Contract renewal and incentives in public procurement*

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Abstract

This paper explores how the government's choice of renewal policy in public procurement programs can be used as a mechanism to provide firms with incentives to supply quality. A public service is produced by several firms. The firms participate in a tournament where they are ranked according to the quality of their services, and rewarded in terms of contract renewals. We analyse the firms' incentives to produce high-quality services, and find that they are maximised if 50 percent of the contracts are renewed. The optimal renewal policy trades off incentive provision (which requires that a relatively large fraction of the firms are replaced each period) against the entry costs of new firms.

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1 Introduction

An important challenge in public procurement programs is how to avoid quality dumping of the procured services. There may be quality aspects of the public service that the government does not anticipate at the procurement stage, and even anticipated aspects may be of a nature that makes them hard to verify for a third party. These sources of incomplete procurement contracts create problems by leaving room for disputes between the firm and the government concerning service quality. Firms may exploit incompleteness by reneging on the quality of the service in order to save costs (see e.g. Hart, Shleifer and Vishney (1997)).

By threatening not to renew the contract when it finds that the quality provided has been unsatisfactory in the past, the government may give the firms stronger incentive to provide quality. Most franchising schemes give the government means of sanctions that allow for suspensions or termination of the contractual relationship if there are serious service failures. For instance, when describing franchise bidding of cable television contracts in the US, Viscusi et al. (1992) claim that (what could be referred to as) *long-term, non-exclusive contracts* are the contracts most commonly used. They were typically 15 years in length, and gave the local government the right to re-auction the contract if the current franchise owner did not perform satisfactorily. In Britain, the Rail Franchising Director employs a system of warnings and loss of franchise (see Baldwin and Cave (1999, ch. 20). Minor infractions by the rail companies trigger a "breach" in the regulation, which is made public and can lead to fines being levied, whereas more serious infractions, known as "defaults", can lead to the loss of a franchise.

The aim of this paper is to explore further how the renewal process - present in all government procurement programs - may be used to create incentives to provide quality. Rather than choosing the actual duration of a contractual relationship *ex ante,* we suggest that this could be a function of the firms' performance, thus enabling the possibility of contract renewal to act as an incentive device.

As already pointed out, quality performance is not easily verifyable, making clear cut definitions of quality defaults difficult to establish. Even observing quality (without having to verify) may prove problematic. However, since there are usually several firms that produce the same types of services for the government, it may still be possible to obtain ordinal information on the relative performance of these firms. What seems realistic is that the government, or an independent party, can undertake - with noise - a ranking of the firms' quality level.

Having access to rank order information from a third party, the government may initiate a rank order contest. Rewards to the firms are based on the their ordinal positions alone and not on the actual level of quality provided[1]. The reward for the winners of the quality contest is simply continuation of the contract (or repeated purchase). Thus, a firm's franchise is renewed as long as the firm's quality ranking is sufficiently high.

We show that the government, by its choice of the proportion of "winners", controls the expected duration of procurements contracts, and in effect the incentives to provide quality (up to some limit). Incentives to provide quality increase with the fraction of replaced firms up to the point where 50 per cent of the firms are replaced each period, after which it falls symmetrically. As there is a sunk cost associated with entry, the government will never replace more than 50 percent of the firms. Furthermore, increasing incentives to provide quality comes at a cost, as it requires that more firms are replaced each period. The optimal design of such a procurement contract, therefore, implies a lower level of quality than the first-best quality level with full information.

Rewards in terms of contract renewal may be considered as an alternative to monetary rewards, in which a high ranking leads to a bonus. A monetary reward system has the advantage that it economizes on entry costs. On the other hand, rewards based on contract renewal rather than pecuniary awards may be less vulnerable to collusion among the suppliers. If collusion takes place, all firms agree (implicitly or explicitly) to choose a low effort level and thereby realize higher expected payoffs. If the rewards are pecuniary, the firms know that they will meet each other in future tournaments. This makes it possible for firms that adhere to the collusion strategy to punish firms that deviate from it. Hence, the collusion strategy may be sustainable. If rewards are based on contract renewal, this punishment strategy will be more difficult to implement, as firms that adhere to the collusion strategy will to a large extent be replaced. Consider for instance the case with two firms. If one firm deviates and the other adheres to the collusion strategy, the latter will most likely lose its contract and thus have no opportunity to punish the deviator in future rounds. As a result, collusion is less likely to occur.

An example that ties in well with our model is found in the so-called Job Network in Australia. The publicly funded Job Network was established in 1998 and provides services to unemployed workers. The Job Network Model has been documented and evaluated in a report by the Australian Productivity commission (2002). The services in the Job Network are supplied by

[1]See e.g. Lazear and Rosen (1981), Nalebuff and Stiglitz (1983), and the more recent work by Clark and Riis (1998) and Moldovanu and Sela (2001).

independent, often private enterprises, and allocated to suppliers after a competitive tendering process. The initial round of tendering was probably the biggest in Australia's history, with more than 1000 participants and 5300 bids. In order to give the suppliers incentives to deliver high-quality services, a star rating model was introduced. Scores are distributed between one and five stars such that 70 per cent of providers in a region are rated at three stars or better. Those providers with high star rating are generally assured future contracts.

