Performance pay and adverse selection[1]

Espen R. Moen[2] and Åsa Rosén[3]

November 3, 2004

[1]We thank the editor and two anonymous referees for valuable comments. Financial support from the Swedish Research Council is gratefully acknowledged.

[2]Norwegian School of Management (BI) and CEPR. Adress: Norwegian School of Management. Box 580 N-1301 Sandvika, Norway E-mail: espen.moen@bi.no

[3]Swedish Institute for Social Research, Stockholm University, S106-91 Stockholm, Sweden. E-mail: asa.rosen@sofi.su.se

Abstract

It is well known in personnel economics that firms may improve the quality of their work-force by offering performance pay. We analyse an equilibrium model where worker productivity is private information and show that the firms' gain from worker self-selection may not be matched by a corresponding social gain. In particular, the equilibrium incentive contracts are too high-powered inducing the more productive workers to exert too much effort and increasing agency costs stemming from the misallocation of effort.

Key words: Performance pay, selection, efficiency.

JEL codes: D82, J30

1 Introduction

In his book on personnel economics, Lazear (1998) stresses the importance of performance pay for firm profitability. He argues that the gains from performance pay are twofold. First, and most obviously, performance pay mitigates moral hazard problems. Second, performance contracts affect the quality of workers applying to a firm. When workers have private information about their productivity at the hiring stage, firms can attract higher quality workers by offering more high-powered contracts. That is, good workers self-select into jobs offering more performance sensitive compensations, e.g., large bonus packages (Lazear 1986).

Incentive contracts may also give rise to agency costs. Agency costs can broadly be divided into three categories: suboptimal risk sharing (Hart and Holmstrom 1987), rent extraction (Laffont and Tirole 1993, Moen and Rosén 2004), and misallocation of effort across multiple tasks (Holmstrom and Milgrom 1991). In this paper, we focus on misallocation of effort across tasks, which is relevant if not all aspects of a worker's output can be adequately measured and compensated for. The workers then concentrate too much on tasks that give rise to performance pay while neglecting the tasks that do not. Standard examples include too much focus on quantity relative to quality, neglect of cooperation, inadequate maintenance of productive assets, and possibly too much focus on short-term rather than long-term performance.[1]

The contribution of this paper is to analyze the welfare properties of markets with heterogenous workers in which firms offer incentive contracts, as often recommended by personnel economists. We argue that the private gains associated with the selection effect may not reflect social gains, because the number of talented people in the economy is limited. We examine to what extent a rat-race between firms for talented workers may lead to excessive use of performance pay, too high agency costs, and thus an inefficient allocation of resources.

Our central proposition is that the incentive power of the equilibrium wage contract exceeds its socially efficient level. The excessive incentive power of the equilibrium wage contracts can be dampened by a tax on high incomes. We derive these results in a simple adverse selection model à la Rothschild and Stiglitz (1976).

The paper is organized as follows. Section 2 describes the model and derives the equilibrium with observable worker types. Section 3 analyses the equilibrium outcome when the workers' type is private information and

[1] The latter point requires that the workers have a shorter time horizon than the firm, which is typically the case due to worker turnover.

firms offer linear wage-contracts. Section 4 discusses the robustness of the results when allowing for non-linear wage contracts, and section 5 concludes. Mathematical derivations and proofs are provided in the appendix.

2 Model and Benchmark

In this section we present the basic features of the model with two observable types of workers. We deliberately construct the model in such a way that the optimal contract is linear over the relevant intervals, as in Holmstrom and Milgrom (1987).

2.1 Framework

There are many firms and many employees in each firm. All agents are risk neutral. Each worker undertakes two tasks. The total value of production of any given worker is given by

$$y = e_1 + e_2 + \varepsilon \tag{1}$$

where e_1 and e_2 is the effort spent on task one and two, respectively. Task 1 may for instance be related to product quality, and task 2 to quantity. The term ε reflects a random influence on output such as the difficulty of the particular task in question, or a worker-firm specific productivity component. The distribution of ε is continuous and symmetric on the interval $[-\overline{\varepsilon}, \overline{\varepsilon}]$. Firms do not observe the realization of ε but they know its distribution. A worker observes ε after the contract is signed, but before he chooses his effort levels.

The existence of ε implies that a simple, non-linear contract with a bonus for output values above a certain threshold is not efficient. As will be made clear below, optimal effort provision requires that the worker has the same incentives on the margin for all realizations of ε. This can only be achieved by a linear contract.

The firm can not observe e_1, e_2, nor y, but only a distorted measure $\widetilde{y}$ of their output, given by

$$\begin{aligned} \widetilde{y} &= (1-\gamma)e_1 + (1+\gamma)e_2 + \varepsilon \\ &= \widetilde{e} + \varepsilon \end{aligned}$$

where $\widetilde{e} = (1-\gamma)e_1 + (1+\gamma)e_2$, and where $\gamma > 0$ reflects the measurement error. The firm thus observes a distorted measure of e_1 and e_2. Due to the measurement problems, e_1 carries less weight than e_2 in the performance evaluation.

Effort above a certain level is costly for workers, and this cost depends on the type of the worker, which is either high (h) or low (l). The expected utility of type $k = \{l, h\}$ is

$$u^k = Ew - C^k(e_1, e_2)$$

where w is the wage and C^k the effort cost, the latter given by

$$C^k(e_1, e_2) = \frac{(e_1 - e^{0k})^2}{2} + \frac{(e_2 - e^{0k})^2}{2} \tag{2}$$

for $e_1, e_2 \geq e^{0k}$. We assume that $e^{0l} < e^{0h}$, reflecting that the latter type is more productive.

The sequence of the moves is as follows:

1. The firm signs contracts with all its employees (individually).
2. Each worker learns the realization of ε which are independent across workers. Due to unmodelled costs of changing jobs, a worker does not want to quit even when a low value of ε realizes.[2]
3. Each worker chooses effort levels e_1 and e_2.
4. Firms observe the distorted output measure $\widetilde{y} = \widetilde{e} + \varepsilon$ for each of their employees and remunerate them accordingly.

2.2 Optimal contracts with observable types

With observable worker types, the optimal contract leads to a segmentation into submarkets, one for each type. In order to simplify notation we supress the superscript k.

An optimal wage contract $f(\widetilde{y})$ maximizes the firm's expected profit subject to the following constraints:

1. Incentive compatibility constraint: Workers choose effort levels so as to maximize their utility.

[2] Switching costs could be endogenized in a search and matching context, see for instance Pissarides (2000) or Moen (1997).

