

# Equilibrium Incentive Contracts and Efficiency Wages<sup>1</sup>

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December 20, 2004

<sup>1</sup>We have benefited from comments by seminar participants at the the Norwegian School of Management in Oslo, University of Helsinki, Tinbergen Institute (Amsterdam and Rotterdam), the Norwegian School of Economics and Business Administration in Bergen, SED annual meeting in Stockholm, Stockholm University (IIES and SOFI), IZA in Bonn, University of Essex, Umeå University, Uppsala University, the CEPR conference "Incentives and Beyond", and The Trade Union Institute (Stockholm). Financial support from The Swedish Research Council is gratefully acknowledged.

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## **Abstract**

We analyse the optimal (efficiency) wage contract when output is contractible but firms neither observe the workers' effort nor their match-specific productivity. Firms offer wage contracts that optimally trade off effort and wage costs. As a result, employed workers enjoy rents, which in turn creates unemployment. Nonetheless, the incentive power of the equilibrium wage contract is constrained efficient in the absence of taxes and unemployment benefits. We also show that more high-powered incentive contracts tend to be associated with a higher equilibrium unemployment rate.

**Key words:** incentives, contracts, unemployment, efficiency

**JEL codes:** E24, J30, J41

# 1 Introduction

Efficiency wage theory is a prominent explanation for unemployment. Its core idea is that wages play other roles than clearing the market. In particular, firms may set wages above the market clearing wage in order to motivate workers (Shapiro and Stiglitz, 1984 and Akerlof, 1982), to recruit high-quality workers (Weiss, 1980), or to retain workers (Salop, 1979). In all cases there is unemployment in equilibrium.

A key feature of the Shapiro-Stiglitz shirking model is that firms cannot condition wages on output. Although such performance independent remuneration may be an appropriate description for some labour markets it is less so for others. Indeed, empirical studies document that performance pay, broadly interpreted, is common practice.<sup>1</sup> Contractible output eliminates unemployment in the Shapiro-Stiglitz framework, and it is therefore a common perception that rent-based unemployment cannot exist if output is contractible.<sup>2</sup> The present paper challenges this view, and demonstrates that efficiency wages and unemployment may arise in equilibrium also when output is contractible.

Our starting point is to show how the (partial) procurement model of Laffont and Tirole (1993) can be applied in an equilibrium model of the labour market. In the Laffont-Tirole model, the regulator offers a contract to a firm that has private information about both its type and its effort choice. As a result, the optimal contract leaves (information) rents to firms with low production costs. In our model, firms offers wage contracts to workers who have private information about their match-specific productivity and their effort choice. Firms face a trade-off between inducing more effort and conceding larger rents. Because hiring is costly, firms choose a contract such that also workers with a below maximum match-specific productivity remain employed. The inframarginal workers obtain information rents, and these rents translate into equilibrium unemployment.

Clearly, the equilibrium is inefficient as unemployment is a waste of resources. More interesting is the question whether market contracts differ from those that maximize welfare, given the firms' entry decision and the

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<sup>1</sup>Based on the National Longitudinal Survey of Youth, MacLeod and Malcomson (1998) report that 24 percent of workers in the US in 1990 received performance pay when bonuses and commissions are included. According to Millward et al. (1992), the fraction of workers in the United Kingdom that received some kind of performance-related pay was 34 % in 1990. Still, the fraction may be even higher, when also promotions based on performance and fixed salaries based on past performance are included.

<sup>2</sup>See e.g., Cahuc and Zylberberg (2004, ch 6), Yang (2003), MacLeod and Malcomson (1998), and Weiss (1990, pp 10-11).

workers' behavior. We show that the market equilibrium outcome is constrained efficient. This may at first glance seem surprising, as firms do not internalize the rents of their employees when they choose wage contracts. However, worker rents have no social value in equilibrium as they are offset by a corresponding social cost of unemployment.

We further explore the welfare properties of the market outcome in two directions. First, the constraint efficiency result also holds in the setting where firms can choose among production technologies that entail different amounts of worker rents. Second, unemployment benefits do not influence the welfare of unemployed workers, as they also lead to higher unemployment. This is true even if the income when unemployed represents a wage in a secondary sector. Third, in the presence of unemployment benefits, firms choose too high-powered incentive contracts, and the resulting unemployment rate is higher than in the constrained efficient allocation.

Our model allows us to identify three possible factors that lead to more high-powered incentive contracts; an increased importance of unobservable effort, lower marginal income taxes, and lower importance of worker heterogeneity in a given job category. The likely effect of an increase in the incentive power due to such changes in any of these three factors is an increase in the equilibrium unemployment rate. This result suggests that the perceived increase in the use of performance pay (Tower and Perrin 1999) may lead to higher equilibrium unemployment.

Efficiency wage theory has also been applied to explain inter-industry wage differentials (Dickens and Katz, 1987 and Gibbons and Katz, 1992). Our efficiency wage model gives the new prediction that industries with a strong relationship between individual performance and wages should pay higher wages.

The combination of asymmetric information and hiring costs implies that firms offer heterogeneous workers the same performance-based wage contracts (or the same menu of linear contracts to choose from). This feature is crucial for all our results. Empirical evidence in support of this feature is discussed in section 7. Note, however, that our results also hold if the firms observe the workers' abilities perfectly, but choose not to make the wage contract fully contingent on their ability (in addition to their performance).<sup>3</sup>

The relevant literature on the relationship between performance related pay and unemployment is thin. Foster and Wan (1984) study information rents in the labour market and show that with an exogenously given num-

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<sup>3</sup>For instance, it may be considered unfair that workers with the same performance are paid differently. This may be particularly true if the differences are not due to "objective" criteria like age, tenure or education but due to more vague criteria like ability.

ber of firms there may be unemployment. However, their model is not an equilibrium model, as the utility of unemployed workers is exogenous and independent of the number of firms. If entry of firms were introduced in their model, there would be no unemployment. MacLeod and Malcomson (1998) analyze the firms' choice between performance pay (bonuses) and efficiency wage as means to motivate workers. While performance pay dominates efficiency wages in their symmetric information setting, firms cannot commit to actually pay out the bonus. Thus, if it is easy to replace workers firms would do so *ex post* rather than to pay the bonus. Hence performance pay can only be used as a motivating device if the labour market is sufficiently tight, i.e., when the unemployment rate is low. In contrast to our model, performance pay is therefore associated with small (no) worker rents in their model.

In a recent paper, Schmitz (2004) analyses a partial model where a single firm extracts worker rents by terminating the employment relationship inefficiently early for low-type workers. Schmitz uses this result as a rationale for strict job protection laws. According to our analysis, by contrast, worker rents have now social value in labour market equilibrium, and laws that protect worker rents will not increase welfare.

The paper is organized as follows. Section 2 presents the model. We solve for the optimal wage contract in Section 3 and derive the labour market equilibrium outcome, in Section 4. Section 5 examines the efficiency and welfare properties of the equilibrium outcome. In Section 6 we derive comparative statics results. In Section 7 we discuss the empirical implications. Section 8 concludes.

## 2 Model

There exist several different types of optimal contract models that balance the costs and benefits of stronger incentives. Typically, stronger incentives induce the agent to exert more effort. The costs (or agency costs) associated with stronger incentives may vary: First, stronger incentives may give rise to a misallocation of risk, as the agent bears more risk than an optimal risk-sharing agreement would imply. Second, stronger incentives may induce the agent to allocate effort across different tasks inefficiently when the output from some of the tasks cannot be measured adequately. Third, stronger incentives implies that the agent captures larger rents, when he has private information about his type.

In this paper we focus on the last type of models where the agent has private information about his ability. A seminal paper within this strand of the literature is Mirlees (1971), which characterizes the optimal tax regime

under asymmetric information. Maskin and Riley (1984) analyze the optimal price discrimination strategy of a monopolist, and Baron and Myerson (1982) the optimal regulation of a monopolist. Laffont and Tirole (1993) study in detail the trade-off between incentives and rent extraction in the context of procurement. In our model, the optimal wage contracts is closely related to the optimal contract in the standard model in their book (1993).

