Incentives in Competitive Search Equilibrium and Wage Rigidity*

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Abstract

This paper examines the competitive search equilibrium when the workers' effort choice and "type" are private information. We derive a modified Hosios rule determining the allocation of resources and analyze how private information influences the responsiveness of the unemployment rate to changes in macroeconomic variables. Most importantly, private information increases the responsiveness of the unemployment rate to changes in the general (type- and effort independent) productivity level. If the changes also affect the information structure, the responsiveness of the unemployment rate may be large even if the changes in expected productivity are small.

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1 Introduction

In this paper we derive the competitive search equilibrium when workers have private information regarding effort and a match-specific "type". We then investigate how private information influences the responsiveness of wages and unemployment to aggregate shocks.

In any search market, the resource constraint implies that there exists a trade-off between high wages and a high exit rate from unemployment. In the competitive search equilibrium, a market maker optimally balances this trade-off. The resulting wage, or equivalently, employment rent ensures that the agents on both sides of the market have the correct incentives to enter the market and search for a trading partner.

In a pure contractual setting with asymmetric information, rents play a different role. In a standard principal-agent model (Laffont and Tirole, 1993), where output is contractible and the agent has private information about his type, this information makes him better off. He receives information rents. The stronger incentives the principal gives the agent to exert effort, the higher will this information rent typically be. The principal thus faces a trade-off between rent extraction and effort provision, and chooses the wage contract so as to optimally balance these two considerations.

Our model combines the principal agent model and the competitive search model. When the market maker trades off employment rents and a high exit rate from unemployment, he or she takes into account the that employment rents ease the constraints imposed by workers' private information and thereby enhances efficiency. We derive a modified Hosios rule determining the constrained efficient resours allocation. When the value of relaxing the private information constraints is large, employment rents are large while few resources are used to create new jobs.

We believe that our model sheds some light on the issue of how wages respond to aggregate shocks. Recent studies by Shimer (2004, 2005a,) document empirical regularities of the business cycle that the standard matching model of the labor market cannot account for. He finds that fluctuations in the unemployment rate predicted by the model as a response to the observed productivity shocks are much smaller than the actually observed fluctuations in the unemployment rate. The reason is that wages are too flexible, and thereby absorb too much of the shock. He also finds that a low job creation rate rather than a high job destruction rate is responsible for the high unemployment rate during recessions. Similar findings are reported in Hall (2005).

A seemingly robust prediction of our model is that a negative shock which reduces the productivity of all matches by the same amount tightens the constraints imposed by the workers' private information. Consequently, employment rents become more important relative to creating new jobs. Therefore, wages become less responsive and unemployment more responsive to such shocks than in the standard search model. We interpret such a shock as an increase in input prices (oil prices). If, in addition, worker effort is more crucial after a negative shock (for instance because effort and other inputs with higher prices are substitutes), the responsiveness of the unemployment rate is further increased. The same may be true if a negative shock is associated with (or caused by) more private information to workers.

By contrast, private information may actually stabilize the unemployment rate for other kinds of macroeconomic shocks. After a negative shock to the matching technology, private information dampens the responsiveness of the unemployment rate.

A closely related paper is Shimer and Wright (2004). They consider a competitive search model where firms (not workers) have private information about productivity and workers have private information about effort. They show how private information may distort trade, and thereby increasing unemployment. They do, however, not analyze how the allocation of rents between workers and firms may influence these distortions.

Hall (2004a and 2004b) shows that the volatility of the unemployment rate increases dramatically if wages are sticky. As a rationale for wage stickiness, Hall refers to social norms.[1] Wage stickiness implies that a larger share of the match surplus is allocated to the workers in a recession than in a boom. Our model gives an alternative micro-foundation as to why this may be the case. Furthermore, in our model this counter-cyclical surplus allocation is an optimal response by the market maker in the presence of private information.

Kennan (2004) also studies the effect of information rents on unemployment fluctuations. In his model, workers and firms bargain over wages once they meet. Firms have private information in booms but not in recessions and thus earn information rents in booms. This increases the profits in booms, and thus also unemployment volatility. Nagypál (2004) and Krause and Lu-

[1] In Hall (2005) it is also shown that wage stickiness may be the result of alternative specifications of the bargaining procedure or of self-selection among workers.

bik (2004) allow for on-the-job search in a matching model and show how this may amplify the effects of productivity shocks on the unemployment rate. Menzio (2004) shows that firms with private information, may find it in their interest to keep wages fixed if hit by high-frequency shocks. Again, this increases volatility.

Another related model is developed in Faig and Jerez (2005). They analyze trade in a retail market with search frictions when buyers have private information about their willingness-to-pay. Although their paper is similar in the sense that they study private information in a competitive search environment, both model and emphasis are different from ours. Moral hazard is absent in their model, and their focus is on welfare analysis. They do not derive the modified Hosios condition, and they do not study the effects of changes in macroeconomic variables.

Our model is also related to a large literature on efficiency wage models (see for instance Weiss 1980 and Shapiro and Stiglitz 1984), and notably those papers that examines the impact of efficiency wages on unemployment volatility. Strand (1992) finds that efficiency wages reduce unemployment volatility. He assumes that firms may be tempted to fire worker after a negative aggregate shock. As a result, firms are reluctant to hire more workers during a boom as this increases the wage necessary to deter shirking. Thus, if the productivity differences are relatively small, employment does not change over the cycle. Dantine and Donaldson (1990) argue that efficiency wages may exacerbate the effect of productivity shocks on the unemployment rate if the shocks are short-lived compared to the time it takes to fire shirking workers. Ramey and Watson (1997) analyze how contractual fragility caused by the firms' inability to commit to a wage contract may increase the volatility of the unemployment rate. Rocheteau (2001) introduces shirking in a search model and shows that the non-shirking constraint forms a lower bound on wages. Finally, MacLeod, Malcomson and Gomme (1994) find that efficiency wages may lead to wage rigidity if workers face a higher probability of being fired for exogenous reasons during a recession than during a boom.

The paper is organized as follows: Section 2 presents the model. The optimal incentive contracts are derived in section 3 and the labour market equilibrium in section 4. In section 5 we analyze whether private information influence the responsiveness of wages to shocks. In section 6 we discuss alternative formulations of the incentive problem. Section 7 concludes.

2 The model

Our model incorporates optimal incentive contracts in a labour market equilibrium model with search frictions. The matching of unemployed workers and vacancies is modelled using the Diamond-Mortensen-Pissarides framework (Diamond 1982, Mortensen 1986, Pissarides 1985) with competitive wage setting as in Moen (1997). The asymmetric information model builds on the procurement model of Laffont and Tirole (1993), see also Moen and Rosén (2005) for an interpretation of this model in a frictionless labor market.