Ranking and quality grading are often important in higher education. In the US, rankings of colleges and universities have become very important, and are used extensively as a guide for students and their parents when choosing between universities. In the UK, a governmental inspectorate (HMI) assesses quality of tuition carried out in the public sector. HMI's evaluation of quality results in the publication of a broad set of grading for the quality of tuition in each institution.[2]

As a curiosity, our incentive mechanism is very similar to mechanisms used in sports. In soccer for instance, the national leagues are typically arranged as tournaments where all teams play against each other, and after the season the teams are ranked according to the results of these matches. In the premier league, typically the three teams with the lowest ranking are replaced, hence their "contracts" are not renewed. The other teams are rewarded by being allowed to play in the premier league in the next period as well. Our model, taken literally, suggests that in order to maximize the incentives for the teams to play well, half of the teams should be replaced each period.

There are a number of studies that address issues of quality and procurement. Che (1993) analyses multidimensional auctions where firms submit bids on price and quality. This obviously requires that quality is observable. Laffont and Tirole (1993) analyse procurement from a firm that produces what they refer to as an "experience good", which has to be purchased before its quality is learned. They derive the optimal contract with heterogenous firms, and show that this contract provides lower incentives for cost reduction (and thus lower information rents) in order to increase the incentives to provide high quality. In Laffont and Tirole's model, as in our model, a motivating factor for providing quality is an increased probability of servicing the government next period. However, in Laffont and Tirole, the probability of keeping the contract next period is not under direct control of the government, and can therefore not be used as an instrument to manipulate the firms' incentives to provide quality. In our paper, by contrast, the probability

[2]See Cave et al. (1995).

of continued service is the only tool available for the regulator. Finally, as a mechanism to provide incentives to maintain quality, our model resembles the repeat-purchase mechanism first studied by Klein and Leffler (1981). The value of future profits motivates firms to maintain quality. Our paper differs from their paper in that our contracts are based on ordinal information only.

Our paper is organized as follows. Section 2 presents the basic model. Section 3 investigates the incentive properties of the renewal policy when only noisy rank order information is available to the government. Section 4 derive the optimal renewal policy. Sections 5 concludes.

2 The model

A public service is produced by several firms or contractors (e.g. nursing homes or local bus transportation). The level of production in these firms is perfectly controlled by the local government, and is normalized to one. The quality of service is only imperfectly controlled, as explained below.

2.1 Technology and preferences

The model is set in discrete time, and all agents have a common discount factor δ. In each period, the government write contracts with n different firms to perform the same task, i.e. to produce a public service. The government's per period preferences are given by the welfare function:

$$W = \sum_{i=1}^{n} [q_i - B_i] \tag{1}$$

where $q_i \geq 0$ is the quality of service supplied by firm i and B_i the payment to firm i.[3]

A large number of identical firms in the market are competing for a contract. A firm's cost of undertaking a contract for the government consists of two parts. First, there is an entry cost C. Second, there is a per period cost c that depends on the quality level q, $c = c(q)$. All firms are identical and risk neutral. Costs and preferences are the same in each period.

Prior to the first period, the government announces its contract renewal policy. This policy will be explained in detail below. The firms then bid on the n contracts. The bids specify an annual payment B. As firms are

[3] We thus assume that the government attaches no value to firm profit. Our results will still hold if the government also takes into account firm profit, but gives it less weight than public funds. The latter may be rationalised by the dead-weight loss associated with taxes.

identical, and there are more firms than contracts, the winning firms just break even in expected terms.

After each period, a fraction of the contracts will not be renewed. These contracts will be auctioned out, following the same procedure as in the initial round. The replaced firms are not allowed to participate in this auction, and the newcomers have to incur the fixed cost C.

The firms maximize expected discounted profit, given by the Bellman equation

$$E\pi = B - c(q) + P\delta E\pi', \tag{2}$$

where P is the probability of contract renewal. As will be seen below, P depends on the firm's choice of q. The last term $E\pi'$ is the expected discounted profit from the next period and onward. Let q^* denote the value of q that maximizes $E\pi$. Due to the stationary of the problem, it follows that

$$E\pi = \frac{B^* - c(q^*)}{1 - P\delta}. \tag{3}$$

Due to entry costs, denoted by C, there must always be a strictly positive per period operating surplus, $B - c(q)$. The zero profit condition then implies that

$$E\pi = C. \tag{4}$$

The first best quality level q^{FB} is such that $c'(q^{FB}) = 1$. With verifiable quality, the first best quality level can be implemented by conditioning B on q. In this paper, by contrast, quality level is not observable, nor is the cost c (as the government knows $c(q)$, observing c and q are equivalent). However, after each period the government hires an independent third party to undertake - subject noise - a ranking of the firms' quality level. By linking a firm's rank - or listing - to the renewal decision, the government is able to affect the firms' incentives to supply quality.