The model is set in continuous time. While the measure of jobs is endogenously determined, the measure of workers in the economy is constant and normalized to one. Workers leave the market for exogenous reasons at a rate  $s$  and are replaced by new workers who enter the market as unemployed. Unemployed workers search for jobs and firms with vacant jobs search for workers. There is no on-the-job search. Workers and firms discount at a common rate  $r$ .

At the hiring stage the expected productivity (given the effort level) is the same for all workers in all firms. We are thus studying a segment of the market in which workers have the same observable characteristics.<sup>4</sup> Once employed, the productivity of a given worker also depends on a match-specific productivity term  $\epsilon$ . For any worker-firm pair the value of  $\epsilon$  is continuously distributed on the interval  $[\epsilon_{\min}, \epsilon^{\max}]$  with the cumulative distribution function  $F$ .<sup>5</sup> The corresponding density function  $f$  has an increasing hazard rate.

The timing is as follows:

1. The firm incurs a job creation cost  $K$ .
2. The firm advertises a wage contract.<sup>6</sup>
3. The firm receives job applications from unemployed workers.
4. One of the applicants is hired.

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<sup>4</sup>Observable differences in productivity would not change our results, as the optimal wage contract is contingent on all observable characteristics. The important aspect of the assumption is that workers and firms are symmetrically informed about the worker's productivity at the stage when the worker decides which job to apply for. This is admittedly a strong assumption, as self-selection mechanisms may be empirically important (see for instance Lazear, 2000). Self-selection by "informed" workers give raise to mechanisms that differ substantially from those analyzed in the present paper. Therefore, we abstract from effects of self-selection issues and refer the interested reader to Moen and Rosén (2004b).

<sup>5</sup>This specification implies that the match-specific productivity of a worker is independent across firms. Thus, a worker's outside option does not depend on his match-specific productivity in that firm. We conjecture that our main results also hold when a worker's productivity term  $\epsilon$  is correlated across firms as long as the correlation not perfect.

<sup>6</sup>It is possible to show that the equilibrium would be unchanged if the firm proposes the wage contract to the worker after he is hired.

5. The worker learns his match-specific productivity,  $\epsilon$  and decides whether to stay or not.
6. Production starts and continues until the worker leaves the market.

The time delay associated with the hiring process is assumed to be small relative to the duration of the employment relationship and is therefore ignored.<sup>7</sup>

The match specific term is revealed to the worker after he is hired. We assume that the time it takes before the worker learns his match specific productivity term is sufficiently long so that other applicants for the job are not available at that point in time. Thus, if the worker leaves at this point, the firm has to incur the job creation cost  $K$  over again to hire a new worker. Still, we assume that this time lag is relatively short compared to the expected duration of the employment relationship. As this paper does not focus on the behavior during the learning process, we assume that  $\epsilon$  is revealed to the worker immediately after he is hired and before production starts. By contrast,  $\epsilon$  remains unobservable to the firm.

The job creation cost  $K$  may be given various interpretations. The most direct interpretation is that  $K$  denotes the cost of advertising a vacancy. It may also include costs associated with evaluating and testing workers. More generally,  $K$  may consist of any costs that is incurred by the firm (not the worker) before the worker's productivity is revealed, and that is wasted if the worker quits. Thus,  $K$  may also include firm-specific training costs during the initial phase of the employment relationship.

As in Laffont and Tirole (1993), we assume that the production function is linear. More specifically, the (flow) value of production of a worker with match specific productivity  $\epsilon$  is

$$y = \bar{y} + \alpha\epsilon + \gamma e, \tag{1}$$

where  $e$  is the worker's effort (unobservable to the firm) and  $\bar{y}$  is a constant. The parameter  $\alpha$  reflects the importance of the match-specific component for the output and  $\gamma$  the importance of worker effort. Output is contractible, and wage contracts may therefore be made contingent on  $y$ . The profit flow of a firm with a worker of match-specific productivity  $\epsilon$  is given by

$$\pi = \bar{y} + \alpha\epsilon + \gamma e - \hat{w}(y), \tag{2}$$

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<sup>7</sup>Note, though, that  $K$  may partly reflect costs associated with time delays caused by a time-consuming hiring process.

where  $\widehat{w}(y)$  denotes the wage as a function of output  $y$ . A worker's utility flow is given by

$$u = \widehat{w}(y) - c(e), \quad (3)$$

where  $c(e)$  denotes the effort costs. We assume that  $c'(e) > 0$  for  $e > 0$ ,  $c'(0) = 0$ ,  $c''(e) \geq 0$ , and  $c'''(e) \geq 0$ . The associated asset values of worker utility and firm profit are  $U = u/(r + s)$  and  $E[\Pi] = E[\pi]/(r + s)$ , respectively.

While we do not impose any restrictions on the shape on  $\widehat{w}(y)$  we do not allow firms to charge an up-front hiring fee (bonding). This assumption is not innocuous, as such fees would eliminate unemployment.<sup>8</sup> The absence of bonding may be rationalized in several ways. First, an entrance fee would have to be paid before the worker learns his match specific productivity. Once the worker knows the match-specific productivity, it is optimal to leave rents to "high-type" workers and bonds would not increase firm profit. Thus, as long as the workers learns  $\epsilon$  relatively quickly, implicit bonding, like deferred wage compensation or seniority wages, as in Lazear (1979), do not work. A bond must be interpreted literally as an up-front payment (or at least as a payment that proceeds the revelation of  $\epsilon$ ).

Second, a worker may be reluctant to pay his employer an up-front fee sufficient to eliminate all expected rents. Ritter and Taylor (1994) show that if firms have private information regarding their bankruptcy probability, a bond can be interpreted as a signal of a high bankruptcy probability. As a result, firms with a low bankruptcy probability leave rents to their employees. More generally, up-front fees may induce firms to fool workers in various ways by hiring and collecting bonds from too many employees, or by prematurely replacing workers (to collect new bonds). By requiring a low bond or no bond, a firm may signal that it has no such intentions.<sup>9,10</sup>

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<sup>8</sup>For a discussion of the so-called bonding critique see e.g. Carmichael (1985, 1990), Dickens et al. (1989) and Akerlof and Katz (1989).

<sup>9</sup>Suppose for instance that firms may choose to open a "fake" vacancy at cost  $\widetilde{K} < K$ . A firm with a fake vacancy collects an entrance fee, and then fires the worker. If the workers cannot distinguish between a firm with a fake vacancy and a firm with an ordinary vacancy, the equilibrium entrance fee cannot exceed  $\widetilde{K}$ , as the market then would be overflowed with fake vacancies. If  $\widetilde{K}$  is not too high, there would still be (an endogenous amount of) rents in the economy.

<sup>10</sup>Also entrance fees are prohibited in some countries. For example, the Norwegian legislation does not allow for entrance fees paid to firms. The contracts act of 31th of May 1918 no 4, §36, in effect deems up-front payments as illegal.



### 3 Optimal contracts

An optimal wage contract maximizes the firm's expected present discounted profits  $E[\Pi]$  subject to the worker's incentive compatibility (IC) constraint and to his individual rationality (IR) constraint. As shown in Baron and Besanko (1984), the optimal dynamic contract repeats the optimal static contract provided that the firm can commit not to renegotiate.<sup>11</sup> We therefore solve for the optimal static contract.

The optimal contract is derived using the revelation principle.<sup>12</sup> In order to induce a worker to report his true "type"  $\epsilon$ , the following incentive compatibility condition must be satisfied (Appendix 1)

$$u'(\epsilon) = c'(e(\epsilon))\alpha/\gamma. \quad (4)$$

Worker utility is increasing in  $\epsilon$ , as long as  $e > 0$ . This reflects that a high-type worker can produce the same output as a worker of lower type by exerting less effort. More specifically, if a worker's type increases by one unit, the worker can reduce his effort by  $\alpha/\gamma$  units and still obtain the same output, and thereby increasing his utility by  $c'(e(\epsilon))\alpha/\gamma$  units. Furthermore, higher effort levels  $e(\epsilon)$  imply larger  $u'(\epsilon)$ , and hence larger rents for the workers. Thus, the firm faces a trade-off between incentive provision and rent extraction from the worker.

