# Incentives in Competitive Search Equilibrium*

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June 27, 2010

**Abstract**

This paper proposes a labor market model with job search frictions where workers have private information on match quality and effort. Firms use wage contracts to motivate workers. In addition, wages are also used to attract employees. We define and characterize competitive search equilibrium in this context, and show that it satisfies a simple modified Hosios rule. We also analyze the interplay between macroeconomic variables and optimal wage contracts. Finally we show that private information may increase the responsiveness of the unemployment rate to changes in the aggregate productivity level and, in particular, to changes in the information structure.

**Key words**: Private information, incentives, search, unemployment, wage rigidity

**JEL classification**: E30, J30, J60

# 1 Introduction

There exists a large literature analyzing the effects of search frictions in the labor market. In this literature, firms are typically modeled in a parsimonious way, with exogenous output per worker. In particular, agency problems between workers and firms are ignored. The focus is thus solely on the effects of search frictions on the flows into and out of employment.

*We would like to than Editor Bruno Biais and three anynomous referees, as well as Per Engstrom, Matthew Lindquist, Marta Lochowska, Dale Mortensen, Robert Shimer, and seminar participants at Northwestern University, University of Pennsylvania, University of Chicago, University of Albany, Federal Reserve Bank of Richmond, ESSLE, Stockholm University, Göteborg University, Humboldt University in Berlin, University of Bonn, the Research Institute of Industrial Economics and the Swedish Institute for Social Research for valuable comments. Financial support from the Norwegian Research Council and the Swedish Research Council is gratefully acknowledged.

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In the present paper, we allow a firm's output to depend on the wage contracts firms offer their workers. A worker's output depends on both her effort and a match-specific component. The firm observes total output, but cannot disentangle output into its different components. The firm acts as a principal and chooses a wage contract that maximizes profits given the information constraints. Our aim is to analyze the interplay between search frictions in the market place and agency problems created by workers' private information. The search frictions and agency problems interact through the amount of "rents" that accrue to the worker.

A worker's private information gives her an information rent, which is larger the closer the wage is linked to her output. Without search frictions, a firm, when setting the wage contract, trades off incentives for the worker to provide effort and rent extraction from the worker. This trade-off is also present when there are search frictions in the labor market. However, with search frictions, rents that accrue to the worker have an additional effect. More rents to the worker when hired also benefit the firm in the recruiting process, as it speeds up the hiring rate. Hence, it is less costly for a firm to provide workers with incentives when it operates in a competitive, frictional market than in a frictionless market.

We show that the resulting search equilibrium, which we refer to as generalized competitive search equilibrium, has a simple form. The agency problem and the wage posting problem can be disentangled into two separate maximization problems. The solution to the firms' problem satisfies a modified Hosios rule, which determines constrained efficient resource allocation. When the information constraints are tight in a well-defined sense, the optimal wage contract prescribes that a large share of the match surplus is allocated to the employees. As a result, profits will be lower, and fewer resources are used to create new vacancies as compared to the equilibrium without agency problems. We also show that macroeconomic variables influence the tightness of the information constraints, and hence the optimal contracts. In particular, high search frictions tend to increase the incentive power of the wage contracts, while a higher value of leisure /unemployment benefit tends to reduce it.

We then analyze the effect of private information on the responsiveness of the unemployment rate to productivity changes, motivated by findings in Shimer (2005) and Hall

(2004). They document that fluctuations in the unemployment rate predicted by the standard Diamond-Mortensen-Pissarides (DMP) model (Diamond 1982, Mortensen 1986, Pissarides 1985) in response to observed productivity shocks are much smaller than actual fluctuations in the unemployment rate, as wages in the model absorb much of the shock.

First, we analyze the effects of negative productivity changes where all firms are hit equally hard (pure productivity shocks). Such a negative shock tightens the constraints imposed by workers' private information, and workers' share of the match surplus *increases*. Therefore, the unemployment rate becomes more responsive to such shocks than in the standard search model. Theoretical considerations imply that an upper bound on the effects of pure productivity shifts on unemployment volatility is the volatility obtained with rent rigidity (the worker's expected gain from finding a job stays constant, see Brugemann and Moscarini (2007) and Brugemann (2008)). Our numerical analysis indicates that the responsiveness of the unemployment rate to pure output shifts is close to this upper limit. We also analyze the effects of changes in the dispersion in the match-specific productivity term. This has a large effect on the unemployment rate and a small effect on output. A combination of changes in productivity and dispersion may give rise to an elasticity of the unemployment rate to observed output per worker consistent with the empirical findings.

Our private information model builds on the procurement model by Laffont and Tirole (1993) and its adoption to a frictionless labor market by Moen and Rosén (2006). As the emphasis in the present paper is on the interplay between search frictions and wage contracts, the analysis differs radically from that of Moen and Rosén (2006).

In a related model, Faig and Jerez (2005) analyze a retail market with search frictions when buyers have private information about their willingness-to-pay. Although their paper studies private information in a competitive search environment, their model and its emphasis differ from ours. Faig and Jarez focus on welfare analysis and abstract from moral hazard problems. They do not derive the modified Hosios condition, nor do they analyze the impact of macroeconomic variables on sharing rules and incentives. Guerrieri (2008) studies the welfare effects of including non-pecuniary aspects of a match which are private information to workers. She finds that the resulting allocation is inefficient out of steady state.

Several recent studies seek to make the search model consistent with Shimer and Hall's empirical findings. Our paper belongs to a small subset of this literature that focuses on private information.[1] Our paper is perhaps most closely related to Kennan (2007). In his paper, firms have more private information about the productivity of the match. Workers and firms bargain over wages, and the bargaining game is set up in such a way that the increase in average productivity associated with a boom is allocated to the firm. This dramatically increases unemployment volatility. Although asymmetric information is the driving force in both models, the mechanisms are very different. In Kennan's model, firms are motivated to create more vacancies during booms because their profit is then disproportionately higher. In our model, private information leads to agency problems within the firm and thus lower output. The firms respond to this by advertising higher expected wages as this reduces the agency problems. As a result, relatively small inefficiencies in the worker-firm relationship may lead to a large increase in the unemployment rate.

Menzio (2005) also studies bargaining between workers and firms with private information, and shows that firms may find it optimal to keep wages fixed if hit by high-frequency shocks. Guerrieri (2007) studies a competitive search model where workers have private information about non-pecuniary aspects of a match. Private information only plays a role at the hiring margin. She finds, in calibrations, that this amendment to the standard search model does not help to amplify unemployment volatility.

Our model is also related to the literature on efficiency wage models (e.g. Weiss, 1980; Shapiro and Stiglitz, 1984). Some of these papers examine the comparative static properties of efficiency wage models (Strand, 1992; Danthine and Donaldson, 1990; Ramey and Watson, 1997; MacLeod, Malcomson and Gomme, 1994; MacLeod and Malcomson, 1998). In a static model, Rocheteau (2001) introduces shirking in a search model and shows that the non-shirking constraint forms a lower bound on wages.

The paper is organized as follows: Section 2 presents the model. In section 3, we study

[1] There are also several other approaches. Nagypál (2006) and Krause and Lubik (2007) show that on-the-job search in a matching model may amplify the effects of productivity shocks on the unemployment rate. In Rudanko (2008), the effect of risk averse workers and contractual incompleteness on volatility is explored. Reiter (2007) shows that the responsiveness of the unemployment rate to productivity shocks may be increased if one allows for technological change that is embodied into the match. Gertler, Sala and Trigari (2007) explain wage rigidity by staggered wage contracts while Hall (2005a) do it by social norms. For a recent survey on this literature see Mortensen and Nagypál (2006).

the full-information benchmark. In section 4, we introduce and characterize the generalized competitive search equilibrium. Section 5 contains the quantitative analysis. Section 6 offers final comments. Unless otherwise stated all proofs are relegated to the appendix.

# 2 The model

The matching of unemployed workers and vacancies is modeled using the Diamond-Mortensen-Pissarides (DMP) framework (Diamond, 1982; Mortensen, 1986; Pissarides, 1985) with competitive wage setting. The economy consists of a continuum of *ex ante* identical workers and firms. All agents are risk neutral and have the same discount factor $r$. Workers live for ever and the measure of workers is normalized to one.

Let $u$ denote the unemployment rate and $v$ the vacancy rate in the economy. Firms are free to open vacancies at no cost, but maintaining a vacancy entails a flow cost $c$. The number of matches per unit of time is determined by a concave, constant return to scale matching function $x(u, v)$. Let $p$ denote the matching rate of workers, showing the rate at which unemployed workers meet a vacancy. Let $q$ denote the matching rate of firms, showing the rate at which firms with a vacancy meet an unemployed worker. The probability rates $p$ and $q$ can be written as $p = x(u, v)/u = x(1, \theta) = \widetilde{p}(\theta)$ and $q = x(u, v)/v = x(1/\theta, 1) = \tilde{q}(\theta)$, where $\theta = v/u$. We assume that $\lim_{\theta \to 0} p(\theta) = 0$ and $\lim_{\theta \to 0} q(\theta) = \infty$. The matching technology can be summarized by a function $q = \tilde{q}(\theta) = \tilde{q}(\widetilde{p}^{-1}(p)) = q(p)$, with $q'(p) < 0$.

Our model brings two new elements into the standard DMP model, both common in other parts of labor economics. First we assume that the output of a match depends on worker effort, $e$. Second, we include stochastic job matching (Jovanovic (1979, Pissarides 2000, ch. 6), i.e., the productivity of a given worker-firm pair is match-specific. The output $y$ of a worker-firm pair is given by

$$y(e, \varepsilon) = \overline{y} + \varepsilon + \gamma e, \tag{1}$$

where $\overline{y}$ is a constant, $\varepsilon$ the match-specific term (or stochastic matching term) and $e$ is worker effort. We assume that $\varepsilon$ is i.i.d across all worker-firm matches. For any given match, $\varepsilon$ is constant over time and continuously distributed on an interval $[\underline{\varepsilon}, \overline{\varepsilon}]$ with the cumulative distribution function $H$ and density function $h$. We further assume that $H$ has an increasing hazard rate. Employed workers who receive a wage $w$ and exert an effort $e$ obtain an instantaneous utility flow $w - \psi(e)$, where $\psi(e)$ is the disutility of effort. In what follows we assume that $\psi'(e) > 0$, $\psi''(e) > 0$ and $\psi'''(e) \geq 0$ and that $\psi(0) = \psi'(0) = 0$.[2] The match-specific term may relate to what sort of tasks the job consists of, organization of work, degree of flexibility etc for which workers may have different comparative advantages or preferences.[3]

Output $y$ is observable and contractible. Still the employee (the agent) has an information advantage over the employer (the principal) as the employee can decompose output $y$ into effort $e$ and the stochastic matching term $\varepsilon$ while the employer only observes $y$. Note that although there are two variables the firm cannot observe, the information problem facing the firm is one-dimensional since the firm observes the sum of the two variables.

Firms advertise wage contracts, and can commit not to renegotiate the contract. We describe the wage contracts as direct revelation mechanisms designed so that workers truthfully reveal their match-specific term, $\varepsilon$. When a worker and a firm meet, the worker learns $\varepsilon$ and reports it to the firm. If the contract prescribes that a match should not be formed for the reported $\varepsilon$, workers and firms continue to search. Formally, a contract is given by a triple $\phi = (w(\varepsilon), e(\varepsilon), \varepsilon_c)$, where $\varepsilon_c \geq \underline{\varepsilon}$ denotes the threshold value of $\varepsilon$ below which a match is not formed. Below we show that the optimal contract indeed has the cut-off property that a worker is employed with probability 1 if $\varepsilon > \varepsilon_c$ and with 0 if $\varepsilon < \varepsilon_c$, both with and without private information (see lemma 2). We do not consider tenure-dependent contracts. Below we show that the optimal contract, allowing for time dependence, is indeed tenure-independent.

Before we continue, we would like to make two comments regarding the set-up, both related to the match-specific term $\varepsilon$. First, we assume that a worker learns $\varepsilon$ before the

[2] An equivalent representation of the model is to write output as $y = \overline{y} + \gamma e$ and the effort cost as $\psi(e - \varepsilon/\gamma)$, $e \geq \varepsilon/\gamma$. The match-specific component is then related to the cost of effort.

A more general formulation is obtained by setting $y = \overline{y} + e$ and write the cost of effort as $\psi(e, \varepsilon)$, where $\psi$ is convex in $e$, $\psi_{e\varepsilon} < 0$, and $\psi_{\varepsilon} < 0$. We conjecture that our results also holds with this specification of effort costs.

[3] Nagypal (2007) finds that match-specific productivity differences are empirically important.

contract is signed. This rules out up-front payments from the worker to the firm before the worker learns $\varepsilon$.[4] Second, our assumption that $\varepsilon$ is iid over workers implies that workers are *ex ante* identical. If workers are *ex ante* heterogenous, and worker type is contractible, different worker types would search in different submarkets. If workers are *ex ante* heterogenous and worker type is private information to the workers, they would self-select on contracts, see Moen and Rosen (2006) or Guerrieri, Shimer and Wright (2009).