2. Individual rationality: The contract provides each worker with at least his reservation expected utility, denoted by u^0.

While the firm can affect a worker's measured effort level ($\widetilde{e}$), it cannot control how the worker allocates effort across the two tasks. Among all effort combinations that yield the worker the same wage w, the worker always chooses the pair (e_1, e_2) that minimizes his effort costs. The Lagrangian associated with this minimization problem is

$$L = (e_1 - e^0)^2/2 + (e_2 - e^0)^2/2 - \lambda[e_1(1-\gamma) + e_2(1+\gamma) - \widetilde{e}].$$

Minimizing L with respect to e_1 and e_2 gives

$$\frac{e_2 - e^0}{e_1 - e^0} = \frac{1+\gamma}{1-\gamma} \tag{3}$$

which is independent of measured total effort $\widetilde{e}$ and the chosen wage contract.

The gain to the firm of the worker's effort is $e_1 + e_2$. In the appendix we show that

$$e_1 + e_2 = 2e^0\frac{\gamma^2}{1+\gamma^2} + \frac{1}{1+\gamma^2}\widetilde{e}. \tag{4}$$

The firm's (gross) benefit from a higher measured effort is largest in the absence of distortion ($\gamma = 0$), and its marginal benefit decreases proportionally with $1 + \gamma^2$.

Equations (3) and (4) have several interesting implications. The production function (equation 1) and the effort cost (equation 2) imply that optimal effort allocation across tasks is $e_1 = e_2$. If $\widetilde{e} = 2e^0$, the worker sets $e_1 = e_2 = e^0$, and there is no misallocation on tasks. For larger values of $\widetilde{e}$, equation (3) shows that effort in excess of e^0 is distorted in the direction of e_2. Equation (4) shows that the productivity $e_1 + e_2$ is less than proportional to $\widetilde{e}$. In relative terms, the misallocation of effort increases with $\widetilde{e}$. Wage contracts with a high incentive power give rise to a high value of $\widetilde{e}$ and thus large distortions in effort allocation across tasks.

The gain to the firm of measured effort is given by equation (4). On the margin, the gain to the firm of measured effort is constant and equal to $\frac{1}{1+\gamma^2}$. Consider a linear wage contract $w = \alpha + \beta\widetilde{y}$ and choose $\beta = \frac{1}{1+\gamma^2}$. For any realization of ε, this contract gives the worker the right incentives: on the margin, the worker is paid the entire marginal gain from his effort, given by $\frac{1}{1+\gamma^2}$. As the firm can extract all the rent by adjusting α in such a way that the participation constraint binds, this linear contract is optimal.

Due to the stochastic term ε, the firm cannot implement the constrained optimal measured effort level (which we denote by $\widetilde{e}^*$) with a trigger contract in which the wage is discontinuous at some point. In order to implement the optimal effort level for all values of ε, the wage contract must be such that the marginal effect of increased $\widetilde{y}$ on wages must be equal to $\frac{1}{1+\gamma^2}$ over the entire interval $[\widetilde{e}^* - \overline{\varepsilon}, \widetilde{e}^* + \overline{\varepsilon}]$. Thus, any optimal contract is linear with slope $\frac{1}{1+\gamma^2}$ on this interval.[3] Finally, the zero-profit condition determines the equilibrium value of α.

Proposition 1. *The unique optimal contract is linear (on the relevant intervals) and can be written as*

$$w = \alpha + \beta\widetilde{y}$$

where

$$\alpha = 2e^0 \frac{\gamma^2}{1+\gamma^2} \tag{5}$$

$$\beta = \frac{1}{1+\gamma^2}. \tag{6}$$

The value of α is derived in the appendix, where we also show that the equilibrium expected utility u^0 is given by

$$u^0 = 2e^0 + \frac{1}{1+\gamma^2}. \tag{7}$$

3 Unobservable worker types

When firms cannot observe worker types, they may induce workers to self-select just as in insurance markets with adverse selection (Rothschild and Stiglitz 1976). For expositional simplicity, we continue to consider linear contracts. In the next section we will argue that our main result also holds with non-linear contracts.

An equilibrium in this market must satisfy the following conditions:[4]

[3] Note the resemblance with Holmstrom and Milgrom (1987). They study the trade-off between incentive provision and risk sharing in a continuous time model in which workers have more information about the state of the world. In this setting the optimal contract is linear if the workers have constant absolute risk aversion.

[4] We assume that each firm offers at most one contract. This assumption is not important, as firms have constant returns to scale production technology.

1. Workers apply to firms that offer them the best contract.
2. Firms choose contracts to maximize their profits, given the workers' behavior.
3. Free entry of firms.

We derive the competitive equilibrium outcome with market clearing, and do not allow for rationing as a sorting mechanism (see Gale 1992). With observable types, firms offer equally high-powered incentive components to both types ($\beta^h = \beta^l$), but pay the high-type a higher fixed salary ($\alpha^h > \alpha^l$). This is no longer an equilibrium when types are not observable, as low-type workers would have an incentive to take jobs intended for high-type workers.

A pooling equilibrium does not exist in the present setting. To see this, suppose firms offer a pooling contract. Consider a firm that deviates slightly and offers a contract with stronger incentives and lower fixed pay. This firm can attract only high-type workers. The reason is that high-type workers are more willing to accept a lower fixed salary component in return for stronger incentives (higher production related bonuses). Thus, by increasing β slightly above $\frac{1}{1+\gamma^2}$, and lowering α so that low-type workers are marginally better off with the initial contract, the firm attracts high-type workers but not low-type workers.

Lemma 1: *Suppose the firms in the market offer a pooling contract* (α^p, β^p)*. Then there exists another contract, arbitrarily close to* (α^p, β^p) *that attracts high-type workers only.*

The next step is to show that for a given contract, firms prefer to attract high-type workers. From equation (A12) in the appendix it follows that

$$\frac{\partial \pi}{\partial e^0} = 2(1-\beta) \tag{8}$$

where π denotes expected profits. Thus, as long as $\beta < 1$, firms strictly prefer to hire high-type workers rather than low-type workers on a given contract. But this rules out any pooling contracts in which $\beta < 1$.. Furthermore, a pooling equilibrium with $\beta = 1$ does not exist either, as low-type workers would prefer a contract with $\beta = \frac{1}{1+\gamma^2}$ and α set according to proposition 1.

