On-the-job search and moral hazard.*

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Abstract

We analyze on-the-job search when moral hazard among employees calls for incentive schemes that include deferred compensation. While deferred compensation improves the workers' incentives to exert effort, it distorts the workers' on-the-job search decisions. We show that deferred compensation is less attractive when overall turnover in the market is high. Moreower, there exists feedback effects between the firms' choice of wage contracts and the labor market tightness in the on-the-job search market. This may lead to multiple equilibria: a low-turnover equilibrium where firms use deferred compensation to motivate workers, and a high-turnover equilibrium where they do not. Our model provides an explanation for observed differences in turnover rates between countries, e.g. US and Europe or Japan, and between regions e.g. Silicon Valley and Route 128.

Key Words: On-the-job search, Moral Hazard, Deferred Compensation, Multiple Equilibria .

JEL Codes: J41, J63.

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1 Introduction

The high worker turnover rates in the economy has spurred a significant literature on on-the-job search. In this literature, the focus is on the role of search. In particular, the models in this literature abstract from any agency problems that may exist between workers and firms along other dimensions than on-the-job search. In the present paper we argue that optimal incentive schemes that motivate workers to provide effort may include an intertemporal element, and this element may interfere with on-the-job search decisions.

Our starting point is that reallocation of workers on firms is necessary in order to obtain an efficient allocation of resources, as experienced workers may have comparative advantage at different tasks and in different firms than inexperienced workers. To capture this we set up an on-the-job search model where experienced workers search for new jobs. Efficient on-the-job search then requires that the experienced workers' wage in the original firm should equal their future production value in that firm (like in Moen and Rosén 2004).

The new feature of our model is that we include moral hazard caused by imperfect monitoring. As in Shapiro and Stiglitz (1984), firms can only imperfectly monitor worker effort. We follow Lazear (1979, 1981) and allow firms to use deferred compensation to provide incentives for workers to provide effort. With deferred compensation, an experienced worker's wage exceeds her productivity. As a result we get a tension: A moral hazard problem which calls for deferred compensation and optimal on-the-job search which calls for wages equal to marginal productivity.

We first show that incentives systems based on deferred compensation become less attractive when turnover is more important for economic efficiency. More interestingly there are feedback effects between the wage contracts used by firms and the number of firms searching for employed workers. These feedback effects may lead to multiple equilibria: A high-effort /low-turnover equilibrium in which firms use deferred compensation to motivate workers, and a low effort - high-turnover equilibrium in which they do not. Furthermore, the larger are the search frictions in the market, the more likely is it that the high-effort/low turnover equilibrium emerges.

As an extension we show that firms in a low-turnover equilibrium with deferred compensation are more reluctant to use piece rate payments to motivate workers, and more inclined to invest in firm-specific human capital, than are firms in a high-turnover equilibrium.

Our paper offers a new explanation for the large variations in turnover rates across countries and regions. For instance, in 1999 the median tenure among employees in 1991 was 3.0 years in the US and 4.4 years in UK, while it was 7.5 years in Germany and 8.2 years in Japan. The percentage of workers with a tenure of less than one year was 28.8 percent in the U.S. and 18.6 % in the U.K., 12.8 % in Germany and only 9.8% in Japan (OECD 1993). Large differences in turnover rates also exist between regions of the same country. For instance, turnover rates are extremely high in Silicon Valley, but much lower along "Route 128" in Massachusetts, another prosperous area with well developed high-technology industry (Saxenian, 1994).

Our model predicts that firms in countries (or regions) with lower turnover rates rely more on long-term wage contracts with deferred compensation (seniority-based wages, promotions etc.) and less on short-term performance-based systems than do firms in countries (regions) with higher turnover rates. This implication is in accordance with popular conceptions of the differences between the US and Japan and between Silicon Valley and Route 128.[1] The prediction of a negative relationship between deferred compensation and short-term performance pay is supported by Bayo-Moriones et.al. (2004). They document that firms which use deferred compensation less than other firms tend to use short-term performance pay as an incentive mechanism.

Related literature Our paper proposes an explanation for differences in turnover between countries and regions. In a recent paper, Pries and Rogerson (2005) argues that the differences in worker turnover between the US and Europe may be explained by institutional factors. There also exist papers that analyze multiple equilibrium turnover rates. Acemoglu and Pischke (1998) develop a model where adverse selection may lead to multiplicity in quit rates. Related arguments are made in Chang and Wang (1995), Owan (2004), Saint-Paul (1995), and Moene and Wallerstein (1997). Morita (2001) shows how multiple turnover

[1] We are not aware of any systematic evidence on the relationship between overall turnover and deferred compensation.

rates may arise as a result of firms' choice of production technology and learning-by-doing. Our paper differs from this literature in several ways. First, multiplicity in our model is caused by incentive contracts and worker moral hazard. Second, our paper is the only one that explicitly model on-the-job search as an equilibrium outcome in the presence of search frictions.

The second contribution of our paper is that we introduce private information into a model of on-the-job search. There is currently a small, but thriving literature on private information in search models. Moen and Rosen (2009) introduce moral hazard and Guerrieri (2008) asymmetric information in competitive search equilibrium. Guerrieri, Shimer and Wright (2009) analyze self-selection of heterogenous workers in a in search environment, and Rudanko (2009) and Menzio and Moen (2009) analyze optimal insurance with limited commitment in a search context. We contribute to this literature by analyzing the relationship between (intertemporal) wage contracts and on-the-job search.