*Asset value equations*

The asset value equations define the parties' payoffs for a given contract, $\phi = (w(\varepsilon), e(\varepsilon), \varepsilon_c)$. Let $U$ denote the expected discounted utility of an unemployed worker and $\widetilde{W}(\varepsilon)$ the expected discounted utility of an employed worker with a match-specific productivity term $\varepsilon$, hereafter somewhat imprecisely referred to as her type. Then $\widetilde{W}(\varepsilon)$ is defined as

$$r\widetilde{W}(\varepsilon) = w(\varepsilon) - \psi(e(\varepsilon)) - s(\widetilde{W}(\varepsilon) - U),$$

where $s$ is the exogenous separation rate. The utility flow when employed is equal to the wage less the effort cost and less the expected capital loss associated with losing the job. Rearranging the above equation gives

$$\begin{aligned} (r+s)\widetilde{W}(\varepsilon) &= w(\varepsilon) - \psi(e(\varepsilon)) + sU \\ &\equiv \omega(\varepsilon) + sU, \end{aligned} \tag{2}$$

where $\omega(\varepsilon)$ is the wage net of effort costs. The expected discounted value of a worker being matched is thus

$$\begin{aligned} W &= \int_{\varepsilon_c}^{\overline{\varepsilon}} \widetilde{W}(\varepsilon) dH + H(\varepsilon_c)U. \\ &= \int_{\varepsilon_c}^{\overline{\varepsilon}} \frac{\omega(\varepsilon) + sU}{r+s} dH + H(\varepsilon_c)U. \end{aligned}$$

The expected discounted utility of an unemployed worker is given by

[4] If up-front payments are not admitted, it is sufficient that the worker learns $\varepsilon$ after exerting effort and observing $y$.

$$rU = z + p(W - U),$$

where $z$ is the utility flow when unemployed.

Let $\widetilde{J}(\varepsilon)$ denote the expected discounted value of a filled job with a worker of type $\varepsilon$. Assuming that an abandoned firm has no value, $\widetilde{J}(\varepsilon)$ is given by

$$(r+s)\widetilde{J}(\varepsilon) = y(e(\varepsilon), \varepsilon) - w(\varepsilon).$$

Let $V$ denote the expected discounted value of a firm with a vacancy. The expected value $J$ to a firm of being matched is thus

$$\begin{aligned} J &= \int_{\varepsilon_c}^{\overline{\varepsilon}} \widetilde{J}(\varepsilon) dH + H(\varepsilon_c)V \\ &= \int_{\varepsilon_c}^{\overline{\varepsilon}} \frac{y(e(\varepsilon), \varepsilon) - w(\varepsilon)}{r+s} dH + H(\varepsilon_c)V. \end{aligned} \tag{3}$$

The value of a vacancy can be written as

$$rV = -c + q(J - V). \tag{4}$$

For our subsequent analysis, it is convenient to use the concept of worker rents associated with a match. The rents from a match reflect the workers' expected "capital gain", or expected income (net of effort costs) in excess of $U$, of being matched to a vacancy. The expected worker rents of a match can be expressed as

$$\begin{aligned} R &\equiv W - U \\ &= \int_{\varepsilon_c}^{\overline{\varepsilon}} [\frac{\omega(\varepsilon) + sU}{r+s} - U] dH. \end{aligned} \tag{5}$$

Using the definition of worker rents, the expected utility of an unemployed worker takes a particularly simple form

$$rU = z + pR. \tag{6}$$

That is, the flow value of an unemployed worker is equal to the utility flow when unemployed plus the expected gain from search, which is equal to the matching rate times the expected rent associated with a match. The total expected surplus of a match is $S \equiv J - V + R$, or (using equations (3) and (5))

$$(r+s)S = \int_{\varepsilon_c}^{\overline{\varepsilon}} [y(e(\varepsilon),\varepsilon) - \psi(e(\varepsilon)) - rU - (r+s)V]dH. \tag{7}$$

Let $u$ denote the equilibrium unemployment rate. In equilibrium, the inflow to unemployment, $s(1-u)$, is equal to the outflow $up(1-H(\varepsilon_c))$. Thus, we have

$$u = \frac{s}{s + p(1-H(\varepsilon_c))}. \tag{8}$$

# 3 Equilibrium with full information

In this section we derive the equilibrium outcome in the special case where $\varepsilon$ and $e$ are observable and contractible. This will serve as a benchmark for later analysis. Our equilibrium concept is the competitive search equilibrium (Moen 1997). One of its core elements is that it postulates a unique relationship between the attractiveness of the offered wage contract and the expected rate at which the vacancy is filled. This relationship can be derived in several alternative settings.[5] In the present paper, we choose the interpretation that firms advertise wage contracts.

Although the contracts advertised by firms may be complex, the relevant variable for an unemployed worker is the expected value of being matched. The more attractive contract a firm offers, the more workers will be attracted to that firm. Generically, let $U^e$ denote the equilibrium utility of a searching worker. For any value of the expected rent $R$ a firm offers, the queue length of workers adjusts so that the applicants obtain their equilibrium expected utility. It follows that

$$z + pR = (r+s)U^e, \tag{9}$$

[5] Moen (1997) assumes that a market maker creates submarkets and shows that the same equilibrium can be obtained if firms advertize wages. The market maker interpretation is further developed in Mortensen and Wright (2002). Mortensen and Pissarides (1999, section 4.1) interpret the market maker as a "middle man" (like a job center) that sets the wage. In Acemoglu and Shimer (1999), the labor market is divided into regional or industrial submarkets offering potentially different wages. Galenianos and Kircher (2009) give a game-theoretic foundation for competitive search equilibrium.

which defines $p$ as a decreasing function of $R$, $p = p(R)$ (the dependence of $U^e$ is suppressed).

In equilibrium, firms choose wage contracts so as to maximize profits. In addition, free entry of firms implies that the value $V$ of a vacancy is zero.

**Definition 1** *The competitive search equilibrium under full information is a contract* $\phi^F = (w^F(\varepsilon), e^F(\varepsilon), \varepsilon_c^F)$*, a vector of asset values* $(S^F, R^F, U^F)$*, and a job finding rate* $p^F$ *such that the following holds:*

1. *Profit maximization.* $\phi^F, S^F, R^F, p^F$ *solves the program P1 given by*

$$rV^{\max}(U^F) = \max_{\phi,S,R,p} -c + q(p)(S - R)$$

*s.t.*

$$\begin{aligned} rU &= z + pR && \textit{(C1)} \\ (r+s)R &= \int_{\varepsilon_c}^{\overline{\varepsilon}} [w(\varepsilon) - \psi(e(\varepsilon)) - rU] dH(\varepsilon) && \textit{(C2)} \\ (r+s)S &= \int_{\varepsilon_c}^{\overline{\varepsilon}} [y(e(\varepsilon), \varepsilon) - \psi(e(\varepsilon)) - rU - (r+s)V] dH(\varepsilon). && \textit{(C3)} \end{aligned}$$

2. *Free entry:*

$$V^{\max}(U^F) = 0. \tag{10}$$

Note that the equilibrium does not explicitly include an *ex post* participation constraint for employed workers. Below we show that this constraint can easily be satisfied.

Note that when setting $R$, the firms take into account that a high wage bill implies a higher arrival rate of workers. There is typically only one value of $R$ advertised in equilibrium (see below). Nonetheless, when setting $R$, firms expect that the arrival rate of workers to their firm, $\widehat{q}$, for out-of equilibrium $R$ offers will be given by $\widehat{q} = q(p(R))$, where $p(R)$ satisfies (9).[6] In addition the firm takes into account that the design of the wage contract influences the value $S$ of a match.

We solve the profit maximization program P1 in two steps.

[6]Note that the expectations are rational in the following sense; Suppose that a small set of firms deviates and advertises an out-of equilibrium rent $R'$. Applications would then flow to these firms up to the point at which the applicants obtain exactly their equilibrium expected income $U^*$, in which case $q(R') = \widehat{q}(R')$ holds (see Moen 1997 for details).

1. For a given $U$, find $S^{\max}(U)$ such that

$$(r+s)S^{\max}(U) = \max_{e(\varepsilon),\varepsilon_c} \int_{\varepsilon_c}^{\overline{\varepsilon}} [y(e(\varepsilon),\varepsilon) - \psi(e(\varepsilon)) - rU - (r+s)V]dH(\varepsilon). \qquad (11)$$

2. For a given $U$ and $S^{\max}(U)$, find $V^{\max}(U)$ such that

$$rV^{\max}(U) = \max_{R} -c + q(p(R))[S^{\max}(U) - R], \qquad (12)$$

where $p(R)$ is defined by (9).

The first-order condition for optimal effort levels reads:

$$\psi'(e(\varepsilon)) = \gamma \text{ for all } \varepsilon. \qquad (13)$$

Note that effort level is independent of worker type, reflecting that the gain from effort is the same for all workers. The optimal cut-off level is given by either $\varepsilon_c = \underline{\varepsilon}$ or (with $V = 0$)

$$\overline{y} + \varepsilon_c + \gamma e(\varepsilon_c) - \psi(e(\varepsilon_c)) = rU. \qquad (14)$$

The above equation uniquely defines the optimal cut-off level, which equalizes the worker's net productivity with her outside option. The solutions for both $\varepsilon_c$ and $e$ are independent of $R$.

Then we turn to the second step. The first order condition for the maximization problem (12) is given by

$$q'(p)p'(R)(S-R) - q = 0. \qquad (15)$$

Let $\eta$ denote the absolute value of the elasticity of $q$ with respect to $\theta = v/u$. In the appendix we show that the first order condition can be rewritten as

$$\frac{R^F}{S^F - R^F} = \frac{\eta}{1-\eta}. \qquad (16)$$

In the appendix we also show that the second order conditions are satisfied provided that $d\eta/d\theta \geq 0$. This is always the case with a Cobb-Douglas matching function, $x(u,v) = Au^{\beta}v^{1-\beta}$, in which case $\eta = \beta$ (constant).

The profit-maximizing level of $R$, which optimally trades off a short waiting time to get a worker and a high expected payment to the worker, is thus obtained if the worker is given a share $\eta$ of the expected surplus $S^F$. Equation (16) is identical to the so-called Hosios condition for socially efficient resource allocations in search models (Hosios, 1990).

Finally, the free-entry condition (10) pins down $U^F$. Given $U^F$, equations (13) and (14) define $e^F$ (which is independent of both $\varepsilon$ and $U$, see equation 13) and $\varepsilon_c^F$, while equation (16) determines $R^F$ and indirectly $p^F$ through equation (9) The equilibrium does not pin down a unique wage schedule, $w^F(\varepsilon)$. The equilibrium wage schedule may, for instance, be a fixed, type-independent wage. Since the effort cost is the same for all worker types hired, the participation constraint is then satisfied for all types.

# 4 Generalized competitive search equilibrium

We now consider the situation where $e$ and $\varepsilon$ are private information. We require that $e \geq 0$. Let $\widetilde{\omega}(\varepsilon, \widetilde{\varepsilon})$ denote the utility flow of a worker of type $\varepsilon$ who reports type $\widetilde{\varepsilon}$, defined as

$$\widetilde{\omega}(\varepsilon, \widetilde{\varepsilon}) = w(\widetilde{\varepsilon}) - \psi(e(\widetilde{\varepsilon}) - \frac{\varepsilon - \widetilde{\varepsilon}}{\gamma}) \tag{17}$$

The first term shows the wage and the second term the effort cost of a worker of type $\varepsilon$ who reports a type $\widetilde{\varepsilon}$. To understand the last term, suppose the contract prescribes effort level $e(\widetilde{\varepsilon})$. A worker of type $\widetilde{\varepsilon}$ then produces $\overline{y} + \widetilde{\varepsilon} + \gamma e(\widetilde{\varepsilon})$. A worker of type $\varepsilon$ can reach this output requirement by exerting an effort $e(\widetilde{\varepsilon}) - \frac{\varepsilon - \widetilde{\varepsilon}}{\gamma}$.

The worker's incentive compatibility constraint can now be formulated as

$$\widetilde{\omega}(\varepsilon, \varepsilon) \geq \widetilde{\omega}(\varepsilon, \widetilde{\varepsilon}), \qquad \text{for all } \varepsilon, \widetilde{\varepsilon}. \tag{C4}$$

Let $\omega(\varepsilon) \equiv \arg\max_{\widetilde{\varepsilon}} \widetilde{\omega}(\varepsilon, \widetilde{\varepsilon})$. The participation constraint requires that $\widetilde{W}(\varepsilon) \geq U$. From equation (2) it thus follows that the participation constraint can be written as

$$\omega(\varepsilon) \geq rU, \qquad \text{for all } \varepsilon \geq \varepsilon_c. \tag{C5}$$

**Definition 2** *The generalized competitive search equilibrium (GCS-equilibrium) is a contract* $\phi^* = (w^*(\varepsilon), e^*(\varepsilon), \varepsilon_c^*)$ *where* $e^*(\varepsilon) \geq 0$ *for all* $\varepsilon$, *a vector of asset values* $(S^*, R^*, U^*)$, *and a job finding rate* $p^*$ *such that the following holds:*

*1. $\phi^*, S^*, R^*, p^*$ solves program P1 for $U = U^*$ with (C4) and (C5) as additional constraints. We refer to this as program P2.*

*2. Free entry:*

$$V^{\max}(U^*) = 0. \tag{18}$$

In the appendix (in the proof of proposition 1) we show that the utility functions of two different worker types satisfies the single-crossing property, and that a sufficient condition for truth-telling is that $e(\varepsilon)$ is monotonically increasing in $\varepsilon$ whenever $e(\varepsilon) > 0$.[7] We will assume that $e(\varepsilon)$ is monotonically increasing in $\varepsilon$, and then verify afterwards that the resulting solution indeed is so. It follows from monotonicity that $e(\varepsilon)$ is continuous and differentiable almost everywhere. In the appendix we show that the wage $w(\varepsilon)$, and thus also $\omega(\varepsilon)$, is differentiable at all points $\varepsilon$ where $e(\varepsilon)$ is differentiable. From the envelope theorem it follows that the first order conditions for truth-telling when $e > 0$ can be written as

$$\begin{aligned} \omega'(\varepsilon) &= \frac{\partial \widetilde{\omega}(\varepsilon, \widetilde{\varepsilon})}{\partial \varepsilon}|_{\widetilde{\varepsilon}=\varepsilon} & (19) \\ &= \psi'(e(\varepsilon))/\gamma, & (20) \end{aligned}$$

(from 17). If a worker's type increases by one unit, she can reduce her effort by $1/\gamma$ units and still obtain the same output, thereby increasing her utility by $\psi'(e(\varepsilon))/\gamma$ units. Incentive compatibility requires that the worker obtains the same gain by reporting her type truthfully.

