Does poaching distort training?*

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Abstract

We analyse the efficiency of the labour market outcome in a competitive search equilibrium model with endogenous turnover and endogenous general human capital formation. We show that search frictions do not distort training decisions if firms and their employees are able to coordinate efficiently, for instance, by using long-term contracts. In the absence of efficient coordination devices there is too much turnover and too little investment in general training. Nonetheless, the number of training firms and the amount of training provided are constrained optimal, and training subsidies therefore reduce welfare.

Keywords: Matching, Training, Poaching, Efficiency

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1 Introduction

The positive relationship between wages and experience is well documented in the empirical labour literature. This stylised fact indicates that on-the-job training is one essential determinant of worker productivity. Accordingly, the extent to which the market induces firms to invest in general and specific training is crucial for economic welfare. In addition, turnover is important for allocational efficiency, to ensure that workers are optimally allocated across firms at any given time. It is well known from Becker (1964) that perfect competition leads to an efficient market outcome with respect to investment in training and turnover, provided that there are no credit constraints or minimum wage regulations.

This paper analyses the conditions under which the labour market outcome is efficient in a model with endogenous human capital formation and endogenous turnover in the presence of *search frictions*. To this end, we develop a directed search model in which turnover is necessary to obtain an efficient allocation of workers. More precisely, there exists two types of firms; training firms which have a comparative advantage in providing general training, and poaching firms which have a comparative advantage in utilising general human capital. Workers with different productivities are assumed to search in different submarkets. Within this setting we analyse whether training firms have the right incentives to enter the market and to provide the optimal amount of general training. In contrast to the existing literature, we treat worker's on-the-job search intensity and the number of poaching firms as endogenous variables.

Our first main result is that internal efficiency is a sufficient condition for an efficient allocation of resources in this economy, both with respect to the allocation of workers to firms and with respect to investment in general training. Internal efficiency refers to the resolution of co-ordination problems within each firm such that the employer and his employees maximise their joint expected income. Internal efficiency can be obtained if workers and firms are able to write long-term binding contracts, or if they are able to bargain efficiently.

This efficiency result contrasts sharply with Acemoglu (1997). He finds that turnover in the presence of search frictions creates positive training externalities for future employers. As a result, there is underinvestment in general training even though firms and workers can write long-term contracts. He attributes the inefficient outcome to the workers' inability to contract with future employers. As we argue below, Acemoglu's result hinges (among other things) on his assumption that workers with different productivities search in the same search market. As a result, low-productivity workers create congestion effects for high-productivity workers, thereby reducing the return from training investments.

Our efficiency result also serves as a convenient benchmark when introducing imperfections other than search frictions and clarifies why such imperfections may give rise to inefficiencies. We focus on the case where internal efficiency does not hold because training firms set wages for trained workers so as to maximise their *ex post* profit. In this case, wages for trained workers in training firms are too low, the equilibrium turnover rate is too high, and investment in general training tends to be too low compared to the socially optimal level.

Our second main result is that this amount of human capital formation is still constrained efficient. Given the search behaviour of workers and the entry behaviour of poaching firms, the social and the private returns from general training coincide. Thus, subsidising general training reduces welfare. More complex policy measures may, however, increase welfare.

This second result also contrasts with the existing literature. Stevens (1994) argues that poaching creates a wedge between the social and the private returns from general training, as long as wages are set below worker productivity. For similar reasons, Booth and Snower (1995, page 345) propose that market failures caused by poaching should be mitigated by subsidising general training, for instance, by letting the government pay a fixed proportion of the firms' training expenditures. Acemoglu and Pischke (1999) are also sympathetic to training subsidies. Moreover, this view influences the policy debate. For instance, the OECD (1995, Chapter 7) argues that poaching externalities lead to underinvestment in general training, thereby providing a rationale for government subsidies, such as tax breaks for training expenses. Another example is the Swedish parliamentary investigation on individual human capital formation (Sveriges Riksdag, Direktiv 1999:106), which explicitly refers to the poaching externality as a rational for subsidising investments in general training. Our paper questions this widely held view.

The paper is organised as follows. Section 2 describes the model. Section 3 analyses the equilibrium outcome with internal efficiency. Section 4 examines the case when wages for trained workers are set so as to maximise *ex post* profits. Section 5 discusses robustness issues, and section 6 concludes. Mathematical proofs are provided in the appendix.

2 The model

In this section we describe the basic structure of our model and discuss wage formation in some detail. The model is set in continuous time. Workers enter the labour market as unemployed and leave at an exogenous death rate s. New workers enter the market at the same rate, keeping the total measure of workers constant.

There are two types of firms in the economy, training firms and poaching firms, and for most of the analysis there is free entry of firms. Each firm hires at most one worker. Since only training firms invest in general training, all workers start their career in a training firm. A worker that is hired by a training firm stays untrained for a period until he eventually becomes trained. Within a continuous-time framework the natural way to model a period of time is to let the period length be stochastic: an untrained worker (a novice) employed in a training firm becomes trained at a rate γ. The investment is made when the worker is a novice, and the return accrues once the worker is trained. The structure of the model is illustrated in figure 1.

The productivity of a novice is y^n. The productivity of a trained worker with human capital level h in a training firm is $y^t(h)$ and in a poaching firm $y^p(h)$. Poaching firms can utilise trained workers better than training firms. This assumption implies that turnover is necessary for an efficient allocation.[1] This also holds under the less restrictive assumption that only some rather than all trained workers are more efficient in poaching firms and that these workers engage in on-the-job search. Naturally, novices are less productive than trained workers, and we have the following ranking $y^n < y^t < y^p$ for all $h \geq 0$. The costs of creating a training vacancy and a poaching vacancy are K^t and K^p, respectively.

There are two distinct search markets in the model, one for employed workers and one for unemployed workers. In both markets, the number of matches between searching workers and firms is determined by a constant return to scale matching function $x(eu, v)$. This matching function maps a measure of workers u who search with an average intensity e for a measure of v vacancies into a flow x of new matches. Let p denote the probability rate that a worker finds a (new) job per unit of search intensity and q denote the probability rate that a firm with a vacancy finds a worker. The arrival rates p and q are interrelated, as both depend on the labour market tightness θ defined as v/eu. Note that p and q only depend on e through θ. Due to constant returns to scale, the matching function can therefore be summarised as $q = q(p)$.[2]

[1] If we instead assumed that $y^t = y^p$ our main results still hold. With internal efficiency (section 3), there would be no turnover in equilibrium. This would also be the efficient solution, as turnover has no social value. With *ex post* wage setting (section 4), the analysis presented here would be directly applicable.

[2] The probability rates p and q can be written as $p = x(eu, v)/eu = x(1, \theta) = \widetilde{p}(\theta)$ and $q = x(eu, v)/v = x(1/\theta, 1) = \tilde{q}(\theta)$. The matching technology can thus be summarised by a function $q = \tilde{q}(\theta) = \tilde{q}(\widetilde{p}^{-1}(p)) = q(p)$.

