	c	0.150
Maximum vacancies per firm	$v*$	0.150
matching function parameter	A	1.000
matching function elasticity	β	0.500
Equilibrium Values		
Joint Income 1	rM_1	1.0088
Joint Income 2	rM_2	1.0183
unemployment flow value	rU	0.9964
unempl. job finding rate in low type	p_{01}	0.7180
on the job finding rate	p_{12}	0.1754
unempl. job finding rate directly to high type	p_{02}	0.0000
Equilibrium Quantities		
Unemployment	n_0	0.0771
Employment in Low productivity type	n_1	0.2352
Employment in High productivity type	n_2	0.6877
Proportion of unemployed in submkt 01	k_{01}	1.0000
Number of Firms	f	0.3133
Proportion of high type firms in submarket 12	τ	1.0000
Worker Flows		
Unemployment Flows	$n_0 * (p_{01} + p_{02})$	0.0554
Job to Job Flows	$n_1 * p_{12}$	0.0413
Firm Size, PDV Wages and Profits		
Profits in submarket 01	Π_{01}	3.5790
Profits in submarket 12	Π_{12}	12.8061
Firm Size in submarket 01	N_{01}	0.8874
Firm Size in submarket 12	N_{12}	14.2524
Wages in submarket 01	rW_{01}	1.0026
Wages in submarket 12	rW_{12}	1.0107
Source: Authors' calculation		