Using equation (20), the utility flow to a worker of type $\varepsilon \geq \varepsilon_c$ can be written as

$$\omega(\varepsilon) = \omega(\varepsilon_c) + \int_{\varepsilon_c}^{\varepsilon} \frac{\psi'(e(x))}{\gamma} dx. \tag{21}$$

Note that contracts that prescribe more effort from low-type workers must give higher utility to high-types to keep the incentive compatibility constraint satisfied.

A first question that arises is whether the full information equilibrium $(S^F, R^F, U^F, p^F, \phi^F)$ is still feasible. The next lemma addresses that question.

**Lemma 1** *a) For $\varepsilon_c^F > \underline{\varepsilon}$, the GCS-equilibrium with full information is not feasible when $\varepsilon$ and $e$ are private information.*

[7] The optimal contract may prescribe that $e = 0$ for some types (pooling).

*b) For $\varepsilon_c^F = \underline{\varepsilon}$, the GCS-equilibrium with full information is feasible with private information if and only if $R^F \geq \overline{R}$, where*

$$\overline{R} = \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \frac{\varepsilon - \underline{\varepsilon}}{r+s} dH(\varepsilon). \tag{22}$$

The lemma thus states that with interior cut-off, the full information equilibrium can never be implemented when information is private. If all worker types are hired in the full information case, and the search rent is sufficiently large ($R^F \geq \overline{R}$), the full information equilibrium can be implemented with private information. In what follows we assume that $R^F \geq \overline{R}$.

To solve the firms' maximization program P2, we use the standard method of integrating up the incentive compatibility constraint using integration by parts. As rent is valuable, firms do not leave rents to the marginal worker, $\omega(\varepsilon_c) = rU$. From equation (21), we then get

$$\begin{aligned}
\int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \omega(\varepsilon) dH(\varepsilon) &= rU + \int_{\varepsilon_c}^{\overline{\varepsilon}} \int_{\varepsilon_c}^{\varepsilon} \frac{\psi'(e(x))}{\gamma} dx dH(\varepsilon) \\
&= rU + \int_{\varepsilon_c}^{\overline{\varepsilon}} \frac{\psi'(e(\varepsilon))}{\gamma} \frac{1-H(\varepsilon)}{h(\varepsilon)} dH(\varepsilon).
\end{aligned}$$

Using (5) thus gives

$$(r+s)R = \int_{\varepsilon_c}^{\overline{\varepsilon}} \frac{\psi'(e(\varepsilon))}{\gamma} \frac{1-H(\varepsilon)}{h(\varepsilon)} dH(\varepsilon). \tag{C6}$$

Equation (C6) incorporates both the incentive compatibility constraint and the participation constraint. As for the full-information equilibrium, program P2 is solved in two steps:

1. (Optimal contracts) For a given $U$ and $R$, find the maximum match surplus $S^{\max}(R,U)$ defined as

$$\begin{aligned}
(r+s)S^{\max}(R,U) &= \max_{e(\varepsilon),\varepsilon_c} \int_{\varepsilon_c}^{\overline{\varepsilon}} [y(e(\varepsilon),\varepsilon) - \psi(e(\varepsilon)) - rU - (r+s)V] dH(\varepsilon) \quad (23) \\
&\text{s.t.} \\
(r+s)R &= \int_{\varepsilon_c}^{\overline{\varepsilon}} \frac{\psi'(e(\varepsilon))}{\gamma} \frac{1-H(\varepsilon)}{h(\varepsilon)} dH(\varepsilon).
\end{aligned}$$

   Denote the associated contract by $\phi^{\max}(R,U)$.

2. (Optimal sharing rule) For a given $U$ and $S^{\max}(R,U)$, find the expected rent $R$ that maximizes the value of a vacancy $V^{\max}(U)$, defined as

$$rV^{\max}(U) = \max_{R} -c + q(p(R))[S^{\max}(R,U) - R], \tag{24}$$

where $p(R)$ is defined by (9).

We write $S^* = S^{\max}(R^*, U^*)$ and $\phi^* = \phi^{\max}(R^*, U^*)$.

**Step 1: Optimal contracts** Denote the Lagrangian parameter associated with the rent constraint (C6) by $\alpha$. The Lagrangian is given by

$$\begin{aligned} L \;=\; & \int_{\varepsilon_c}^{\overline{\varepsilon}} [\overline{y} + \varepsilon + \gamma e(\varepsilon) - \psi(e(\varepsilon)) - rU - (r+s)V] dH(\varepsilon) \\ & -\alpha\Big[\int_{\varepsilon_c}^{\overline{\varepsilon}} \frac{\psi'(e(\varepsilon))}{\gamma}\frac{1-H(\varepsilon)}{h(\varepsilon)} dH(\varepsilon) - (r+s)R\Big]. \end{aligned} \tag{25}$$

**Proposition 1** *(Solution to the first step).*

*i) For given $R$ and $U$, the optimal contract satisfies the following conditions:*

*1. The first-order condition for the effort level. The optimal effort is either $e = 0$ or given by*

$$\gamma - \psi'(e(\varepsilon)) = \alpha \frac{1-H(\varepsilon)}{h(\varepsilon)} \psi''(e(\varepsilon))/\gamma. \tag{26}$$

*2. The first-order condition for the optimal cut-off level. The optimal cut-off level is either $\varepsilon_c = \underline{\varepsilon}$ or (with $V = 0$)*

$$[\overline{y} + \varepsilon_c + \gamma e(\varepsilon_c) - \psi(e(\varepsilon_c)) - rU] h(\varepsilon_c) = \alpha(1 - H(\varepsilon_c)) \frac{\psi'(e(\varepsilon_c))}{\gamma}. \tag{27}$$

*3. The rent-constraint defined by equation (C6).*

*ii) The first order conditions have a unique solution, and solve the first step of the maximization problem P2.*

The proof of ii) is given in the appendix. There we also show that if $e(\varepsilon) = 0$ for some $\varepsilon'$, then $e(\varepsilon) = 0$ for all $\varepsilon < \varepsilon'$. For convenience we assume that the optimal effort level is strictly positive for all $\varepsilon$. From (26) it follows that $e(\varepsilon)$ is continuously differentiable in $\varepsilon$ for $e > 0$.[8]

Before we explain the first-order conditions in some detail, note that $\alpha$ is the shadow flow value of worker rents for the match surplus $S^{\max}(R; U)$. From (25) it follows that $\frac{\partial L}{\partial R} = (r + s)\alpha$, or

$$S_R^{\max} = \alpha, \tag{28}$$

where subscript $R$ denotes the derivative with respect to $R$.

Equation (28) captures a fundamental role of private information. In the full-information equilibrium derived in the previous section, the match surplus $S^{\max}$ was independent of $R$. With private information, the total amount and the division of surplus are interrelated. The higher the expected rent/wage the firm pays to the worker, the higher is the expected output.

The two first-order conditions generalize optimal contracts with private information (as in e.g. Laffont and Tirole, 1993) to a setting with search frictions. In the contracting model of Laffont and Tirole, the principal maximizes (in our terminology) $S(R) - R$, and the maximum is obtained for $S'(R) = 1$. With search friction, the rent paid to the worker also influences the arrival rate of workers, hence it is less costly for the firm to give rents to the worker. As will be clear below, it is always true that $S'(R) < 1$ (or $\alpha < 1$) in *GCS*-equilibrium. From (26) it follows that the incentive power of the contract is lower in our case than in the standard Laffont-Tirole environment, where the agent (or the set of potential agents) is present from the outset.

[8] With the more general model formulation given in footnote 2, the truth-telling condition reads $\omega'(\varepsilon) = -\psi_\varepsilon(e(\varepsilon), \varepsilon)$. By doing exactly the same steps as in the main text, it follows that optimal effort is given

$$\gamma - \psi_e(e, \varepsilon)) = \alpha \frac{1 - H(\varepsilon)}{h(\varepsilon)} \psi_{e\varepsilon}(e, \varepsilon).$$

The optimal cut-off is given by

$$\overline{y} + \gamma e(\varepsilon_c) - \psi(e, \varepsilon_c) - rU = \alpha \frac{1 - H(\varepsilon_c)}{h(\varepsilon_c)} \psi_\varepsilon(e, \varepsilon_c).$$

Consider the optimal effort equation (26) and suppose that the effort level of a type $\hat{\varepsilon}$ worker increases by one unit. The left-hand side of equation (26) captures the resulting efficiency gain $\gamma - \psi'(e(\hat{\varepsilon}))$. The right-hand side captures the costs associated with an increase in effort. A one unit increase in effort of a type $\hat{\varepsilon}$ worker increases the rents of all workers above $\hat{\varepsilon}$ by $\psi''(e(\hat{\varepsilon}))/\gamma$ units (from equation 21) and the shadow value of this rent is $\alpha$. The likelihood of obtaining a worker of type $\hat{\varepsilon}$ is reflected in $h(\hat{\varepsilon})$, while the measure of workers with higher match-specific productivity is $1 - H(\hat{\varepsilon})$. This explains the factor $(1 - H(\hat{\varepsilon}))/h(\hat{\varepsilon})$. Note that $e(\bar{\varepsilon}) = e^F$ (no distortion at the top). Since $h$ has an increasing hazard rate and $\psi'''(e) \geq 0$, $e(\varepsilon)$ is increasing in $\varepsilon$ (hence the second order conditions for truth-telling are satisfied, see appendix for details).

The left-hand side of the cut-off equation (27) shows the net productivity loss of increasing $\varepsilon_c$. The right-hand side represents the gain in terms of reduced rents, which have a shadow flow value $\alpha$.

Let $(a, b)$ denote a linear contract of the form $w = a + by$. The optimal non-linear contract can be represented by a menu $(a(\varepsilon), b(\varepsilon))$ of linear contracts.[9] For any $b$, the worker chooses the effort level such that $\psi'(e) = b\gamma$. Inserting this condition into equation (26), we obtain

$$b(\varepsilon) = 1 - \alpha \frac{1 - H(\varepsilon)}{h(\varepsilon)} \frac{\psi''(e)}{\gamma^2}. \tag{29}$$

We refer to $b(\varepsilon)$ as the incentive power of the optimal contract. The value of $a(\varepsilon)$ is set so that (C6) is satisfied. For later reference, we also express the expected rent in terms of $b(\varepsilon)$. Inserting $\psi'(e) = b\gamma$ into equation (C6) gives

$$(r + s)R = \int_{\varepsilon_c}^{\bar{\varepsilon}} b(\varepsilon) \frac{1 - H(\varepsilon)}{h(\varepsilon)} dH(\varepsilon). \tag{30}$$

**Proposition 2** *The optimal contract* $\phi^{\max}(R, U)$ *and match surplus* $S^{\max}(R, U)$ *have the following properties:*

*a) The effort level* $e(\varepsilon)$ *is strictly increasing in* $R$ *for all* $\varepsilon < \bar{\varepsilon}$ *and the cut-off level* $\varepsilon_c$ *is decreasing in* $R$.

*b) The match surplus* $S^{\max}(R, U)$ *is strictly increasing and concave in* $R$.

[9] See, e.g., Laffont and Tirole, 1993.

*c) If all types are hired ($\varepsilon_c = \underline{\varepsilon}$), then*

*i) a shift in $U$ shifts $a(\varepsilon)$ but leaves $b(\varepsilon)$ unchanged for all $\varepsilon$.*

*ii) a shift in $U$ does not influence the marginal value of rents, i.e., $S^{\max}_{RU} = 0$.*

When the principal has more rents to dole out, she can afford to give stronger incentives to all workers. Furthermore, as the expected rent is decreasing in the cut-off level, a higher $R$ also implies that the principal can afford to hire workers of a lower type, by reducing $\varepsilon_c$. Proposition 1a) states that the principal does both.

The first part of b), that the match surplus, $S^{\max}$, increases in $R$, follows directly from the fact that the rent constraint is binding. The second part of b), that $S^{\max}$ is concave in $R$, follows from the convexity of the maximization problem, i.e. that the marginal return from a higher effort or a lower cut-off level is decreasing.

Result c) states that if all workers are hired, the workers' outside option $U$ neither influences the incentive power of the contract nor the shadow value of rents. Intuitively, for a given cut-off, a change in $U$ (for a given $R$) only implies that more income is transferred to the worker and the effort level remains constant for all types.