The government thus constructs a rank order contest, which means that rewards to firms are based on the their relative performance.[4] The reward in our model is a certain probability of contract renewal. The government assigns a probability for having the contract renewed to each of the n possible positions as a function of their ranking. We denote the probability of contract renewal for a firm with ranking j by P_j, $j = 1, .., n$. The tournament can thus

[4] It is well-known from the literature on rank order contests, in which the incentives to perform well are affected through the prices associated with each ranking, that a first best outcome may be feasible. See Lazear and Rosen (1981), Green and Stokey (1983) and Nalebuff and Stiglitz (1983)).

be characterized by a vector $[P_1, ..., P_n]$ of probabilities for contract renewal. We assume that the government chooses the same tournament in each period.

2.2 Quality tournaments

The ranking process is subject to noise. Formally, we assume that the third party in effect ranks a vector $(q_1 + \epsilon_1, q_2 + \epsilon_2, ..., q_n + \epsilon_n)$, where the error terms $\epsilon_1, ..., \epsilon_n$ are independently drawn from the same distribution. In order to simplify the exposition and obtain closed-form solutions, we assume that the errors are normally distributed with expectation 0 and variance σ^2. [5]

Technically, it is convenient to rank the error terms. Denote by F the cumulative distribution function of the normal distribution and $G_j(\epsilon)$ the probability that a given error term will have rank j as a function of its realized value (that is, that $j-1$ error terms are higher than ϵ and $n-j-1$ are lower than ϵ). From Ross (2003, p. 60) we have that[6]

$$G_j(\epsilon) := \frac{(n-1)!}{(n-j)!(j-1)!}[1-F(\epsilon)]^{j-1}F(\epsilon)^{n-j}. \tag{5}$$

Let the probability that firm i obtains rank j, given that the other firms' quality levels are all equal to $\overline{q}$, be denoted by $\mu_j(q_i, \overline{q})$.Then it follows that

$$\mu_j(q_i, \overline{q}) = \int_{-\infty}^{\infty} G_j(q_i - \overline{q} + \epsilon) f(\epsilon) d\epsilon.$$

Firm i's profit is thus (from equation 2 and the fact that $E\pi' = C$)

$$E\pi = B_i - c(q_i) + \delta \left[\sum_{j=1}^{n} \mu_j P_j \right] C.$$

The first order condition defining the optimal choice of quality is given by

$$-c'(q_i) + \delta \left[\sum_{j=1}^{n} \frac{\partial \mu_j(q_i, \overline{q})}{\partial q_i} P_j \right] C = 0. \tag{6}$$

The first term is the marginal cost of increasing quality, while the second term represents the marginal gain - which is equal to the marginal increase in the probability of renewal of the contract times the value of renewal. Note

[5] Unless otherwise stated, it is sufficient to assume that the distribution of the error terms is symmetric, unimodal and (if warranted) with finite support.

[6] Note that a rank of k in Ross (2003) denotes the k'th *lowest* ranking, not highest ranking as in this paper.

that $\frac{\partial \mu_j(\overline{q},\overline{q})}{\partial q_i}$ is independent of $\overline{q}$, and will be denoted by μ'_j . In what follows we will focus on the symmetric equilibrium in which $q_i = \overline{q}$ for all i. A discussion of second order conditions is given in the appendix.

The next step is to evaluate $\frac{\partial \mu_j(\overline{q},\overline{q})}{\partial q_i}$. Let e_j denote the expected value of the j-th highest of n independent draws from the standard normal distribution. In the appendix we show that when the error terms are normally distributed, then

$$\frac{\partial \mu_j(\overline{q},\overline{q})}{\partial q_i} = \frac{e_j}{n\sigma} \tag{7}$$

In what follows, we let $m \equiv (n+1)/2$ denote the median ranking.[7] By definition $e_1 > e_2, ... > e^n$. Furthermore, since $e_m = 0$, the following lemma follows directly:

Lemma 1 *The ranking probability function $\mu_j(q_i, \overline{q})$, $j = 1, ..., n$, evaluated at $q_i = \overline{q}$, has the following properties:*

a) Nonresponsivness at the median: $\frac{\partial \mu_m}{\partial q_i} = 0$

b) Symmetry around the median: For any pair $\{j, j'\}$ such that $j' = n + 1 - j$, we have that $\frac{\partial \mu_j(\overline{q},\overline{q})}{\partial q_i} = -\frac{\partial \mu_{j'}(\overline{q},\overline{q})}{\partial q_i}$.

c) $\frac{\partial \mu_j}{\partial q j}$ is decreasing in j.