Lemma 2. *There exists no pooling equilibrium*

Thus, any equilibrium of the model has to be a separating equilibrium. Denote the equilibrium wage contracts by (α^k, β^k), with $k = l, h$. In a separating equilibrium, the contract offered to low-type workers coincides with the optimal contract with observable types. If not, a firm that offers a contract with $\beta = \frac{1}{1+\gamma^2}$ and α slightly below $2e^{0l}\frac{\gamma^2}{1+\gamma^2}$ would attract workers and earn positive profits. It follows that $\beta^l = \frac{1}{1+\gamma^2}$ and $\alpha^l = 2e^{0l}\frac{\gamma^2}{1+\gamma^2}$..

The contracts for high-type workers must be such that low-type workers do not apply for the jobs intended for high-type workers. To ensure separation, the fixed salary must be smaller and the performance component larger than in the optimal contract with observable types. The indirect utility function of a low-type worker is given by (see equation A8 in the appendix)

$$u^l(\alpha, \beta) = \alpha + \beta 2e^{0l} + \beta^2(1+\gamma^2). \tag{9}$$

In a separating equilibrium, only high-type workers apply for high-type jobs. In the appendix (see equation A13), we show that the zero-profit condition for firms implies the following relationship between α^h and β^h

$$\alpha^h = 2(1-\beta^h)e^{0h} + 2\beta^h(1-\beta^h(1+\gamma^2)). \tag{10}$$

By combining equations (9) and (10), we obtain the expected utility of a low-type worker who applies for a high-type job

$$u^l(\alpha^h, \beta^h) = 2e^{0l} + 2(1-\beta^h)(e^{0h} - e^{0l}) + [2\beta^h - (\beta^h)^2(1+\gamma^2)]. \tag{11}$$

The first term of this expression is the same as in equation (7). The second term in equation (11) reflects the additional income due to higher average productivity of high-type workers. When $\beta^h = 1$, this additional income vanishes. Due to the distorted measure of effort, there are, however, costs associated with providing so strong incentives. The last term in equation (11) is maximized for $\beta^h = \frac{1}{1+\gamma^2}$, in which case it takes the value of $\frac{1}{1+\gamma^2}$, and is equal to the last term of equation (7). For $\beta^h > \frac{1}{1+\gamma^2}$, the term decreases in β^h reflecting that it is costly to increase the incentives above the optimal level. Due to the free entry of firms, this cost is ultimately borne by the workers.

In a separating equilibrium $u^l(\alpha^l, \beta^l) \geq u^l(\alpha^h, \beta^h)$ must hold. In the appendix, we show that this condition is equivalent to (see equation A18):

$$(\beta^h - \frac{1}{1+\gamma^2})^2(1+\gamma^2) \geq 2(1-\beta^h)(e^{0h} - e^{0l}). \tag{12}$$

Since the incentive compatibility constraint binds in a separating equilibrium, β^h lies in the interval $(\frac{1}{1+\gamma^2}, 1)$.[5]

[5] If we solve this equation, we find that β^h is given by

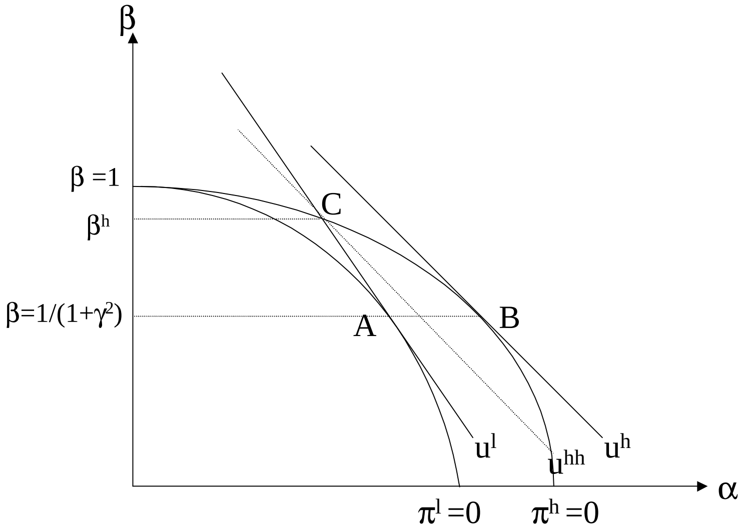


Figure 1 shows the indifference curves of low-type workers and of high-type workers in the α, β space (given by equation A8). The iso-profit curves $\pi^h = 0$ and $\pi^l = 0$ are the combinations of α and β that yield zero profits to a firm attracting high-type and low-type workers (equation A13). If the worker type were observable, the equilibrium contract would be given by point A for low-type workers and point B for high-type workers. With unobservable types, the equilibrium contract for high-types is given by point C.

We know from Rothschild and Stiglitz (1976) that a separating equilibrium may not exist for all parameter values. The reason is that an efficient contract that attracts both types may be more profitable than the optimal contracts in a separating equilibrium. This happens if the share of low-productivity workers in the economy is sufficiently low. In this case no equilibrium exists, since pooling can never be an equilibrium.

Let us consider the existence of equilibrium in some more detail. Consider a separating equilibrium candidate, where the low-type workers choose a contract (α^l, β^l) with β^l equal to $1/(1+\gamma^2)$, while the high-type workers

---

$$\beta^h = \sqrt{\Gamma^2[(1-\Delta)^2 - 1] + 2\Delta\Gamma} + \Gamma(1-\Delta),$$

with $\Gamma = \frac{1}{1+\gamma^2}$ and $\Delta = e^{0h} - e^{0l}$.

choose a contract (α^h, β^h). Consider a firm that deviates and offer a pooling contract (α^p, β^p). The optimal pooling contract is characterized by an incentive power $\beta^p = \frac{1}{1+\gamma^2}$. The constant $\alpha^p > \alpha^l$ is set so as to satisfy the participation constraint of the high-type workers $(u^h(\alpha^p, \beta^p) = u^h(\alpha^h, \beta^h))$. The firm offering this pooling contract earns a positive profit if it hires a high-type worker, but a negative profit if it attracts a low-type worker (since $\alpha^p > \alpha^l$). Following Rothschild and Stiglitz (1976), we assume that the probability of attracting a given type is equal to the proportion of that type in the market. Let a denote the share of low-type workers in this economy. In the appendix we show that the expected gain $\Delta\pi^d$ from deviating and offering the pooling contract is equal to

$$\Delta\pi^d = (1-a)(1+\gamma^2)(\beta^h - \frac{1}{1+\gamma^2})^2 - 2a(\beta^h - \frac{1}{1+\gamma^2})(e^{0h} - e^{0l}). \quad (13)$$