Agents and information

The economy consists of a continuum of ex ante identical workers and firms. All agents are risk neutral and have the same discount factor r. The measure of workers is normalized to one. Workers leave the market at an exogenous rate s and new workers enter the market as unemployed at the same rate. A firm is either matched with a worker and produces or unmatched and searches for a worker.

Output y of a worker-firm pair is observable and contractible and given by

$$y = \overline{y} + \varepsilon + \gamma e.$$

The variable ε reflects a match-specific productivity term, which is observable to the worker but not to the firm. It is I.I.D. over all worker-firm matches and continuously distributed on some interval $[\underline{\varepsilon}, \overline{\varepsilon}]$ with cumulative distribution function H.[2] The corresponding density function h has an increasing hazard rate. The variable e denotes worker effort, also unobservable to the firm.[3] The cost of effort is denoted by $\psi(e)$. We assume that $\psi(e)$ is increasing and $\psi'(e)$ is increasing and convex in e.

Firms offer performance contracts linking wages to output, and the worker observe these before he approaches the firm. When a worker and a firm meet, the worker first learns ε and then decides whether to accept the contract. If he rejects the offer he starts searching again, and the vacancy remain vacant.

Matching and asset value equations

[2] Note that ε also may reflect idiosyncraticies of the job in question which can easily be observed by the worker but not by the firm.

[3] We have thus not included cross effects between effort and worker type. Adding a cross term εe will complicate the expressions but will not bring new insights.

Let u denote the unemployment rate and v the vacancy rate in the economy. The number of contacts in the economy is determined by a concave, constant return to scale matching function $x(u, v)$. Let p denote the contact rate for workers and q the contact rate of firms. Since the matching function has constant return to scale we can write $q = q(p)$, with $q'(p) < 0$. We assume that the matching function is Cobb Douglas, $x(u, v) = Au^{\beta}v^{1-\beta}$, and hence that $p = q^{-\frac{1-\beta}{\beta}}$.

Suppose that all contacts with a match-specific productivity term that exceeds a cut-off level ε^* leads to employment. Furthermore, let W denote the expected discounted utility of a worker hired in a firm. The expected discounted utility U of an unemployed worker is given by

$$(r + s)U \equiv z + p(1 - H(\varepsilon^*))(W - U). \tag{1}$$

where z is utility flow when unemployed. The expected rent for a worker associated with a contact is defined as

$$R \equiv (1 - H(\varepsilon^*))(W - U).$$

There is a flow cost, c, associated with maintaining a vacancy. Let V denote the expected value of a firm with a vacancy and J the expected value to a firm that gets in contact with a worker. We can write

$$rV \equiv -c + qJ.$$

There is free entry of vacancies, hence the equilibrium value of V is equal to zero. Let S denote the expected surplus of a contact, defined as

$$S \equiv J + R.$$

Finally, let Y denote the expected discounted output net of worker effort of a contact, given that the contact leads to employment. It follows that

$$S = (1 - H(\varepsilon^*))(Y - U).$$

3 Optimal incentive contracts

An important building block for deriving the equilibrium is to characterize the solution to the problem of maximizing the contact surplus S with respect to the wage contract $w(y)$ under the condition that the expected rents R to the worker is below a threshold $\overline{R}$. (This is equivalent to the problem of maximizing expected firm profit J given that $R \leq \overline{R}$). We do not allow for transfers that take place before the worker learns ε. In principle, the market maker may condition the contract on tenure or past behavior. However, as shown in Baron and Besanko (1984) and Moen and Rosén (2005), the optimal dynamic contract repeats the optimal static contract provided that the firm can commit not to renegotiate. See footnote 4 for more on this.

In order to derive the optimal wage contract we use the revelation principle. A mechanism is a triple $(\varepsilon, e(\varepsilon), w(\varepsilon))$ that obeys the workers' incentive compatible (IC) constraints (workers choose effort so as to maximize utility) and the individual rationality (IR) constraints (workers only accept a contract if it gives an expected utility that is higher than that of continuing searching).

A worker of type ε that reports type $\widetilde{\varepsilon}$ receives a utility flow given by

$$\omega(\varepsilon, \widetilde{\varepsilon}) = w(\widetilde{\varepsilon}) - \psi(e(\widetilde{\varepsilon}) - \frac{\varepsilon - \widetilde{\varepsilon}}{\gamma}).$$

Truth-telling requires that $\varepsilon = \arg\max_{\widetilde{\varepsilon}} \omega(\varepsilon, \widetilde{\varepsilon})$, and from the envelope theorem it follow that the incentive compatibility constraint is given by $\frac{\delta\omega}{\delta\widetilde{\varepsilon}} = \psi'(e(\varepsilon))/\gamma$ (evaluated at $\widetilde{\varepsilon} = \varepsilon$). Let $R(\varepsilon)$ denote the rent to a worker of "type" ε. It follows that $R(\varepsilon) = \frac{\omega(\varepsilon)}{r+s} - U$. The incentive compatibility constraint can then be written as

$$(r+s)R'(\varepsilon) = \psi'(e(\varepsilon))/\gamma.. \tag{2}$$

The right-hand side expresses the gain in terms of reduced effort from deviating and reporting one unit lower value of ε. The worker must gain the same by telling the truth. The individual rationality constraint requires that $R(\varepsilon^*) \geq 0$.

The expected (flow) surplus of a contact can be written as

$$(r+s)S = \int_{\varepsilon^*}^{\overline{\varepsilon}} [\overline{y} + \varepsilon + \gamma e(\varepsilon) - \psi(e(\varepsilon))] - (r+s)U]dH, \tag{3}$$

which is maximized subject to the IR constraint, the IC constraint, and the maximum expected rents given to workers:

$$\begin{aligned} R(\varepsilon^*) &= 0 \qquad (4) \\ (r+s)R'(\varepsilon) &= \psi'(e(\varepsilon))/\gamma \\ \int_{\varepsilon^*}^{\overline{\varepsilon}} R(\varepsilon)dH(R) &= \overline{R}. \qquad (5) \end{aligned}$$

The associated Hamiltonian is

$$\begin{aligned} \mathcal{H} &= [\overline{y} + \gamma\varepsilon + \gamma e(\varepsilon) - \psi(e(\varepsilon)) - (r+s)U]h(\varepsilon) \\ &\quad + \lambda\psi'(e(\varepsilon))/\gamma \ - \alpha[\int_{\varepsilon^*}^{\overline{\varepsilon}} R(\varepsilon)dH(R) - \overline{R}]. \end{aligned}$$

The first order conditions for $e(\varepsilon)$ can be written as

$$(\gamma - \psi'(e(\varepsilon))h(\varepsilon) = \lambda\psi''(e(\varepsilon))/\gamma. \qquad (6)$$

Furthermore,

$$\lambda'(\varepsilon) = \delta\mathcal{H}/\delta R(\varepsilon) = -\alpha h(\varepsilon).$$

Since $\overline{\varepsilon}$ is free it follows that $\lambda(\overline{\varepsilon}) = 0$. Thus, $\lambda = \alpha(1 - H(\varepsilon))$. Inserted, this gives

$$\gamma - \psi'(e(\varepsilon)) = \alpha\frac{1 - H(\varepsilon)}{h(\varepsilon)}\psi''(e(\varepsilon))/\gamma. \qquad (7)$$

Consider a worker of type ε' The left hand side is the gain of increasing effort for a type ε'. The right hand side the cost in terms of increased rents to all workers with a type higher than ε', divided by the density of ε'-type workers.