2.1 Asset values

Let W^u and W^n denote the expected discounted income, or "asset value" of an unemployed and of an untrained worker (novice), respectively. The asset value of an unemployed worker is given by

$$(r+s)W^u = e^u p^u (W^n - W^u) - c(e^u). \tag{1}$$

Here r denotes the discount rate, and $c(e^u)$ is the search effort cost of the worker. The latter is increasing, convex and $c(0) = c'(0) = 0$. We normalise the value of leisure to zero. The asset value of a novice is given by

$$(r+s)W^n = w^n - \mu ah + \gamma(W^t - W^n), \tag{2}$$

where w^n is the wage of a novice, μ the share of the training cost paid by the worker, ah the flow training cost, and W^t the asset value of an experienced worker in a training firm with human capital level (training level) h. For expositional ease the dependence on h is suppressed. Analogously, W^t is given by

$$(r+s)W^t = w^t + e^t p^t (W^p - W^t) - c(e^t), \tag{3}$$

where w^t is the wage of a trained worker with human capital h in a training firm, p^t the probability rate that a trained worker with human capital h finds a job in a poaching firm per unit of search intensity e^t, and W^p the expected income to the worker in a poaching firm. The expected income in a poaching firm is given by

$$(r+s)W^p = w^p,$$

where w^p is the wage in a poaching firm for a worker with human capital level h.

Turning to the asset value equations of firms, J^i , $i \in \{n, e, p\}$ denotes the expected discounted value of a firm with an employee. A firm that is abandoned by its employee has no value.

$$
\begin{aligned}
(r+s)J^n &= y^n - w^n - (1-\mu)ah + \gamma[J^t - J^n], \\
(r+s)J^t &= y^t - w^t - e^t p^t J^t, \\
(r+s)J^p &= y^p - w^p.
\end{aligned} \tag{4}
$$

Denote the joint expected income of a firm and its employee by $Y^i \equiv W^i + J^i$, $i \in \{n, e, p\}$. The joint asset values are

$$(r+s)Y^n = y^n - ah + \gamma(Y^t - Y^n), \tag{5}$$

$$(r+s)Y^t = y^t + e^t p^t (W^p - Y^t) - c(e^t), \tag{6}$$

$$(r+s)Y^p = y^p. \tag{7}$$

Finally, the asset value equations for training vacancies (V^n) and poaching vacancies (V^p), are given by

$$rV^n = q(p^u)(Y^n - W^n - V^n), \tag{8}$$

$$rV^p = q(p^t)(Y^p - W^p - V^p). \tag{9}$$

2.2 Competitive search equilibrium

Competitive search equilibrium combines competitive price determination and search frictions and is thus a useful benchmark when analysing the impact of search frictions. As workers are assumed to know the wages in all firms prior to searching, frictions are due to other aspects of the search process than collecting information on wages. Examples are the costs and time delays associated with writing and processing applications, with identifying firms with vacancies, or with testing applicants.

A core element of the competitive search equilibrium concept is the unique relationship between the advertised wage and the expected rate at which the vacancy is filled. The relationship can be derived in several settings.[3] Moen (1997) considers an economy in which a market maker creates submarkets, each characterised by a single wage. Workers and firms are free to choose which submarket to enter. As shown by Moen, wage advertisements by firms, or reputation about their wages, is sufficient to ensure that the same equilibrium wage prevails. In this paper we follow this wage advertisement approach. Mortensen and Pissarides (1999, section 4.1) interpret the market maker similarly, by assuming that a "middle man" (like a job centre) sets the wage. In Acemoglu and Shimer (1999 a) and b)) the labour market is divided into regional or industrial submarkets offering potentially different wages. Alternatively, the matching technology may be derived from the urn-ball process (Montgomery (1991), Peters (1991), and Burdett et al. (2001)).

We first define equilibrium in the unemployed-search market. Firms advertise wage contracts that may be rather complex, including wages for novices and experienced workers, and possibly also conditioned on human capital, h. The specifics of the advertised wage contract are discussed in some detail below. From the workers point of view, the attractiveness of the job can be summarised by the expected income W^n if employed by the firm. A training firm can always set W^n optimally simply by varying w^n, the wage for a novice. For now we therefore treat Y^n as given and W^n as a choice variable of the firm. From equations (1) and (8) it follows that we can write the asset value of an unemployed worker as $W^u = W^u(W^n, p^u, e^u)$, and the asset value of a training firm as $V^n = \widetilde{V}^n(W^n, q(p^u)) = V^n(W^n, p^u)$. The equilibrium in this search market is a vector (W^{n*}, p^{u*}, e^{u*}) that satisfies the three following conditions.

1. Optimal search effort

$$e^{u*} = \arg\max_{e^u} W^u(W^{n*}, p^{u*}, e^u).$$

[3] In this paragraph we borrow some arguments from Acemoglu and Shimer (1999b).

2. Profit maximisation

$$(W^{n*}, p^{u*}) = \arg \max_{W^n,\, p^u} V^n(W^n, p^u) \quad \text{subject to} \quad W^u(W^n, p^u, e^{u*}) \geq W^{u*}.$$

3. Zero profit condition

$$V^n(W^{n*}, p^{u*}) = K^t.$$

The profit maximisation condition can be given the following interpretation: All submarkets (or firms) that attract workers must offer these workers their equilibrium expected income W^{u*}. There is typically only one wage advertised in equilibrium (see below). Nonetheless, when setting the wage, firms expect that the arrival rate of workers to their firm $\hat{q}^u$ for out-of equilibrium wage offers will be given by $\hat{q}^u(W^n) = q(p^u(W^n))$, where $p^u(W^n)$ satisfies

$$W^u(W^n, p^u; e^{u*}) = W^{u*}.$$

Firms choose W^n so as to maximise profits given these expectations. This yields the profit maximisation condition. Note that the expectations are rational in the following sense. Suppose that a small set of firms deviates and advertises an out-of equilibrium wage W'. Applications would then flow to these firms up to the point at which the applicants obtain exactly their equilibrium expected income W^{u*}, in which case $q^u(W') = \hat{q}^u(W')$ holds (see Moen (1997) and Acemoglu and Shimer (1999a) for details).

The competitive search equilibrium allocation is such that V^n is maximised given W^{u*}, while free entry ensures that $V^n = K^t$. It is straightforward to show that in equilibrium W^u is maximised given that $V^n = K^t$. To be more precise, define the *feasible set* of pairs (W^n, p^u) as $\Phi^t = \{(W^u, p^u) | V^n(W^u, p^u) \geq K^t\}$.

Lemma 1 *In the competitive search equilibrium,* $W^u(W^n, p^u, e^u)$ *is maximised given that* $(W^n, p^u) \in \Phi^t$*.*[4]

[4] A similar result is derived in Acemoglu and Shimer (1999a).

Moen (1997) shows that there may be multiple equilibria that all yield the same value of W^u. For our purposes the value of W^u is the relevant characteristic, and we therefore abstract from uninteresting technicalities and assume that the equilibrium is unique. It follows that the competitive search equilibrium vector (W^{u*}, p^{u*}, e^{u*}) can be defined as the solution to the maximisation problem

$$\max_{W^n, p^u, e^u} W^u(W^n, p^u, e^u), \quad \text{given that } (W^n, p^u) \in \Phi^t \tag{10}$$

We now turn to the search market for employed workers. Employed workers may (potentially) be heterogenous, both with respect to human capital levels and current wages. Workers with different characteristics search in different markets. The separation of workers with different human capital levels into different search markets may be due to production technology, say because a worker's training level determines what kind of tasks he can do (and will do in his next job). If training increases productivity, without affecting the range of job tasks that the worker can perform, the production technology by itself does not create separation. Still, firms may separate workers into different submarkets by advertising the required human capital level for their position (in addition to wages), thus mimicing a market maker that separates workers with different productivity into different submarkets.[5] The issue of separate search markets is discussed further in section 5.