The first term reflects the expected gain of hiring a high-type worker, and the second term the expected loss of hiring a low-type worker. A separating equilibrium exists whenever $\Delta\pi^d \leq 0$. Since β^h is independent of the proportion of low-type workers a, the next lemma follows directly:

Lemma 3.*For any parameter constellation e^{0h}, e^{0l} and γ there exists an a', $0 < a' < 1$ such that a separating equilibrium exists whenever $a \geq a'$ and does not exist whenever $a < a'$.*

In what follows we assume that the parameter values are such that the separating equilibrium exists.[6]

Welfare analysis

We define welfare as the sum of the workers' and firms' payoffs,

$$W = au^l + (1-a)u^h + \pi \quad (14)$$

where π is the firms' expected profit per worker (the measure of workers is normalized to 1). We now consider a social planner who chooses contracts so

[6]Alternatively, the existence of an equilibrium can be ensured by refining the equilibrium concept. Cho and Kreps (1987), Riley (1979), Hellwig (1987), Mailath et al. (1993) and Asheim and Nilssen (1996) are contributions to the literature on equilibrium refinements in signalling games. For instance, Riley (1979) develops an equilibrium concept (reactive equilibrium) that constrains the set of admissible deviating strategies that can break an equilibrium. A set of contracts is a reactive (Riley) equilibrium if no other contract exists that remains profitable even after yet another new (deviating) contract is offered. It can be shown that the separating equilibrium derived in the paper is the unique reactive equilibrium for all parameter values.

as to maximize welfare, subject to the workers' incentive compatibility and individual rationality constraints.

Suppose first that there is only one worker type in the economy. As wages cancel out in equation (14), it follows that $W = e_1 + e_2 - C(e_1, e_2)$, or (from equation 4)

$$W = 2e^0 \frac{\gamma^2}{1+\gamma^2} + \frac{1}{1+\gamma^2}\widetilde{e} - C(\widetilde{e})$$

where $C(\widetilde{e})$ is given by equation (A4) in the appendix. The first order condition is thus that $C'(\widetilde{e}) = 1/(1+\gamma^2)$. In equilibrium workers choose $\widetilde{e}$ such that $C'(\widetilde{e}) = \beta = 1/(1+\gamma^2)$. Hence, the equilibrium with one worker type is constrained efficient.

With two worker types, the constrained optimal values of $\widetilde{e}^h$ and $\widetilde{e}^l$ maximize W given by

$$W = a[2e^{0l}\frac{\gamma^2}{1+\gamma^2} + \frac{1}{1+\gamma^2}\widetilde{e}^l - C^l(\widetilde{e}^l)] + (1-a)[2e^{0h}\frac{\gamma^2}{1+\gamma^2} + \frac{1}{1+\gamma^2}\widetilde{e}^h - C^h(\widetilde{e}^h)]$$

The first order conditions are given by $C^{h\prime}(\widetilde{e}^h) = C^{l\prime}(\widetilde{e}^l) = 1/(1+\gamma^2)$. Hence, when types are unobservable, the planner can obtain the optimal allocation of resources by offering a pooling contract with $\beta = \frac{1}{1+\gamma^2}$ and set α in accordance with the zero profit condition in a pooling equilibrium.

Proposition 2. *The separating equilibrium is not efficient, as the high-type workers are offered contracts that are too high powered (β^h is too high).*

The overall output in the separating equilibrium exceeds the output in the efficient allocation. From a welfare point of view, this additional output comes at a too high (effort) cost. Compared to the equilibrium with observable types, the high-type workers suffer: In order to obtain separation, high-type workers are offered contracts that provide them with too strong incentives. They receive high salaries, but have to exert excessive effort, which reduces their utility.[7]

Note that proposition 2 does not depend on our assumption of free entry. Suppose first that firms incur a cost F of opening a job. It is straight-forward to show that this only reduces the fixed payment α^h and α^l by F units,

[7] When using the refinement concept of "undefeated equilibrium" (Mailath et al. 1993), a pooling equilibrium exists if the fraction a of low-type workers is sufficiently small. The equilibrium pooling contract is such that the utility of high-type workers is maximized. Applied to our setting, this refinement concept implies that the value of β in the pooling equilibrium exceeds the constrained efficient level.

otherwise leaving the equilibrium unchanged. Suppose then that the number of firms is exogenously given. If there are more firms than workers, the zero profit condition still applies. If there are more workers than firms, but fewer high-type workers than firms, low-type workers obtain zero utility and α^l adjusts accordingly, while β^l and β^h remain unchanged. Thus, proposition 2 still applies. Proposition 2 only breaks down if there are fewer firms than high-type workers, in which case only high-type workers are employed and receive zero utility.

As shown, the constrained welfare maximizing contract is the pooling contract with $\beta = 1/(1+\gamma^2)$. One way to implement this contract for all workers is to have unions negotiate wages at the industry level.[8] If the union maximizes the expected (or average) utility of its members, i.e., $au^l + (1-a)u^h$, efficient bargaining results in a single wage contract with $\beta = 1/(1+\gamma^2)$.[9]

Absent such wage negotiations at the industry level, progressive taxes may improve welfare.

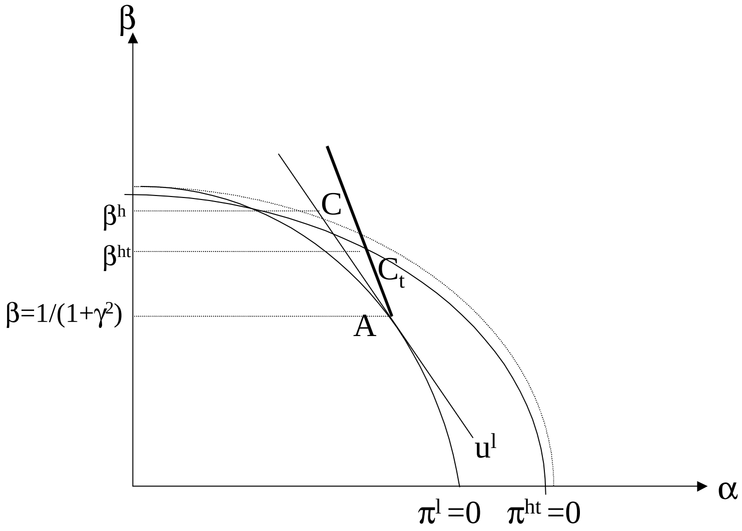
Figure 2 illustrates the effect of a marginal tax on incomes exceeding w^l, the low-type equilibrium income. First note that an increase in β along the indifference curve (i.e., matched by a decrease in α) leads to higher worker income as well as higher effort. The tax affects the low-type workers' indifference curve only for β values above $\beta^l = 1/(1+\gamma^2)$, which therefore shifts out (though not necessarily in a linear way). In addition, the iso-profit curve of firms attracting high-type workers shifts inwards, as the tax lowers effort (for a given β). But this implies that the equilibrium value of β falls from β^h to β^{ht}. Thus the excessive incentives for high-type workers are reduced. For small values of the tax rate it thus follows that welfare increases.[10] In the next section we will argue that this result does not crucially depend on our restriction to linear contracts.