The difference between the first order condition (7) and the analogous first order condition in Laffont and Tirole (1993) (and Moen and Rosén, 2005) regards α. As will be clear below, α reflects the shadow value of worker rents for the contact surplus. In the standard model, $\alpha \equiv 1$, as the firm does not attach any value to worker rents. In our model, α is endogenous and depends on $\overline{R}$.

Let (a, b) denote a linear contract of the form $w = a + by$. It is well known that the optimal non-linear contract can be represented by a menu $(a(\epsilon), b(\epsilon))$ of linear contracts (see, e.g., Laffont and Tirole, 1993). For any b, the worker chooses the effort level such that $\psi'(e) = b\gamma$. Using the condition $\psi'(e) = b\gamma$ in equation (7), we obtain

$$b(\epsilon) = 1 - \alpha \frac{1 - H(\epsilon)}{h(\epsilon)} \frac{\psi''(e)}{\gamma^2}. \tag{8}$$

Henceforth, we refer to b as the incentive power of the associated linear contract.

The optimal cut-off value ε^* is obtained by setting $\mathcal{H} = 0$:

$$-[\overline{y} + \varepsilon^* + \gamma e(\varepsilon^*) - \psi(e(\varepsilon^*)) - (r+s)U] h(\varepsilon^*) + \alpha(1 - H(\varepsilon^*)) \frac{\psi'(e(\varepsilon^*))}{\gamma} = 0. \tag{9}$$

The first term is the direct loss in (flow) profits of increasing the cut-off value (value of production less of the wage of an ε^*-type worker). The second term is the gain due to decreased rents to the remaining workers. If $\mathcal{H}(\underline{\varepsilon}) < 0$, it is optimal to hire all workers, and we set $\varepsilon^* = \underline{\varepsilon}$.[4]

For sufficiently high values of $\overline{R}$, the constraint (5) does not bind and hence $\alpha = 0$. From the first order conditions (7) and (9) it then follows that $\psi'(e(\varepsilon)) = \gamma$ (full incentives for all types) and that $\overline{y} + \varepsilon^* + \gamma e(\varepsilon^*) - \psi(e(\varepsilon^*)) = (r+s)U$ (net productivity equal outside option for an "ε^*-type" worker). We refer to this as the first best production level. Let R^* denote the lowest value of $\overline{R}$ such that constraint (5) does not bind. When $\psi'(e(\varepsilon)) = \gamma$ equation (2) implies that $(r + s)R'(\varepsilon) = 1$ and, since $R(\varepsilon^*) = 0$, that $R(\varepsilon) = \frac{\varepsilon - \varepsilon^*}{r+s}$ and hence that [5]

[4] As the workers' outside option is constant over time, a time dependent contract cannot improve efficiency. To gain intuition for this, suppose to the contrary that the firm offered the worker a contract with an effort level $e_1(\varepsilon)$ for the first t periods, and then effort level $e_2(\varepsilon)$, with $e_1(\varepsilon) \neq e_2(\varepsilon)$ for some ε. The firm can always improve by smoothing the effort levels. From the incentive compatibility constraint it follows that the flow value of the information rent is convex in e (since $\psi'''(e) > 0$). At the same time, the net output flow is concave in e. Smoothing effort levels therefore both increases the net output flow and reduces the information rents, and thereby surely increases efficiency. A mathematical proof is given in Moen and Rosén (2005).

[5] More intuitively, when the worker is given full incentives he is the residual claimant and receives the entire production value in excess of $(r + s)U$.

$$R^* = \int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{\varepsilon - \varepsilon^*}{r+s} dH(\varepsilon).$$

For $\overline{R} > R^*$, the optimal value of S is independent of $\overline{R}$, and we write $S = S^*(U)$. Note also that as long as $\alpha > 0$, it follows from (6) that the effort level is strictly decreasing in α for any ε. We refer to α as the shadow (flow) value of worker rents.

Lemma 1 *Suppose $R < R^*$. Then the following holds:*
a) The cut-off level ε^* is increasing in α (for a given U)
b) The shadow value α of worker rents is decreasing in R (for given U).
c) If $\varepsilon^* > \underline{\varepsilon}$, an increase in U reduces the shadow value α of worker rents. If $\varepsilon^* = \underline{\varepsilon}$, α is independent of U.

Proof. See Appendix ■

At first glance, result a) may seem surprising. An increase in α tends to reduce e, which again reduces the cut-off. However, a reduction in e also reduces the value of hiring a person. Given the shadow cost α of worker rents, the value of $e(\varepsilon^*)$ is optimally set, and the envelope theorem thus applies.

We write the optimal S as a function of U and $\overline{R}$, $S = S(\overline{R}, U)$. Furthermore, let $S_R = \frac{\partial S(R,U)}{\partial R}$, and define S_U, S_{RR} and S_{RU} analogously. Note that

$$(r+s)S_R = \delta\mathcal{H}/\delta R = \alpha.$$

From Lemma 1 the next Lemma follows directly

Lemma 2 *The following holds for the contract surplus $S(R,U)$*
a) $S(R,U)$ is increasing and concave in R for $R < R^*$ ($S_{RR} < 0$).
b) If all types are hired ($\varepsilon^* = \underline{\varepsilon}$), then $S_{RU} = 0$.

Proof. Property a) is shown in Appendix. Property b) follows directly from Lemma 1 c). In this case we can write S_R as a function of R only, $S_R(R)$. ■

4 Equilibrium

Our equilibrium solution concept is the competitive search equilibrium (Moen, 1997). In the competitive search equilibrium, the expected utility of unemployed workers is maximized subject to the resource constraint of the economy (essentially the free entry condition of firms).[6] As in Mortensen and Wright (2002), the equilibrium can be interpreted as follows: A market maker determines the wage contract in his market. Free entry of market makers ensures that the only market maker that survives in the market is the one that maximizes the utility of unemployed workers given the free entry condition of firms.