As noted above, we derived the optimal contract under the assumption that it has the cut-off property that a worker is hired with probability 1 if $\varepsilon > \varepsilon_c$ and with probability 0 if $\varepsilon < \varepsilon_c$. With a larger contract space, the hiring probability of a worker, $\sigma$, is a general function of $\varepsilon$, $\sigma = \sigma(\varepsilon)$.

**Lemma 2** *The optimal contract has the cut-off property that there exists a value $\varepsilon_c \geq \underline{\varepsilon}$ such that $\sigma(\varepsilon) = 1$ for $\varepsilon > \varepsilon_c$ and $\sigma(\varepsilon) = 0$ for $\varepsilon < \varepsilon_c$.*

Above, we have derived the optimal static (tenure independent) contract. In the appendix we set up a more general contracting problem, where effort and wages may be time dependent, and show the following result:

**Lemma 3** *The optimal dynamic contract repeats the static contract.*

Providing incentives is costly for firms, as it yields information rents to inframarginal workers. Deferred compensation or other time dependent wage contracts do not reduce this information rent, as they do not reduce the rent high types can obtain by pretending to be low types. Furthermore, deferred compensation does not influence the participation constraint at the hiring stage. It may loosen the participation constraint for tenured workers, but this has no value to the firm as the worker's outside option is time independent.

**Step 2: Optimal sharing rules** In the appendix we derive the first- and second order condition for the maximization problem (24). With the equilibrium value of $U^*$ inserted, the first-order condition reads

$$[1 - S_R^{\max}(R^*, U^*)]\frac{R^*}{S^* - R^*} = \frac{\eta}{1-\eta}, \tag{31}$$

where, as before, $\eta$ denotes the absolute value of the elasticity of $q$ with respect to $\theta = v/u$. The second order condition is satisfied as long as $\eta$ is non-decreasing in $\theta$, exactly as in the full-information case.

We refer to (31) as the modified Hosios condition. The modified Hosios condition states that the workers' share of the match surplus, $R^*/(S^* - R^*)$, increases with the marginal value of worker rents, $S_R^{\max}$. Thus, a smaller fraction of the match surplus is allocated to job creation. When $S_R^{\max} = 0$, equation (31) is identical to the Hosios condition with full information given by equation (16). With full information, a wage increase is purely redistributional. It reduces the value of a match for the firm by exactly the same amount as it increases its value for the worker. With private information, this no longer holds. A one-unit increase in $R$ increases the match surplus $S^{\max}$ by $S_R^{\max}$ units, thereby reducing the firm's profit $J$ by $1 - S_R^{\max}$ units only.

Equation (31) can be rewritten as

$$R^* = \beta^{eff} S^*$$

where

$$\beta^{eff} = \frac{\eta}{1 - (1-\eta)S_R^{\max}} \tag{32}$$

Hence $\beta^{eff}$ is the share of the surplus allocated to the worker, i.e., her effective "bargaining power". It follows that $\beta^{eff} = \eta$ when $S_R^{\max} = 0$, $\beta^{eff}$ is strictly increasing in $S_R^{\max}$, and $\beta^{eff}$ approaches 1 if $S_R^{\max}$ approaches 1 (in which case it is costless to provide incentives). Note that with a Cobb-Douglas matching function $x(u,v) = Au^{\beta}v^{1-\beta}$, it follows that $\eta = \beta$, independently of $\theta$. This is very convenient when analyzing how private information influences $\beta^{eff}$.

**Proposition 3** *Suppose $z < \overline{y} + \gamma e^F - \psi(e^F) + \overline{\varepsilon}$. Then the generalized competitive search equilibrium exists. If $\eta$ is non-decreasing in $\theta$, the equilibrium is unique.*

The competitive search equilibrium with full information maximizes the asset value of unemployed workers, given that firms break even (Acemoglu and Shimer, 1999, Moen and Rosen, 2004). This property also holds for the GCS-equilibrium:

**Proposition 4** *The generalized competitive search equilibrium maximizes $U$ given the free entry constraint $V = 0$ and the relevant information constraints.*

An interesting question is how the optimal contract is influenced by policy variables. To some extent $z$ is a variable controlled by the government. The vacancy costs $c$ and the efficiency of the matching process ($A$ in the Cobb-Douglas case) may also depend on institutional arrangements.

**Proposition 5** *Suppose $\varepsilon_c = \underline{\varepsilon}$. Then an increase in $z$ and $A$ or a reduction in $c$ strictly reduces the incentive power of the wage contract $b(\varepsilon)$ for all $\varepsilon$.*

An increase in $z$ tends to reduce the match surplus, and hence the amount of rents available to incentivize the worker. Firms thus have to cut back on the incentive power of the wage contract. For the same reason firms will lower the incentive power of the contract if the matching process becomes more efficient ($c$ falls or $A$ increases).

We can also show that when $\varepsilon_c = \underline{\varepsilon}$, the equilibrium with private information has a higher unemployment rate than the equilibrium without private information

**Proposition 6** *Suppose $\varepsilon_c = \underline{\varepsilon}$. Then the unemployment rate is strictly higher with private information than with full information.*

For a given cut-off, the unemployment rate increases for two reasons. First, for a given rent sharing rule, the expected productivity of a worker falls, and as a result fewer firms enter the market. Second, the workers' share of the surplus is higher with private information, hence fewer firms enter the market compared to the full information case. This in turn also increases the unemployment rate.

Finally, it is particularly interesting to see how the workers' share of the match surplus depends on macroeconomic conditions, as this has consequences for the responsiveness of the unemployment rate to macroeconomic shocks. As pointed in the introduction, work by Shimer (2005) has demonstrated that the standard Diamond-Mortensen-Pissarides search model cannot easily explain the observed fluctuation in unemployment and vacancy rates, the observed volatility of the unemployment rate relative to that of aggregate productivity is much larger than the DMP-model predicts. Since we are comparing equilibria with different values of $\theta$, it is convenient to assume that the matching function is Cobb-Douglas, as $\eta$ then does not depend explicitly on $\theta$.

**Proposition 7** *Suppose $\varepsilon_c = \underline{\varepsilon}$, and $R^* \leq \overline{R}$. Then $\frac{d\beta^{eff}}{d\overline{y}} < 0$.*

A negative shift in $\overline{y}$ decreases the equilibrium surplus $S^*$, which for a given sharing rule reduces $R^*$. Since $S_{RR}$ is negative this increases the shadow value of rents, so that $\beta^{eff}$ increases and $R^*$ falls less than if the sharing rule was constant. It follows that a negative shift in $\overline{y}$ increases the share of the expected match surplus allocated to the worker. As a result, private information tends to make wages less responsive and unemployment more responsive to shifts in $\overline{y}$.

Our result relates to the discussion in Hall (2005a). Hall argues that due to social norms, the worker's share of the match surplus is counter-cyclical.[10] Our model generates a counter-cyclical sharing rule as an optimal response to changes in aggregate variables in the presence of private information.

Intuitively, changes in the information structure (changes in $\gamma$ and the distribution of $\varepsilon$) may have large impacts on the unemployment rate. However, as the effects of such shifts are

[10] Hall (2005b) also shows that wage rigidity may be the result of alternative specifications of the bargaining procedure or self-selection among workers.

hard to analyze qualitatively, we defer this to the next section.

# 5 Numerical analysis

Shimer (2005) demonstrates that a standard Diamond-Mortensen-Pissarides search model indicates an unemployment-output elasticity of around 1, data suggests an elasticity that is significantly larger, around 10. In this subsection we will analyze whether private information in our model may enhance the responsiveness of the unemployment rate to aggregate shocks. To simplify the analysis we focus on comparative statics analysis. We assume that the matching function is Cobb-Douglas, $x(u,v) = Au^{\beta}v^{1-\beta}$. We first analyze the effects of shifts in $\overline{y}$, and then on changes in the distribution of $\varepsilon$.

Our numerical analysis follows Kennan (2009), who also studies a model with private information and two states. Kennan in turn builds on Shimer (2005). We set $r = 0.012$ and $s = 0.1$ (with a quarter as the time unit) and $\beta = 0.5$. The search cost $c$ is set equal to 0.4, and $A$ is calibrated so that the job finding rate initially is $p = 1.35$.

We assume that $\varepsilon$ is uniformly distributed on an interval $[-\widehat{\varepsilon}, \widehat{\varepsilon}]$, where $\widehat{\varepsilon}$ is to be determined below. The cost of effort is written as $\psi(e) = ge^2/2$, where $g$ is a constant. In our baseline case we set $g = 0.3$ and the value of effort, $\gamma$, equal to 0.6. Without private information and with $\varepsilon_c = -\widehat{\varepsilon}$, the average output per worker net of effort cost is $\overline{y} + 0.6$, while the average *measured* output per worker (effort costs not subtracted) is $\overline{y} + 1.2$. We set the initial value/benchmark of $\overline{y}$ to be 1, in which case observed first best output is 2.2.

Kennan (2009) argues that the variance in productivity estimated in Shimer's is analogous to a drop in productivity in a two-state model of 3 percent. With output equal to 2.2, this corresponds to a drop in $\overline{y}$ by 0.066 with first-best output. Note though that we are focusing on the elasticity of the unemployment rate to output changes, hence the size of the shift is of second order importance. For the income while unemployed we follow Shimer and set it equal to 40 percent of initial output, $z = 0.4 * 2.2 = 0.88$ (see comments below).

**Changes in productivity $\overline{y}$**

We will first study the effects of changes in the deterministic productivity term $\overline{y}$. Recall that $\beta^{eff}$ defined by (32) denotes the equilibrium share of the surplus allocated to the worker.

| Table 1. Unemployment volatility without effort costs | | | |
|---|---|---|---|
| y | u | % change | Elasticity |
| 2.2 | 6.9 | | |
| 2.134 | 7.08 | 2.6087 | 0.8696 |

Note that $\beta^{eff}$ increases after a negative shift in $\overline{y}$ because $R^*$ decreases. Hence, an upper bound on the volatility that our model can deliver after shocks in $\overline{y}$ is what obtains if $R^*$ stays constant after the shock. Brugemann and Moscarini (2007) and Brugemann (2008) refer to constant worker rent as rent rigidity. They show that if rent rigidity is imposed on the standard DMP model after a shock, this is not sufficient to account fully for the lacking volatility in the unemployment rate. They find that the standard matching model with rent rigidity can explain at most around 20-30 percent of the observed unemployment volatility. Our main issue when looking at changes in $\overline{y}$ is therefore how close we can come to the upper bound for unemployment volatility defined by the volatility with rent rigidity, not whether we can "explain" the Shimer puzzle.

Table 1 shows the effect of a three percent reduction in output in our model if all workers have constant productivity and there is no effort cost.

The elasticity of the unemployment rate to changes in y (measured as the percentage increase in $u$ to the percentage decrease in y) is below .9, much lower than Shimer's estimate of 10.

We then include private information. We calibrate the model such that $R = \overline{R}$ for $\overline{y} = 1$. This is done by solving the model in the full information case. Recall from (22) that with uniform distribution, $(r+s)\overline{R} = \widehat{\varepsilon}$. We therefore set $\widehat{\varepsilon} = (r+s)\beta S^*$, in which case $R = \overline{R}$ initially. A negative shock to $\overline{y}$ will then drive $R$ below $\overline{R}$. In the appendix we show that if $R = \overline{R}$ at $y = \overline{y}$, there always exists an interval $[y^a, \overline{y}]$ at which $\varepsilon_c = -\widehat{\varepsilon}$. The critical value of $y^a$ is reported below.

We analyze three different cases; one without private information, one with private information, and one with rent rigidity. In the model simulations without private information we simply fix the expected output per worker at its first best level and the worker's share of

| Table 2. Unemployment volatility with private info | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| z=0.88 (0.4*2.2), β=0.5 | | | | | | | | |
| | Without priv. Info | | | With private info | | | | |
| $y^{bar}$ | u | % increase in u | Elasticity | u | % increase in u | Rel. to no ›rivate inf‹ | ‹ reductio› in output | Elasticity |
| 1.00 | 6.8966 | | | 6.8966 | | | | |
| 0.934 | 7.2370 | 4.9358 | 1.6453 | 7.5585 | 9.5975 | 1.9445 | 3.2091 | 2.9907 |
| Relative to no private info shows the increase in u with private information relative to that of no private information. | | | | | | | | |

the surplus at $\beta$. Rent rigidity is obtained by fixing $R$ at $\overline{R}$.

The first part of Table 2 shows the responsiveness of the unemployment rate without private information. Note that even without private information the elasticity of the unemployment rate with respect to output is almost twice as big as in table 1. The reason is that the cost of effort is now included. The cost of effort increases the attractiveness of unemployment relative to employment. Recall that the unemployment benefit is 40% of observed first-best output, $z = .4 * 2.2 = .88$. The first best output level net of effort cost is only 1.6. Hence the effective replacement ratio is now $0.88/1.6 = 0.55$.

The second part of table 2 shows the effect of the same shock in $\overline{y}$ with private information. The unemployment rate increases by 9.6 percentage points, almost double that without private information (column "Rel. to no private info"). Expected output per worker drops by more than 3%, as we are no longer in first best and the effort level decreases. Still the elasticity of the unemployment rate with respect to output is almost 3, and 82 percent larger than without private information.

Table 3 compares the solutions with private information and with rent rigidity.