We denote the worker's flow utility while unemployed by  $u^0$ . Individual rationality requires  $u(\epsilon) \geq u^0$  for any worker who stays with the firm. Let  $\epsilon_c$  denote the associated cut-off level, where  $u(\epsilon_c) = u^0$ . Inserting  $u(\epsilon) = w(\epsilon) - c(e)$  into equation (2) gives  $\pi(\epsilon) = \bar{y} + \alpha\epsilon + \gamma e(\epsilon) - c(e(\epsilon)) - u(\epsilon)$ . The optimal contract thus solves the problem

$$Max E[\pi] = \max_{e(\epsilon), \epsilon_c} \int_{\epsilon_c}^{\epsilon^{\max}} [\bar{y} + \alpha\epsilon + \gamma e(\epsilon) - c(e(\epsilon)) - u(\epsilon)] e^{-rt} dF \quad (5)$$

$$\begin{aligned} \text{subject to } u'(\epsilon) &= c'(e(\epsilon))\alpha/\gamma \\ u(\epsilon_c) &\geq u^0. \end{aligned}$$

The first order condition for the optimal  $e$  can be written as (Appendix 1)

$$\gamma - c'(e(\epsilon)) = \frac{1 - F(\epsilon)}{f(\epsilon)} c''(e(\epsilon))\alpha/\gamma. \quad (6)$$

<sup>11</sup>For an instructive proof see Fudenberg and Tirole (1991), p. 299 ff.

<sup>12</sup>Since the contract is advertised, and thus constructed before the worker is hired, the revelation principle cannot be interpreted literally.

The intuition for this condition is as follows: Suppose the effort level of a worker with a match-specific productivity  $\epsilon'$  increases by one unit. The resulting efficiency gain is  $\gamma - c'(e(\epsilon'))$ , while the cost in terms of larger rents for all workers with a match specific productivity above  $\epsilon'$  is  $c''(e(\epsilon'))\alpha/\gamma$  (equation 4). The likelihood of obtaining a worker of type  $\epsilon'$  is reflected in  $f(\epsilon')$ , while the measure of workers with a higher match-specific productivity is  $1 - F(\epsilon')$ . Equation (6) thus ensures that the gains from effort and rent extraction are balanced at the margin. Given  $f$  has an increasing hazard rate ( $(1 - F(\epsilon))/f(\epsilon)$  decreasing in  $\epsilon$ ) and  $c'''(e) \geq 0$  equation (6) implies that  $e(\epsilon)$  is increasing in  $\epsilon$ .

The optimal cut-off value solves the equation (Appendix 1)

$$\bar{y} + \alpha\epsilon + \gamma e - c(e) - u^0 = \frac{1 - F(\epsilon_c)}{f(\epsilon_c)} c'(e)\alpha/\gamma. \quad (7)$$

This equation uniquely determines  $\epsilon_c$  (Appendix 2). The expected profit of the firm can be written as a function of  $u^0$ , i.e.,  $E[\pi] = E[\pi(u^0)]$ , or equivalently  $E[\Pi] = E[\Pi(U^0)]$  where  $U^0$  denotes the asset value of an unemployed worker. The function  $E[\Pi]$  is strictly decreasing in  $U^0$ .

Let  $(a, b)$  denote a linear contract of the form  $w = a + by$ . It is well known that the optimal non-linear contract can be represented by a menu  $(a(\epsilon), b(\epsilon))$  of linear contracts (see for instance Laffont and Tirole, 1993). For any  $b$ , the worker chooses the effort level such that  $c'(e) = b\gamma$ . Henceforth, we refer to  $b$  as the incentive power of the associated linear contract. Using the condition  $c'(e) = b\gamma$  in equation (6) we obtain

$$b(\epsilon) = 1 - \frac{1 - F(\epsilon)}{f(\epsilon)} c''(e)\alpha/\gamma^2. \quad (8)$$

Thus, the optimal wage contract involves distortion for all workers, except the one with maximum match-specific productivity ( $b(\epsilon^{\max}) = 1$ ).<sup>13</sup>

In what follows, we are interested in comparing different wage contracts. We call wage contract  $A$  *more incentive powered* than a wage contract  $B$  if  $b^A(\epsilon) \geq b^B(\epsilon)$  for all  $\epsilon$  with a strict inequality for some  $\epsilon$ .

Since  $u'(\epsilon) = \alpha c'(e(\epsilon))/\gamma = \alpha b$  the rent for a worker with match-specific productivity  $\epsilon'$  is given by

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<sup>13</sup>We have not imposed any restrictions on  $b$ . A natural restriction would be that  $b$  (or  $e$ ) are non-negative. This is always the case if  $c''(0) = 0$ .

$$\begin{aligned}
\rho(\epsilon') &= \int_{\epsilon_c}^{\epsilon'} u'(\epsilon) d\epsilon \\
&= \int_{\epsilon_c}^{\epsilon'} \alpha b(\epsilon) d\epsilon.
\end{aligned} \tag{9}$$

Let  $\tilde{F} = F/(1 - F(\epsilon_c))$  denote the distribution of  $\epsilon$  conditional on being above  $\epsilon_c$ . The expected rent of a hired worker that remains with the firm is (Appendix 3)

$$\begin{aligned}
E[\rho] &= \int_{\epsilon_c}^{\epsilon^{\max}} \int_{\epsilon_c}^{\epsilon'} \alpha b(\epsilon) d\epsilon d\tilde{F}(\epsilon') \\
&= \frac{\alpha}{1 - F(\epsilon_c)} \int_{\epsilon_c}^{\epsilon^{\max}} b(\epsilon)(1 - F(\epsilon_c) - F(\epsilon)) d\epsilon.
\end{aligned} \tag{10}$$

The expected income flow of an employed worker can thus be written as  $E[u(\epsilon)] = u^0 + E[\rho]$ .

**Lemma 1** *Given that  $\epsilon_c < \epsilon^{\max}$ , employed workers receive a strictly positive expected rent  $E[\rho]$ .*

Proof: No worker with  $\epsilon \geq \epsilon_c$  receives a negative rent, as it would violate the individual rationality constraint. From equation (10) it thus follows that  $E[\rho]$  is zero if and only if  $b$  is zero almost everywhere. However, equation (8) implies that  $b(\epsilon)$  is strictly positive for all  $\epsilon$  sufficiently close to  $\epsilon^{\max}$ . QED

## 4 Labour Market Equilibrium

To focus on how efficiency wages lead to unemployment, we assume that there is no time delay in the hiring process.<sup>14</sup>

Free entry of firms ensures that the expected profit  $E[\Pi]$  of a firm equals the job creation cost  $K$ . Our first equilibrium condition (entry condition) can thus be written as

$$E[\Pi(U^0)] = K. \tag{11}$$

Since  $E[\Pi]$  is strictly decreasing in  $U^0$  this equation determines the equilibrium value of  $U^0$  uniquely. We denote this equilibrium value by  $U^{0*}$ .

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<sup>14</sup>In an earlier version, we show how the labour market equilibrium can be derived as the limit equilibrium of an urn-ball model when the frictions go to zero, (Moen and Rosén, 2004c).

Let  $z$  denote the utility flow of unemployed workers and  $p$  the transition rate from unemployment to employment in steady state. The transition rate  $p$  equals the rate at which workers are hired times  $1 - F(\epsilon_c)$ . The relationship between  $U^0$  and  $p$  is then given by

$$(r + s)U^0 = z + p(W - U^0), \quad (12)$$

where  $W$  is the expected discounted income when employed. Let  $R$  denote the asset value of the expected rents ( $R = E[\rho]/(r+s)$ ). As  $R$  is by definition equal to  $W - U^0$  we can rewrite equation (12) as

$$(r + s)U^0 = z + pR. \quad (13)$$

The equilibrium in the labour market is defined as a pair  $(p, U^0)$  satisfying equations (11) and (13).