As a benchmark, suppose the planner had full information about ε, enabling him to implement the first best production level. In this case, $S = S^*(U)$, defined above. The competitive search equilibrium p^c, R^c, U^c then solves

$$\begin{aligned}\max_R (r+s)U &= z + pR \\ &\quad \text{S.T.} \\ \frac{c}{q(p)} &= S^*(U) - R.\end{aligned}$$

If R^* is less than the "search rent" R^c, the market maker can implement the full-information competitive search equilibrium (FICSE) even in the presence of private information:

Lemma 3 *If the search rent R^c exceeds the information rent R^*, the market maker can implement the full information competitive search equilibrium.*

Proof. Omitted ■

In the remainder of the paper we assume that $R^* > R^c$. This assumption is discussed in some detail in section 6.2. FICSE can also be obtained when $R^* > R^c$ if the market maker can cross subsidize entry, by collecting an entry fee from workers and a subsidy for vacancies.

Proposition 4 *(Irrelevance of private information) Suppose the market maker can collect an entry fee from the workers, and subsidize vacancies that enter their market. Then the first best competitive search equilibrium is always feasible.*

Proof. See Appendix. ■

Cross subsidization between workers and firms breaks the link between the workers' rent when employed and the firms' incentives to enter the market.

[6] See Acemoglu and Shimer (1999) and Moen and Rosén (2004).

Thus, the market maker can solve for the optimal trade-off between wages and job finding rate without influencing worker productivity once hired. A similar result is derived in Faig and Jarez (2005).

A sign-on fee paid by the worker to the firm may play the same role as an entry fee. When the worker has private information the sign-on fee must be agreed upon before the private information is revealed to the worker.

In what follows we do not allow for cross-subsidization between workers and firms. As the market maker has to obey the individual rationality constraint and the incentive compatibility constraint of workers, he faces a relationship $S(R,U)$ between productivity and worker rents. The constrained competitive search equilibrium then solves

$$\max_R (r+s)U = z + pR \tag{10}$$

$$\text{S.T. } \frac{c}{q(p)} = S(R,U) - R. \tag{11}$$

For any given R, there exists a corresponding value of p and U, hence we can write $p = p(R)$ and $U = U(R)$. By definition, $U'(R) = 0$ in optimum. From equation (10) it follows that

$$el_R p = -1, \tag{12}$$

where $el_R p$ denotes the elasticity of p with respect to R. From equation (11) it follows that

$$el_R[\frac{c}{q(p(R))}] = -(1 - S_R)\frac{R}{S-R}. \tag{13}$$

Substituting in for $el_R p = -1$ gives

$$\begin{aligned} el_R[\frac{c}{q(p(R))}] &= -el_p q(p) el_R p(R) = el_p q(p) \\ &= -\frac{\eta}{1-\eta}, \end{aligned}$$

where $\eta = |el_\theta \widetilde{q}(\theta)|$ denotes the absolute value of the elasticity of q with respect to $\theta = v/u$. [7] The equilibrium in the search market is thus given by

[7] To see that $-el_p q(p) el_R p(R) = el_p q(p)$, let $p = \widetilde{p}(\theta)$. Then

$$(1-S_R)\frac{R}{S-R}=\frac{\eta}{1-\eta}. \tag{14}$$

When $S_R=0$, the equation is identical to the Hosios condition for efficiency in search models (Hosios 1990). We will refer to this equation as the modified Hosios condition.

Proposition 5 *The constrained competitive search equilibrium satisfies the modified Hosios condition (14).*

The modified Hosios condition states that the share of the match surplus that is allocated to the worker increases, as the marginal value of worker rents, S_R, increases. Thus, a smaller fraction of the match surplus is allocated to job creation. With Cobb-Doulgas matching function, $\eta=\beta$, and the modified Hosios condition is

$$(1-S_R)\frac{R}{S-R}=\frac{\beta}{1-\beta}. \tag{15}$$

5 Comparative statics

As mentioned in the introduction, an important issue is whether private information influences the responsiveness of wages to economy-wide shocks. We address this question by analyzing how a *change* in parameter values (for instance productivity) changes the unemployment rate.

It is well known that the comparative statics with endogenous cut-off levels are notoriously difficult in search models. Therefore, we first assume that all worker types are hired, i.e. $\varepsilon^*=\underline{\varepsilon}$. In this case we have from lemma 1 that $S_{RU}=0$.. We return to the case with an interior cut-off level in section 5.3.

We say that private information stabilize the unemployment rate whenever a negative shift (in, say, productivity) leads to a reduction in S_R (or,

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$$\begin{aligned} el_p q(p) &= el_p \widetilde{q}(\widetilde{p}^{-1}(p)) \\ &= \frac{el_\theta \widetilde{q}(\theta)}{el_\theta \widetilde{p}(\theta)} \end{aligned}$$

Since $el_\theta \widetilde{q}(\theta)=-\eta$ and $el_\theta \widetilde{p}(\theta)=el_\theta[\theta\widetilde{q}(\theta)]$, it follows that $el_p q(p)=-\frac{\eta}{1-\eta}$.

equivalently, in α) and thus to a larger fraction of the match surplus being allocated to job creation. In the opposite case, the private information destabilize the unemployment rate.

In general, a shift in parameters may influence the relationship between S and R. However, some shocks will typically not influence this relationship, and we refer to them as contract-neutral shifts. These shifts are:

- Changes in the value of leisure (or unemployment benefit).
- Changes in the matching function.
- Changes in general (type- and effort-independent) productivity, here changes in $\overline{y}$. This may be interpreted as changes in input prices (e.g. oil prices).

By contrast, shifts in the distribution of ε and the importance of effort, γ, have a direct influence on the relationship between S and R, and are referred to as contract-affecting shifts.

5.1 Contract-neutral shifts

Contract-neural shocks can be analyzed by the following figure (formal proofs are given after the Proposition below).

The equilibrium is described in figure 1. At R^0, $1 - S_R(R) = 0$. For values at or below R^0, there is no trade-off between R and p, as an increase in R increases p. The equilibrium thus has to be to the right of R^0. By assumption, the equilibrium value of R is below R^*.

The decreasing curves show the ratio $\frac{S-R}{R}$ as a function of R for a given value of U. For $R > R^0$ we have that $S_R(R;U) - R$ is decreasing in R.[8] Note that for a given R, S is decreasing in U. Thus, a positive shift in U shifts the $\frac{S-R}{R}$-curve downwards.