As shown in the appendix, result a) and b) also hold under the less restricitve assumption that F is symmetric and unimodal. For property c) this turns out to be insufficient, and we cannot rule out that there may exist distributions such that an increase in effort increases the probability of rank j by less than the increase in the probability of rank $j+1$. However, we have not been able to find such a distribution.

3 Quality and the renewal policy

The symmetry around the median rank implies that the increase in the probability of getting position j *from the top* exactly balances the decrease in the probability of getting position k up *from the bottom (*i.e. having position j'*)*. This symmetric effect on the probability distribution over the n positions allows us to write the first order condition in the following way

[7] We refer to the median ranking, although, strictly speaking, the median ranking only exists if n is an odd number.

$$-c'(\overline{q})+\delta\left[\sum_{j=1}^{m}(P_j-P_{j'})\mu'_j\right]C=0,\quad \text{with } j'=n+1-j. \tag{8}$$

Consequently, incentives to supply quality can only be provided by treating firms at the j-th and the j'-th positions differently at renewal stages. Increasing P_j and $P_{j'}$ by the same number Δ (keeping all the other Ps constant) will not change the overall incentives to supply quality.

Two tournaments are *incentive equivalent* if they give rise to the same incentives for all $\overline{q}$. A tournament is *more incentive powered* than another tournament if it gives rise to higher incentives for all $\overline{q}$. It follows that the marginal gain of quality in the tournament can be written as $\delta\beta C$, where β is given by

$$\beta=\sum_{j=1}^{m}(P_j-P_{j'})\mu'_j. \tag{9}$$

β is controlled by the government, and will be referred to as the incentive power of the tournament.

The following lemma holds:

Lemma 2 *Let $[P_1^1,...,P_n^1]$ and $[P_1^2,...,P_n^2]$ be two tournaments. If $P_j^1-P_{j'}^1=P_j^2-P_{j'}^2$ for all j, then the two tournaments are incentive equivalent.*

As an example, consider the following two tournaments:

$$\begin{array}{ll} (i) & [P_1,..,P_j,P_{j+1},..,P_m,..,P_{j'},P_{j'+1},..,P_n]=[1,..,1,0,..,0,..,0,0,..,0] \\ (ii) & [P_1,..,P_j,P_{j+1},..,P_m,..,P_{j'},P_{j'+1},..,P_n]=[1,..,1,1,..,1,..,1,0,..,0] \end{array}$$

The first tournament implies that the j highest ranked firms will get a contract renewal with certainty, while the rest will be replaced with certainty. The second tournament implies the j-th lowest ranked firms will be replaced with certainty, while the rest will get a contract renewal with certainty. The lemma implies that the two are incentive equivalent.

In what follows we assume that the tournaments in question do not lead to collusion, as explained in the introduction. When choosing between two such tournaments, and the tournaments are incentive equivalent, the government will always prefer the one in which most firms get their contracts renewed. This will reduce entry costs and thus also the per period payments B. This *maximum renewal principle* implies that the government must make sure that $\max\{P_j,P_{j'}\}=1$ for all j.If $\max\{P_j,P_{j'}\}<1$, the government can reduce the probability of setting up new auctions by increasing both P_j and $P_{j'}$ (i.e. choose a Δ) without changing incentives to provide quality.[8]

An *optimal tournament* is a tournament which, for a given incentive power β, obeys the maximum renewal principle. The fact that $\max\{P_j, P_{j'}\} = 1$ gives rise to the following lemma:

Lemma 3 *Any optimal tournament is of the form* $[1, ..., 1, P_k, 0, ...0]$*, where* $0 \leq P_k \leq 1$ *and where* $m \leq k$*.*

Note that any optimal tournament gives rise to a unique incentive power β of the tournament. Furthermore, for any given incentive power β there exists at most one optimal tournament. Let $P = \Sigma_{j=1}^{n} P_i/n$ denote the equilibrium probability of contract renewal, which is the same for all firms. From lemma 3 it follows that for any P in the interval $[1/2, 1]$ there is a corresponding tournament that obeys the maximum renewal principle. We write the incentive power β of a tournament as a function of P, $\beta = \beta(P)$. The next proposition then follows:

Proposition 4 *The incentive power* β *of the tournament can be written as an decreasing function of* P *on* $[0.5, 1]$*. Maximum quality is obtained at* $P = 1/2$*,n i.e., when half of the firms get their contract renewed after each period. Furthermore,* $\beta'(P) < 0$*.*

The claim that β is maximized at $P = 1/2$ follows from the fact that μ_i' is positive for ranking above the median, zero at the median, and negative below the median. That $\beta'(P) < 0$ for $P \geq 1/2$ follows from the fact that μ_i'.