Of course, progressive taxes have other effects that are not captured in our model. Still, self-selection of high-type workers on high-powered incentive contracts provide a new argument in favor of progressive taxes.

[8] This point was suggested to us by an anonymous referee.

[9] Although this contract would be socially efficient, high-type workers lose compared to the separating equilibrium.

[10] A similar result can be found in Moen (2003), where search frictions lead to excessive wages for high-type workers that can be dampened by marginal taxes.



4 Non-linear contracts

We know from section 2 that (constrained) efficiency requires that the contract is linear on the relevant interval. However, a firm that attracts high-type workers may try to prevent low-type workers from applying to their firm by specifying a very low wage if output falls below a certain threshold $\widetilde{y}^t$. To avoid distortions for the high-type workers, such a threshold must satisfy the condition $\widetilde{y}^t \leq \widetilde{e}^h(\frac{1}{1+\gamma^2}) - \overline{\varepsilon}$, where $-\overline{\varepsilon}$ is the lower bound of the support of ε.

Consider a firm that offers a contract $(\alpha^h, \frac{1}{1+\gamma^2})$, where α^h satisfies the zero-profit constraint with high-type workers. In addition, the firm sets a wage w' if output is less than $\widetilde{y}^t$. Even in the absence of a lower bound on w' (which seems unreasonable as long as slavery is forbidden), this may not be sufficient to satisfy the low-type workers' individual rationality constraint, because a low-type worker can always obtain $\widetilde{y}^t$ by working sufficiently hard.

Define $\Delta e^0 \equiv e^{0h} - e^{0l}$. Our last proposition shows that if Δe^0 is not too large, there exists no contract for high-type workers that implements first best:

Proposition 3. *There exists a value $\overline{\Delta e} > 0$ such that if $\Delta e^0 < \overline{\Delta e}$, there is no contract for high-type workers that implements a constrained efficient effort level for high-type workers and at the same time satisfies the low-type workers' incentive compatibility constraint.*

The value of $\overline{\Delta e}$ depends on the distribution of ε. In order to do comparative statistic, we define $\varepsilon = \sigma\widehat{\varepsilon}$, where $\widehat{\varepsilon}$ is a single-peaked stochastic variable and σ is a shift parameter (noise term). In the appendix we show that $\overline{\Delta e}$ goes to infinity as σ goes to infinity. Thus, if the support of the error term is unbounded, the inefficiency result holds for all parameter values. Note, however, that the value of $\overline{\Delta e}$ remains strictly positive also when σ goes to zero.

Thus, our result that the separating equilibrium is inefficient carries over to the general case with non-linear contracts, although the welfare loss may be smaller. For large differences between worker types, separation may be possible without distorting the incentives for high-type workers. However, this may require an unreasonably low (negative) wage if output falls below $\widetilde{y}^t$.

We conjecture that the welfare improving effect of progressive taxes also carries over to the case with non-linear contracts. Given that first best contracts do not implement separation, the logic underlying lemma 2 implies that the equilibrium (if it exists) is a separating equilibrium. In order to obtain separation, firms attracting high-type workers must lower base pay and increase performance pay (at least on some intervals). Again, progressive taxes relax the incentive compatibility constraint of low-type workers, reduce the excessive effort among high-type workers, and may thus improve welfare.

5 Concluding remarks

At the firm level, performance pay may lead to a positive selection effect, as highly productive workers are more attracted by performance pay than less productive workers. This paper demonstrates that the private gains from selection are not matched by social gains. The equilibrium incentive contracts provide too strong incentives, inducing high-type workers to exert too much effort and increasing agency costs resulting from the misallocation of effort. A tax on high incomes may reduce the incentive power of the contract targeted at high-type workers and thereby increase welfare.

The economic significance of this market failure depends on the prevalence of incentive contracts and on the importance of self-selection. Based on the National Longitudinal Survey of Youth, Lazear (2000) reports that the fraction of employees working on piece rate contracts is quite small, only 3.3% among young workers in the US in 1990. Using the same data source, MacLeod and Malcomson (1998) conclude that this number rises to 24% when bonuses and commissions are included. According to Millward et al. (1992), the fraction of workers in the United Kingdom that received some kind of merit pay was 34 % in 1990.[11] Still, the relevant fraction may be even higher, as our notion of performance pay also includes both promotions based on performance and fixed salaries based on past performance. Thus, performance pay, broadly interpreted, seems to be common.

How important is self-selection on contracts? Lazear (1986) was the first to propose that firms offering performance pay can attract high-ability workers. This reason for performance pay has received considerable attention in economic theory, as well as in the management and organization literature (e.g. Lazear 1998) and in the financial press (The Economist 1998). As regards the empirical evidence, a number of papers examine the effects of introducing performance pay on worker productivity (see Prendergast 1999 for a survey). In most of these studies the data does, however, not allow to disentangle the incentive and the selection effects.

A notable exception is Lazear (2000), who analyses the effects of introducing piece rate payments in one particular firm. Lazear finds that the selection effect accounts for almost 50 percent of the improvement in worker productivity. However, when the firm introduced performance pay, it simultaneously increased average wages (for a given performance). Hence, the selection effect may reflect the effect of higher average salaries rather than of performance pay.

Another exception is a study by Sørensen and Grytten (2003). They study differences in performance among Norwegian physicians, who choose to either work on a fixed wage contract or on a performance based contract. Sørensen and Grytten find that physicians on performance pay have around 35 percent more patients than those on fixed pay and attribute around 1/3 of the difference to the selection effect. A later study by Grytten et al. (2004) examines a reform in the Norwegian health system that imposed a performance based remuneration on all physicians. After the reform, the average number of patients per physicians was lower than the average number per physician who prior to the reform voluntarily chose to be on performance pay. The authors estimate that 30 percent of the productivity increase associated with performance pay is due to the selection effect, confirming the previous

[11] Merit pay is defined as payment that depends on subjective judgement by a supervisor or a manager of the individual's performance.

findings in Sørensen and Grytten (2003).