Consider the search market for employed workers with human capital level h and wage w^t in the training firm. From an identical argument as used in the proof of Lemma 1, it follows that the equilibrium allocation (W^{t*}, p^{t*}, e^{t*}) in this submarket maximises the utility of the searching worker W^t given the feasibility constraint $(W^p, p^t) \in \Phi^p$, where $\Phi^p = \{(W^p, p^t) | V^p(W^p, p^t) \geq K^p\}$. That is, the equilibrium allocation solves the problem

[5] Inderst (2000) analyses competitive search equilibrium with hetrogeneous agents by applying the market maker approach. He shows that it is indeed optimal for a market maker to separate agents with different characteristics into different submarkets.

$$\max_{W^p, p^t, e^t} W^t(W^p, p^t, e^t; w^t), \quad \text{given that } (W^p, p^t) \in \Phi^p(h) \tag{11}$$

where the notation $\Phi^p(h)$ captures the dependence of the feasibility constraint on h. Below we find that in equilibrium, all firms choose the same training level and wages for employed workers. Thus, there is only one on-the-job search market in equilibrium. However, the firms' out-of equilibrium beliefs are crucial, and we therefore have to specify the firms' beliefs regarding the turnover rate of trained workers as a function of w^t and h. We model these beliefs analogously as when considering out-of equilibrium wage offers in the unemployed search market. When considering out-of equilibrium trained-worker wages w' and human capital levels h', a firm expects that the arrival rate of job offers and search intensity are given as the solution to the maximisation problem

$$\max_{W^p, p^t, e^t} W^t(W^p, p^t, e^t; w'), \quad \text{given that } (W^p, p^t) \in \Phi^p(h'). \tag{12}$$

These expectations are rational in the sense that if a small set of firms deviates by choosing h' and w', the expectations are fulfilled (this follows from equation 11)). In an earlier version of this paper (Moen and Rosén (2001)), we derive the equilibrium when the investment cost a has a discrete distribution and training levels are discrete. In this case, all possible training levels are actually chosen in equilibrium. The equilibrium in this paper can be derived as the limit when the distribution of training costs converges to a mass point (without reducing the support) and when the difference between two adjacent investment levels becomes arbitrarily small.[6]

It is well known that the competitive search equilibrium, under a given set of assumptions, is efficient, in the sense that an optimal amount of resources are

[6] A similar argument can be made for wages to searching workers. If firms "tremble" and choose wages other than the equilibrium training wage, non-empty submarkets for other wages than the equilibrium wage exist. The competitive search equilibrium can be defined as the limit obtained when the measure of deviating firms converges to zero. A formal treatment of this argument can be found in Moen (1994).

used in order to get workers into jobs. We elaborate on this result and study the efficiency of the search markets for a given inflow of workers with a given productivity (level of human capital investment). The welfare criterion is to maximise the present value of aggregate production net of search costs and net of the vacancy cost. In the appendix, we prove the following lemma:[7]

Lemma 2 *Suppose that the income flow to searching workers is equal to their productivity, and that the social and the private values of a match coincide. Then the following holds:*
a) The socially optimal allocation maximises the expected discounted income of the searching workers given the feasibility constraint. That is, the socially optimal allocation solves equation (10) for the unemployment-search market and equation (11) for the employed-search market.
b) The social and the private values of a worker entering the search market are equal.
c) Property b) still holds when the number of firms in the market is exogenous.

The prerequisite that the income flow to the searching workers is equal to their productivity requires that $w^t = y^t$ in the employed-search market. In section 4 we study the equilibrium when this condition is not satisfied. Result b) states that the asset values W^u and W^t in the unemployed-search and employed search markets also reflect the social value of one additional worker in these markets.

Lemma 2 states conditions under which a search market functions efficiently for given investments in training and a given inflow of searching workers. In the next two sections we address the issue of efficiency of the human capital investments level and of the number of trained workers entering the employed search market.

[7]Result a) is stated and proved in Acemoglu and Shimer (1999b)

3 Internal efficiency

In this section, we define internal efficiency and then derive and evaluate the equilibrium of the model with internal efficiency. We also show how internal efficiency can be implemented through various contractual arrangements. Finally, we discuss whether the firm has an incentive to pay for the worker's general training.

In a training firm, the choice of training level h and the worker's on-the-job search behaviour influence their joint income Y^n. We refer to a training firm as internally efficient if its co-ordination problems are resolved, such that the joint expected income Y^n is maximized. Internal efficiency requires that the following two conditions are satisfied:

1. The on-the-job search behaviour that maximises W^t also maximises Y^t. (Internal efficiency *ex post*)

2. The training level h is set so as to maximise Y^n. (Internal efficiency *ex ante*)

One way to implement internal efficiency *ex post* is to set the wage of a trained worker equal to his productivity, in which case $Y^t \equiv W^t$. In this case it follows trivially that the first prerequisite of Lemma 2 (that searching workers income flow are equal to their productivity) holds. Moreover, Lemma 2 also applies if internal efficiency *ex post* is implemented in alternative ways, since the search behaviour still is the same as above.

3.1 Equilibrium with internal efficiency

With internal efficiency *ex post*, it follows from equation (11) that for any equilibrium training level h, the associated employed-search market equilibrium solves the problem

$$\max_{W^p, p^t, e^t} Y^t(W^p, p^t, e^t; h), \text{ given that } (W^p, p^t) \in \Phi^p(h).$$

Define the function $Y^t = Y^{t*}(h)$ as the associated maximum. Trivially, $Y^{t*}(h) = Y^{t*}$ for any equilibrium value of h. Furthermore, for out-of-equilibrium values of h, $Y^{t*}(h)$ is the agents' perceived value of Y^t if they choose this training level, as this is consistent with (12). *Ex ante* internal efficiency requires that h maximises Y^n. The equilibrium value of h, h^*, thus solves the problem

$$\max_h Y^n(h) \quad \text{given that } \; Y^t = Y^{t*}(h).$$

Denote the maximum by Y^{n*}. The equilibrium in the unemployed-search market then solves the problem (from equation (10))

$$\max_{W^n, p^u, e^u} W^u(W^n, p^u, e^u), \quad \text{given that } (W^n, p^u) \in \Phi^{t*},$$

where the feasibility constraint Φ^{t*} incorporates that $Y^n = Y^{n*}$. As is standard, an equilibrium exists if the economy is productive. This holds in our case, if the value of hiring a novice is sufficiently larger than the entry cost of training firms. In addition, an equilibrium with poaching firms requires that trained workers are sufficiently more productive in poaching firms.[8]

We now turn to the welfare properties of this equilibrium. Solving the social planner's maximisation problem in full is rather complex, and therefore deferred to the appendix. For any given h, we know from Lemma 2 that for a given inflow of workers to the employed search market, the competitive search equilibrium in this market is efficient. Also the social and private value of an additional worker in the employed search market coincide. When the worker-firm pairs decide on h, they do so on the basis of their expectations $Y^{t*}(h)$, which equal the social value of investing this amount. Since the relationship between Y^t and Y^n is

[8]Formally, such an equilibrium exists if $y^p - y^t > (r+s)K^p$ and $\frac{(r+s)(y^n - ah^*) + \gamma y^t(h^*)}{r+s+\gamma} > (r+s)K^t$. (Proof available on request.)

mechanical (for a given h), the social and (perceived) private value of $Y^n(h)$ coincides for all h. Accordingly, the planner when choosing training level solves the same maximisation problem as the agents in the market. The training level is therefore efficient. Lemma 2 part a then ensures efficiency of the unemployed-search market as well. Finally, part c) of lemma 2 indicates that efficiency holds even with a given number of poaching firms in the market.

Proposition 1 *With internal efficiency, the labour market equilibrium outcome is efficient. In particular, the following holds:*

a) The level of general human capital investments is socially optimal.

b) The numbers of training firms and of poaching firms entering the market are socially optimal.

c) When the number of poaching firms is exogenous the equilibrium allocation is still efficient in the sense that aggregate net output is maximised given the number of poaching firms.