In order to gain intuition for this result, consider first a situation where n is high. Suppose all firms have a quality level $\overline{q}$. The law of large numbers then ensures that the median firm has an error term $\epsilon \approx 0$. Let ΔP_k denote the increased probability of contract renewal obtained by increasing q from $\overline{q}$ to $\overline{q} + \Delta q$, when in total k firms obtain renewal. If all firms above a median ranking get their contract renewed, $\Delta P_m = f(0)\Delta q$. If firms need a ranking $k > m$ in order to obtain a new contract, the corresponding increase in probability is $f(\hat{\epsilon}_k)\Delta q$, where $\hat{\epsilon}_k < 0$ is the error term associated with obtaining ranking k. Since $f(0) > f(\hat{\epsilon}_k)$ it follows that the effect of increased

[8] If collusion is an issue, the government may - for given incentive power - choose a tournament with a higher renewal frequency. Thus, the threat of collusion may render the maximum renewal principle invalid, and defines a lower bound on the number of renewals for which our analysis is valid.

effort in terms of higher probability of contract renewal is higher at the median, hence also the incentives to provide effort. For lower values of n, we cannot apply the law of large number, and the error term for the k-th best firm is stochastic. Still, the density of the error term for the median firm will be highest at 0, and the flavor of the argument still holds.

4 Optimal tournaments

In this section we will derive optimal tournaments, and analyze the welfare loss of using contract renewal rather than pecuniary prices in order to promote quality.

The first question we address is in what situations is it possible to implement the first-best quality level defined by $c'(q^{FB}) = 1$? As we have seen, the tournament with the highest incentive power is obtained when $P = 1/2$. From (6) and the definition of β in (9) it thus follows that first best can be obtained if and only if

$$\delta\beta(1/2)C \geq 1.$$

Thus, q^{FB} is most likely to be obtainable if

1. The entry cost C is high.

2. The discount factor δ is close to 1.

3. The incentive power $\beta(1/2)$ is high, that is, the noise term σ is low.

However, high quality level comes at a higher cost than in the full information case. In order to increase incentives, more firms will have to be replaced each period, and hence each firm has a smaller number of periods to capitalize their sunk entry cost. The regulator must compensate for this through a higher value of B.

To be more specific, note that since all firms choose the same quality level in the symmetric equilibrium, the probability of contract renewal is equal to P for all firms. From equations (3) and (4) we know that $B = C(1 - P\delta) + c(q)$. We have seen that $q = \widetilde{q}(\beta(P))$ is strictly decreasing in P on the relevant interval. The government chooses P so as to maximize $q - B$, and thus solves

$$\max_P \widetilde{q}(P) - C(1 - P\delta) - c(q(P)).$$

The corresponding first order condition can be written as

$$c'(q^r) = 1 + \frac{\delta C}{\partial q^r / \partial P}. \tag{10}$$

Since $c''(q) > 0$ and $\frac{\partial q^r}{\partial P} < 0$, $q^r < q^{FB}$. Note also that as $\partial q^r / \partial P = \widetilde{q}'(\beta)\beta'(P) = 0$ at $P = 0.5$, it follows that it is never optimal to induce maximum quality by setting $P = 1/2$:

Lemma 5 *It is never optimal to give the firms maximum incentives by setting $P = 1/2$.*

The intuition for this result is that the incentive power β becomes insensitive to P as P approaches $1/2$, while costs in terms of increased entry costs are proportional to P.

We will now define an upper bound for the welfare loss associated with the use of contract renewal rather than pecuniary prices. Let P^{FB} denote the value of P necessary to obtain first-best quality given by $c'(q^{FB}) = 1$. From equation (3) it follows that the additional per period costs of implementing q^{FB} of using contract renewal as the incentive mechanism instead of a pecuniary mechanism (in which $P = 1$) is given by

$$\Delta W = \delta C(1 - P^{FB}).$$

As the optimal policy with contract renewal is to implement a quality that is strictly below q^{FB}, our calculated welfare loss ΔW is an upper bound on the actual welfare loss.

To derive the relationship between the welfare loss and the parameters of the model, we first look closer at the relationship between P and β. Note that equation (6) and lemma 3 implies that

$$\beta(P) = \sum_{j=1}^{k-1} \mu'_j + P_k \mu'_k \tag{11}$$

where $P = (k - 1 + P_k)/n$. As an increase in P_k increases P with $1/n$ units, it follows that $\beta'(P) = \mu'_k \cdot n$. As P increases, so do k, and from lemma 1 it thus follows that $\beta'(P)$ decreases (increases in absolute value). Hence, $\beta(P)$ is concave in P. Since $\beta(1) = 0$ it follows that

$$\beta(P) \geq -\beta'(P)(1 - P) \tag{12}$$

with strict inequality unless $k = n$.

We are now able to show the following result:

Proposition 6 *The following holds*

a) Reduced noise σ (reduced $f(0)$ with more general distributions) in the ranking process reduces the welfare loss ΔW.

b) An increase in δ or C reduces the welfare loss ΔW.

c) The welfare loss goes to zero when the entry cost C and the number of firms n both go to infinity.