Eriksson and Villeval (2004) report results from a laboratory experiment in which fixed salaries and performance pay are offered by firms and chosen by low-skilled and high-skilled workers. The participants in the experiment are randomly assigned the roles of firm, high-skilled worker, and low-skilled worker. The experiment shows a concentration (self-selection) of high skilled workers in firms offering performance pay.

Our results crucially depend on some kind of agency costs associated with performance pay. In this paper the agency cost of performance pay is a misallocation of effort across tasks, and that increases with the incentive power of the wage contract. As actual output grows less than proportional to measured output (see the discussion in section 2), firms reduce the variable part and increase the fixed part of the salary compared to the situation with no agency costs. As a result, firms prefer to hire high-type workers to low-type workers on a given contract. If there were no misallocation of effort ($\gamma = 0$), then $\beta = 1$ and $\alpha = 0$ would be optimal, independent of worker type. In this case, the firm is indifferent as to which type of worker it hires, and there is no welfare loss due to excessive effort by high-type workers.

We conjecture that similar inefficiency results obtains with other kinds of agency costs that also reduce the optimal incentive power below 1. For instance, if workers are risk averse and firms risk neutral, the optimal contract trades off incentives and worker insurance, and the resulting contract provides workers with less than full incentives. As a result, the firm prefers to hire a high-type worker to a low-type worker on a given contract. We conjecture that the resulting equilibrium is separating with excessive effort provision and too little insurance for high-type workers.

To sum up, our analysis suggests that inefficiencies created by competition among firms to attract high-type workers are most prevalent when performance pay is present but when the incentive power of the contracts is reduced due to agency costs.

6 Appendix

6.1 Derivation of equation (4)

From the definition of $\widetilde{e}$, it follows that

$$(e_1 - e^0)(1-\gamma) + (e_2 - e^0)(1+\gamma) = \widetilde{e} - 2e^0.$$

Substituting $e_2 - e^0$ by using equation (3) gives

$$(e_1 - e^0)[1 - \gamma + \frac{(1+\gamma)^2}{1-\gamma}] = \widetilde{e} - 2e^0$$

or

$$e_1 - e^0 = \frac{(\widetilde{e} - 2e^0)}{2}\frac{1-\gamma}{1+\gamma^2}. \tag{A1}$$

It thus follows that

$$e_2 - e^0 = \frac{(\widetilde{e} - 2e^0)}{2}\frac{1+\gamma}{1+\gamma^2} \tag{A2}$$

$$\begin{aligned} e_1 + e_2 &= 2e^0 + \frac{(\widetilde{e} - 2e^0)}{2}[\frac{1-\gamma}{1+\gamma^2} + \frac{1+\gamma}{1+\gamma^2}] \\ &= 2e^0 + (\widetilde{e} - 2e^0)[\frac{1}{1+\gamma^2}] \\ &= 2e^0\frac{\gamma^2}{1+\gamma^2} + \widetilde{e}\frac{1}{1+\gamma^2}. \end{aligned} \tag{A3}$$

6.2 Deriving equations (5) and (7)

The costs (for workers) of providing measured effort:

Let us denote the cost function by $C(\widetilde{e})$. From equations (A1) and (A2), it follows that

$$\begin{aligned} C(\widetilde{e}) &= [\frac{\widetilde{e} - 2e^0}{2}\frac{1-\gamma}{1+\gamma^2}]^2/2 + [\frac{\widetilde{e} - 2e^0}{2}\frac{1+\gamma}{1+\gamma^2}]^2/2 \\ &= \frac{(\widetilde{e} - 2e^0)^2}{8}[\frac{(1-\gamma)^2}{(1+\gamma^2)^2} + \frac{(1+\gamma)^2}{(1+\gamma^2)^2}] \\ &= \frac{(\widetilde{e} - 2e^0)^2}{4}\frac{1}{1+\gamma^2}. \end{aligned} \tag{A4}$$

Choice of measured effort

We first derive a worker's choice of measured effort $\widetilde{e}$ for an arbitrary contract. The worker chooses $\widetilde{e}$ to maximize

$$\begin{aligned} u(\varepsilon) &= \alpha + \beta\widetilde{e} - C(\widetilde{e}) + \varepsilon \\ &= \alpha + \beta\widetilde{e} - \frac{(\widetilde{e} - 2e^0)^2}{4}\frac{1}{1+\gamma^2} + \varepsilon \qquad \text{(A5)} \end{aligned}$$

and the solution is

$$\widetilde{e} = 2e^0 + 2\beta(1+\gamma^2). \qquad \text{(A6)}$$

With an optimal contract ($\beta = \frac{1}{1+\gamma^2}$) this expression simplifies to $\widetilde{e} = 2e^0 + 2$. Using equations (A6) and (A3) yields

$$e_1 + e_2 = 2e^0 + 2\beta. \qquad \text{(A7)}$$

The (expected) indirect utility function

Inserting the optimal value of $\widetilde{e}$ (equation A6) into the maximand (A5), gives

$$\begin{aligned} u &= \alpha + \beta(2e^0 + 2\beta(1+\gamma^2)) - \frac{[2\beta(1+\gamma^2)]^2}{4}\frac{1}{1+\gamma^2} \\ &= \alpha + \beta(2e^0 + 2\beta(1+\gamma^2)) - \beta^2(1+\gamma^2) \\ &= \alpha + \beta 2e^0 + \beta^2(1+\gamma^2). \qquad \text{(A8)} \end{aligned}$$

For an optimal contract, it follows that u is given by

$$u = \alpha + 2e^0\frac{1}{1+\gamma^2} + \frac{1}{1+\gamma^2}. \qquad \text{(A9)}$$

Reservation utility u^0 and the value of α:

The firm sets α such that $u = u^0$, the equilibrium value of u. From equation (A8), it therefore follows that the firm sets α such that

$$\alpha = u^0 - \beta 2e^0 - \beta^2(1+\gamma^2). \qquad \text{(A10)}$$

Given the optimal value of β, $\beta = \frac{1}{1+\gamma^2}$, we thus have that

$$\alpha = u^0 - \frac{2e^0 + 1}{1 + \gamma^2}. \tag{A11}$$

Free entry and the value of α:

For any given contract, equations (A6) and (A3) imply that expected profits is given by

$$\begin{aligned} \pi &= e_1 + e_2 - \beta\widetilde{e} - \alpha \\ &= 2e^0 + 2\beta - \beta(2e^0 + 2\beta(1+\gamma^2)) - \alpha \\ &= 2(1-\beta)e^0 + 2\beta(1 - \beta(1+\gamma^2)) - \alpha. \end{aligned} \tag{A12}$$

Thus, the zero profit condition amounts for a given contract to

$$\alpha = 2(1-\beta)e^0 + 2\beta(1 - \beta(1+\gamma^2)). \tag{A13}$$

and for the optimal contract to

$$\alpha = 2e^0 \frac{\gamma^2}{1+\gamma^2}. \tag{A14}$$

We have thus derived equation (5). Combining equation (A14) and equation (A9) gives equation (7).