In the absence of unemployment, workers find a job immediately, which implies that  $p$  is infinite. However, this leads to a contradiction, as  $U^0$  defined by equation (13) then goes to infinity and thus exceeds  $U^{0*}$  as defined by equation (11).

**Proposition 1** *The equilibrium unemployment rate is strictly positive.*

*Proof:* Given  $U^{0*}$ , equation (7) determines  $\epsilon_c^*$ . Furthermore,  $\epsilon_c^* < \epsilon^{\max}$ , otherwise the firm would not recoup  $K$ . It then follows from Lemma 1 that  $E[\rho]$  is strictly positive. But then it follows from equation (13) that  $U^0$  goes to infinity if  $p$  does. Thus,  $p$  is finite. QED

As being unemployed is the outside option for a worker, rents imply that it is strictly better (in expected terms) to be employed than to be unemployed. But this is inconsistent with full employment.

The transition rate to employment is such that the rents are dissipated. Inserting  $U^0 = U^{0*}$  into equation (13) and re-arranging gives

$$p = \frac{(r + s)U^{0*} - z}{R}. \quad (14)$$

Let  $x$  denote the unemployment rate in the economy. Using the fact that  $(p + s)x = s$  holds in steady state yields

$$x = \frac{s}{r + s} \frac{R}{U^{0*} - Z + \frac{s}{r+s}R}, \quad (15)$$

where  $Z = z/(r + s)$  is the asset value of staying unemployed forever.

## 5 Efficiency

Obviously, the equilibrium outcome is not first-best, as this requires full employment and  $c'(e) = \gamma$ . (Almost) full employment can be obtained by an arbitrarily high negative unemployment benefit, while the efficient level of effort can be approximated by a negative income tax schedule on labour income. (We discuss the impact of taxes on the wage contract in the next section). For reasons outside of our model, these policy recommendations are unlikely to be taken seriously by any government.

In our view a more interesting question is whether the wage contracts chosen by the firms are socially optimal, given the behavior of the workers and the entry decisions of firms. Or putting it differently, what wage contract would a social planner choose given that all other decisions are still taken by the market participants?

We call the equilibrium wage contracts chosen by the firms *constrained efficient* if they maximize welfare subject to the workers' incentive compatibility and individual rationality constraints and to the entry condition (equation 11).

The social planner maximizes overall production less the job creation and effort costs. Let  $V(\Phi)$  denote the expected discounted production value of a worker-firm pair net of the effort costs as a function of the wage contract  $\Phi$ . This contract also specifies a cut-off level  $\epsilon_c$ . For each formed worker-firm pair, the vacancy creation cost  $K$  is incurred  $1/[1 - F(\epsilon_c^*(\Phi))]$  times. Finally, assume that the social value of the utility flow of an unemployed worker is equal to  $z$ . The transition rate to employment is determined by equation (14), and can thus be written as a function of  $\Phi$ . Hence, the planner's objective function is

$$S(\Phi) = \int_0^\infty [zx + xp(\Phi)[V(\Phi) - \frac{K}{1 - F(\epsilon_c^*(\Phi))}]]e^{-rt} dF. \quad (16)$$

The social planner maximizes  $S$  given the constraint that  $\dot{x} = s - (p + s)x$ .

Within a search context, Acemoglu and Shimer (1999), Moen and Rosén (2004a), and Pissarides (2000) show that the social planner chooses the vacancy rate to maximize the welfare of the unemployed workers. An analogue result holds here.

**Lemma 2** *The social planner's problem is equivalent to maximizing the unemployed workers' expected discounted utility.*

Proof: See Appendix 4.

We are now in the position to show that the equilibrium wage contract is constrained efficient. The equilibrium contract  $\Phi^*$  satisfies the two following conditions

1.  $\max_{\Phi} E[\Pi(\Phi)]$  subject to  $u'(\epsilon) = c'(e(\epsilon))\alpha/\gamma$   
 $u(\epsilon_c) \geq u^0$ .
2.  $E[\Pi(\Phi)] = K$ .

The constrained efficient contract  $\Phi'$  solves the "dual" maximization problem (since maximizing  $U^0$  is equivalent with maximizing  $u^0$ )

1.  $\max_{\Phi} u^0(\Phi)$  subject to  $u'(\epsilon) = c'(e(\epsilon))\alpha/\gamma$   
 $u(\epsilon_c) \geq u^0$ .
2.  $E[\Pi(\Phi)] = K$ .

**Proposition 2** *Provided that the income  $z$  reflects the social value of staying unemployed, the equilibrium wage contract is constrained efficient.*

Proof: Suppose the proposition does not hold. Then  $U^0(\Phi') > U^0(\Phi^*)$ . However, since  $E[\Pi(\Phi')] = K$  when  $U = U^0(\Phi')$ , it follows that  $E[\Pi(\Phi')] > K$  when  $U = U^0(\Phi^*)$ . But then  $\Phi^*$  cannot be an equilibrium contract, a contradiction. QED.

The firms choose the contracts to balance rent extraction and worker effort. Increasing the incentive power of the contract  $b$  for some types gives rise to a positive externality, as this tends to increase worker rent. One may therefore expect that the incentive contracts are too low powered (too low values of  $b$ ). However, this is not correct. The rents of employed workers have no social value in equilibrium, because the unemployment rate is determined so as to dissipate all rents. That is, larger worker rents lead to a higher unemployment rate, leaving the asset value of an unemployed worker constant.

To explore this argument, suppose that firms can choose among production technologies that give rise to different levels of worker rents. For instance, output with different production technologies may depend to a varying extent on unobserved effort. Ceteris paribus, technologies that are more sensitive to effort lead to more high-powered incentives and higher worker rents than technologies for which effort is less important. Similarly, production technologies may differ in their sensitivity to worker-firm specific productivity differences.

**Corollary 1** *Firms choose the constrained efficient production technology.*

*Proof:* See Appendix 5.

When firms choose between different technologies, worker rents do not enter their objective function. However, as worker rents are dissipated in labour market equilibrium worker rents do not enter the objective function of the social planner either. Thus, the firms' choice of technology is constrained efficient. As mentioned in the introduction, this finding contrasts the results in Schmitz (2004). He analyses a partial model where a single firm extracts worker rents by terminating the employment relationship inefficiently early for low-type workers. Schmitz uses this result as a rationale for strict job protection laws. According to our findings this is not the case, firms have the correct incentives to reduce worker rents as worker rents have no social value.

Proposition 2 relies on the assumption that the income  $z$  reflects the social value of being unemployed. Thus,  $z$  may reflect the value of leisure, of home production, or alternatively wages in a secondary labour market. An interesting result follows from equation (11) and from the fact that  $E[\Pi]$  is independent of  $z$ .

**Corollary 2** *Social welfare is independent of  $z$ , irrespective of whether  $z$  reflects the social value of being unemployed, or unemployment benefits, or the wage in a secondary sector.*

A higher  $z$  makes it more time-consuming to dissipate rents and thus increases unemployment (or employment in the inferior secondary sector) exactly by the amount such that unemployed workers obtain the same utility level.

Suppose that  $z$  (partly) consists of government transfers (unemployment benefit), and hence does not reflect the social value of being unemployed. Corollary 2 implies that the unemployment benefits are a waste of resources, as they do not influence the well-being of unemployed workers. Consequently, the unemployment rate does not influence the equilibrium wage contract or the equilibrium cut-off value either. However, the unemployment rate increases with the unemployment benefit, as it takes more time to dissipate the rents associated with employment.

**Corollary 3** *With unemployment benefits the equilibrium wage contract is more high-powered than the constrained efficient wage contract.*

*Proof:* See Appendix 5.