From equation (7) and (9) it follows that a reduction in σ implies an increase in P for a given β. It then follows directly that ΔW falls, as fewer firms are replaced each period

We then turn to b). To retain q^{FB} requires that (from (6) and the definition of β in (9))

$$\delta\beta(P^{FB})C = 1. \tag{13}$$

It thus follows that

$$\frac{dP^{FB}}{dC} = \frac{-\beta(P^{FB})}{C\beta'(P)} \geq \frac{1-P^{FB}}{C}.$$

For the welfare loss we thus find that

$$\begin{aligned}\frac{d\Delta W}{dC} &\sim (1-P^{FB}) - C\frac{dP^{FB}}{dC} \\ &< (1-P^{FB}) - C\frac{1-P^{FB}}{C} \\ &= 0\end{aligned}$$

Part b) thus follows.

Part c) follows from the fact that as C and n grow larger, $\frac{dP^{FB}}{dC}$ becomes extremely large. Formally, suppose C is sufficiently large so that $P_k = 0$ for all k except $k = n$. From (11) and (13) we find that

$$\delta C\left[\sum_{j=1}^{n-1}\mu'_j + P_n\mu'_n\right] = 1.$$

Since $\sum_{j=1}^{n}\mu'_j = 1$ we can write this as

$$-\delta C(1-P_n)\mu'_n = 1.$$

In this case, $1 - P = (1 - P^n)/n$, thus $\delta C(1 - P)\mu_n' = n$. Thus

$$\begin{aligned} \Delta W &\sim (1 - P)C \\ &= -\frac{n}{\mu_n' \delta} \\ &= \frac{1}{e_n \delta} \end{aligned}$$

where we used lemma 1. Note that the loss is independent of C. Now we let n go to infinity, and simultaneously increase C so that P_k remains zero for all $k < 1$. As e_n goes to infinity with n this proves c).

5 Concluding remarks

In this paper, we have investigated how the government in public procurement programs may give the suppliers incentives to provide quality. Such incentives are established by applying a mechanism of contract renewal, where firms that fail to perform well compared to the other suppliers are replaced. In effect, the government's renewal policy forms a tournament, where firms are ranked according to the quality of their services by an independent third party, and the rewards come in terms of contract renewal. The incentive power (with respect to quality) of such a tournament is shown to depend on the proportion of firms that are offered contract renewals. In this paper we have characterized the relationship between renewal policy and incentive power, and deriving the optimal policy. Our first set of findings are:

1. The incentive power of the tournament is highest when the contract is renewed for half of the firms.

2. Maximum incentive power is never optimal to implement. More than 50 per cent of the firms are offered contract renewals.

An alternative to the renewal mechanism would be to offer monetary bonuses and penalties, in which a high ranking leads to a bonus and a low ranking leads to a penalty. A monetary reward system has the advantage that it economizes on entry costs. On the other hand, as emphasized in the introduction, rewards based on contract renewal rather than pecuniary awards may be less vulnerable to collusion among the suppliers. If collusion takes place, all firms agree (implicitly or explicitly) to choose a low effort level and thereby realize higher expected payoffs. If the rewards are pecuniary, the firms know that they will meet each other in future tournaments. This makes it possible for firms that adhere to the collusion strategy to punish firms that deviate from it. However, bringing collusion in to the analysis is left for future research. Instead, we have derived a measure of the welfare loss associated with the use of contract renewal instead of pecuniary prices, taking into account excessive entry costs only. This leads to the second set of findings:

3. Welfare loss is reduced when (i) noise regarding the ranking of firms' performance is reduced, and (ii) when entry costs increase.

4. Welfare loss converges to zero as entry costs increase without a bound.

Our analysis rests on several simplifying assumptions. We have assumed that firms that compete for contracts are identical, and that there is always perfect competition when the government puts contracts up for auction. Competition for the contracts may, however, vary from time to time due to capacity constraints or asymmetric productivity growth between firms. Long-term, non-exclusive contracts may be vulnerable to such variations in the degree of competition. If a firm happens to win a contract at a time of weak competition, its operating profit will be greater than what is "normal". Consequently, such a firm has more interest in getting a renewal of its contract than other firms.