To gain further intuition, suppose *ex post* internal efficiency is obtained by paying trained workers in training firms a wage equal to their productivity. In this case, the workers' search behaviour has no externality on their employers. Since trained workers do not generate any profits for training firms, these firms do not care whether their trained workers stay or leave.

The entry decision of a firm with a vacancy gives rise to search externalities, a positive externality for workers and a negative externality for other firms with vacancies. In the competitive search equilibrium, these externalities offset each other (as the Hosios condition is met), and therefore an optimal number of vacancies exists in the market.

A worker that enters the on-the-job search market creates a negative externality for other workers and a positive externality for poaching firms (for a given number of vacancies). In the competitive search equilibrium, these two externalities exactly cancel out, and the social and private value of entering the market coincide. This is true in the search markets associated with all possible values of h (although only one of them is active in equilibrium). Thus the training choice of worker-firm pairs has no net externalities on other agents. As training is determined so as to maximise their joint surplus, the training decision is socially optimal, and thus the social and private values of a worker-training firm match coincide. This in combination with Lemma 2 implies that the unemployed search market is efficient as well.

3.2 Implementing internal efficiency

In this subsection, we discuss how internal efficiency can be obtained if the firms advertise long-term wage contracts. We then briefly discuss other ways of implementing internal efficiency.

One set of contracts that ensure internal efficiency is a long-term contingent contract $(w^n, w^t(h))$ in which the wage of an experienced worker equals his productivity, $w^t(h) = y^t(h)$. As such a wage schedule makes the worker a residual claimant on the return from human capital, the efficient investment is undertaken if the worker bears the entire investment cost. The wage w^n is set so as to implement the desired level of W^u. Trivially, the same outcome can be implemented allowing only two levels for w^t, a low level if investment is below h^*, and a wage equal to $y^t(h^*)$ if investment is at or above h^*, where h^* is the optimal training level.

If wage contracts in which w^t is contingent on h are difficult to enforce, internal efficiency can be obtained by a non-contingent wage contract $w^t = y^t(h^*)$. In order to achieve internal efficiency, h^* could then be advertised. Alternatively, the firm can advertise a share μ of the investment costs that the worker has to bear. For a given w^t, the firm receives the increase in y^t associated with a higher h, while the worker gains by increasing his prospects in the on-the-job search market. By the envelope theorem (on $W^t(h^*)$), the worker's share of the total gain is

$$\mu = \frac{\frac{e^{t*}p^{t*}}{r+s+e^{t*}p^{t*}}\frac{dY^p(h^*)}{dh}}{\frac{dY^t(h^*)}{dh}}. \tag{13}$$

Thus, if the worker (the firm) finances shares μ and $(1-\mu)$ of the costs, the first best investment level is reached. Again, w^n should be adjusted so as to implement the desired level of W^u.

Alternatively, the firm can use *quitting fees* to ensure optimal on-the-job search behaviour. With quitting fees and wages below marginal product, an efficient training level can be implemented if the firm partly finances training or if the training level h^* is advertised.

Finally, as there is symmetric information between the worker and the employee, standard Nash bargaining leads under quite general assumptions to an internally efficient outcome. As long as the efficient outcome is in the opportunity set and utility (income) is transferable, internal efficiency prevails. Suppose, for instance, that the firm advertises an unconditional wage $\overline{w}$ only. Internal efficiency can still be obtained if the worker and the firm bargain over the wage contract and the training level once the worker is employed.

3.3 Who pays for training?

Several recent papers address the issue of why and when firms have incentives to invest in training. One finding in this literature, surveyed by Acemoglu and Pischke (1999), is that firms have incentives to invest in general training when wages increase less than productivity. In our model with internal efficiency, the extent to which firms pay for training depends on the contractual arrangement. For instance, when firms advertise long-term contracts that condition a trained worker's wage on his level of training, workers pay the full cost of training. If the long-term contracts do not condition a trained worker's wage on his training level, firms pays part of the training cost. At the margin, the firm finances a share

$1-\mu$ of the training, where μ is given by equation (13). Rewriting equation (13) gives[9]

$$\mu = \frac{e^{t*}p^{t*}\frac{dY^p(h^*)}{dh}}{e^{t*}p^{t*}\frac{dY^p(h^*)}{dh} + \frac{dy^t(h^*)}{dh}}.$$

Hence, the larger the search frictions (measured as a low optimal turnover rate $e^t p^t$ for a given Y^p) the smaller the share μ that is paid by the worker. The reason is that the longer the worker stays in the firm, the larger is the share of the return on training that accrues to the firm at the margin.

4 *Ex post* determination of wages

In this section we address the common concern found in the literature that there may be excessive turnover and too little investment in general training because wages for trained workers are below the workers' productivity (e.g., Stevens (1994), OECD (1995, Chapter 7), and Booth and Chatterji (1998)).

We therefore modify our framework and assume that firms cannot commit *ex ante* to the wage that they will pay a worker once he is trained. In addition we rule out quitting fees. In this case, training firms set wages for trained workers so as to maximise *ex post* profit. That is, a firm trades off a low wage bill against a high turnover rate.[10] We keep our assumption of *ex ante* internal efficiency, as our focus is on how excessive turnover (and not contractual difficulties regarding h) may distort the training decision.[11]

[9] The expression for μ follows from $Y^t(h) = W^t(h) + \frac{y^t(h)-w^t}{r+s+e^t p^t}$, $\frac{\partial W^t}{\partial e^t} = \frac{\partial W^t}{\partial p^t} = 0$, and $y^t(h^*) - w^t = 0$.

[10] The trade-off between turnover and wage costs has been studied by several authors, (e.g. Salop (1979), Stiglitz (1985), Burdett and Mortensen (1998)). Our paper differs from these papers in several respect, most notably, in our choice of a directed search model and in our focus on the efficiency of the level of general training provided by the market.

[11] One may also argue that firms more easily can commit to a training level than to *ex post* wages because training is undertaken earlier in the relationship. Furthermore, if the training level is determined through bargaining, relatively simple arrangements may be sufficient to obtain *ex ante* efficiency.

4.1 Equilibrium with *ex post* wage setting

A high wage, w^t, reduces the turnover rate for two reasons. First, it implies that the worker applies for jobs offering high wages with long job queues, thereby reducing p^t. Second, the worker reduces his on-the-job search effort e^t.

Formally, the firm chooses w^t so as to solve the problem (from equation (4))

$$\max_{w^t} y^t - w^t - e^t p^t J^t,$$

given that p^t, e^t solves

$$\max_{W^p, p^t, e^t} W^t(W^p, p^t, e^t; w^t), \qquad \text{given that } (W^p, p^t) \in \Phi^p, \tag{14}$$

where Φ^p is defined as in section 2.2 and $W^t(W^p, p^t, e^t; w^t)$ denotes the asset value of a searching worker with income flow w^t while searching, (see equation (11) and the following discussion). We are not able to find a closed form solution to this problem, even when we parameterise the matching function. It is, however, clear that the firm always sets $w^t < y^t$. At $w^t = y^t$ the firm earns zero profit while it obtains a strictly positive profit for all $w^t < y^t$. The maximisation problem given by (14) defines the equilibrium in the on-the-job search market $(W^{p*}, p^{t*}, e^{t*}; w^t)$.