A positive unemployment benefit makes the government bear part of the burden associated with being unemployed. This is not taken into account when the incentive contracts are determined. Hence, equilibrium contracts result in a too high unemployment rate. By contrast, taxes on labour income tend to reduce the equilibrium incentive power of the contracts below its constrained optimal level. We return to this point shortly.

## 6 Determinants of the unemployment rate

In this section we analyze the effects of changes in the wage contract on the equilibrium unemployment rate. These effects may depend on which of the structural parameters in the model triggers the change in the wage contract.

Since we are not able to characterize the effects of a change in the cut-off level  $\epsilon_c$  on the worker rents in the general case, we subsequently assume that the cut-off level is below  $\epsilon^{\min}$ . Thus, the firms retain all workers, which may in fact be a good approximation of their actual behavior. Given that a firm has spent resources on hiring and possibly training a worker, the likelihood of dismissal may be fairly low by the time the match-specific productivity component is revealed.

We first analyze the effects of changes in  $b(\epsilon)$  around the optimal schedule  $b^*(\epsilon)$ . As we have seen,  $u^0(b)$  is maximized at  $b^*(\epsilon)$ . By the envelope theorem  $u^0$  is approximately constant for wage contracts close to  $b^*(\epsilon)$ . From equations (10) and (15) it follows that stronger incentives yield, *ceteris paribus*, more rents to the workers and thereby higher unemployment rate.

For a given rent, equation (15) implies that an increase in  $u^0$  reduces unemployment. The effect of any parameter change may therefore be disentangled into a direct effect on unemployment (keeping  $u^0$  constant) and a welfare effect (the effect of an increase in  $u^0$ ). In what follows we analyze the direct effect of changes in private information, importance of effort, and marginal taxes.

The direct effect thus measures the effect of changes in parameter values for a given welfare level in the economy. In a dynamic setting the direct effect is an appropriate measure in the following sense: Suppose  $\tau_1$  and  $\tau_2$  are two alternative technologies. At any point in time, we know that firms in equilibrium selects the technology that maximizes  $u^0$ . Suppose  $\tau_1$  dominates  $\tau_2$  initially, and furthermore that technological changes imply that  $\tau_2$  gradually improves relative to  $\tau_1$ . At some point in time  $\tilde{t}$ , firms switch to technology  $\tau_2$ . At this point,  $u^0$  is still continuous in time although technology is discontinuous at  $\tilde{t}$ .



### Reduced relative importance of private information

The (direct) effect of a reduced relative importance of private information is defined as a reduction in  $\alpha$  together with an increase in  $\bar{y}$  such that  $u^0$  remains unchanged. Such a reduction in  $\alpha$  has two opposing effects on worker rents. On the one hand, a lower  $\alpha$  leads to lower expected rents for a given wage contract, (equation 10). On the other hand, a reduction in  $\alpha$  leads to an increase in  $b$  for all  $\epsilon$  types (equation 8), which tend to increase expected rents. In order to obtain clear-cut results, we assume that  $c(e)$  is quadratic. We define the average value of  $b$  as  $\bar{b} = \int_{\epsilon^{\min}}^{\epsilon^{\max}} b(\epsilon) / (\epsilon^{\max} - \epsilon^{\min})$ .

**Proposition 3** *A reduction in the importance of ex post heterogeneities leads to more high-powered wage contracts and more (less) unemployment if  $\bar{b} < 1/2$  ( $\bar{b} > 1/2$ ).*

Proof: See Appendix 6

The average value  $\bar{b}$ , corresponds to the expected value of  $b(\epsilon)$  only when  $\epsilon$  is uniformly distributed. Since  $b'(\epsilon) > 0$ ,  $\bar{b}$  may be less than  $1/2$  even if  $E[b(\epsilon)]$  is greater than  $1/2$  if most of the probability mass is located in the upper part of the distribution. Finally, if  $c'''(e)$  is strictly positive, the responsiveness of the optimal contract to changes in  $\alpha$  is reduced, making it more likely that a reduction in  $\alpha$  leads to less unemployment than indicated in the proposition.

### Increased importance of unobservable effort

We want to measure the effect of increased importance of unobservable effort, keeping the relative importance of private information constant. We therefore examine the joint effects of an increase in  $\gamma$  and a reduction in  $\bar{y}$  that leave  $u^0$  unchanged.<sup>15</sup>

On the one hand, an increase in  $\gamma$  tends to increase the right-hand side of equation (8), and thus  $b^*(\epsilon)$  for a given effort level. On the other hand, an increase in  $\gamma$  tends to increase  $e$  for a given  $b$ . Since,  $c'''(e) \geq 0$  this tends to reduce incentives, because rent extraction becomes more important. While the net effect is therefore indeterminate, mild assumptions on the cost-function ensure that the first effect dominates. More specifically,  $b$  increases in  $\gamma$ , if  $c''(e)/c'(e)$  is non-increasing. All polynomials of the form  $x^n$  as well as the exponential function satisfy this restriction.<sup>16</sup>

<sup>15</sup>Note that  $u^0$  is constant if the deterministic part of output,  $\bar{y} + \gamma e$ , is held constant given that the initial wage contract is in place (this follows from the envelope theorem).

<sup>16</sup>As will be clear from the proof, it is actually sufficient to assume that  $c''(e)/c'(e)^2$  is decreasing.

**Proposition 4** *An increase in the importance of unobservable effort leads to more high-powered wage contracts and more unemployment.*

Proof: See Appendix 7.

If unobserved effort becomes relatively more important, firms provide their workers with stronger incentives, as incentive provision becomes more important relative to rent extraction. As a result, the expected rent associated with employment increases, and thus also unemployment.

### **Reduction in marginal taxes**

Suppose the income tax  $T$  is given by

$$T = tw - A, \tag{17}$$

where  $A$  is a constant and  $t$  the marginal tax rate. To study the direct effect of an increase in  $t$ , we simultaneously reduce  $A$  to keep  $u^0$  unchanged. We also maintain the mild restriction that  $c''(e)/c'(e)$  is decreasing in  $e$ .

**Proposition 5** *A reduction in the marginal tax rate leads to more high-powered wage contracts and more unemployment.*

Proof: See Appendix 8.

If marginal taxes are reduced, firms provide their workers with stronger incentives. As a result, the expected rent associated with employment increases, and thus also unemployment.

Propositions 3, 4 and 5 illustrate the direct effects that changes in the structural parameters have on equilibrium wage contracts and unemployment. First, the unemployment rate may increase if workers have less private information provided that the average value of  $\bar{b}$  is less than  $1/2$ . Similar features are not present in the Shapiro-Stiglitz (1984) model. Second, higher unemployment may be the result of increased importance of unobservable effort. This result can be replicated in the Shapiro-Stiglitz model. Finally, higher marginal taxes give rise to lower unemployment. By contrast, the structure of taxes does not affect unemployment in the Shapiro-Stiglitz model, (Pissarides, 1998).

## 7 Empirical Evidence

In this section we discuss the empirical evidence in support of the model's assumptions and implications. As pointed out in the introduction two crucial assumptions are that heterogeneous workers are offered the same wage contract (or choose from the same menu of contracts) and that hiring is costly. These assumptions are supported by Lazear's (2000) case study on a shift from a flat wage scheme to performance pay in a large corporation (Safelite). Lazear had access to data concerning individual productivity before and after the shift in pay structure. He documents substantial productivity differences among workers offered the same wage contract. Controlling for time and tenure, the variances of workers' productivity differences in percentage of the means are 24 percent before and 20 percent after the switch to piece rate payments. The difference in productivity between the 90th and the 10th percentile of the workers in percent of the means are 47 percent before and 35 percent after the switch to piece rate payments.

The Safelite case also give support to our second assumption that it is costly to replace workers. When introducing piece rate payments, Safelite included a wage floor approximately equal to the earlier fixed wage. According to Lazear, Safelite did this "*in order to avoid massive turnover*". Thus, Safelite regarded it as being in its interest to keep the workers at the lower end of the productivity scale. Furthermore, many workers ended up in the guarantee range.