Appendix

A1: Proof of equation (7)

By using integration by parts, we find that

$$\begin{aligned}\frac{\partial \mu_j(q_i,\overline{q})}{\partial q_i} &= \int_{-\infty}^{\infty} \frac{dG_j(q_i-\overline{q}+\epsilon)}{dq_i} f(\epsilon)d\epsilon \\ &= -\int_{-\infty}^{\infty} G_j(q_i-\overline{q}+\epsilon)f'(\epsilon)d\epsilon.\end{aligned}$$

Inserting for G_i, using (5), gives

$$\frac{\partial \mu_j(q_i,\overline{q})}{\partial q_i} = -\frac{(n-1)!}{(n-j)!(j-1)!}\int_{-\infty}^{\infty} [1-F(q_i-\overline{q}+\epsilon)]^{j-1}F(q_i-\overline{q}+\epsilon)^{n-j}f'(\epsilon)d\epsilon. \tag{14}$$

For the normal distribution with zero mean we have that $f'(\varepsilon) = -\frac{1}{\sigma^2}\varepsilon f(\varepsilon)$. Inserting this into (14) gives

$$\frac{\partial \mu_j(q_i,\overline{q})}{\partial q_i} = -\frac{(n-1)!}{(n-j)!(j-1)!}\int_{-\infty}^{\infty} [1-F(q_i-\overline{q}+\epsilon)]^{j-1}F(q_i-\overline{q}+\epsilon)^{n-j}\frac{1}{\sigma^2}\varepsilon f(\varepsilon)d\epsilon. \tag{15}$$

Denote by $F_{(j)}(\varepsilon)$ the operability that the j'th largest drawing out of n drawings from the distribution $F(\varepsilon)$ is below ε. From Ross (2003) p. 60ff it follows that

$$f_{(j)}(\varepsilon) = \frac{n!}{(n-j)!(j-1)!}(1-F(\varepsilon))^{j-1}F(\varepsilon)^{n-j}f(\varepsilon).$$

The expected j'th largest drawing is

$$E\varepsilon_{(j)} = \int_{-\infty}^{\infty} \varepsilon f_{(j)}(\varepsilon)d\varepsilon = \frac{n!}{(n-j)!(j-1)!}\int_{-\infty}^{\infty}(1-F(\varepsilon))^{j-1}F(\varepsilon)^{n-j}\varepsilon f(\varepsilon)d\varepsilon. \tag{16}$$

Combining (15) and (16) gives that

$$\frac{\partial \mu_j(\overline{q},\overline{q})}{\partial q_i} = \frac{1}{n\sigma^2}E\varepsilon_{(j)}.$$

As $E\varepsilon_{(j)} = e/\sigma$, equation (7) follows.

A2: Proof of lemma 1a) and 1b) under less restrictive assumptions on ϵ.

Suppose the error term is symmetric and unimodal but not normally distributed, and with finite support. We have that (repeating equation 5 for $j = m$)

$$\frac{\partial \mu_j(q_i,\overline{q})}{\partial q_i} = -\frac{(n-1)!}{(n-j)!(j-1)!}\int_{-\infty}^{\infty}[1-F(q_i-\overline{q}+\epsilon)]^{j-1}F(q_i-\overline{q}+\epsilon)^{n-j}f'(\epsilon)d\epsilon. \tag{17}$$

For any function $F(z)$, we know that $F^m(z)(1-F(z))^m$ is symmetric around $z = 0$. The derivative of a symmetric function is odd (antisymmetric) around zero). It thus follows that $-\int_{-\infty}^{0}[1-F(\epsilon)]^{j-1}F(\epsilon)^{n-j}f'(\epsilon)d\epsilon = \int_{0}^{\infty}[1-F(\epsilon)]^{j-1}F(\epsilon)^{n-j}f'(\epsilon)d\epsilon$. Hence $\frac{\partial \mu_j(\overline{q},\overline{q})}{\partial q_i} = 0$. We have thus proved part a).

For all j, we have that (again by repeating equation 5)

$$\frac{\partial \mu_j(\overline{q}_i,\overline{q})}{\partial q_i} = -\frac{(n-1)!}{(n-j)!(j-1)!}\int_{-\infty}^{\infty}[1-F(\epsilon)]^{j-1}F(\epsilon)^{n-j}f'(\epsilon)d\epsilon.$$

Since the distribution is symmetric it follows that $F(\epsilon) = 1 - F(-\epsilon)$, and hence that

$$\begin{aligned}
&\int_{-\infty}^{\infty}[1-F(\epsilon)]^{j-1}F(\epsilon)^{n-j}f'(\epsilon)d\epsilon \\
=& \int_{-\infty}^{\infty}[F(-\epsilon)]^{j-1}(1-F(-\epsilon)^{n-j})f'(\epsilon)d\epsilon \\
=& -\int_{-\infty}^{\infty}[F(\epsilon)]^{j-1}(1-F(\epsilon)^{n-j})f'(\epsilon)d\epsilon.
\end{aligned}$$

Since $f'(\epsilon) = -f'(-\epsilon)$ as f' is odd. Part b) thus follows.