It follows that the change in the unemployment rate under private information is 95% of the change with rent rigidity. If we look at elasticities, the number is slightly smaller, since output falls by more than 3 percent with private information (with rent rigidity, first best

| Table 3. Private information vs rent rigidity | | | | | |
|---|---|---|---|---|---|
| | Rent rigidity | | | Private info rel. to rent rigidity | |
| y | u | % increase in u | Elasticity | Change in unempl. | Elasticity |
| 1.00 | 6.8966 | | | | |
| 0.934 | 7.5943 | 10.1166 | 3.3722 | 94.8689 | 88.6873 |

output is achieved also after the shift). Still the elasticity under private information accounts for 89 percent of the elasticity with rent rigidity. Thus, in this case private information increases the responsiveness of the unemployment rate to shocks close to its upper bound defined by rent rigidity. To understand the result, note that the cost of increasing the worker's share of the surplus from its initial value of $\beta$ is of second order, as it is the solution to the firms' maximization problem. Thus, as the rent constraint starts to bite, firms are reluctant to reduce the rent paid to workers', $R$, they instead create fever vacancies.

We also calculated $y^a$ (the lowest value of $\overline{y}$ at which $\varepsilon_c = -\widehat{\varepsilon}$), and found it to be .77. Thus there is a large interval of $\overline{y}$ at which the rent constraint binds and at the same time all workers are hired. In this simulation $\widehat{\varepsilon} = 0.0512$. The support of $\varepsilon$ thus has measure 0.1024.

We have repeated the exercise with $\beta = .28$, the same parameter as Shimer uses. This increases the volatility without private information, the elasticity of $u$ with respect to $\overline{y}$ is 2.68. Private information increases the elasticity with 38.2 percent, which is less than with $\beta = .5$. However, more importantly the elasticity with private information is now 95.3 percent of the elasticity with rent rigidity, which is higher than in the $\beta = .5$ case.

Hall and Milgrom (2008) argue that if the value of leisure is included, a value of $z$ equal to 0.71 times initial output may be a better estimate of the value of leisure. The value of leisure is arguably analogous to the absence of effort cost when working, hence including both would imply double accounting of the disutility of working. Still we have carried out the same calculations as above using a higher unemployment benefit, by setting $z = 1.136$ ( 0.71 times the expected productivity of a worker *net of* effort cost, 0.52 times observed output.

| Table 4. Changes in the support of ε. | | | | | |
|---|---|---|---|---|---|
| z=0.88 (0.4*2.2), β=0.5 | | | | | |
| % of eps | u | y | % matches not accepted | % reduction in y | Elasticity of u |
| 100 | 6.8966 | 2.20 | 0 | 0 | |
| 110 | 7.4698 | 2.1911 | 0 | 0.4045 | 20.5449 |
| 120 | 8.0368 | 2.1828 | 0 | 0.7818 | 21.1466 |
| 130 | 8.4589 | 2.1788 | 2.1484 | 0.9636 | 23.5080 |
| 140 | 8.7583 | 2.178 | 5.7059 | 1.0000 | 26.9945 |

Not unexpectedly, the unemployment rate becomes more responsive to shifts in output. In addition, the difference between the model with private information and without private information grows slightly, while the difference between the model with private information and with rent rigidity drops slightly. The responsiveness of the unemployment rate with private information is now 2.01 times the responsiveness without private information and 96.6 percent of the response with rent rigidity. The elasticity of the unemployment rate with respect to output is 5.0, which is 90.0 percent of the elasticity under rent rigidity.

In our calibration, effort is important. The value added from optimal effort, net of effort cost, is 0.6, and the value gross of effort cost is 1.2. It is therefore interesting to see what happens if we reduce the importance of effort. To this end we reduce $\gamma$ to .3. In this case the optimal effort level is 1, the gross value of effort is only .3, while the net value is only .15. We increase the benchmark value of $\overline{y}$ to 1.45, so that the equilibria without private information and with rent rigidity are unaltered (the first best expected output level net of effort costs remains 1.6). Note that the net value of effort is now less than 10 percent of total output net of effort costs. Even in this case the responsiveness of unemployment is 84 percent of the response with rent rigidity. The elasticity of the unemployment rate to changes in output is 80 percent of that with rent rigidity.

**Changes in the distribution of $\varepsilon$**

We now study the effects of changes in $\widehat{\varepsilon}$. We calibrate the model exactly in the same way as above, but now we keep $\overline{y}$ equal to 1 and instead study the effects of an increase in $\widehat{\varepsilon}$. We also report the resulting changes in $y$, the observed average productivity.

| Table 5. Simultaneous shifts in y and epsilon | | | | | |
|---|---|---|---|---|---|
| z=0.88, β=.5 | | | | | |
| Parameters | u | % increase in u | % matches not accepted | % reduction in output | Elasticity |
| Baseline | 6.8966 | 0 | 0 | 0 | 0 |
| y: 1%, eps: 40% | 8.9651 | 29.993 | 6.56 | 1.991 | 15.06 |
| y:2%, eps 30% | 8.8713 | 28.633 | 3.87 | 3.259 | 8.79 |
| y:3%, eps 20% | 8.7596 | 27.013 | 0.83 | 3.905 | 6.92 |
| y:4%, eps 10% | 8.463 | 22.713 | 0 | 4.664 | 4.87 |

The first column in table 4 shows the value of $\widehat{\varepsilon}$ in percent of the initial value which gives $R = \overline{R}$. The initial value of $\widehat{\varepsilon}$ is 0.0512, as above. The first line in the figure shows the baseline case. In the second line, $\widehat{\varepsilon}$ is increased by 10 percent, in the third line by 20 percent and so forth. The second column shows the unemployment rate, and the third column shows average observed productivity per match. The fourth column shows the percentage of matches that are rejected (that do not lead to employment). The fifth column shows the reduction in expected output relative to the full-information case. Finally, the last column shows the elasticity of the unemployment rate to observed output (still measured as the ratio of the percentage increase in the unemployment rate to the percentage reduction in output).

We see that the elasticity of the unemployment rate is around 20 and slightly increasing as the shifts become larger. An increase in $\widehat{\varepsilon}$ by 40 percent increases the unemployment rate by more than 1.8 percentage points, and reduces output per employee by slightly more than 1.3 percent. Note that all worker types are hired except in the last two cases, where a small proportion of matches are rejected. However, as only matches that lead to employment are observable and included in $y$, an increasing cut-off dampens the reduction in $y$. Similar results are obtained using $z = 1.136$.

It can be interesting to see the effects of combined changes in $\overline{y}$ and $\widehat{\varepsilon}$, after all the shifts may be likely to occur in tandem.

The baseline case in table 5 is the same as in table 4. The next row shows the effects of a reduction in $\overline{y}$ of 1% and an increase in $\widehat{\varepsilon}$ of 40 percent, the next row shows a decrease in $\overline{y}$ of 2 percent and an increase in $\widehat{\varepsilon}$ of 30 percent, and so forth. Clearly, the effect is particularly large when the change in $\widehat{\varepsilon}$ is large. Still a two percent decrease in $\overline{y}$ coupled with a 30

| Table 6. Cut-off binds initially | | | | | | |
|---|---|---|---|---|---|---|
| z=0.88, β=.5 | | | | | | |
| Parameters | u | % increase in u | % matches not accepted | Output | % reduction in output | Elasticity |
| Baseline | 6.8966 | 0 | 0 | 2.1825 | 0 | 0 |
| y: 3% | 7.4162 | 7.534 | 2.5824 | 2.117 | 3.001 | 2.51 |
| eps 20% | 7.519 | 9.025 | 8.74 | 2.1809 | 0.073 | 123.10 |
| eps 40% | 8.0789 | 17.143 | 15.51 | 2.1794 | 0.142 | 120.69 |

percent increase in $\widehat{\varepsilon}$ increases the unemployment rate by around 2 percentage points and gives an elasticity of the unemployment rate to output of 8.79.

We have also analyzed the effects of shifts using lower value of effort, that is, with $\gamma = 0.3$. As described in the subsection on shifts in $\overline{y}$, this reduces the value of effort net of search effort from .6 to .15. We calibrate the model as above and find that this reduces the responsiveness of the unemployment rate to shifts in $\widehat{\varepsilon}$, but it is still large. A 20 percent increase in $\widehat{\varepsilon}$ increases the unemployment rate from 6.8966 to 7.7295, and the elasticity of the unemployment rate to changes in observed output is 9.1.

**Internal cut-off**

Finally, we analyze the effects of shifts in $\overline{y}$ and $\widehat{\varepsilon}$ if $R < \overline{R}$ initially. We have calibrated the model by choosing $A$ and $\widehat{\varepsilon}$ so that 1) the first order conditions for optimal cut-off level has $\varepsilon_c = -\widehat{\varepsilon}$ as the solution initially, and 2) $p = 1.35$. This was obtained for $\widehat{\varepsilon} = 0.0535$ and $A = 1.3088$. A reduction in $\overline{y}$ below 1 or an increase in $\widehat{\varepsilon}$ above 0.0535 implies that we have interior cut-off. It follows that $\alpha > 0$ initially and worker effort is lower than its first-best level from the outset. The results are shown in table 6.

The baseline case shows the initial equilibrium. Note that expected output is less than 2.2 initially, as we do not have first best effort levels. The next line shows the effects of a reduction in $\overline{y}$ of 3 percent of first best output ($\overline{y}$ is reduced to 0.934). The elasticity of the unemployment rate is still substantially larger than the elasticity without private information as calculated above. Still the effect is smaller than above, when the initial equilibrium satisfied $R^* = \overline{R}$. To understand why, note that if $\varepsilon_c$ stays constant at $-\widehat{\varepsilon}$, firms can only adjust to a lower level of $R$ by cutting back on worker effort. When $\varepsilon_c > -\widehat{\varepsilon}$, the firm can cut back both by reducing effort and increasing the hiring threshold, and as a result the shadow value of rent, $\alpha$, and thus $\beta^{eff}$ increases less rapidly when $R$ is reduced. On the other hand, increasing $\varepsilon_c$ has a direct, negative impact on the unemployment rate. It turns out that the first effect dominates, and that the responsiveness of the unemployment rate is lower with interior cut-off.[11]

The two last rows show the effects of increasing $\widehat{\varepsilon}$ by 20 and 40 percent, respectively. Compared to the results obtained when $R^* = \overline{R}$ initially, we see that also for changes in $\widehat{\varepsilon}$ the effects in terms of increase in the unemployment is lower with interior cut-off. However, as firms become more selective, the decrease in observed output associated with an increase in $\widehat{\varepsilon}$ falls, and the elasticity of the unemployment rate to observed expected output actually increases.[12]

# 6 Final comments

In this paper, we define and characterize what we refer to as the generalized competitive search equilibrium, in which workers have private information regarding their effort and "type". In our model, the firms face a trade-off between extracting rents from workers and providing incentives to exert effort. Search frictions with competitive wage setting imply that the cost of leaving rents to the worker are lower than in the standard frictionless model, as worker rents save on search costs for the firms. We show that the resulting equilibrium satisfies what we refer to as the modified Hosios condition. The incentive power of the wage contracts is positively related to high productivity, low unemployment benefits and high search frictions, and private information increases the unemployment rate.

We then analyze analytically and numerically to what extent our model is able to reconcile the high volatility of the unemployment rate relative to the volatility of output per worker observed in the data. Theoretical considerations imply that an upper bound on the effects

[11] We have also calculated the effect on $u$ of a fall in $\overline{y}$ with the same initial conditions, but with $\varepsilon_c$ locked at $-\widehat{\varepsilon}$. The unemployment rate is then 7.5695 after the shock in $\overline{y}$, which is larger than the rate of 7.4162 reported in the table.

[12] Also for changes in $\widehat{\varepsilon}$ we have calculated the effect on the unemployment rate with $\varepsilon_c$ locked at $-\widehat{\varepsilon}$. A 20 percent increase in $\widehat{\varepsilon}$ then leads to an unemployment rate of 8.0789, approximately the same as in the case where $R^* = \overline{R}$ initially.

of pure productivity shifts on unemployment volatility is the volatility obtained with rent rigidity (the worker's expected gain from finding a job stays constant). Our numerical analysis shows that the volatility of the unemployment rate is close to this upper bound with reasonable parameter values. If the negative shifts are associated with greater variance in output per worker, the model can easily reproduce the volatility of the unemployment rate observed in the data.

It is our belief that developing search models with a richer structure than the standard Diamond-Mortensen-Pissarides model may add new insights, both within macroeconomics and different subfields of labor economics. In previous studies, the inclusion of human capital in search models has improved our understanding of human capital formation. The present paper addresses questions that are relevant for both macroeconomic fluctuations and personnel economics within a search framework. Adding more structure to search models may therefore be a fruitful avenue for future research.

# Appendix

**Optimal sharing rules, full information**

From (15) it follows by simple manipulation that

$$el_p q(p) el_R p(R) = \frac{R}{S^F(R) - R}. \tag{33}$$

From equation (9) it follows that $el_R p(R) = -1$. We want to show that $el_p q(p) = -\frac{\eta}{1-\eta}$. To see this, let $p = \widetilde{p}(\theta)$ and $q = \widetilde{q}(\theta)$. Then

$$\begin{aligned} el_p q(p) &= el_p \widetilde{q}(\widetilde{p}^{-1}(p)) \\ &= \frac{el_\theta \widetilde{q}(\theta)}{el_\theta \widetilde{p}(\theta)}. \end{aligned}$$

Since $el_\theta \widetilde{q}(\theta) = -\eta$ and $el_\theta \widetilde{p}(\theta) = el_\theta[\theta \widetilde{q}(\theta)] = 1 - \eta$, it follows that $el_p q(p) = -\frac{\eta}{1-\eta}$. The result thus follows.