A main result in our paper is that performance pay gives rise to worker rents. Lazear's study also supports this result. In Safelite, workers were better off after the introduction of performance pay. 92 percent of the workers experienced a wage increase, and a quarter received a wage increase exceeding 28 percent. Since workers within the guarantee wage range were no worse off than before the shift, all workers experiencing a wage increase must have gained from the shift. Furthermore, the numbers suggests that many workers substantially benefitted from the introduction of the piece rate payment scheme.

Weaker, but still interesting evidence in support of this result is provided in Booth and Frank (1999) and Eriksson (2001). Booth and Frank find that male (female) employees with a performance related wage-scheme receive an average wage premium of 9 (6) percent. Eriksson compares compensation in firms that pay fixed salaries and firms that have adopted "new practises" (that is, offers incentives in the form of team bonuses, individual bonuses, stock options and profit sharing). He finds that firms offering performance-based compensations are more likely to claim that they are paying higher wages than other firms in the same local labor market. Note, however that

in these two studies higher payment may partly reflect compensation for higher effort.

Furthermore, some type of firms, like law firms, consulting firms and universities often have an up-or-out promotion system, which is extremely high-powered incentive schemes: The employee is either promoted to a more lucrative (senior) position or fired after a period of time. According to popular perceptions, employment in these firms is very attractive, suggesting that the employees obtain rents in expected terms. (Note though that the alternative to employment in such firms is typically not unemployment but employment in less attractive firms).

As mentioned in the introduction, we challenge the view that efficiency wages due to unobservable effort are exclusively associated with non-verifiable output. Chen and Edin (2002) adopt this view when evaluating efficiency wages as an explanation for inter-industry wage differentials. More precisely, they test the hypothesis that "these pay differences should be less sizeable and have less explanatory power for piecework than for timework". For their sample of male blue-collar workers in the Swedish metal industries in 1985 their findings are mixed. In line with our model, but contrary to earlier theoretical work, this suggest that workers receiving piece-rate pay do not necessarily earn less rents than workers on time-pay.

## 8 Conclusion

We show that efficiency-wage based unemployment may arise in a model with contractible output if workers have private information about their match-specific productivity. Worker heterogeneity at the firm level and the absence of entry fees imply that wage contracts trade-off incentive provision and rent extraction. Moreover, the incentive power of the equilibrium wage contract is constrained efficient in the absence of unemployment benefits. At first glance, this may seem surprising because firms do not internalize the value of their employee's rents. Wage contracts are nonetheless constrained efficient because equilibrium rents lead to higher unemployment, which reduces welfare.

The unemployment level is determined by the amount of rents that the optimal wage contract leaves to the workers. The expected worker rents are larger if non-observable effort is relatively important, or if the marginal tax rate is lower, or if the heterogeneity among workers is neither too large nor too small. In all these circumstances the equilibrium unemployment rate is higher as well.

Finally, our model generates new predictions regarding the likely impor-

tance of efficiency wages and worker rents that are consistent with the existing empirical literature.

## Appendix

### Appendix 1: Deriving equations 4, 6 and 7

When pretending to be type  $\tilde{\epsilon}$  a  $\epsilon$  type worker obtains a (flow) utility

$$\tilde{u}(\epsilon, \tilde{\epsilon}) = w(\tilde{\epsilon}) - c(e(\tilde{\epsilon})) + \frac{\alpha}{\gamma}(\tilde{\epsilon} - \epsilon).$$

where  $w(\tilde{\epsilon})$  and  $e(\tilde{\epsilon})$  denote the wage and the effort level as a function of the reported type  $\tilde{\epsilon}$ . The indirect flow utility can be written as  $u(\epsilon) = \max_{\tilde{\epsilon}} \tilde{u}(\epsilon, \tilde{\epsilon})$ . The incentive compatibility constraint requires that  $\epsilon = \arg \max_{\tilde{\epsilon}} \tilde{u}(\epsilon, \tilde{\epsilon})$ , or from the envelope theorem equivalently that

$$u'(\epsilon) = \left. \frac{\partial \tilde{u}(\epsilon, \tilde{\epsilon})}{\partial \epsilon} \right|_{\tilde{\epsilon}=\epsilon}$$

i.e., that

$$u'(\epsilon) = c'(e(\epsilon))\alpha/\gamma.$$

The current-value Hamiltonian associated with the firm's maximization problem can be written as

$$H = [\bar{y} + \alpha\epsilon + \gamma e - c(e) - u]f(\epsilon) + \lambda(c'(e)\alpha/\gamma),$$

where  $\lambda$  is the adjoint function. For a given cut-off  $\epsilon_c$ , the first order conditions for a maximum are

$$\begin{aligned} (\gamma - c'(e))f(\epsilon) + \lambda c''(e)\alpha/\gamma &= 0, \\ \dot{\lambda}(\epsilon) &= f(\epsilon). \end{aligned}$$

Since there are no terminal conditions at  $\epsilon^{\max}$  it follows that  $\lambda(\epsilon^{\max}) = 0$ , and  $\lambda(\epsilon) = -(1 - F(\epsilon))$ . The first order condition for the optimal  $e$  can thus be written as

$$\gamma - c'(e(\epsilon)) = \frac{1 - F(\epsilon)}{f(\epsilon)} c''(e(\epsilon))\alpha/\gamma.$$

The optimal cut-off value solves the equation  $H(\epsilon_s) = 0$ , or

$$\bar{y} + \alpha\epsilon + \gamma e - c(e) - u^0 = \frac{1 - F(\epsilon_c)}{f(\epsilon_c)} c'(e)\alpha/\gamma.$$

## Appendix 2: Unique cut-off level

Define

$$\Psi(\epsilon) = \bar{y} + \alpha\epsilon + \gamma e - c(e) - u^0 - \frac{1 - F(\epsilon)}{f(\epsilon)} c'(e) \alpha / \gamma$$

Equation (7) determines a unique  $\epsilon_c$  iff  $\Psi(\epsilon) = 0$  is uniquely defined.

$$\frac{d\Psi(\epsilon)}{d\epsilon} = \alpha + \gamma \frac{de}{d\epsilon} - c'(e) \frac{de}{d\epsilon} - c'(e) \alpha / \gamma \frac{d}{d\epsilon} \frac{1 - F(\epsilon)}{f(\epsilon)} - \frac{1 - F(\epsilon)}{f(\epsilon)} c''(e) \alpha / \gamma \frac{de}{d\epsilon}$$

Inserting  $-\frac{1-F(\epsilon)}{f(\epsilon)} c''(e) \alpha / \gamma = c'(e) - \gamma$  (equation 6) yields

$$\frac{d\Psi(\epsilon)}{d\epsilon} = \alpha - c'(e) \alpha / \gamma \frac{d}{d\epsilon} \frac{1 - F(\epsilon)}{f(\epsilon)} > 0$$

Hence,  $\Psi(\epsilon) = 0$  is uniquely defined. QED

## Appendix 3: Deriving equation 10

The integral

$$E[\rho] = \int_{\epsilon_c}^{\epsilon^{\max}} \int_{\epsilon_c}^{\epsilon'} \alpha b(\epsilon) d\epsilon d\tilde{F}(\epsilon')$$

can be simplified using integration by parts. We use that  $\int_a^b u(x)v'(x)dx = |^b_a u(x)v(x) - \int_a^b u'(x)v(x)dx$ . Let  $v = 1 - \tilde{F}$ ,  $v' = -d\tilde{F}$ ,  $u = \int_{\epsilon_c}^{\epsilon'} \alpha b(\epsilon) d\epsilon$  and  $u' = \alpha b(\epsilon)$ . This allows us to rewrite the above integral as

$$\begin{aligned} E[\rho] &= -|^{\epsilon^{\max}}_{\epsilon_c} (1 - \tilde{F}) \int_{\epsilon_c}^{\epsilon'} \alpha b(\epsilon) d\epsilon + \int_{\epsilon_c}^{\epsilon^{\max}} \alpha b(\epsilon) (1 - \tilde{F}) d\epsilon \\ &= \int_{\epsilon_c}^{\epsilon^{\max}} \alpha b(\epsilon) (1 - \tilde{F}) d\epsilon \\ &= \frac{\alpha}{1 - F(\epsilon_c)} \int_{\epsilon_c}^{\epsilon^{\max}} b(\epsilon) (1 - F(\epsilon_c) - F(\epsilon)) d\epsilon. \end{aligned}$$