A3: A note on the second-order conditions

It is straightforward to show that the conditions for a local maximum are satisfied in the solution described by Proposition 1. We can write the first order condition as follows,

$$-c'(q_i) + \delta \frac{(n-1)!}{(n-k)!(k-1)!} \int S(q_i - \overline{q} + \epsilon; n, k) f(\epsilon) d\epsilon C = 0.$$

where

$$\begin{aligned} S(q_i - \overline{q} + \epsilon; n, k) \quad : \quad &= [P_k (n-k) F(q_i - \overline{q} + \epsilon)^{n-k-1} (1 - F(q_i - \overline{q} + \epsilon))^{k-1} + \\ &(1 - P_k)(k-1) F(q_i - \overline{q} + \epsilon)^{n-k} (1 - F(q_i - \overline{q} + \epsilon))^{k-2}] f(q_i - \overline{q} + \epsilon). \end{aligned}$$

Using integration by parts, we find the second order condition for a local maximum:

$$-c''(q_i) - \delta \frac{(n-1)!}{(n-k)!(k-1)!} \int S(q_i - \overline{q} + \epsilon; n, k) f\prime(\epsilon) d\epsilon C < 0.$$

Since $k > m = (n+1)/2$ it follows that the function $S(q_i - \overline{q} + \epsilon; n, k)$ is tilted to the left (note that $F(1-F)$ is symmetric). As $f\prime(\epsilon)$ is odd (and positive for $\epsilon < 0$), the integral is clearly positive. Hence, the *local* conditions for a maximum holds. However at low values of q_i the firm is certain to be ranked in the n'th position. Therefore, as providing quality is costly, $q_i = 0$ is a local maximum as well. Comparing the two local maxima, then $q_i = \overline{q}$ is the *global* maximum iff $B - c(\overline{q}) + \delta P E\pi \geq B - c(0) + \delta P_n E\pi$. Inserting $E\pi = C$, and rearranging yields the condition

$$c(\overline{q}) - c(0) \leq \delta C(P - P_n). \tag{18}$$

The nonconvexity problem pointed out by Nalebuff and Stiglitz (1983) arises in our setting only at noise levels so low that even the lowest ranked firm may get his contract renewed, $P_n > 0$. Observe the following non-monotonicity: as noise in the ranking process decreases, the marginal gain from providing higher quality increases. To off-set this effect on the incentives (in order to keep quality unchanged), the number of renewals must increase (P up). As long as $P_n = 0$, higher P means that the net surplus obtained by setting $q_i = \overline{q}$ increases compared to the surplus obtained by zero quality. Hence, the non-convexity problem arises only if the noise is so small that P_n becomes positive and, eventually, converges to P, in which case the symmetric pure strategy equilibrium breaks down.

Finally, we will show that the symmetric solution is a global maximum also in the limit, i.e. as C goes to infinity. This requires that condition (11), $c(\overline{q}) - c(0) \leq \delta C(P - P_n)$, still holds as C goes to infinity (the concern is that P_n converges to P, hence $P - P_n$ goes to zero). We know that P_n is zero for low values of C. Furthermore P is strictly increasing in C, and eventually P_n becomes positive. In that case we can write the first order condition and the condition for global maximum as follows:

$$c'(q_i) = \delta \left[\sum_{j=1}^{n} P_j \mu'_j \right] C = \delta \left[\sum_{j=1}^{n-1} \mu'_j + P_n \mu'_n \right] C = \delta \left[-\mu'_n + P_n \mu'_n \right] C = -\delta \mu'_n \left[1 - P_n \right] C$$

$$c(\overline{q}) - c(0) \leq \delta C(P - P_n) = \delta C(1 - P_n)(1 - 1/n). \tag{19}$$

Hence, the increase in C exactly cancels out the effect of the adjustment in P_n which is necessary in order to keep the incentive power unchanged. The third point can be shown as follows. As shown in section 2 , when quality is observable, a first best situation can be implemented by denying renewal of the contracts for those firms that provide insufficient quality, on the condition that the continuation payoff dominates the short run gain from reduced quality - that is, $c(q^{FB}) - c(0) < \delta C$. Assume that first best quality is achievable using our non-exclusive contracts (as we already have shown this is certainly true if the noise in ranking is not too large). Furthermore, we know from point a) that reduced noise reduces the welfare loss since P increases, and disappears as P converges to one (hence a first best quality is realized). However as P_n becomes positive, the condition for a global maximum, $c(\overline{q}) - c(0) \leq \delta C(P - P_n) = \delta C(1 - P_n)(1 - 1/n)$, may break down. Consider the term $(1 - P_n)(1 - 1/n)$. As shown above, when P_n is positive, the first order condition can be written $c'(q^{FB}) = -\delta \mu'_n \left[1 - P_n \right] C$ where $\mu'_n \equiv \int [1 - F(\epsilon)]^{n-1} f'(\epsilon) d\epsilon$. It is straight forward to show that μ'_n is strictly increasing in n and strictly decreasing in the level of noise. By continuity, at a given n, a level of noise exists such that $P_{n-1} = 1$ and $P_n = 0$. This yields a decreasing sequence n^ϵ as a function of the level of noise such that $P_n = 0$ and $P = 1 - 1/n^\epsilon$. As the level of noise converges to zero, n^ϵ goes to infinity. Hence $(1 - P_n)(1 - 1/n)$ converges to one.

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