Consider first an increase in z, the value of leisure. For a given wage contract, an increase in z increases the expected discounted utility U for unemployed workers. For a given R this reduces the match surplus S, and

[8] By definition, $\frac{dU}{dR} = 0$ in equilibrium. Thus, the slope of the $\frac{S(R;U)-R}{R}$ curve (with U constant) is equal to the slope of the $\frac{S(R,U(R))-R}{R}$-curve at the equilibrium point.

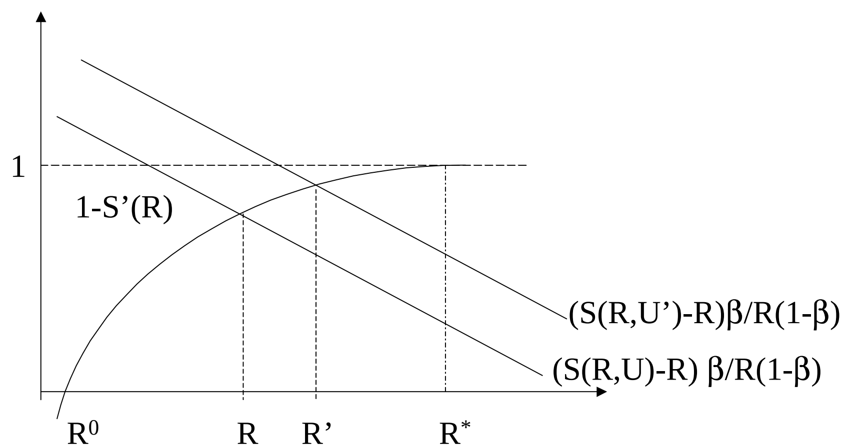

Figure 1

the $\frac{S-R}{R}$-curve shifts down. It follows that R falls, and hence that $S'(R)$ increases in equilibrium. Thus, a smaller share of the surplus is allocated to job creation, and this increases the unemployment rate further. Thus, for changes in the value of leisure, private information tends to destabilize the unemployment rate.

Consider now a negative shift in $\overline{y}$. This reduces both Y and U. Due to the envelope theorem, it follows from (1) that $(r+s)dU = p(dW - dU)$ and $dW = dY$. Thus,

$$\frac{\partial U}{\partial \overline{y}} = \frac{p}{r+s+p}\frac{\partial Y}{\partial \overline{y}} < \frac{\partial Y}{\partial \overline{y}}. \tag{16}$$

Thus, after a negative shock to $\overline{y}$, S falls for a given R. It follows that the $\frac{S-R}{R}$ curve shifts down, and hence that R falls and $S'(R)$ increases. Thus, a larger share of the surplus is allocated to the worker after the negative shock, and hence destabilizes the unemployment rate.

Finally, consider shocks to the matching technology measured by A. A negative shock to the matching technology increases the unemployment rate. At the same time U shifts down, and hence $\frac{S-R}{R}$ shifts up. It follows that R increases and S_R decreases. Thus, private information stabilize the unemployment rate after a shift in A. The same holds for an increase in the cost of search, c.

Proposition 6 *Consider a shock to the economy. Then the following holds*

a) Private information destabilize the unemployment rate after shocks to the value of leisure.

b) Private information destabilize the unemployment rate after contract-independent changes in productivity (the same for all worker "types")

c) Private information stabilize the unemployment rate after shocks to the matching technology and to the cost of search.

Proof. See Appendix ■

5.2 Contract-affecting shifts

Consider first the effects of shifts in the importance of effort. We want a negative shift in the product function to be associated with increased importance of effort. If input prices drive the shock, this may be interpreted as that worker effort and other inputs being substitutes. We therefore rewrite the production function slightly to

$$y = \widetilde{y} + \varepsilon + \gamma(e - e^0), \tag{17}$$

where $e^0 > e^*$, the equilibrium value of e. This is equivalent to our initial formulation with $\widetilde{y} = \overline{y} + \gamma e^0$. Suppose that a negative shock is driven by an increase in γ (the importance that the worker exert effort).

Proposition 7 *Consider a positive shift in γ. Suppose $\psi''/(\psi')^2$ is non-increasing in e, then the private information destabilizes the unemployment rate*

Proof. See Appendix ■

The destabilizing effects may be particularly strong in this case, as it consists of two components. An increase in γ increases S_R *for a given value of U*, and this was not the case in the other shifts we have been studying so far. This comes *in addition* to the effects through a reduction in R induced by the fall in productivity.

The restriction imposed on $\psi(e)$ is rather mild, and is satisfied for most convex functions. For instance, any polynomial of the form $\psi(e) = e^n$ satisfies this condition, as well as the exponential function e^e.

Consider then a shift in the amount of private information the worker's possesses, modeled as an increase in the spread of ε. To this end, write $\varepsilon = ka$, where a is a stochastic variable and k a scalar. On the one hand, an increase in k increases the amount of private information the workers posses, and for a given contract this increases worker rents. Thus, for a given R the incentive power of the contract has to be reduced. This tends to increase the marginal value of effort and thus S_R, the marginal value of worker rents. On the other hand, an increase in k implies that more rents are needed to increase worker incentives, and this tend to reduce the value of S_R. It turns out that if the private information problems are moderate, the first effect dominates, and an increase in k increases S_R. If private information problems are more severe, an increase in k may reduce α. To get sharper result, assume that the cost of effort function $\psi(e)$ is quadratic, and define $\overline{b}$ as

$$\overline{b} = \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} b(\varepsilon)/(\overline{\varepsilon} - \underline{\varepsilon})d\varepsilon.$$

The following then holds:

Lemma 8 *For a constant U, the following holds: If $\overline{b}$ is above $1/2$ initially, an increase in k increases S_R. If $\overline{b}$ is below $1/2$ initially, an increase in k increases S_R*

Note that if R is close to R^*, then b is close to 1 for all ε, and $\overline{b}$ is certainly above $1/2$.

What then about the effects caused by changes in U. From the previous section we know that an increase in U, cet par, increases S_R. As long as $\varepsilon^* = \underline{\varepsilon}$, an increase in k is associated with a fall in U, as the effort level falls. In this case, the effects through U and through the change in contract goes in the same direction after a shock if $b < 1/2$. However, if $\varepsilon^* > \underline{\varepsilon}$, an increase in k will increase the average ε among hired workers, and it follows that U may increase. In this case the two effects go in the same direction if $b > 1/2$ (for a given cut-off).

The important insight here is that changes in workers' private information may give rise to substantial changes in the unemployment level even if the changes in U and average productivity are fairly small.