Lemma 3 *Compared to the equilibrium with internal efficiency, the following holds in the equilibrium with ex post wage determination*

1. *For a given level of training h:*

   (a) *Too many poaching firms enter the market relative to the number of training firms (p^t is higher).*

   (b) *The on-the-job search intensity is higher (e^t is higher).*

2. *Fewer training firms enter the market.*

With *ex post* wage setting, the equilibrium value of Y^n is lower than with internal efficiency and thus fewer training firms enter the market. With respect to the amount of training in each firm, the impact of *ex post* wage determination is by no means clear cut. The reason is that we have no control over the relationship between w^t and h; it may even be discontinuous. If a small increase in h leads to a large increase in w^t, investments in h may be considered as a *commitment device*. By increasing h by a small amount the firm may find it in its own interest *ex post* to set substantially higher wages, thereby reducing the inefficiencies created by excessive turnover. Therefore, we can not rule out that *ex post* wage determination actually increases the amount of training undertaken compared to the first best.

In order to derive more clear-cut results, further restrictions must be imposed on the model. As an example, assume for a moment that there are only two levels of human capital, zero and one, and that only workers with human capital $h = 1$ engage in on-the-job search. In this case, excessive turnover due to *ex post* wage setting reduces $Y^t_{h=1}$, but has no effect on $Y^t_{h=0}$, and the joint private return from investing in human capital unambiguously falls. Furthermore, assume that the workers differ with respect to the cost of acquiring human capital, a, where each worker's a is independently drawn from a known distribution and the draw takes place after the worker is hired but prior to the investment decision. Then there exists a cut-off value a^* such that all workers with $a < a^*$ invest in training. This cut-off level may then be compared with the corresponding socially optimal cut-off level:

Result *Given* $h \in \{0, 1\}$, $e^t_{h=0} = 0$, *and* $e^t_{h=1} > 0$ *with internal efficiency, the amount of training (the cut-off level of* a*) with ex post wage setting is lower than the first best level obtained with internal efficiency.*

(Proof omitted.) Similar results have often been used in the literature to rationalise training subsidies. We will now show that training subsidies do not increase welfare in our model. To this end, we introduce the concept of constrained efficiency. Consider the case where the social planner has discresion over the number of training firms and the level of training h, while all other decisions are determined in the market.[12]

Definition of Constrained Efficiency: *The level of training and the number of training firms entering the market are constrained efficient if the social planer chooses the same outcome as the one that prevails in the market.*

Proposition 2 *The training level and the number of training firms are constrained efficient.*

The point is that the social and private values of an additional worker entering the on-the-job search market coincide. Although excessive turnover reduces the private returns from training, *it also reduces the social returns by the same amount*. A reduction in the training level and the number of training firms are thus rational responses to the excessive turnover created by low wages for trained workers in training firms.

The social and private values of training coincide because training has no net externalities for other agents in the market. Too see this, suppose a small group of firms deviates from the equilibrium value h' and instead chooses a training level h''. In response to this deviation, fewer poaching firms will enter the submarket for h'-workers, and the equilibrium values of p^e, W^e, and e^e stay constant. Thus, workers in the h' -submarket are not affected. Since p^e and e^e are also unaffected, so are the training firms. It follows that the training level and the number of training firms entering the market are both constrained efficient (a more formal argument is provided in the appendix).

[12] We thus do a similar exercise as in Stevens (2001), where it is assumed that the planner can only overrule the investment decisions of firms.

Corollary: *A training subsidy reduces welfare.*

A training subsidy will increase the training level in each firm. However, a training subsidy does not influence the equilibrium equations (10) or (14) for given Y^n and h, respectively. Proposition 2 thus applies and the corollary follows. Similar results can be derived for regulations of training or subsidised entry of training firms, which both will reduce welfare.

By contrast, the literature discussed in the introduction tends to conclude, without further discussion, that underinvestment due to excessive turnover calls for training subsidies. In fact, much of this literature focuses on circumstances in which there is underinvestment in training and then conclude that subsidies/regulation (in the absence of governmental failures) are welfare improving.

An exception is Stevens (2001), who explicitly analyses the impact of subsidies and regulation. In her model, both the number of firms and the number of workers trained in each firm are endogenously determined. Due to high turnover, firms train too few workers and the equilibrium is not constrained efficient. Consequently, the government can improve welfare by forcing firms to train more workers. Stevens' model differs from ours in several respects. For instance, search frictions are not explicitly modelled and there is no free entry of firms in the on-the-job search market. This latter feature makes her model similar to our model with free entry of training firms but with a fixed number of poaching firms. With an exogenous number of poaching firms, the equilibrium of our model may not be constrained efficient either. To see this, consider the example above with only two training levels $h = 0$ and $h = 1$ and with no on-the-job search when $h = 0$. If firms train more workers (higher cut-off value a^*), the arrival rate p^e for trained workers falls, which may affect the incentives to invest in training in the first place.

In Stevens' model, first best (although achievable with direct regulation of training) cannot be achieved by subsidies alone, because a training subsidy distorts the *entry decision* of training firms. In order to reach efficiency, a tax on firms has to be imposed. As will become clear below, first best can be implemented by a mix of taxes and training subsidies also in our model. Taxes play a different role in our model. They are used to avoid excessive turnover, while in Stevens' model they reduce the profitability of entering the market for training firms.

4.2 Combined policy measures

While training subsidies or regulation of training alone cannot improve welfare, they may do so if combined with policy measures aimed at reducing the turnover rate, such as taxes on poaching firms. Moreover, these policy measures may by themselves improve welfare on their own. We discuss the effects of profit and pay-roll taxes, alone and in combination with training subsidies.[13]

As the trade-off that training firms face when setting the wage w^t is rather complex, it is extremely difficult to characterise the impact that taxes on poaching firms have on w^t. For instance, in response to less entry by poaching firms training firms may reduce or increase w^t, depending on the functional form of the matching function. As argued in Moen and Rosén (2002), there exist sets of combined policy measures that implement first best. These can e.g. be a combination of pay-roll taxes and entry taxes on poaching firms, training subsidies/taxes, and entry taxes/subsidies for training firms. It is, however, not possible to determine whether these combined policy measures entail a tax or subsidy on training. In addition, the implementation of first best typically requires that taxes and subsidies discriminate between poaching and training firms.

To provide an understanding of these claims, we fix the wage for experienced workers in training firms and consider first the effect of taxes on poaching firms. The turnover rate is a function of both the workers' on-the-job search and entry

[13] The subsequent discussion is based on the more detailed analysis in Moen and Rosén (2002).

by poaching firms. Hence, implementing first best requires more than one policy instrument. In Moen and Rosén (2002) we argue that there exists a combination of pay-roll and entry (or profit) taxes that implements first best levels of e^t and p^t (for any given h).

Given that pay-roll and entry taxes on poaching firms induce first best levels of p^t and e^t, how can the optimal level of training be achieved? The constrained efficiency result (Proposition 2) no longer holds once poaching firms are taxed. An increase in h increases the number of poaching firms in the market and, under reasonable assumption on the matching function, also the wages they offer. Thus, there is a positive tax externality from training, and efficiency can be improved and a first best training level obtained by a training subsidy.

Finally, consider entry by training firms. Taxes on poaching firms tend to decrease the joint private expected income Y^n (as taxes decrease the value of turnover to the worker), while a training subsidy tends to increase Y^n. Hence, we cannot determine whether it is optimal to subsidise or to tax entry of training firms.

With an endogenous wage w^t, it may not be optimal to subsidise the level of training. As discussed earlier we cannot determine how w^t responds to changes in h, leaving the possibility open that investment in training reduces turnover at the margin. In this case, training gives rise to a negative tax externality, i.e., lower tax revenues. As a result, training should be taxed and not subsidized.