6.3 Proof of lemma 1

Proof. Let u^{0h} and u^{0l} be the expected income of high-type workers and low-type workers in the pooling equilibrium. Let $\delta > 0$ be arbitrarily small. We want to show that for any δ there exists a k such that the contract $(\alpha^* - k\delta, \beta^* + \delta)$ attracts high-type workers only, i.e., such that $u^h(\alpha^* - k\delta, \beta^* + \delta) \geq u^{0h}$ and $u^l(\alpha^* - k\delta, \beta^* + \delta) < u^{0l}$. From the envelope theorem, it follows that $\frac{\partial u^k}{\partial \beta} = \widetilde{e}^k$, $k = l, h$.. From equation (A6), we know that $\widetilde{e}^h = \widetilde{e}^l + 2(e^{0h} - e^{0l})$ for any given contract, and thus that $\frac{\partial u^h}{\partial \beta} = \frac{\partial u^l}{\partial \beta} + 2(e^{0h} - e^{0l})$. Thus, increasing β by δ and reducing α by $\widetilde{e}^h\delta$ yields the same expected utility to the high-type workers, while low-type workers are strictly worse under this modified contract. ■

6.4 Derivation of equation (12)

In the high-type market, we know that the zero profit condition holds. Let $u^k(\alpha, \beta)$ denote the expected utility of a k type worker applying for a job with a contract (α, β). Combining equation (A8) and equation (A13) implies that a high-type worker applying for a high-type contract may expect a utility

$$\begin{aligned} u^h(\alpha^h, \beta^h) &= 2(1-\beta^h)e^{0h} + 2\beta^h(1-\beta^h(1+\gamma^2)) + \beta^h 2e^{0h} + \beta^{h2}(1+\gamma^2) \\ &= 2e^{0h} + 2\beta^h - \beta^{h2}(1+\gamma^2) \qquad \text{(A15)} \end{aligned}$$

which (of course) is maximized for $\beta^h = \frac{1}{1+\gamma^2}$. The expected utility of a low-type worker applying for a high-type job is given by (from equation A8, then replacing α with equation A13),

$$\begin{aligned} u^l(\alpha^h, \beta^h) &= \alpha^h + \beta^h 2e^{0l} + \beta^{h2}(1+\gamma^2) \\ &= 2(1-\beta^h)e^{0h} + 2\beta^h(1-\beta^h(1+\gamma^2)) + \beta^h 2e^{0l} + \beta^{h2}(1+\gamma^2) \\ &= 2(1-\beta^h)e^{0h} + \beta^h 2e^{0l} + 2\beta^h - \beta^{h2}(1+\gamma^2). \qquad \text{(A16)} \end{aligned}$$

The expected utility of a low-type worker applying for a low-type job is (the same formula as equation A15 with $\beta = \frac{1}{1+\gamma^2}$ and e^{0l} instead of e^{0h})

$$u^l(\alpha^l, \beta^l) = 2e^{0l} + \frac{1}{1+\gamma^2}. \qquad \text{(A17)}$$

In order to obtain separation, we must have that

$$u^l(\alpha^l, \beta^l) \geq u^l(\alpha^h, \beta^h)$$

or

$$2e^{0l} + \frac{1}{1+\gamma^2} \geq 2(1-\beta^h)e^{0h} + \beta^h 2e^{0l} + 2\beta^h - \beta^{h2}(1+\gamma^2)$$

or

$$\frac{1}{1+\gamma^2} - [2\beta^h - \beta^{h2}(1+\gamma^2)] \geq 2(1-\beta^h)(e^{0h} - e^{0l}).$$

As the left-hand side can be written as $(\beta^h - \frac{1}{1+\gamma^2})^2(1+\gamma^2)$, the inequality reduces to

$$(\beta^h - \frac{1}{1+\gamma^2})^2(1+\gamma^2) \geq 2(1-\beta^h)(e^{0h} - e^{0l}). \qquad \text{(A18)}$$

In equilibrium, (A18) holds with equality and hence β^h lies in the interval $(\frac{1}{1+\gamma^2}, 1)$.

6.5 Derivation of equation (13)

When offering a pooling contract, the firm gains if it hires a high-type worker and looses if it hires a low-type worker. Expected profits from hiring a high-type worker is

$$\pi^{ph} = e_1 + e_2 - Ew = e_1 + e_2 - C^h(e_1, e_2) - u^h(\alpha^h, \beta^h)$$

For any given β, equations (A4) and (A6) imply that the cost of effort is $\beta^2(1+\gamma^2)$. Hence, expected profit is $\pi^{ph} = e_1 + e_2 - \beta^2(1+\gamma^2) - u^h(\alpha^h, \beta^h)$, or using equation (A7)

$$\pi^{ph} = 2e^{0h} + 2\beta^p - \beta^{p2}(1+\gamma^2) - u^h(\alpha^h, \beta^h)$$

where β^pdenotes the incentive power of the pooling contract. Inserting for $u^h(\alpha^h, \beta^h)$ from equation (A15) gives

$$\pi^{ph} = 2\beta^p - \beta^{p2}(1+\gamma^2) - (2\beta^h - \beta^{h2}(1+\gamma^2))$$

We now derive the loss if a the firm attracts a low-type worker. From equation (A8) it follows that the difference in the utilities of a high- and low-type under the same arbitrarily chosen linear contract, is given by

$$u^h - u^l = 2\beta(e^{0h} - e^{0l})$$

By definition, the low-type worker is indifferent between the two separating contracts, $u^l(\alpha^l, \beta^l) = u^l(\alpha^h, \beta^h)$. Since the high-type is indifferent between the high-type separating contract and the pooling contract, $u^h(\alpha^h, \beta^h) = u^h(\alpha^p, \beta^p)$, it follows that the low-type obtains a utility of $u^l(\alpha^l, \beta^l) + 2(\beta^h - \beta^p)(e^{0h} - e^{0l})$ in a deviating pooling contract. We thus have that

$$\alpha^p = \alpha^l + 2(\beta^h - \beta^p)(e^{0h} - e^{0l})$$

Since firms make zero profit in the separating equilibrium, it follows that a firm earns a negative profit of $2(\beta^h - \beta^p)(e^{0h} - e^{0l})$ if it offers a pooling contract and hires a low-type worker. Using that $\beta^p = 1/(1+\gamma^2)$ equation (13) follows.