5.3 Effects through the cut-off level

The analysis above is made under the assumption that all types are hired. In this subsection we briefly discuss the effect of the same shifts on the cut-off level when $\varepsilon^* > \underline{\varepsilon}$. To facilitate reading we repeat the first-order condition for optimal cut-off level ε^*.

$$\overline{y} + \varepsilon^* + \gamma e - \psi(e(\varepsilon^*)) - (r+s)U = \alpha \frac{(1 - H(\varepsilon^*))}{h(\varepsilon^*)} \frac{\psi'(e(\varepsilon^*))}{\gamma} \tag{18}$$

The following Proposition holds:

Proposition 9 *For contract-neutral productivity shocks:*
a) A fall in $\overline{y}$ increases the cut-off level ε^*.
b) A rise in z increases the cut-off level ε^*.
c) A fall in A decreases the cut-off level ε^*.

Proof. See Appendix ■

A fall in $\overline{y}$ implies that the left-hand side falls (since $\overline{y}$ falls more than $(r+s)U$). This tends to increase the cut-off level ε^*. Furthermore, we know that an increase in α also increases ε^*. A similar argument holds for shifts in z and A.

Thus, in all cases the effects through the cut-off level seems to accesarbate our previous findings. For instance, a negative shift in $\overline{y}$ will increase the cut-off level, and this will further destabilize the unemployment level. However, there is one caveat here: As ε^* shifts up after a fall in $\overline{y}$, this tends to dampen the increase in S_R, and S_R may even fall. However, this typically happens when the increase in ε^* (and thus its effect on the unemployment rate) is large.

Consider a change in γ. With the re-specification of the product function (equation (17)), the cut-off level is given by

$$\overline{y} + \varepsilon^* + \gamma(e(\varepsilon^*) - e^0) - \psi(e(\varepsilon^*)) - (r+s)U = \alpha \frac{(1 - H(\varepsilon^*))}{h(\varepsilon^*)} \frac{\psi'(e(\varepsilon^*))}{\gamma}. \quad (19)$$

Again, the direct effect of an increase in α is an increase in ε^*. The fall in U is smaller the fall in average productivity. The fall in productivity is, for a given e, higher the lower is the worker type (since his effort is lower), which in isolation implies that the left-hand side falls even more. However, e also increases, making the results more uncertain.

We have not been able to show robust results for changes in the amount of private information, measured by k. Intuition suggests that the cut-off level should increase, as it become more important to get good matches. Furthermore, if $\overline{b}$ is relatively large, α goes up for a given ε^*, and this will also tend to increase ε^*. However, it is hard to prove these results analytically.

6 Generalization and discussion

In this section we first discuss alternative formulations asymmetric information problem the agents are facing. Then we discuss the requirement that $R < R^*$ in some detail.

6.1 Alternative formulations of the incentive problem

The shirking model

In the shirking model (Shapiro and Stiglitz 1984), workers are identical, but both worker effort and output is private information to the worker. Effort is either 0 or 1, and output is y if the worker exerts effort and zero otherwise.[9] The effort cost of is ψ. Let g denote the probability rate that a shirking worker is detected, in which case he is fired. The non-shirking condition is then given by

$$\psi \leq gR$$

That is, the cost of effort should be less than the probability rate of being detected when shirking times the cost of losing the job. Let $R^{ns} = \psi/g$ denote the lowest rent that prevents the worker from shirking. Define the constrained competitive search equilibrium as the allocation that maximizes U given the non-shirking constraint. It follows that $R = \max[R^c, R^{ns}]$. Suppose we are in a region where the non-shirking constraint binds. A fall in y then has no impact on R. Since the contact surplus S decreases, this requires that α increases, a larger fraction of the match surplus is given out as employment rents. Thus, shirking destabilize the economy. [10]

Non-pecuniary aspects of employment

Suppose workers obtain non-pecuniary gains from the employment relationship, and that these gains are private information to the workers and thus cannot be contracted upon. In all other respects the workers have symmetric information.

To be more specific, suppose the utility flow of a match for a worker who is paid a wage w is equal to $w + \tau$, where τ can take a high value τ^h or a low value τ^l. We assume that τ is I.I.D. over all worker-firm pairs. Worker productivity is the same for both types of workers, and equal to y. Efficient matching requires that a contact leads to a match whenever $S(\tau) \geq 0$. Workers, by contrast, only accept jobs for which $R(\tau) \geq 0$. Suppose that initially, $R(\tau^l) \geq 0$ in the unconstrained equilibrium. Thus, both types of workers accept the job and there are no information problems. In this case, $\alpha = 0$.

Consider a fall in y. For a given value of α, this leads to a fall in R. Thus, after the shock we may have that $R(\tau^l) < 0 < S(\tau^l)$ if the same surplus-sharing rule is applied. Thus, in order to motivate workers to stay

[9]Note that S_R is not defined at $R = R^{ns}$. Thus, we cannot set $\alpha = S_R$ in this case.

[10]Rocheteau (2001) incorporates the shirking condition into a standard search model where workers and firms bargain over the wage. However, he does not analyze the effects of economywide shocks.

after a low realization of τ, the market maker may increase the share of the surplus that is allocated to the workers so that workers accept all job offers. This will increase α and thus destabilize the unemployment rate.

6.2 More on the requirement that $R^c < R^*$

In the analysis above we have assumed that $R^c < R^*$. Here we discuss requirements making this condition to hold. The first thing to note is that if not all workers are hired in the first best equilibrium, then $R^c < R^*$:

Lemma 10 *Suppose $\overline{y} + \underline{\varepsilon} + \gamma e(\underline{\varepsilon}) - \psi(e(\underline{\varepsilon})) < U^c$. Then the first best competitive search equilibrium is infeasible.*

Proof. The proof is done by contradiction. Suppose the first best competitive search equilibrium did exist. In this equilibrium, let ε^* denote the (optimal) cut-off productivity, given by the equation $\overline{y} + \varepsilon^* + \gamma e(\varepsilon^*) - \psi(e(\varepsilon^*)) = U^c$. The marginal worker must be paid a wage equal to his productivity. Furthermore, as $w'(y) = 1$ for all other workers, first best implies zero profit to the firm. Thus, no vacancies enter the market and no workers are employed. This is inconsistent with equilibrium. ■

It follows that as long as the distribution of ε is non-degenerate, the first best competitive search equilibrium is infeasible, provided that the search frictions measured by the search costs c are sufficiently small:

Corollary 11 *The first best competitive search equilibrium is infeasible if the search costs c are sufficiently small (provided that the distribution of ε is not degenerate).*

Proof. In competitive search equilibrium, $p \to \infty$ as $c \to 0$... As a result, the optimal cut-off approaches $\overline{\varepsilon}$. From Lemma 10 it then follows that the first best competitive search equilibrium is infeasible. ■

However, it may well be that $R^c < R^*$ even if all worker "types" are hired. Suppose $\varepsilon = ka$, where a is a stochastic variable and k a scalar. Then the following holds

Lemma 12 *For any given combination of parameters and any distribution H of a with finite support, there exists an interval $k \in (\underline{k}, \overline{k})$ such that for any $k\varepsilon$ the following holds: 1) first best is not feasible, and 2) the cut-off level is equal to $\underline{\varepsilon}$.*

Proof. See Appendix. ■

7 Conclusion

In this paper we derive the competitive search equilibrium when workers have private information regarding effort and "type". Wage contracts are used to enhance efficiency. We then investigate the effects of economy-wide shocks on the unemployment- and vacancy rates.