Policies to promote training have been introduced in several countries. For instance, in Australia the government imposed a pay-roll tax of 1% on firms that provided insufficient training (OECD (1995), Chapter 7). From the analysis above it follows that such a tax alone cannot implement first best. For a given w^t, the pay-roll tax on poaching firms, which *ceteris paribus* reduces turnover, tends to increase welfare. To achieve the optimal level of training a subsidy is needed in addition. With endogenous wages, we cannot make predictions regarding the welfare effects of this policy measure. In France and earlier also in U.K, a pay-roll tax for all firms is coupled with a subsidy for training (Steven (2001)). In Moen and Rosén (2002), we argue that a combination of pay-roll taxes and training subsidies does not implement first best, but may improve welfare by reducing turnover.

5 Discussion

In this section, we discuss some important features and assumptions of our model, in the matching technology and the wage determination process. Before doing so, we like to point out that our paper is also a contribution to the literature on the broader issue of search and efficiency. In this literature, Acemoglu and Shimer (1999b) is closest related to our paper. They study the firms' incentives to invest in physical capital when search frictions are present. Our model differs from theirs, most importantly in this context by altering the side of the market that undertakes the investment. In Acemoglu and Shimer (1999b), the agents who invest also advertise the wages, while in our model the agents on the other side of the market invest. Furthermore, in our model a third party (the training firm) may influence the search process through the wage it sets for searching workers.

Also related is the literature on efficient investments in a matching context without search frictions (e.g., Cole et al. (2001)). Cole et al. (2001) find that even when the parties cannot contract with each other before the investment is undertaken, an equilibrium with efficient investments can be sustained. They do, however, abstract from the workers' search behaviour, from firm entry, and from turnover, which are all key components in our analysis.

5.1 Wage bargaining under the Hosios condition

In this subsection we study to what extent our results remain valid when wages are determined by bargaining. We keep the assumption that workers with different characteristics search in separate markets. We also assume internal efficiency.

With wage bargaining, the search market is generally inefficient, even with homogeneous workers, due to search externalities. The equilibrium outcome is efficient only if the sharing rule is such that the Hosios condition is met (Hosios 1990). The Hosios condition is satisfied whenever the absolute value of the elasticity of q (the arrival rate of workers to firms with a vacancy) with respect to the labour market tightness θ is equal to the worker's bargaining power, and when the parties' outside option in the bargaining is their "asset value" prior to the match.

Thus, if the relevant disagreement point for a worker bargaining with a poaching firm is to remain in the training firm, our conjecture is that the Hosios condition ensures an efficient allocation in the on-the-job search market. The Hosios condition implies that the negative search externality for agents on the same side of the market and the positive search externalities for agents on the other side exactly balance in all submarkets. Wages and labour market tightness are the same as in a competitive search market in all on-the-job search markets (for all training levels). Hence, the expected income for a trained worker and thus the incentives to invest are the same as in a competitive search equilibrium model. The efficient outcome of the on-the-job search markets implies that the unemployed search market is also efficient (given that the Hosios condition holds).

If wages are frequently renegotiated the relevant outside option for the worker is unemployment (see Pissarides (1994)). In this case, the wage level in poaching firms under the Hosios condition is too low. As in our model with *ex post* wage setting both the number of training firms and welfare certainly fall short of the efficient level.

Acemoglu (1997) also considers investments in on-the-job training in a setting with enforceable long-term contracts and bargaining. In his model, turnover is a result of an exogenous job destruction process after which the worker becomes unemployed and starts searching for a new job. Acemoglu identifies a positive externality from training on future employers, and as a result there is underinvestment in training. Within his model, we conjecture that efficiency can be obtained if one allows for separated search markets for employed workers with different training levels combined with wage advertisements or bargaining under the Hosios condition.

5.2 Matching technology

Crucial for our efficiency result (Proposition 1) is the assumption that workers with different training levels search in different submarkets. If workers with different characteristics were searching in the same submarket, efficiency would no longer prevail. Suppose a subset of workers improve their training. As long as wages increase less than their productivity, more vacancies enter this market. If the search markets are not separated, this benefits all workers in the market. Thus, there exists a positive externality from training (the firms, by definition, earn zero profit in any case), and underinvestment in training results.[14]

The critical issue is therefore to what extent our assumption that different worker types search in separate search markets is plausible. To be clear, we do not necessarily argue that *complete* market separation is the most accurate description of the real world. Still we believe that this is an interesting benchmark, as is the complete-market competitive model without search frictions. Furthermore, there are compelling reasons that market separation takes place at least to some extent. As discussed in section 2, workers are separated into submarkets if, in addition to wages, firms advertise the human capital level required for the job. We have also noticed that a market maker finds it optimal to separate the market into

[14]See Acemoglu and Shimer (1999b) for a similar result with physical investments by firms.

submarkets. Furthermore, a somewhat counter-intuitive implication of a non-separated search market is that workers with different productivities have the same probability of finding a job in a poaching firm.

In a setting where firms invest in physical capital, Acemoglu and Shimer (1999b) argue that even if firms cannot advertise wages, workers have an incentive to direct their search towards firms with high investments, as they anticipate that the bargaining outcome in such a firm will be attractive. Thus, even if wages are determined by wage bargaining, the market may endogenously separate into submarkets. This mechanism seems less realistic in our setting with investments in human capital. Firms usually hire a large number of workers, and it is therefore more plausible to assume that workers know the capital level in firms rather than the other way around.

The discussion concerning separated search markets points at a weakness of the Diamond-Mortensen-Pissarides search framework, namely the exogenity of the matching process. It would therefore be of interest to analyse the training decision in a framework in which the matching process is explicitly modelled. A natural starting point is the urn-ball process (Montgomery (1991), Peters (1991), Moen (1999) and Burdett, et al. (2001)). We conjecture that within the urn-ball matching framework, a sufficient condition for an optimal training decision is that firms are able to advertise wages contingent on worker productivity.

6 Conclusion

This paper analyses the incentives to invest in general training in a matching model with endogenous worker turnover and with wages set in a competitive fashion. As long as employers and employees are able to resolve within-firm co-ordination problems (internal efficiency), search frictions do not induce inefficiencies and the resulting resource allocation is optimal. In the absence of internal efficiency, there may be underinvestment in training as a result of excessive turnover. As excessive turnover reduces both the private and social returns from training, the level of investment in training is, however, still constrained efficient. Training subsidies alone, therefore, reduces welfare. In combination with additional policy measures aimed at reducing turnover, subsidies may increase welfare.

Appendix

A. Proof of Lemma 1

Suppose Lemma 1 does not hold. Then there exists a triple $(W^{n\prime}, p^{u'}, e^{t\prime})$ such that $W^u(W^{n\prime}, p^{u\prime}, e^{u\prime}) > W^{u*}$ and $V^n(W^{n\prime}, p^{u\prime}) \geq K^n$. By continuity of the problem, there exists another triple $(W^{n\prime\prime}, p^{u\prime\prime}, e^{u\prime\prime})$ such that $W^u(W^{n\prime\prime}, p^{u\prime\prime}, e^{u\prime\prime}) > W^{t*}$ and $V^n(W^{n\prime\prime}, p^{n\prime\prime}) > K^n$. This implies that the profit maximisation condition is not satisfied, violating an equilibrium condition.