## Appendix 4: Proof of Lemma 2

The current-value Hamiltonian associated with the social planner's maximization problem can be written as

$$H^c = zx + xp(\Phi) \left[ V(\Phi) - \frac{K}{1 - F(\epsilon_c(\Phi))} \right] + \lambda [s - (p(\Phi) + s)x]$$

where the only state variable is  $x$ . The first order conditions are given by

$$r\lambda = \frac{\partial}{\partial x} H^c = z + p(\Phi)[V(\Phi) - \frac{K}{1 - F(\epsilon_c(\Phi))}] - \lambda(p(\Phi) + s) \quad (18)$$

$$\begin{aligned} \Phi &= \arg \max_{\Phi} H^c \\ &= \max_{\Phi} [zx + xp(\Phi)[V(\Phi) - \frac{K}{1 - F(\epsilon_c(\Phi))}] + \lambda[s - (p(\Phi) + s)x] \end{aligned} \quad (19)$$

Equation (18) implies that

$$(r + s)\lambda = z + p(\Phi)[V(\Phi) - \frac{K}{1 - F(\epsilon_c(\Phi))}] - \lambda. \quad (20)$$

In the maximization problem (19), the state variable  $x$  and the adjoint variable  $\lambda$  are regarded as constant, and the maximization problem can equivalently be expressed as

$$\max_{\Phi} p(\Phi)[V(\Phi) - \frac{K}{1 - F(\epsilon_c(\Phi))}] - \lambda.$$

This problem is equivalent to maximizing  $\lambda$  as defined by equation (20).

We want to show that this is equivalent to maximizing  $U^0$  defined by equation (12), given equation (14) and the entry condition  $E[\Pi] = K$ . The entry condition implies that the expected profit of a firm with a worker who remains in the firm is equal to  $K/(1 - F(\epsilon_c))$ . The rest of the surplus accrues to the worker. Thus,  $W(\Phi) = V(\Phi) - K/(1 - F(\epsilon_c))$ . Inserted into equation (12), we find that

$$(r + s)U^0 = z + p(\Phi)[V(\Phi) - \frac{K}{1 - F(\epsilon_c(\Phi))}] - U^0. \quad (21)$$

The expression for  $U^0$  in equation (21) is formally identical to the expression for  $\lambda$  in equation (20). We have already shown that the social planner maximizes  $\lambda$  given that  $V = V(\Phi)$  and  $p = p(\Phi)$ . This is equivalent to maximizing  $U^0$  given the same two constraints. QED

## Appendix 5: Proof of welfare corollaries

*Proof of Corollary 1:* Let  $\tau$  denote the technology, and let  $V = V(\tau, \Phi)$  denote the associated expected production value net of effort costs, and let  $R(\tau, \Phi)$  denote the associated expected rent that the contract leaves to the

worker. The firm chooses the value of  $\tau$ ,  $\tau^*$ , that maximizes  $E[\Pi(\tau)] = (1 - F(\epsilon_c(\tau, \Phi)))[V(\tau, \Phi) - R(\tau, \Phi) - U^0]$ . Moreover,  $E[\Pi(\tau^*)] = K$  in equilibrium. Let  $\tau'$  denote the constrained efficient value of  $\tau$  and  $\Phi' = \Phi(\tau')$  the associated optimal contract (which coincides with the equilibrium contract for this technology). Free entry implies that

$$U^{0'} = V(\tau', \Phi') - R(\tau', \Phi') - \frac{K}{(1 - F(\tau', \epsilon_c(\Phi')))}.$$

It follows that  $\tau^*$  is constrained efficient. Otherwise,  $U^{0'} > U^{0*}$  and the firms in the market could improve by choosing  $\tau'$  and the wage schedule given by  $\Phi'$  minus an arbitrarily small constant. QED

*Proof of Corollary 3.* While worker rents have zero social value with zero unemployment benefits, they have strictly negative values with positive unemployment benefit. The planner's maximization problem is thus identical with the maximization problem (5), with  $u(\epsilon)$  replaced by  $ku(\epsilon)$  in the integrand, where  $k > 1$  is a constant. The first-order condition is thus given by

$$b(\epsilon) = 1 - k \frac{1 - F(\epsilon)}{f(\epsilon)} c''(e) \alpha / \gamma^2.$$

As  $b$  decreases in  $k$  this completes the proof. QED.

## Appendix 6: Proof of Proposition 3

From equations (8) and (10) it follows that

$$E[\rho] = \int_{\epsilon^{\min}}^{\epsilon^{\max}} \alpha \left[ 1 - \frac{1 - F(\epsilon)}{f(\epsilon)} \frac{\alpha c''(e)}{\gamma^2} \right] (1 - F(\epsilon)) d\epsilon.$$

Given  $c'''(e) = 0$  the derivative with respect to  $\alpha$  is

$$\begin{aligned} \frac{dE[\rho]}{d\alpha} &= \int_{\epsilon^{\min}}^{\epsilon^{\max}} \left[ 1 - \frac{1 - F(\epsilon)}{f(\epsilon)} \frac{\alpha c''(e)}{\gamma^2} \right] - \alpha \frac{1 - F(\epsilon)}{f(\epsilon)} \frac{c''(e)}{\gamma^2} (1 - F(\epsilon)) d\epsilon \\ &= \int_{\epsilon^{\min}}^{\epsilon^{\max}} [2b(\epsilon) - 1] (1 - F(\epsilon)) d\epsilon. \end{aligned}$$

Hence, a reduction in  $\alpha$  increases (reduces) worker rents if  $\bar{b} \leq (>) 1/2$ , and then from equation (15) implies that the unemployment rate increases (decreases) if  $\bar{b} \leq (>) 1/2$ . QED



## Appendix 7: Proof of Proposition 4

For any given  $b$ ,  $c'(e) = \gamma b$  and hence  $c''(e)/\gamma^2 = b^2 c''(e)/c'(e)^2$ . We can thus write equation (8) as

$$b(\epsilon) = h(\epsilon, e(\gamma), b(\gamma))$$

where

$$h(\epsilon, e(\gamma), b(\gamma)) = 1 - \alpha \frac{1 - F(\epsilon)}{f(\epsilon)} \frac{b(\gamma)^2 c''(e(\gamma))}{c'(e(\gamma))^2}$$

which yields

$$\frac{db}{d\gamma} = \frac{\frac{\partial h}{\partial e} \frac{\partial e}{\partial \gamma}}{1 - \frac{\partial h}{\partial b}}.$$

Since  $c''(e)/c'(e)^2$  decreases in  $e$  and since  $e$  increases in  $\gamma$ ,  $\frac{\partial h}{\partial e} \frac{\partial e}{\partial \gamma} > 0$ . Furthermore,  $\frac{db}{d\gamma} > 0$  as  $\frac{\partial h}{\partial b} < 0$ . From equation (10) it follows that  $E[\rho]$  and thereby also  $R$  increases in  $\gamma$ . Equation (15) then implies that the unemployment rate increases in  $\gamma$ . QED.