In the standard competitive search equilibrium, the planner trades off a high wage (or the rents associated with employment) to employed workers and a high exit rate from unemployment. Private information brings in an additional effect: Worker rents ease the constraints imposed by the workers' private information and thereby enhance efficiency. We derive a modified Hosios rule determining the allocation of resources. When the information problems are more severe, fewer resources are used to create vacancies.

Shocks to the economy may change the productivity-enhancing value of worker rents, and this influences the responsiveness of the wage- and unemployment rate. We find that private information reduces the responsiveness of the unemployment rate to changes in the matching technology. However, it increases the responsiveness of the unemployment rate to changes in the deterministic part of the production function. or in the value of leisure. The responsiveness of the unemployment rate to changes in the information structure may be large even if the changes in expected productivity are small.

8 Appendix

Proof of Lemma 1.

a) It is convenient to rewrite (9) as

$$\overline{y} + \varepsilon^* - (r+s)U = \alpha \frac{1-H(\varepsilon^*)}{h(\varepsilon^*)} \frac{\psi'(e(\varepsilon^*))}{\gamma} - (\gamma e(\varepsilon^*) - \psi(e(\varepsilon^*)))$$

Denote the left-hand side by $X_L(\varepsilon)$ and the right-hand side by $X_R(\varepsilon; \alpha)$. Obviously $X_L'(\varepsilon) = 1 > 0$. As the second order condition must be satisfied locally, we know that $X_L(\varepsilon)$ crosses $X_R(\varepsilon)$ from below. It is therefore sufficient to show that around $\varepsilon = \varepsilon^*$an increase in α shifts $X_R(\varepsilon)$ up. Now

$$\frac{\partial X_R(\varepsilon^*; \alpha)}{\partial \alpha} = \frac{1-H(\varepsilon^*)}{h(\varepsilon^*)} \frac{\psi'(e(\varepsilon^*))}{\gamma} + \alpha \frac{1-H(\varepsilon^*)}{h(\varepsilon^*)} \frac{\psi''(e(\varepsilon^*))}{\gamma} \frac{de}{d\alpha} - (\gamma - \psi'(e(\varepsilon^*))) \frac{de}{d\alpha}$$

Now from (7) $\gamma - \psi'(e) = \alpha \frac{1-H}{h} \frac{\psi''}{\gamma}$. Hence the two last terms cancel out, and

$$\frac{\partial X_R(\varepsilon^*; \alpha)}{\partial \alpha} = \frac{1-H(\varepsilon^*)}{h(\varepsilon^*)} \frac{\psi'(e(\varepsilon^*))}{\gamma} > 0$$

Result a) thus follows.

b) We know from a) that ε^* is increasing in α. From (7) it follows that $\frac{\partial e}{\partial \alpha} < 0$.

Suppose then that α is increasing in R. Then we know that an increase in R implies that

1. e decreases
2. ε^* increases

But then it follows that R falls (from (2)), a contradiction as long as $R < R^*$.

c) U only enters the contract through the cut-off equation, which can be written as

$$\overline{y} + \varepsilon^* - (r+s)U + (\gamma e(\varepsilon^*) - \psi(e(\varepsilon^*))) = \alpha \frac{1-H(\varepsilon^*)}{h(\varepsilon^*)} \frac{\psi'(e(\varepsilon^*))}{\gamma} \tag{20}$$

For all hired workers, the effort level is independent of U.

We first want to show that an increase in U leads to a fall in α if and only if it leads to an increase in ε^*. Consider an increase in ε^*. This leads to a lower value of R, as we are integrating over a shorter interval. Thus, the rent-constraint allows for more incentive-powered contracts. As a result, the shadow value of R (that is, α) falls. If ε^* falls, the opposite holds, and α increases.

Suppose then that ε^* decreases in U. For a given α, an increase in U reduces the gain from hiring workers (reduces the left-hand side of 20), and ε^* increases. From a) we know that ε^* is increasing in α. Thus, for ε^* to fall we must have that α decreases. However, we have just shown that ε^* and α move in opposite directions, and we have thus derived a contradiction.

Proof of Proposition 4

We know that efficiency can be obtained if $R^c \geq R^*$. Suppose therefore that $R^c < R^*$. Then first best can be obtained as follows. When the worker and the firm meets, the worker receives an expected rent R^* so that first best production is ensured. To obtain the optimal vacancy rate, the market maker gives the vacancies a subsidy $D = q(p^c)(R^* - R^c)$ when entering the search market, so that the expected value of entering is $q(Y^* - R^* - U^c) + D = q(Y^* - R^* - U^c) + q(R^* - R^c) = q(Y^* - R^c - U^c)$. It follows that the correct number of firms enter the market. The unemployed workers are charged a fee fee $T = p^c(R^* - R^c)$ when entering. Since $qv = x = pu$, this scheme balances the budget. QED

Proof of Proposition 6

It is convenient to rewrite (15) as

$$(1 - S_R(R, U; X))R - \frac{\beta}{1-\beta}(S(R, U; X) - R) = Q(R, U; X) = 0$$

where $X = \overline{y}, z, A, c$. Total differenciating gives

$$\frac{dR}{dX} = -\frac{\partial Q(R, U; X)}{\partial X} / \frac{\partial Q(R, U; X)}{\partial R}$$

Now,

$$\frac{\partial Q(R, U; X)}{\partial R} = -S_{RR}R + 1 - S_R - \frac{\beta}{1-\beta}(S_R - 1)$$

Since $S_{RR} < 0$ (Lemma 2a) and $1 - S_R > 0$, we know that $\frac{\partial Q(R,U,X)}{\partial R} > 0$. Furthermore, since α is decreasing in R it is sufficient to prove that: a) $\frac{\partial Q}{\partial z} > 0$, b)$\frac{\partial Q}{\partial \overline{y}} < 0$, c) $\frac{\partial Q}{\partial A} < 0$, $\frac{\partial Q}{\partial c} > 0$. First recall that $S_{RU} = 0$ (Lemma 1c), $S_U < 0$, $S_U \frac{\partial U}{\partial \overline{y}} + S_Y \frac{\partial Y}{\partial \overline{y}} > 0$ (equation (16)), Also note that $S_{RA} = S_{Rc} = S_{R\overline{y}} = 0$ (proof analogous to that of $S_{RU} = 0$). Now,

$$\frac{\partial Q(R,U;z)}{\partial z} = -\frac{\beta}{1-\beta} S_U \frac{\partial U}{\partial z} > 0$$

$$\frac{\partial Q(R,U;\overline{y})}{\partial \overline{y}} = -\frac{\beta}{1-\beta} (S_U \frac{\partial U}{\partial \overline{y}} + S_{\overline{y}}) < 0$$

$$\frac{\partial Q(R,U;A)}{\partial c} = -\frac{\beta}{1-\beta} S_U \frac{\partial U}{\partial A} < 0$$

$$\frac{\partial Q(R,U;c)}{\partial c} = -\frac{\beta}{1-\beta} S_U \frac{dU}{dc} > 0$$

Q.E.D.