Let us then turn to the second order conditions. Using (12) gives

$$\begin{aligned}\frac{dV}{dR} &= q'(p)p'(R)(S-R) - q(p(R)) \\ &= q(p(R))\frac{q'(p)p}{q}\frac{p'(R)R}{p}\frac{S-R}{R} - q(p(R)) \\ &= q(p(R))(\frac{\eta}{1-\eta}\frac{S-R}{R} - 1)\end{aligned}$$

(since $el_R p(R) = -1$ and $el_p q(p) = -\frac{\eta}{1-\eta}$). The second order derivative is thus

$$\frac{d^2V}{dR^2} = q'(p)p'(R)(\frac{\eta}{1-\eta}\frac{S^F-R}{R} - 1) + \frac{d\frac{\eta}{1-\eta}}{dR}q(p(R))\frac{S^F-R}{R} + \frac{d\frac{S^F-R}{R}}{dR}q(p(R))\frac{\eta}{1-\eta} \tag{34}$$

The first term is zero at the stationary point. The last term is strictly negative. However, our assumptions on the matching function is not sufficient to sign $\frac{d\frac{\eta}{1-\eta}}{dR}$. A sufficient condition to satisfy the second order conditions and ensure that the Hosios condition has a unique solution is thus $\frac{d\frac{\eta}{1-\eta}}{dR} \leq 0$, i.e., since $\theta$ is decreasing in $R$, that $\frac{d\eta}{d\theta} \geq 0$.

**Proof of Lemma 1**

a) Suppose the full information equilibrium is feasible. Denote the full information output level by $y^F(\varepsilon)$. Truth-telling around $\varepsilon_c$ requires that $\widetilde{W}(\varepsilon_c) = U$, or $\omega(\varepsilon_c) = rU$. In order to implement the first best cut-off it thus follows that $w(\varepsilon_c) = y^F(\varepsilon_c)$ (from 14). Inserting $\psi'(e(\varepsilon)) = \gamma$ into (21) thus gives

$$\begin{aligned}w(\varepsilon) &= y^F(\varepsilon_c) + \varepsilon - \varepsilon_c \\ &= y^F(\varepsilon).\end{aligned}$$

Hence, the profit is zero for all worker types. Since search is costly this implies that the value of a vacancy is negative for all $\theta > 0$, and no firm enters the market. This is inconsistent with equilibrium.

b) For $\varepsilon_c = \underline{\varepsilon}$ we may have that $rU < y^F(\underline{\varepsilon})$ and hence $\omega(\varepsilon_c) < y^F(\varepsilon_c) - \psi(e^F)$. Set $\omega(\varepsilon_c)$ at its lowest possible value that satisfies the participation constraint, $\omega(\varepsilon_c) = rU$. Inserting $\psi'(e(\varepsilon)) = \gamma$ into (21) then gives $\omega(\varepsilon) = rU + \varepsilon - \underline{\varepsilon}$. The lowest possible rent $R$ that implements the full information allocation is thus given by (22).

**Proof of the claim that $\omega(\varepsilon)$ is differentiable.**

Since $e(\varepsilon)$ is strictly increasing it is differentiable almost everywhere. We want to show that $w(\varepsilon)$ is differentiable at all points where $e(\varepsilon)$ is differentiable. It then follows that $\omega(e) = w(\varepsilon) - \psi(e(\varepsilon))$ is differentiable almost everywhere.

Suppose $e(\varepsilon)$ is differentiable at $\varepsilon = \varepsilon^1 > \varepsilon_c$. Consider another type $\varepsilon^2 > \varepsilon_c$. Truth-telling then implies

$$w(\varepsilon^2) - \psi(e(\varepsilon^2)) \geq w(\varepsilon^1) - \psi(e(\varepsilon^1) - \frac{\varepsilon^2 - \varepsilon^1}{\gamma})$$

and

$$w(\varepsilon^1) - \psi(e(\varepsilon^1)) \geq w(\varepsilon^2) - \psi(e(\varepsilon^2) - \frac{\varepsilon^1 - \varepsilon^2}{\gamma})$$

Combining the equations gives

$$\psi(e(\varepsilon^2) - \frac{\varepsilon^1 - \varepsilon^2}{\gamma}) - \psi(e(\varepsilon^1)) \geq w(\varepsilon^2) - w(\varepsilon^1) \geq \psi(e(\varepsilon^2)) - \psi(e(\varepsilon^1) - \frac{\varepsilon^2 - \varepsilon^1}{\gamma})$$

Suppose first that $\varepsilon^2 > \varepsilon^1$. It follows that

$$\frac{\psi(e(\varepsilon^2) - \frac{\varepsilon^1-\varepsilon^2}{\gamma}) - \psi(e(\varepsilon^1))}{\varepsilon^2 - \varepsilon^1} \geq \frac{w(\varepsilon^2) - w(\varepsilon^1)}{\varepsilon^2 - \varepsilon^1} \geq \frac{\psi(e(\varepsilon^2)) - \psi(e(\varepsilon^1) - \frac{\varepsilon^2-\varepsilon^1}{\gamma})}{\varepsilon^2 - \varepsilon^1}$$

We then take the limit as $\varepsilon^2 \to \varepsilon^{1+}$. Both the left-hand side and the right-hand side, and thus also $\frac{w(\varepsilon^2)-w(\varepsilon^1)}{\varepsilon^2-\varepsilon^1}$, converge to $\psi'(e'(\varepsilon^1) + 1/\gamma)$. Suppose then that $\varepsilon^2 < \varepsilon^1$. It follows that

$$\frac{\psi(e(\varepsilon^2) - \frac{\varepsilon^1-\varepsilon^2}{\gamma}) - \psi(e(\varepsilon^1))}{\varepsilon^2 - \varepsilon^1} \leq \frac{w(\varepsilon^2) - w(\varepsilon^1)}{\varepsilon^2 - \varepsilon^1} \leq \frac{\psi(e(\varepsilon^2)) - \psi(e(\varepsilon^1) - \frac{\varepsilon^2-\varepsilon^1}{\gamma})}{\varepsilon^2 - \varepsilon^1}$$

We now take the limit as $\varepsilon^2 \to \varepsilon^{1-}$. Again both the left-hand side and the right-hand side converge to $\psi'(e'(\varepsilon^1) + 1/\gamma)$. It follows that the left and the right limit of $\frac{w(\varepsilon^2)-w(\varepsilon^1)}{\varepsilon^2-\varepsilon^1}$ both exist and are equal, and hence that $w(\varepsilon)$ is differentiable at $\varepsilon = \varepsilon^1$.

**Proofs related to Proposition 1.**

In this appendix we 1) identify sufficient conditions for truth-telling, 2) show that the first order condition has a unique solution, 3) derive properties for $e(\varepsilon)$, $\varepsilon_c$ and $\alpha$, and 4) show that the first order condition solves step 1 of P2.

1. *Sufficient conditions for truth-telling*

We want to show that the second order condition for truth-telling is satisfied if $e(\varepsilon)$ is monotonically increasing in $\varepsilon$ whenever $e > 0$. Recall that

$$\widetilde{\omega}(\varepsilon, \widetilde{\varepsilon}) = w(\widetilde{\varepsilon}) - \psi(e(\widetilde{\varepsilon}) - \frac{\varepsilon - \widetilde{\varepsilon}}{\gamma})$$

The second order condition for truth-telling is satisfied if $\widetilde{\omega}(\varepsilon, \widetilde{\varepsilon})$ satisfies the single crossing property $\widetilde{\omega}_{\widetilde{\varepsilon},\varepsilon}(\varepsilon, \widetilde{\varepsilon}) > 0$. To see this, recall that the first order condition requires that $\widetilde{\omega}_{\widetilde{\varepsilon}}(\varepsilon, \varepsilon) = 0$ for all $\varepsilon > \varepsilon_c$ The single crossing property thus implies that for $\widetilde{\varepsilon} > \varepsilon$, $\widetilde{\omega}_{\widetilde{\varepsilon}}(\varepsilon, \widetilde{\varepsilon}) < \widetilde{\omega}_{\widetilde{\varepsilon}}(\widetilde{\varepsilon}, \widetilde{\varepsilon}) = 0$. Analogously, the single crossing property implies that for $\widetilde{\varepsilon} < \varepsilon$, $\widetilde{\omega}_{\widetilde{\varepsilon}}(\varepsilon, \widetilde{\varepsilon}) > \widetilde{\omega}_{\widetilde{\varepsilon}}(\widetilde{\varepsilon}, \widetilde{\varepsilon}) = 0$. Together these conditions show that it is optimal to report $\widetilde{\varepsilon} = \varepsilon$. Now

$$\begin{aligned}\widetilde{\omega}_{\widetilde{\varepsilon}}(\varepsilon, \widetilde{\varepsilon}) &= w'(\widetilde{\varepsilon}) - \psi'(e(\widetilde{\varepsilon}) - \frac{\varepsilon - \widetilde{\varepsilon}}{\gamma})(e'(\widetilde{\varepsilon}) + \frac{1}{\gamma}) \\ \widetilde{\omega}_{\widetilde{\varepsilon}\varepsilon}(\varepsilon, \widetilde{\varepsilon}) &= \psi''(e(\widetilde{\varepsilon}) - \frac{\varepsilon - \widetilde{\varepsilon}}{\gamma})(e'(\widetilde{\varepsilon}) + \frac{1}{\gamma})\frac{1}{\gamma}\end{aligned}$$

which is surely satisfied if $e'(\widetilde{\varepsilon}) > 0$ (actually it is sufficient that $e'(\widetilde{\varepsilon}) > -1/\gamma$, i.e. that output is increasing in $\varepsilon$).

2. *Unique solution to the first order conditions*

From (25) it follows that

$$\frac{\partial L}{\partial e(\varepsilon)} = \gamma - \psi'(e(\varepsilon)) - \alpha \frac{1 - H(\varepsilon)}{h(\varepsilon)} \psi''(e(\varepsilon))/\gamma \tag{35}$$

The second order derivative is

$$\frac{\partial^2 L}{\partial e(\varepsilon)^2} = -\psi''(e(\varepsilon)) - \alpha \frac{1 - H(\varepsilon)}{h(\varepsilon)} \psi'''(e(\varepsilon))/\gamma < 0 \tag{36}$$

It follows that the first order condition (26) uniquely defines the effort level $e(\varepsilon)$ (for a given $\alpha$), and that $e(\varepsilon)$ is independent of $\varepsilon_c$.

Then we turn to $\varepsilon_c$. Since $e(\varepsilon)$ is independent of $\varepsilon_c$, we can substitute the optimal value of $e(\varepsilon)$ into $L$. Taking derivatives of (25) then gives

$$\frac{\partial L}{\partial \varepsilon_c} = -h(\varepsilon_c)[\overline{y} + \varepsilon_c + \gamma e(\varepsilon_c) - \psi(e(\varepsilon_c)) - rU - \alpha \frac{1 - H(\varepsilon_c)}{h(\varepsilon_c)} \frac{\psi'(e(\varepsilon_c))}{\gamma}] \tag{37}$$

$$\begin{aligned}\frac{\partial^2 L}{\partial \varepsilon_c^2} &= -h(\varepsilon_c) - h(\varepsilon_c)[\gamma - \psi'(e(\varepsilon_c)) - \alpha \frac{1 - H(\varepsilon_c)}{h(\varepsilon_c)} \psi''(e(\varepsilon_c))/\gamma)]e'(\varepsilon_c) \\ &\quad + h(\varepsilon_c)\alpha \frac{d\frac{1 - H(\varepsilon_c)}{h(\varepsilon_c)}}{d\varepsilon_c} \frac{\psi'(e(\varepsilon_c))}{\gamma} \\ &\quad - h'(\varepsilon_c)[\overline{y} + \varepsilon_c + \gamma e(\varepsilon_c) - \psi(e(\varepsilon_c)) - rU - \alpha \frac{1 - H(\varepsilon_c)}{h(\varepsilon_c)} \frac{\psi'(e(\varepsilon_c))}{\gamma} \\ &= -h(\varepsilon_c) + h(\varepsilon_c)\alpha \frac{d\frac{1 - H(\varepsilon_c)}{h(\varepsilon_c)}}{d\varepsilon_c} \frac{\psi'(e(\varepsilon_c))}{\gamma} < 0\end{aligned} \tag{38}$$

where the last equation is obtained by substituting in the first order conditions for $e$ and $\varepsilon_c$ and the fact that the hazard rate $h/(1-H)$ is increasing by assumption. Hence any stationary point must be a local maximum. It follows that the first order condition (27) at most has one stationary point and that this uniquely defines $\varepsilon_c$ for $\varepsilon_c > \underline{\varepsilon}$.

*3. Properties of* $e(\varepsilon)$, $\varepsilon_c$ *and* $\alpha$,

Since $\varepsilon$ by assumption has an increasing hazard rate, an increase in $\varepsilon$ shifts $\partial L/\partial e(\varepsilon)$ in (35) up. Thus, $e(\varepsilon)$ is strictly increasing in $e$ for $e(\varepsilon) > 0$. This has two implications:

1. The second order condition for truth-telling is satisfied

2. If $e(\varepsilon') = 0$, $e(\varepsilon) = 0$ for all $\varepsilon < \varepsilon'$ as claimed in the main text.

Similarly, an increase in $\alpha$ shifts $\partial L/\partial e(\varepsilon)$ in (35) down, hence $e(\varepsilon)$ is decreasing in $\alpha$.