6.6 Proof of proposition 3

Suppose the high-type workers are given a contract that induces efficiency, i.e., $\beta^h = 1/(1+\gamma^2)$ and $\alpha^h = 2e^{0h}\gamma^2\beta^h$. Let $u^{lh}(e^{0h}, e^{0l})$ denote the expected utility of a low-type worker that works under this contract and chooses, for any ε, an effort level that generates exactly the same measurable output as the one of high-type workers. As this choice is not optimal for the low-type worker, his pay-off from choosing the high-type's contract is at least $u^{lh}(e^{0h}, e^{0l})$. Due to the envelope theorem it follows that the derivative of u^{lh} with respect to e^{0h} evaluated at $e^{0h} = e^{0l}$ is equal to $2\gamma^2\beta^h > 0$. Hence, there exists an interval for $e^{0h} - e^{0l}$ such that the individual rationality constraint of the low-type worker cannot be satisfied.

6.7 Proof of claims following proposition 3

As noted in the main text, the firm punishes the worker arbitrarily hard if the measured output falls short of $\widetilde{y}^t$. Thus, a low-type worker chooses his first best effort level if the resulting observed productivity exceeds $\widetilde{y}^t$, and the effort level necessary to reach $\widetilde{y}^t$ otherwise. The maximum cost of excessive effort (obtained when $\widehat{\varepsilon} = -\overline{\varepsilon}$) is equal to $(e^{0h} - e^{0l})^2/(1+\gamma^2)$ (using equation (A4) and the high-type's optimal choice of e_1 and e_2).

The probability that the observable output with first-best effort level of the low-type worker falls short of $\widetilde{y}^t$ is given by

$$\rho(\sigma) = \Pr[\widehat{\varepsilon} \leq -\overline{\varepsilon} + (e^{0h} - e^{0l})/\sigma]$$

The expected cost of excessive effort is bounded from above by $(e^{0h} - e^{0l})^2/(1+\gamma^2)\rho(\sigma)$, which goes to zero as σ goes to infinity. Hence, $\overline{\Delta e}$ goes to infinity as σ goes to infinity.

The claim that $\overline{\Delta e} > 0$ as $\sigma \to 0$ follows directly from the proof of proposition 3. The derivative of u^{lh} with respect to e^{0h} evaluated at $e^{0h} = e^{0l}$ is equal to $2\gamma^2\beta^h > 0$.

References

Asheim, G. B. and Nilssen, T. (1996), Nondiscriminating Renegotiation in a Competitive Insurance Market, *European Economic Review 40*, 1717-1736.

Cho, I. and Kreps, D. (1987), Signalling Games and Stable Equilibria, *Quarterly Journal of Economics 102*, 179-222.

Eriksson, T. and Villeval, M-C. (2004), Other-regarding Preferences and Performance Pay - An Experiment on Incentives and Sorting. Working paper, Aarhus School of Business.

Gale, D. (1992), A Walrasian Theory of Markets with Adverse Selection, *Review of Economic Studies 59*, 229-255.

Grytten, J., Skau, I., Sørensen, R. J. and Aasland, O. G. (2004), Fastlegeordningen og endringer i legenes tjenesteproduksjon, *Tidsskrift for velferdsforskning 7*, 78-89.

Hart, O. and Holmstrom, B. (1987), The Theory of Contracts, In *Advances in Economic Theory*: Fifth World Congress, Econometric Society Monographs series, no. 12, Camebridge University Press.

Hellwig, M. (1987), Some Recent Developments in the Theory of Competition in Markets with Adverse Selection, *European Economic Review 31*, 319-325.

Holmstrom, B. and Milgrom, P. (1987), Aggregation and Linearity in the Provision of Intertemporal Incentives, *Econometrica 55*, 303-328.

Holmstrom, B. and Milgrom, P. (1991), Multitask Principal-Agent Analyses: Incentive Contracts, Asset Ownership, and Job Design, *Journal of Law, Economics, and Organizations 7*, 24-52.

Laffont, J.J. and Tirole, J. (1993), *A Theory of Incentives in Procurement and Regulatio*n. MIT Press, Cambridge, Massachusetts.

Lazear, E.P. (1986), Salaries and Piece Rates, *Journal of Business 59*, 405-431.

Lazear, E.P. (1998), *Personell Economics for Manager*, Wiley, New York.

Lazear, E.P. (2000), Performance Pay and Productivity, *American Economic Review 90*, 1346-1361.

MacLeod, W. B. and Malcomson, J. M. (1998), Motivation and Markets, *American Economic Review 88*, 388-411.

Mailath, J.M., Okuno-Fujiwarwa, M. and Postlewaite, A. (1993), Belief-Based Refinements in Signalling Games, *Journal of Economic Theory 60*, 241-246.

Millward, N., Stevens, M., Smart, D., and Hawes, W.R. (1992) *Workplace industrial relations in transition: The ED/ESRC/PSI/ACAS surveys.* Aldershot, U.K.: Dartmounth.

Moen, E.R. (1997), Competitive Search Equilibrium, *Journal of Political Economy 105*, 385-411.

Moen, E.R. (2003), Do Good Workers Hurt Bad Workers, or is it the Other Way Around? *International Economic Review 44*, 779-799.

Moen, E. R. and Rosén, Å. (2004), Equilibrium Incentive Contracts and Efficiency Wages, mimeo.

Pissarides, C.A. (2000), Equilibrium Unemployment Theory, MIT press, Cambridge, Massachusetts.

Prendergast, C. (1999), The Provision of Incentives in Firms, *Journal of Economic Literature 37*, 7-63.

Riley, J. (1979), Informational Equilibrium, *Econometrica 47*, 331-59

Rothschild, M. and Stiglitz, J. (1976), Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information. *Quarterly Journal of Economics 17*, 557-66.

Sørensen, R. J. and Grytten, J. (2003), Service production and contract choice in primary physician services, *Health Policy 66*, 73-93.

The Economist (1998), Incentives Pay survey. 29th of August