Proof of Proposition 7

Worker rent for any given type ε' is given by

$$R(\varepsilon') = \int_{\underline{\varepsilon}}^{\varepsilon'} \frac{\psi'(e(\varepsilon))}{\gamma} dH = \int_{\underline{\varepsilon}}^{\varepsilon'} b(\varepsilon) dH$$

Suppose an increase in γ increases $b(\varepsilon)$ for all ε for a given α. Then R must increase. Thus, for a given value of R, α increases. Due to the envelope theorem, small changes in R does not influence U. Thus, in keeping R constant, it follows that p and hence $J = S - R$ are constant as well. In order to satisfy (15) both R and α must increase relative to their initial value.

It is thus sufficient to show that $b(\varepsilon)$ is increasing for all ε for a given α. First note that $e(\varepsilon)$ must be increasing, otherwise (7) cannot be satisfied. Denote by b^{old} and b^{new} the value of b before and after the increase in γ. Suppose $b^{old} > b^{new}$. Substituting in $\gamma = b^{new}/\psi'(e^{new})$ into (7) gives

$$
\begin{aligned}
b^{new} &= 1-\alpha\frac{1-H}{h}\frac{b^{new}\psi''(e^{new})}{\psi'(e^{new})^2} \\
&> 1-\alpha\frac{1-H}{h}\frac{b^{old}\psi''(e^{new})}{\psi'(e^{new})^2} \\
&> 1-\alpha\frac{1-H}{h}\frac{b^{old}\psi''(e^{old})}{\psi'(e^{old})^2} \\
&= b^{old},
\end{aligned}
$$

which is a contradiction.

In addition, an increase in γ reduces output, and as for a reduction in $\overline{y}$ this will reduce R for a given α. This will further increase α.

Proof of Proposition 9

It is convenient to rewrite (9) as

$$
\varepsilon^* = -\overline{y} + \varepsilon^* + (r+s)U + \alpha\frac{1-H(\varepsilon^*)}{h(\varepsilon^*)}\frac{\psi'(e(\varepsilon^*))}{\gamma} - (\gamma e(\varepsilon^*) - \psi(e(\varepsilon^*))).
$$

Denote the left-hand side by $X_L(\varepsilon)$ and the right-hand side by $X_R(\varepsilon;\overline{y},z,A,c)$. Obviously $X_L'(\varepsilon) = 1 > 0$. As the second order condition must be satisfied locally, we know that $X_L(\varepsilon)$ crosses $X_R(\varepsilon;\overline{y},z,A,c)$ from below. It is therefore sufficient to show that around $\varepsilon = \varepsilon^*$ a) an increase in $\overline{y}$ shifts X_R up, b) an increase in z shifts X_R down, c) an increase in A or a decrease in c shifts X_R up.

$$
\frac{\partial X_R(\varepsilon;\overline{y},z,A,c)}{\partial \overline{y}} = -1 + (r+s)\frac{\partial U}{\partial \overline{y}} + \frac{(1-H(\varepsilon^*))}{h(\varepsilon^*)}\frac{\psi'(e(\varepsilon^*))}{\gamma}\frac{\partial \alpha}{\partial \overline{y}}
$$

From (16) we know that $-1 + (r+s)\frac{\partial U}{\partial \overline{y}} < 0$, and from Proposition 6b that $\frac{\partial \alpha}{\partial \overline{y}} < 0$. Hence $\frac{\partial X_R}{\partial \overline{y}} < 0$, and result a) thus follows.

$$
\frac{\partial X_R(\varepsilon;\overline{y},z,A,c)}{\partial z} = (r+s)\frac{\partial U}{\partial z} + \frac{(1-H(\varepsilon^*))}{h(\varepsilon^*)}\frac{\psi'(e(\varepsilon^*))}{\gamma}\frac{\partial \alpha}{\partial z}
$$

From Proposition 6b we know that $\frac{\partial \alpha}{\partial z} > 0$. Hence $\frac{\partial X_R}{\partial z} < 0$, and result b) thus follows

$$\frac{\partial X_R(\varepsilon;\overline{y},z,A,c)}{\partial A} = (r+s)\frac{\partial U}{\partial A} + \frac{(1-H(\varepsilon^*))}{h(\varepsilon^*)}\frac{\psi'(e(\varepsilon^*))}{\gamma}\frac{\partial \alpha}{\partial A}$$

$$\frac{\partial X_R(\varepsilon;\overline{y},z,A,c)}{\partial c} = (r+s)\frac{\partial U}{\partial c} + \frac{(1-H(\varepsilon^*))}{h(\varepsilon^*)}\frac{\psi'(e(\varepsilon^*))}{\gamma}\frac{\partial \alpha}{\partial c}$$

Since $\frac{\partial U}{\partial A} > 0$, $\frac{\partial U}{\partial A} > 0$ and $\frac{\partial \alpha}{\partial A} > 0$, $\frac{\partial \alpha}{\partial A} > 0$ (from Proposition 6c). Hence $\frac{\partial X_R}{\partial A} < 0$, and $\frac{\partial X_R}{\partial A} < 0$ and result c) thus follows.

Proof of Lemma 12

For $k = 0$, all workers are hired and first best is obtained. In this case, an increase in k does not influence U. We have to show that the market maker starts reducing the incentive power of the contract before he increases the cut-off. As $R^c > 0$, we must have that $\overline{y} + \underline{\varepsilon} + \gamma e^* - \psi'(e^*) > U^c$ at the point where $R^c = R^*$. At this point, increasing the cut-off level has a first-order effect on expected output. Reducing the incentive power of the contract slightly only gives a second-order effect on expected output. It thus follows that the market maker will reduce the incentive power of the contract before he increases the cut-off level (i.e., for a lower value of k).

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