We also want to show that $\varepsilon_c$ is increasing in $\alpha$. To this end, we take the derivative of (37) with respect to $\alpha$ and find that

$$\begin{aligned}\frac{\partial^2 L}{\partial \varepsilon_c \partial \alpha} &= -h(\varepsilon_c)[(\gamma - \psi'(e(\varepsilon_c))\frac{de}{d\alpha} + \alpha\frac{1-H(\varepsilon_c)}{h(\varepsilon_c)}\frac{\psi''(e(\varepsilon_c))}{\gamma}\frac{de}{d\alpha} + \frac{1-H(\varepsilon_c)}{h(\varepsilon_c)}\frac{\psi'(e(\varepsilon_c))}{\gamma}] \\ &= [1-H(\varepsilon_c)]\frac{\psi'(e(\varepsilon_c))}{\gamma} > 0\end{aligned}$$

From (38) we know that $\partial^2 L/\partial \varepsilon_c^2 < 0$ and it follows that $\partial \varepsilon_c/\partial \alpha > 0$ for $\varepsilon_c > \underline{\varepsilon}$.

*Unique* $\alpha$

We want to show that $\alpha$ is unique, and thus that the first order conditions have unique solutions. Suppose there is more than one solution for $\alpha$, and denote two of the solutions by $\alpha_1$ and $\alpha_2$, $\alpha_1 < \alpha_2$. From above it follows that $\varepsilon_c(\alpha_1) < \varepsilon_c(\alpha_2)$ and that $e(\varepsilon, \alpha_1) \geq e(\varepsilon, \alpha_2)$ with strict inequality whenever $e(\alpha_1) > 0$. But then it follows from (C6) that $R(\alpha_1) > R(\alpha_2)$, a contradiction.

4. *The first order conditions define the global maximum*

We have already seen that $e(\varepsilon)$ is strictly increasing in $\varepsilon$ for $e > 0$, and as stated above this is a sufficient condition for truth-telling.

We want to show that the first order conditions (C6), (26) and (27) solve P2 step 1, and do this by showing that the solution $e(\varepsilon), \varepsilon_c$ maximizes $L$ (with $\alpha$ defined by the first order conditions). The cut-off makes it hard to show that $L$ is concave, still the result follows almost immediately from (35)-(38).

For any given cut-off, it follows from (36) that $e(\varepsilon)$ defined by the first order condition maximizes $L$. Since $e(\varepsilon)$ is independent of $\varepsilon_c$, it is therefore sufficient to evaluate $L$ for different values of $\varepsilon_c$ given the optimal effort $e(\varepsilon)$.However, we have already seen that the second order conditions are always satisfied locally at any stationary point. It follows that $L$ is maximized at $\varepsilon_c$ given by the first order condition (27) if this equation has a interior solution. Otherwise, it is maximized at $\varepsilon_c = \underline{\varepsilon}$. The proof is thus complete.

**Proof of Proposition** 2

We will first show that $\alpha$ is strictly decreasing in $R$ for $R < \overline{R}$. Suppose the opposite, i.e. suppose that $R$ strictly increases and $\alpha$ increases . In the proof of proposition 1 we showed that that $\varepsilon_c$ is increasing in $\alpha$ and that $e(\varepsilon)$ is decreasing in $\alpha$ for all $\varepsilon$. Equation (C6) thus implies that $R$ is decreasing, a contradiction. 2a) and b) then follows directly.

Proof of Proposition 2c). The results in part c) directly follow from the fact that when $\varepsilon_c = \underline{\varepsilon}$, $U$ only influences the maximization problem through the participation constraint $\omega(\underline{\varepsilon}) = rU$. The first-order condition for optimal effort as well as $\alpha$ is then independent of $U$.

**Proof of lemma 2**

We want to prove that the optimal contract has the cut-off property that the worker is hired with probability 1 if her type is above a threshold $\varepsilon_c$ and with probability 0 if her type is strictly below this threshold (unless $\varepsilon_c = \underline{\varepsilon}$, in which case the worker is always hired).

To this end, we extend the contract space by allowing for randomized hiring. Let $\sigma(\varepsilon)$ denote the probability that a worker of type $\varepsilon$ is hired, $0 \leq \sigma(\varepsilon) \leq 1$. The contract is thus a vector $(w(\varepsilon), e(\varepsilon), \sigma(\varepsilon))$.

*Full information*

Although almost trivial, we first show the claim under full information. The optimal contract solves (analogous to 11)

$$(r+s)S^{\max}(U) = \max_{\sigma(\varepsilon),e(\varepsilon)} \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \sigma(\varepsilon)[y(e(\varepsilon),\varepsilon) - \psi(e(\varepsilon)) - rU]dH(\varepsilon).$$

which is maximized given the constraints that $0 \leq \sigma(\varepsilon) \leq 1$. The associated Lagrangian writes

$$L = \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \sigma(\varepsilon)[y(e(\varepsilon),\varepsilon) - \psi(e(\varepsilon)) - rU - (r+s)V]dH(\varepsilon) - \alpha_1(\sigma(\varepsilon) - 1) + \alpha_0\sigma(\varepsilon)$$

The first order conditions for $e(\varepsilon)$ reads (as in 13)

$$\psi'(e^F(\varepsilon)) = \gamma \text{ for all } \varepsilon.$$

The first order condition for $\sigma(\varepsilon)$ reads

$$\begin{aligned} \overline{y} + \varepsilon + \gamma e^F - \psi(e^F) - rU &> 0 \Rightarrow \sigma(\varepsilon) = 1 \\ \overline{y} + \varepsilon + \gamma e^F - \psi(e^F) - rU &< 0 \Rightarrow \sigma(\varepsilon) = 0 \\ \overline{y} + \varepsilon + \gamma e^F - \psi(e^F) - rU &= 0 \Rightarrow \sigma(\varepsilon) \in [0,1] \end{aligned}$$

The last equations is identical to equation (14), which uniquely defines $\varepsilon_c^F$. It thus follows that

$$\begin{aligned} \sigma(\varepsilon) &= 1 \text{ if } \varepsilon > \varepsilon_c^F \\ \sigma(\varepsilon) &= 0 \text{ if } \varepsilon < \varepsilon_c^F \end{aligned}$$

*Private information*

Again we use the revelation principle. Let $\widetilde{W}^\sigma(\varepsilon,\widetilde{\varepsilon})$ denote the expected discounted income of a matched worker of type $\varepsilon$ that claims to be of type $\widetilde{\varepsilon}$. It follows that

$$\widetilde{W}^\sigma(\varepsilon,\widetilde{\varepsilon}) = (1-\sigma(\widetilde{\varepsilon}))U + \sigma(\widetilde{\varepsilon})\frac{w(\widetilde{\varepsilon}) - \psi(e(\widetilde{\varepsilon}) - \frac{\varepsilon-\widetilde{\varepsilon}}{\gamma}) + sU}{r+s}$$

Truth-telling requires that $\widetilde{W}^\sigma(\varepsilon,\widetilde{\varepsilon})$ is maximized for $\widetilde{\varepsilon} = \varepsilon$. The envelope theorem thus implies that a necessary condition for truth-telling is that

$$\frac{d\widetilde{W}^\sigma(\varepsilon,\widetilde{\varepsilon}(\varepsilon))}{d\varepsilon} = \frac{\partial\widetilde{W}^\sigma(\varepsilon,\widetilde{\varepsilon})}{\partial\varepsilon}$$

evaluated at $\varepsilon = \widetilde{\varepsilon}$. It thus follows that

$$\frac{d\widetilde{W}^{\sigma}(\varepsilon, \widetilde{\varepsilon}(\varepsilon))}{d\varepsilon} = \sigma(\varepsilon)\frac{\psi'(e(\varepsilon))/\gamma}{r+s}$$

It follows that we can write

$$\widetilde{W}^{\sigma}(\varepsilon) = \widetilde{W}^{\sigma}(\underline{\varepsilon}) + \int_{\underline{\varepsilon}}^{\varepsilon} \sigma(\varepsilon)\frac{\psi'(e(x))/\gamma}{r+s}dx$$

Integrating over all $\varepsilon$ and using that $\widetilde{W}^{\sigma}(\underline{\varepsilon}) = U$ (as in the main model)

$$\begin{aligned} \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \widetilde{W}^{\sigma}(\varepsilon)dH(\varepsilon) &= U + \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \int_{\underline{\varepsilon}}^{\varepsilon} \sigma(\varepsilon)\frac{\psi'(e(x))/\gamma}{r+s}dxdH(\varepsilon) \\ &= U + \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \sigma(\varepsilon)\frac{\psi'(e(x))/\gamma}{r+s}\frac{1-H(\varepsilon)}{h(\varepsilon)}dH(\varepsilon) \end{aligned}$$

The rent constraint thus reads

$$(r+s)R = \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \sigma(\varepsilon)\psi'(e(x))/\gamma\frac{1-H(\varepsilon)}{h(\varepsilon)}dH(\varepsilon)$$

The only difference from the rent constraint (C6) is the multiplicative factor $\sigma(\varepsilon)$. In order to find the optimal contract we proceed in the same way as above. It follows that the associated Lagrangian reads (analogous to 25)

$$\begin{aligned} L &= \int_{\varepsilon_c}^{\overline{\varepsilon}} \sigma(\varepsilon)[\overline{y} + \varepsilon + \gamma e(\varepsilon) - \psi(e(\varepsilon)) - rU]dH(\varepsilon) \\ & -\alpha[\int_{\varepsilon_c}^{\overline{\varepsilon}} \sigma(\varepsilon)\frac{\psi'(e(\varepsilon))}{\gamma}\frac{1-H(\varepsilon)}{h(\varepsilon)}dH(\varepsilon) - (r+s)R]. \\ & -\alpha_1(\sigma(\varepsilon)-1) + \alpha_0\sigma(\varepsilon) \end{aligned}$$

where the two last terms capture the constraints that $0 \leq \sigma(\varepsilon) \leq 1$ for all $\varepsilon$, and where $\alpha_1$ and $\alpha_0$ are the associated non-negative Lagrangian parameters. The first order conditions for $e(\varepsilon)$ reads

$$\gamma - \psi'(e(\varepsilon)) = \alpha\frac{1-H(\varepsilon)}{h(\varepsilon)}\psi''(e(\varepsilon))/\gamma$$

which is identical to (26). For $\sigma$ we get that

$$\begin{aligned} \overline{y} + \varepsilon + \gamma e(\varepsilon) - \psi(e(\varepsilon)) - rU &> \frac{\psi'(e(\varepsilon))}{\gamma}\frac{1-H(\varepsilon)}{h(\varepsilon)} \Rightarrow \sigma(\varepsilon) = 1 \\ \overline{y} + \varepsilon + \gamma e(\varepsilon) - \psi(e(\varepsilon)) - rU &< \frac{\psi'(e(\varepsilon))}{\gamma}\frac{1-H(\varepsilon)}{h(\varepsilon)} \Rightarrow \sigma(\varepsilon) = 0 \\ \overline{y} + \varepsilon + \gamma e(\varepsilon) - \psi(e(\varepsilon)) - rU &= \frac{\psi'(e(\varepsilon))}{\gamma}\frac{1-H(\varepsilon)}{h(\varepsilon)} \Rightarrow \sigma(\varepsilon) \in [0,1] \end{aligned}$$

The third equation is identical to equation (27) determining $\varepsilon_c$ (unless the left-hand side is strictly greater than the right-hand side for all $\varepsilon$, in which case $\varepsilon_c = \underline{\varepsilon}$). We have already shown in the proof of proposition 1) that $\varepsilon_c$ is unique. It thus follows that

$$\begin{aligned} \sigma(\varepsilon) &= 1 \text{ if } \varepsilon > \varepsilon_c \\ \sigma(\varepsilon) &= 0 \text{ if } \varepsilon < \varepsilon_c \end{aligned}$$

The proof is thus complete.

**Proof of Lemma 3**

We want to show that the optimal time-independent contract is also optimal within the larger class of time-dependent contracts. A similar proof, based on Baron and Besanko (1984), can be found in Fudenberg and Tirole (1991, p. 299). To simplify the proof and avoid uninteresting technicalities, we assume time to be discrete. We first consider the case where the cut-off level is $\underline{\varepsilon}$. This will be modified at the end.

The revelation principle still holds. Hence, it is sufficient to study the set of contracts that maps the worker's (reported) type into a sequence of wages and effort levels $\{w_t(\varepsilon), e_t(\varepsilon)\}_{t=0}^{\infty}$, where $t$ denotes the tenure of the worker in question.

Let $\pi_t(\varepsilon, e_t) = \overline{y} + \varepsilon + \gamma e_t(\varepsilon) - w_t(\varepsilon)$. The expected discounted profit to the firm is given by

$$\Pi = E^{\varepsilon}\Sigma_{t=0}^{\infty}\pi_t(\varepsilon, e_t)\delta^t,$$

where $\delta = \frac{1-s}{1+r}$ is the discount factor, including the exit rate of the worker. The expected discounted utility of a worker of type $\varepsilon$ who announces type $\widetilde{\varepsilon}$ is given by

$$\overline{W}(\varepsilon, \widetilde{\varepsilon}) = \Sigma_{t=0}^{\infty}\left[w_t(\widetilde{\varepsilon}) - \psi(\varepsilon, e(\widetilde{\varepsilon})) + sU\right]\delta^t,$$

where

$$\psi(\varepsilon, e(\widetilde{\varepsilon})) \equiv \psi(e(\widetilde{\varepsilon}) - \frac{\varepsilon - \widetilde{\varepsilon}}{\gamma}).$$

Incentive compatibility requires that $\varepsilon = \arg\max_{\widetilde{\varepsilon}} \overline{W}(\varepsilon, \widetilde{\varepsilon})$. Let $W(\varepsilon) \equiv \overline{W}(\varepsilon, \varepsilon)$.