B. Proof of Lemma 2

The proof is made for an arbitrary search market satisfying 1) the income flow to searching workers is equal to their productivity, and 2) the social and private values of a match coincide. We drop the "superindex" whenever this does not lead to ambiguities, otherwise we use superscript 0 for values corresponding to searching workers and superscript 1 for values connected to successful search. The asset value of a searching worker is

$$(r + s)W^0 = w^0 + ep(W^1 - W^0) - c(e) \tag{15}$$

where w^0 is the income flow of a searching worker. Free entry implies that $W^1 = Y^1 - \frac{r+q}{q}K$. Inserted into equation (15) this gives

$$(r + s)W^0 = w^0 + ep(Y^1 - \frac{r + q(p)}{q(p)}K - W^0) - c(e), \tag{16}$$

where w^0 is the income flow of a searching worker. For a given inflow of workers, we say that a search market is efficient if the net value created in the search market is maximised. This value is equal to the product of the number of matches times the value of each match less the cost of vacancy creation. Denote the number of searching workers N. The number of matches in the market is given by epN, and the value of each match is Y^1. In steady state, the flow value creation in the market is thus $epNY^1 - epNK$. Furthermore, as the expected time before a vacancy is filled is $1/q$ and the flow cost of having an open vacancy stock is rK, the total hiring costs can be written as $epN\frac{r+q}{q}K$. The planner's objective function is

$$R(N) = \int_0^{\infty} [y^0 N + epNY^1 - epN\frac{r+q(p)}{q(p)}K - c(e)N]e^{-rt}dt. \tag{17}$$

The social planner maximises this function with respect to p and e, subject to the constraint

$$\dot{N} = b - (s + ep)N,$$

where b is the exogenous inflow of workers to the search market.

We use optimal control theory to solve the maximisation problem. The associated current-value Hamiltonian is given by

$$H = y^0 N + epNY^1 - epN\frac{r+q(p)}{q(p)}K - c(e)N + \lambda(b - (s + ep)N),$$

where λ is the associated adjunct function. First order conditions for the maximum are as follows

1. p and e maximise H

2. $r\lambda - \dot{\lambda} = \frac{\partial H}{\partial N}$

Condition 1 implies that p and e solve

$$\max_{p,\, e} ep(Y^1 - \frac{r+q(p)}{q(p)}K - \lambda) - c(e). \tag{18}$$

In steady state, condition 2 implies that

$$(r+s)\lambda = y^0 + ep(Y^1 - \frac{r+q(p)}{q(p)}K - \lambda) - c(e). \tag{19}$$

The comparison of (16) and (19) shows, given $w^0 = y^0$, that the expressions for λ and W^0 are equivalent. Furthermore, as the maximisation problem (18) is equivalent to maximising λ in (19), the planner maximises W^0, as in the competitive search equilibrium. This proves part a). Moreover, as $\frac{dR}{dN} = \lambda$, the social value of a worker entering the market is equal to W^0, proving part b).

To prove part c), suppose that the number of firms is exogenously given. (For the unemployed search market the number of training firms and for the employed search market the number of poaching firms.) Consider the associated (steady state) competitive search equilibrium, and denote by V' the equilibrium value of a vacancy. Compare this equilibrium with the equilibrium of a model in which firms may enter at an entry cost V'. By construction, the equilibrium without entry is also an equilibrium with entry. Furthermore, as the equilibrium of the model is unique it follows that the two equilibria coincide. Hence, the asset value of a searching worker W^0 with and without entry must also coincide.

We now want to show that the social value of a searching worker in the economy is the same with and without entry by firms. Let z denote the associated number of jobs in the steady state equilibrium (which is initially equal to the exogenous number of jobs without entry) and write the aggregate discounted income net of entry- and search costs (welfare) as a function $G(N, z)$. Without entry, the shadow price of a worker in this economy is $g_n = \frac{\partial G}{\partial N}$. With entry, the corresponding price is $g_e = \frac{\partial G}{\partial N} + \frac{\partial G}{\partial z}\frac{dz}{dN}$. Since the last term is zero due to the envelope theorem $g_n = g_e$ and $g_n = g_e = W^0$. This completes the proof of part c).

D. Proof of Proposition 1

Denote the number of unemployed workers by N_0, the number of novice workers by N_1, the number of trained workers in training firms by N_2, and the number of workers in poaching firms by N_3. We normalize $N_0 + N_1 + N_2$ to one. The planner's objective function is then given by

$$\begin{aligned} R(N_0, N_1, N_2, N_3) &= \int_0^\infty [N_1(y^n - ah) + N_2 y^t + N_3 y^p - e^u p^u N_0 \frac{r + q(p^u)}{q(p^u)} K^t \\ &\quad - e^t p^t N_2 \frac{r + q(p^u)}{q(p^u)} K^p - N_0 c(e^u) - N_2 c(e^t)] e^{-rt} dt, \end{aligned}$$

which has to be maximised with respect to h, e^u, e^t, p^u and p^t subject to the following constraints:

$$\begin{aligned} \dot{N}_0 &= s - (e^u p^u + s) N_0, \\ \dot{N}_1 &= e^u p^u N_0 - (\gamma + s) N_1, \\ \dot{N}_2 &= \gamma N_1 - (e^t p^t + s) N_2, \\ \dot{N}_3 &= e^t p^t N_2 - s N_3, \end{aligned}$$

We first derive the solution for a given h. The associated current-value Hamiltonian can be written as

$$\begin{aligned} H &= N_1(y^n - ah) + N_2 y^t + N_3 y^p \\ &\quad - [e^u p^u N_0 \frac{r + q(p^u)}{q(p^u)} K^t + e^t p^t N_2 \frac{r + q(p^t)}{q(p^t)} K^p] - N_0 c(e^u) - N_2 c(e^t) \\ &\quad + \lambda_0 (s - (e^u p^u + s) N_0) \\ &\quad + \lambda_1 (e^u p^u N_0 - (\gamma + s) N_1) \\ &\quad + \lambda_2 (\gamma N_1 - (e^t p^t + s) N_2) \\ &\quad + \lambda_3 (e^t p^t N_2 - s N_3). \end{aligned}$$

The first order conditions for maximum are:

1. The Hamiltonian is maximised with respect to e^u, e^t, p^u and p^t.

2. For all i, $r\lambda_i = \frac{\partial H}{\partial N_i}$ (assuming that we are in steady state).

From condition 1 it follows that p^u and e^u solve

$$\max_{p^u, e^u} e^u p^u [\lambda_1 - \lambda_0 - \frac{r + q(p^u)}{q(p^u)} K^t] - c(e^u), \tag{20}$$

and that p^t and e^t solve

$$\max_{p^t, e^t} e^t p^t [\lambda_3 - \lambda_2 - \frac{r + q(p^t)}{q(p^t)} K^p] - c(e^t). \tag{21}$$

From condition 2 it follows that

$$(r+s)\lambda_0 = e^u p^u [\lambda_1 - \lambda_0 - \frac{r + q(p^u)}{q(p^u)} K^t] - c(e^u), \tag{22}$$

$$(r+s)\lambda_1 = y^n - ah + \gamma(\lambda_2 - \lambda_1), \tag{23}$$

$$(r+s)\lambda_2 = y^t + e^t p^t [\lambda_3 - \lambda_2 - \frac{r + q(p^t)}{q(p^t)} K^p] - c(e^t), \tag{24}$$

$$(r+s)\lambda_3 = y^p. \tag{25}$$

We now compare the optimal solution with the market solution. With *ex post* internal efficiency and for a given value of h, the expressions for λ_0-λ_3 are identical to the corresponding expressions for W^u, Y^n ,Y^t, and Y^p. Furthermore, (20) and (21) imply that (p^u, e^u) maximises λ_0, and that (p^t, e^t) maximises λ_2, just as the competitive search equilibrium maximises W^u and Y^t. Thus, for a given value of h the equilibrium and the planner's solution coincide, proving part b).