## Appendix 8: Proof of proposition 5

Worker utility is given by  $u(\epsilon) = w(1 - t) - c(e)$ . Firm profit can thus be written as

$$\pi(\epsilon) = \bar{y} + \alpha\epsilon + \gamma e - \frac{u(\epsilon) + c(e)}{1 - t}.$$

The truth-telling condition is unaffected by taxes and remains  $u'(\epsilon) = c'(e)\alpha/\gamma$ . Hence, the first order conditions for the optimal contract are

$$\begin{aligned} \frac{c'(e)}{1 - t} &= \gamma + \frac{1}{f(\epsilon)} \lambda c''(e)\alpha/\gamma, \\ \dot{\lambda} &= \frac{f(\epsilon)}{1 - t}. \end{aligned}$$

Integrating up  $\lambda$ , inserting and re-arranging gives

$$c'(e) = \gamma(1 - t) - \frac{1 - F(\epsilon)}{f(\epsilon)} c''(e)\alpha/\gamma.$$

In a menu of linear contracts, the corresponding incentive parameter  $b$  is such that  $c'(e) = \gamma(1 - t)b$ , or that  $b = c'(e)/[\gamma(1 - t)]$ . It thus follows that

$$b(\epsilon) = 1 - \frac{1}{1 - t} \frac{1 - F(\epsilon)}{f(\epsilon)} c''(e)\alpha/\gamma^2. \quad (22)$$

For any given  $b$ ,  $c'(e) = \gamma b(1-t)$ , and hence  $c''(e)/[(1-t)\gamma^2] = bc''(e)/c'(e)\gamma\dots$   
 We can thus write equation (22) as  $b(\epsilon) = h(\epsilon, e(t), b(t))$ , where

$$h(\epsilon, e(t), b(t)) = 1 - \alpha \frac{1 - F(\epsilon)}{f(\epsilon)} b(t) c''(e(t)) / c'(e(t)) \gamma.$$

Differentiating  $b(\epsilon)$  with respect to  $t$  yields

$$\frac{db}{dt} = \frac{\frac{\partial h}{\partial e} \frac{\partial e}{\partial t}}{1 - \frac{\partial h}{\partial b}}.$$

Since  $e$  decreases in  $t$  and  $c''(e)/c'(e)$  decreases in  $e$ ,  $\frac{\partial h}{\partial e} \frac{\partial e}{\partial t} < 0$ . Furthermore,  $\frac{db}{dt} < 0$ , as  $\frac{\partial h}{\partial b} < 0$  which proves the first part of the proposition. Since  $\rho(\epsilon') = \int_{\epsilon_c}^{\epsilon'} u'(\epsilon) d\epsilon$  and  $u'(\epsilon) = c'(e)\alpha/\gamma = \alpha(1-t)b$  a reduction in  $t$  increases  $\rho(\epsilon)$  for all  $\epsilon$ , and thereby also  $R$ . But then it follows from equation (15) that a decrease in  $t$  increases the unemployment rate. QED

## References

- Akerlof, G. A. (1982), "Labor Contracts as Partial Gift Exchange ", *Quarterly Journal of Economics*, **97**, 543-69.
- Akerlof, G. A. and Katz, L. F. (1989), "Workers' Trust Funds and the logic of Wage Profiles", *Quarterly Journal of Economics*, **104**, 525-536.
- Acemoglu, D. and Shimer, R. (1999), "Holdups and Efficiency with Search Frictions", *International Economic Review*, **40**, 827-849.
- Baron, D. and Besanko, D. (1984), "Regulation and Information in a Continuing Relationship", *Information Economics and Policy* **1** 447-470. Baron, D. and Myerson, R. (1982), "Regulating a monopolist with unknown costs", *Econometrica*, **50**, 911-930.
- Booth, A.L. and Frank J. (1999), "Earnings, Productivity, and Performance-Related Pay" , *Journal of Labor Economics*, **17**, 447-463.
- Cahuc, P. and Zylberberg, A. (2004), *Labor Economics*, The MIT Press, Cambridge, Massachusetts.
- Carmichael, H. L. (1985), "Can Unemployment be Involuntary? Comment", *American Economic Review*, **75**, 1213-1214.

- Carmichael, H. L. (1990), "Efficiency Wage Models of Unemployment: One View", *Economic Inquiry*, **28**, 269-295.
- Chen, P. and Edin, P.A. (2002), "Efficiency Wages and Industry Wage Differentials: A Comparison Across Method of Pay", *The Review of Economics and Statistics*, **84**, 617-631.
- Dickens, W.T. and Katz, L. (1987), "Inter-Industry Wage Differences and Theories of Wage Differentials", NBER Working Paper No.. 2271.
- Dickens W.T., Katz, L.F., Lang, K. and Summers L.H. (1989), "Employee Crime and the Monitoring Puzzle", *Journal of Labor Economics* **7**, 331–347.
- Eriksson, T. (2001) , "How Common are the New Compensation and Work Organization Practices and Who Adopts them?" Aarhus School of Business, Manuscript.
- Foster, J.E. and Wan, Jr, H.Y. (1984), "Involuntary Unemployment as a Principal Agent Equilibrium", *American Economic Review*, **74**, 476-484.
- Fudenberg, D. and Tirole, J. (1991), *Game Theory*, MIT Press, Cambridge.
- Gibbons, R. and Katz, L. (1992), "Does Unmeasured Ability Explain Inter-Industry Wage Differentials", *Review of Economic Studies*, **59**, 515-535.
- Laffont, J.J. and Tirole, J (1993), *A Theory of Incentives in Procurement and Regulation*, MIT Press, Cambridge.
- Lazear, E.P. (1979), "Why is There Mandatory Retirement?", *Journal of Political Economy*, **87**, 1261–1264.
- Lazear, E.P. (2000), "Performance Pay and Productivity", *American Economic Review*, **90**, 1346-1361.
- MacLeod, W. B. and Malcomson, J. M. (1998), "Motivation and Markets", *American Economic Review* **88**, 388-411.
- Maskin, E. and Riley, J. (1984), "Monopoly with incomplete information", *The RAND Journal of Economics*, **15**, 171-196.
- Millward, N., Stevens, M., Smart, D., and Hawes, W.R. (1992) *Workplace industrial relations in transition: The ED/ESRC/PSI/ACAS surveys*. Aldershot, U.K.: Dartmouth.

- Mirlees, J. (1971), "An exploration in the theory of optimum income taxation", *Review of Economic Studies*, **38**, 175-208.
- Moen, E.R. and Rosén, Å. (2004a), "Does Poaching Distort Training", *Review of Economic Studies*, **71**, 1143-1162.
- Moen E.R and Rosén, Å. (2004b), "Performance Pay and Adverse Selection", CEPR Working Paper no 4511.
- Moen, E.R. and Rosén, Å. (2004c), "Equilibrium Incentive Contracts and Efficiency Wages", Manuscript.
- Pissarides, C.A. (2000), *Equilibrium Unemployment Theory*. MIT press, Cambridge MA.
- Pissarides, C.A. (1998), "The impact of employment tax cuts on unemployment and wages; The role of unemployment benefits and tax structure" *European Economic Review*, **42**, 155-183.
- Ritter, J.A. and Taylor L.J. (1994), "Workers as Creditors: Performance Bonds and Efficiency Wages", *American Economic Review*, **84**, 694 - 704.
- Salop, S.C. (1979), "A Model of the Natural Rate of Unemployment" *American Economic Review*, **69**, 117-125.
- Schmitz, P..(2004), "Job Protection Laws and Agency Problems Under Asymmetric Information" *European Economic Review* Vol. 48 (5), 2004, 1027-1046.
- Shapiro, C, and Stiglitz, J.E. (1984), "Equilibrium Unemployment as a Worker Discipline Device", *American Economic Review*, **74**, 433-444.
- Towers Perrin (1999). "Worldwide Total Remuneration". Report.
- Yang, H. (2003), "Efficiency wages and Performance Pay", University of Pennsylvania, Working Paper
- Weiss, A. W. (1980), "Job Queues and Layoffs in Labor Markets with Flexible Wages", *Journal of Political Economy*, **88**, 525-538.
- Weiss, A., (1990), *Efficiency Wages: Models of Unemployment, Layoffs and Wage Dispersion*, Claredon Press, Oxford..