The optimal dynamic contract solves

$$\max_{\{w_t(\varepsilon), e_t(\varepsilon)\}_{t=0}^{\infty}} E^{\varepsilon}\Sigma_{t=0}^{\infty}\pi_t(\varepsilon, e_t)\delta^t$$

subject to

- Incentive compatibility: $\varepsilon = \arg\max_{\widetilde{\varepsilon}} \overline{W}(\varepsilon, \widetilde{\varepsilon})$.

- Individual rationality: $W(\varepsilon) \geq U$ for all $\varepsilon$. This constraint is only binding for $\underline{\varepsilon}$.

Note that the participation constraint regards the expected discounted utility of all future periods. It does not require that the utility flow of employed workers is higher than the utility flow of unemployed workers in all periods. Thus, deferred compensation with an increasing wage-tenure profile is allowed for.

Let $C^d = \{w_t^d(\varepsilon), e_t^d(\varepsilon)\}_{t=0}^{\infty}$ denote an optimal contract within the larger set of time-dependent contracts, and let $C^* = \{w^*(\varepsilon), e^*(\varepsilon)\}_{t=0}^{\infty}$ denote the time-independent contract. We want to show that $C^d$ is equivalent to $C^*$, in the sense that it implements the same effort level in each period, the same discounted expected profit to the firm and the same expected discounted rents to the workers.

Suppose that $C^d \neq C^*$. Then, $C^d$ cannot implement a time independent effort level, as this contract is, by definition, dominated by the optimal static contract $C^*$. Therefore, suppose that $C^d$ does not implement a time independent effort level. We will show that this leads to a contradiction.

To this end, consider the random time-independent stochastic mechanism $C^{dS}$, defined as follows: in each period, the contract $(w_t^d(\varepsilon), e_t^d(\varepsilon))$ is implemented with probability $\frac{\delta^t}{1-\delta}$. By definition, this contract is both incentive compatible and satisfies the individual rationality constraint. Furthermore, it yields a higher expected profit to the firm than the static contract $(w^*(\varepsilon), e^*(\varepsilon))$, since $C^d$ dominates $C^*$ and thus, contradicts the optimally of the latter mechanism in the class of time-independent contracts. Thus, it follows that $C^d = C^*$.

Finally, the same argument holds for any given cut-off value $\varepsilon_c$ and hence, the optimal cut-off level with time-dependent contracts must be equal to the optimal cut-off level with time-independent contracts.

**Optimal sharing rules with private information, equation (31)**

Taking the first-order condition for the problem of maximizing $V$ defined by equation (24) gives

$$q'(p)p'(R)(S^{\max}(R;U) - R) - q(1 - S_R^{\max}) = 0,$$

or, by simple manipulation,

$$el_p q(p) el_R p(R) = (1 - S_R^{\max}) \frac{R}{S^{\max} - R},$$

analogous to (33). From equation (9) it follows that $el_R p(R) = -1$ and from appendix 1 that $el_p q(p) = -\frac{\eta}{1-\eta}$, thus we have

$$\frac{\eta}{1-\eta} = (1 - S_R^{\max}) \frac{R}{S^{\max} - R}.$$

Let us then turn to the second order conditions. Using (24) we can write

$$\begin{aligned} \frac{dV(R)}{dR} &= q'(p)p'(R)(S^{\max} - R) - q(p(R))(1 - S_R^{\max}) \\ &= q(p(R)) \frac{q'(p)p}{q} \frac{p'(R)R}{p} \frac{S^{\max} - R}{R} - q(p(R))(1 - S_R^{\max}) \\ &= q(p(R)) (\frac{\eta}{1-\eta} \frac{S^{\max} - R}{R} - (1 - S_R^{\max})). \end{aligned}$$

The second order derivative thus writes

$$\begin{aligned} V''(R) &= q'(p)p'(R)(\frac{\eta}{1-\eta} \frac{S^{\max} - R}{R} - (1 - S_R^{\max})) \\ &\quad + \frac{d\frac{\eta}{1-\eta}}{dR} q(p(R)) \frac{S^{\max} - R}{R} + \frac{d\frac{S^{\max}-R}{R}}{dR} q(p(R)) \frac{\eta}{1-\eta} + S_{RR}^{\max} q(p(R)). \end{aligned}$$

The first term is zero at the stationary point, and locally the condition thus writes

$$\frac{d\frac{\eta}{1-\eta}}{dR} q(p(R)) \frac{S^{\max} - R}{R} + \frac{d\frac{S^{\max}-R}{R}}{dR} q(p(R)) \frac{\eta}{1-\eta} + S_{RR}^{\max} q(p(R)). \tag{39}$$

If we compare this expression to the analogous expression (34) for the full information case, it follows that the only difference is the last term in (39), and since $S_{RR}^{\max} < 0$ this term is surely negative. It follows that the sufficient conditions for the second order conditions to be satisfied are weaker with private information than with full information. In particular, the requirement that $\frac{d\frac{\eta}{1-\eta}}{dR} \leq 0$ is a sufficient condition also in the private-information case.

**Proof of proposition 3**

It is trivial to show that $V^{\max}(U)$ is continuous and strictly decreasing in $U$. Clearly $\lim_{U \to \infty} V^{\max}(U) < 0$. To show existence, it is sufficient to show that $V^{\max}(\frac{z}{r}) > 0$. This will always be the case if $\overline{y}$ is sufficiently high. We want to show that a sufficient productivity requirement for $V^{\max}(\frac{z}{r}) > 0$ is that $z < \overline{y} + \gamma e^F - \psi(e^F) + \overline{\varepsilon}$.

Suppose $U = z/r$. Let $\Delta = \overline{y} + \gamma e^F - \psi(e^F) + \overline{\varepsilon} - z$. Consider a firm that offers the following contract: The worker will receive a wage $w = z + \Delta + \psi(e^F) - 2k$ if and only if she delivers an output $y \geq z + \Delta + \psi(e^F) - k$, where $k$ is an arbitrary number in $(0, \Delta/2)$. Since the contract gives workers an expected rent that is strictly positive, all workers in the economy apply to this firm, and the labor market tightness facing the firm is 0. The vacancy is filled immediately and the firm earns an expected profit $k/(r+s) > 0$. Hence $V^{\max}(z/r) > 0$.

Since $V^{\max}(U)$ is strictly decreasing in $U$, the equilibrium value $U^*$ defined by $V^{\max}(U) = 0$ is unique. We have already shown that for a given $U^*$, the optimal contract is unique provided that $\eta(\theta)$ is non-decreasing. The proposition thus follows.

**Proof of proposition 4**

Suppose the contrary, i.e. that there exists a wage contract $\tilde{\phi}$ such that $U(\tilde{\phi}) = \tilde{U} > U^*$ and $V = 0$. By definition, a firm offering $\tilde{\phi}$ breaks even at $U = \tilde{U}$. Thus, the firm makes a strictly positive profit if it advertises this contract when $U = U^* < \tilde{U}$ (recall that $V$ only depends on $U$). But then $\phi^*$ cannot be a profit-maximizing contract, which is a contradiction.

**Proof of proposition 5**

The maximization problem (23), and thus $S^{\max}(R, U)$, is independent of $z$ and $A$. Changes in $z$ and $A$ only influences the optimal contract through $R^*$ and $U^*$.

From (29) it follows that the incentive power $b(\varepsilon)$ is strictly decreasing in $\alpha$. From proposition 2 part c) it follows that $S_{RU} = 0$ (in this proof we suppress the superscript $^{\max}$ for convenience). It follows from 2 part b) that $\alpha = S_R(R^*)$ is increasing in $z$ if and only if $R^*$ is decreasing in $z$. Suppose therefore to the contrary that $R^*$ is increasing in $z$. We want to derive a contradiction.

Since the equilibrium maximizes $U$, it follows from the envelope theorem that $U^*$ is increasing in $z$. Recall that $R^* = \beta^{eff} S^*$. Suppose first that the matching function is

Cobb-Douglas. Then

$$\begin{aligned}\frac{d}{dz}\frac{1-\beta^{eff}}{\beta^{eff}} &= \frac{d}{dz}\frac{S(R^*,U^*)-R^*}{R^*} \\ &= \frac{((S_R-1)\frac{dR^*}{dz}+S_U\frac{dU^*}{dz})R^*-(S(R^*,U^*)-R^*)\frac{dR^*}{dz}}{R^{*2}} \\ &< 0\end{aligned}$$

given that $R^*$ is increasing in $z$, where we have used that $S_U < 0$ (from 23). It follows that $\beta^{eff}$ is increasing in $z$. From (32) it then follows that $S_R(R^*)$ is increasing and hence that $R^*$ is decreasing, a contradiction.

Consider then the general case, in which case $\eta = \eta(\theta)$, $\eta'(\theta) \geq 0$. Suppose again that $R^*$ is increasing in $z$. Since $S^*$ is decreasing, it follows that $\theta^*$ is decreasing and hence that $\eta(\theta^*)$ is decreasing as well, contributing to a further reduction in $\beta^{eff}$. Exactly the same argument as above thus applies, hence $R^*$ is decreasing in $z$. This completes the proof.

**Proof of proposition 6**

It is sufficient to show that $p^F > p^*$. Suppose $p^F \leq p^*$. We want to show that this leads to a contradiction. Suppose first that $\eta(\theta)) = \beta$ (Cobb-Douglas matching function). First recall that

$$\begin{aligned}rU^F &= z + p^F\beta S^F \\ rU^* &= z + p^*\beta^{eff}S^*\end{aligned}$$

Since $U$ is maximized in equilibrium, it follows that $U^F > U^*$. If $p^F \leq p^*$ it follows that $S^F > S^*$ (since $\beta^{eff} > \beta$). But then

$$\frac{c}{q(p^F)} = (1-\beta)S^F > (1-\beta^{eff})S^* = \frac{c}{q(p^*)}$$

Hence $p^F > p^*$, a contradiction.

Consider then the general case, in which case $\eta = \eta(\theta)$, $\eta'(\theta) \geq 0$. With full information, the workers' share of the surplus is $\eta(\theta^F)$. It follows that if $\theta^* \geq \theta^F$, then certainly $\beta^{eff} > \eta(\theta^F)$. Exactly the same argument as above thus applies, and it follows that $p^* < p^F$ also in the general case.

**Proof of proposition 7**

From proposition 2 part c) it follows that $S_{RU} = 0$. It follows that $\alpha = S_R(R^*)$ is increasing in $\overline{y}$ if and only if $R^*$ is increasing in $\overline{y}$.

Consider a positive shift in $\overline{y}$. From Lemma 4, we know that in equilibrium, $U^*$ is maximized; hence, it is trivial to show that $U^*$ is increasing in $\overline{y}$.

Suppose that $S^*$ shifts downwards following an increase in $\overline{y}$. We want to derive a contradiction. First recall that

$$R^* = \beta^{eff} S^*$$

From (32) it follows that $\beta^{eff}$ is decreasing in $R^*$. Differentiating the above equation gives

$$dR^* = \frac{d\beta^{eff}}{dR^*} S^* dR^* + \beta^{eff} dS^*$$

or

$$dR^*(1 - \frac{d\beta^{eff}}{dR^*} S^*) = \beta^{eff} dS^*$$

Hence $dS^* < 0 \Rightarrow dR^* < 0$, and $d\beta^{eff} > 0$. Free entry together with equation (4) implies that

$$\frac{c}{q(p^*)} = (1 - \beta^{eff}) S^*$$

Hence $d\frac{c}{q(p^*)} < 0 \Rightarrow dp^* < 0$. Since $rU^* = z + p^* R^*$, it follows that $U^*$ decreases. We have thus derived a contradiction.

**Proof of existence of $y^a$**

Suppose $R^F = \overline{R}$ for $\overline{y} = y^1$. We want to show that there exists an $y^a < y^1$ such that for any $\overline{y} \in (y^a, y^1)$ the following holds: 1) $\overline{R} > R^F$ and 2) the cut-off level is equal to $\underline{\varepsilon}$.

First note that for $\overline{y} < y^1$, $S^F(\overline{y}) < S^F(y^1)$, and hence $R^F(\overline{y}) = \beta S^F(\overline{y}) < \beta S^F(y^1) = \overline{R}$. Hence the first condition is satisfied.

By assumption, the first best effort level is obtained at $\overline{y} = y^1$. As workers have full incentives, $w'(y) = 1$. Since firms have positive profit, it thus follows that $y(\underline{\varepsilon}) > w(\underline{\varepsilon})$; otherwise firms would obtain zero profits. Thus, increasing the cut-off level has a first-order effect on the expected surplus. Slightly reducing the incentive power of the contract only gives a second-order effect on the expected surplus. Thus, for values of $\overline{y} < y^1$ sufficiently close to $y^1$, firms reduce the incentive power of the contract below the first best and still hire all types. It follows that there exists an $y^a < y^1$ such that for any $\overline{y} \in (y^a, y^1)$ 1) and 2) above are both satisfied.

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