We know from optimal control theory that the adjoint variables are equal to the marginal value of the associated state variables. The planner therefore chooses h so as to maximise the value of an additional worker entering the market. That is, he chooses h so as to maximise λ_0. From (22) it follows that this is equivalent to maximising λ_1. Since h is set so as to maximise Y^n in equilibrium, the planner and the agents in the market solve the same maximisation problem, and the equilibrium value of h is socially optimal, proving part a).

The proof of part c is analogous to the proof of Lemma 2 part c. Suppose the number of poaching firms is given exogeneously, and consider the corresponding equilibrium. Suppose the asset value of a poaching vacancy in this equilibrium is V'. Then consider the equilibrium that emerges with free entry of firms and a cost of creating poaching vacancies equal to V'. We know from the proof of Lemma 2 that this equilibrium will be identical to the equilibrium without entry of poaching firms (as all the asset values and thus also the investments in training will be the same). We want to show that the social value of training is the same in the two equilibria as well. Suppose a small subset of worker-firm pairs deviate and increase their investments in training. The optimal response with free entry will then be to increase the number of poaching firms as well. However, due to the envelope theorem the effect of the latter is of second order. Thus, the marginal social value of level of training is the same in the two equilibria. Thus, since the training level is optimal in the equilibrium with entry it follows that this will also be the case in the equilibrium without entry. The same argument holds for entry of training firms.

E. Proof of Lemma 3

Part (1a): Using equation (9), free entry by poaching firms implies that $W^p = Y^p - \frac{r+q^t}{q^t}K^p$. Hence, for a given w^t, p^t maximises

$$(r+s)W^t(w^t) = w^t + e^t p^t (Y^p - \frac{r+q(p^t)}{q(p^t)}K^p - W^t(w^t)) - c(e^t). \qquad (26)$$

The above equation implies that the equilibrium value p^{t*} maximises $p^t(Y^p - \frac{r+q^t}{q^t}K^p - W^t(w^t)) \equiv f(W^t(w^t), p^t)$ and that the cross derivative $f_{p^t,W^t} < 0$. As the second-order conditions for the maximum are always satisfied locally, $\frac{dp^{t*}}{dW^t} < 0$. From the envelope theorem it follows that $\frac{dW^t(w^t)}{dw^t} = 1/(r+s+e^t p^t) > 0$. Thus, p^{t*} decreases in w^t.

Part (1b): We know that p^t maximises W^t, and from equation (26) that p^t therefore maximises $p^t(Y^p - \frac{r+q^t}{q^t}K^p - W^t(w^t))$. Hence, we can write $W^t(w^t)$ as

$$(r+s)W^t(w^t) = \max_{e^t}\left\{w^t - c(e^t) + e^t \max_{p^t}[p^t(Y^p - \frac{r+q(p^t)}{q(p^t)}K^p - W^t(w^t))]\right\}.$$

Hence, the first order condition for e^t is

$$c'(e^t) = \max_{p^t} p^t(Y^p - \frac{r + q(p^t)}{q(p^t)} K^p - W^t(w^t)).$$

From the envelope theorem it follows that the derivative of the right hand side with respect to w^t is equal to $-p^t \frac{\partial W^t}{\partial w^t} = -p^t/(r + s + e^t p^t) < 0$. Hence, $\frac{de^t(w^t)}{dw^t} = \frac{-p^t/(r+s+e^t p^t)}{c''(e^t)} < 0$. Thus, e^{t*} decreases in w^t.

Part (2): Using equation (8), free entry by training firms implies that $W^n = Y^n - \frac{r+q^u}{q^u} K^t$. Hence, in the unemployed search

$$(r + s)W^u = e^u p^u (Y^n - \frac{r + q(p^u)}{q(p^u)} K^t - W^u) - c(e^u)$$

is maximised with respect to e^u, and p^u. The above equation implies that the equilibrium value p^u maximises $p^u(Y^n - \frac{r+q^u}{q^u} K^t - W^u) \equiv f(Y^n, p^u)$, and that the cross derivative $f_{p^u,Y^n} > 0$. As the second-order conditions for the maximum are always satisfied locally, $\frac{dp^u}{dY^n} > 0$. Since, Y^n is strictly less with ex post wage setting than with internal efficiency fewer training firms are created.

F. Proof of Proposition 2

We first show that the social and the private value of an additional trained worker entering the market coincide, given the workers' search behaviour and entry decisions of firms in the on-the-job search market.

The joint private value of a trained worker in a training firm is given by $Y^t = \frac{y^t - w^t}{r+s+e^t p^t} + W^t(w^t)$, where the first term denotes profits and the second the expected discounted income to workers. From Lemma 2 it follows that the social value of a trained worker with *productivity* w^t in the training firm and y^p in a poaching firm is equal to $W^t(w^t)$. When the productivity exceeds the wage the difference $(y^t - w^t)$ is allocated to the firm. The social value of one more trained worker is thus $\frac{y^t - w^t}{r+s+e^t p^t} + W^t(w^t) = Y^t(w^t)$. That is, the social and the private value coincide.

It follows that at the stage at which human capital investments are made, the social and the private returns from training coincide. As the training firms by assumption behave internally efficient at this stage, it follows that the training levels undertaken by the agents are equal to the investment levels undertaken by the planner, i.e., the equilibrium is constrained efficient. Finally, this implies that the social value of hiring an untrained worker coincides with the private value. Thus, by Lemma 2, the unemployed search market is efficient as well, and the optimal number of training firms enter the market. As the market is constrained efficient at the stage when the entry decision of training firms and their investment decision in training are undertaken, training subsidies reduce the allocative efficiency of the economy.

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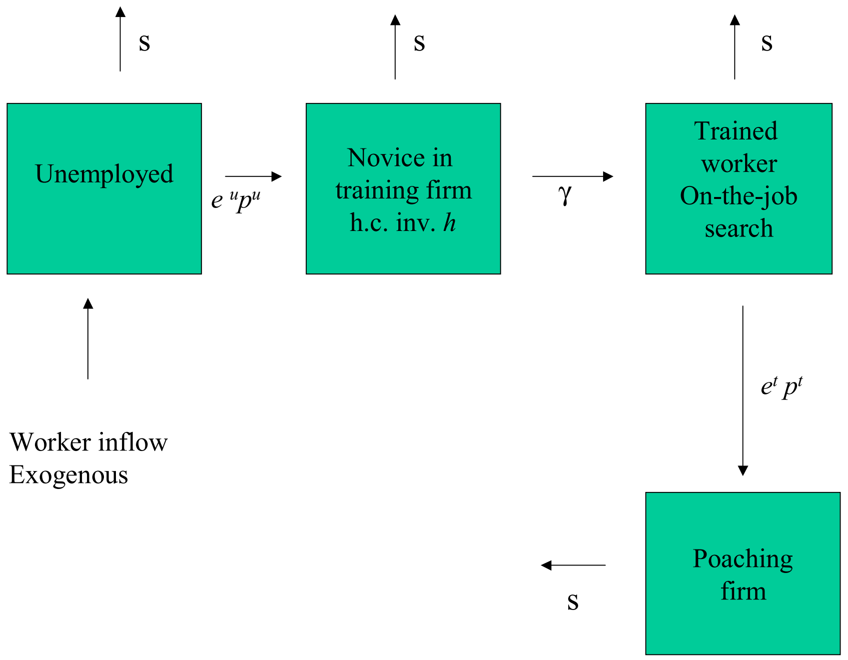

Figure 1:

Figure 1. Worker flows in the economy.