Also related are extensions of the Burdett -Mortensen model (Burdett and Mortensen 1998) which allow for back-loading of wages, see Burdett and Coles (2003) and Stevens (2004). We want to point out that the mechanism at play in these papers is very different from the one in our paper. In their models, search is inefficient from the point of view incumbent firm and the employee, as it reduces their joint income. The employer discourages job quits by back-loading wages (but never to the extent that the wage is higher than output). In our model, by contrast, on-the-job search is efficient, as it increases the joint value of the incumbent firm and the employee. Back-loading is used to motivate workers to exert effort, and implies that wages for senior workers exceed output. Reduced on-the-job search then comes as a costly and unintended by-product of this back-loading.

Finally, as deferred compensation plays an important role in our paper, it is interesting to note that several empirical studies do suggest that deferred compensation is important. Medoff and Abraham (1980) find that pay increases with seniority, although supervisors' rating of performance do not. Lazear and Moore (1984) compare age-income profiles for tenured workers and for self-employed workers, for whom there exists no agency problems. They find that the returns to seniority are higher for tenured workers, and attribute this to deferred compensation. Katlikof and Gokhale (1992) compare wages and productivity of more than 300,000 workers in a Fortune 1000 firm. They find a substantial degree of deferred compensation for all categories of workers. In particular, managers' productivity exceeds compensation by a factor of more than two at the age of 35, while the opposite is true at the age of 57. Barth (1997) documents that workers on piece-rate compensation schemes have neglible returns to seniority, while workers who are not paid by piece-rates earn significant returns to seniority.

The paper is organized as follows. Section 2 describes our model. Section 3 defines equilibrium and section 4 characterize equilibrium. Section 5 analyzes multiple equilibria. In section 6 we study implications for contractible effort, firm-specific human capital and entrepreneurship. Section 7 discusses our main assumptions and section 8 concludes. Proofs are relegated to the appendix.

2 The Model

We study an overlapping generations model where workers live for two periods. The economy consists of two types of firms, ordinary firms and specialized firms. All workers start their career in ordinary firms. After the first period they qualify for a job in a specialized firm, where their productivity is higher. However, finding a specialized job is hard due to search frictions. All agents are risk neutral with zero discount rate. As there is no interaction between the generations, each generation can be studied in isolation.

Ordinary firms may employ both young and old workers. The productivity of a young worker in an ordinary firm is y_1+e, where $e \in \{0, \bar{e}\}$ is her effort level. The cost of effort is ec, $c < 1$. We introduce a moral hazard problem which may call for deferred compensation, and do this in the simplest possible way by assuming that the effort level of a worker first can be observed in the following period. This may reflect that effort is hard and time-consuming to observe, for instance because it takes time to complete the project the worker is participating in and the effort level cannot be observed before the project is completed. Alternative model specifications that give rise to deferred compensation is presented in the discussion section. Old workers make no effort choice, and produce y_2 units in ordinary firms and y_p units in specialized firms, $y_p > y_2$.[2]

For simplicity, we assume that the labor market for jobs in ordinary firms is frictionless, and equivalent to the Walrasian market solution. In the labor market for specialized firms the frictions are non-neglible, so that there are unmatched agents on both side of the market and wages are determined by bargaining. Free entry of both firm types implies zero profits in equilibrium.

Ordinary firms go into a period with a set of existing (old) employees, while specialized firms enter the period with no workers. Specialized firms only hire old workers, and therefore have new workers each period. Each period is divided into four stages, the hiring stage, the production stage, the remuneration stage and the search stage.

Ordinary firms

At the hiring stage, ordinary firms can hire both young and old workers. The firm can thus potentially have three different categories of workers: young (junior) workers, old workers with tenure (senior workers) and old workers without tenure. Junior workers are offered a wage schedule $\omega = \{w_1, w_2(e)\}$, where $w_1 \in R$ denotes the wage in the current period and $w_2(e) : \{0, \bar{e}\} \rightarrow R$ is the wage in the next period, given that the worker is still employed in that firm. Newly hired old workers are offered a wage $w^o \in R$. Senior workers work under the contract signed in the previous period.[3]

At the production stage, junior workers choose effort level e and produce $y_1 + e$ units of output. At the remuneration stage, the workers are paid their wages according to the

[2] Several arguments support that turnover can be efficient. Workers may try out several jobs to determine their comparative advantage (Johnson, 1978) or because of match-specific productivity differences (Jovanovic, 1979). A worker's relative productivity in different firms may also change over time as she gains experience and expertise (Moen and Rosén, 2004). Furthermore, sectorial shocks to the economy may warrant a re-allocation of workers. Finally, with technological progress, efficient dissemination of knowledge may require turnover as workers may learn from each other (Saxenian, 1994). The main results of this paper also hold under the less restrictive assumption that only some rather than all workers have a higher productivity in search firms and that only those workers engage in search.

[3] We assume that firms don't pay workers who have left the firm. This may be because: a) It is hard to verify whether movers had high effort in the first period. b) A firm's reputation may suffer more from breaking the contract if the worker in question is still employed than if she has quitted. c) Deferred compensation may reflect the (expected) gain from promotions. As argued in Carmichael (1983), it may be easier for a firm to commit to promotions than to cash payments not associated with particular positions, e) It may be easier for a worker to retaliate in informal ways after a breach of contract if she is still employed than if she works in another firm.

contract.

At the search stage, junior workers may search for both specialized jobs and ordinary jobs. Searching for specialized jobs is costly. A search intensity of s implies an effort cost of $\gamma s^2/2$. Searching for an ordinary job is costless. Senior /old workers do not search. Ordinary firms post vacancies at no cost.

Specialized firms

At the hiring stage, specialized firms who have found a worker bargain with her over the wage. The resulting bargained wage is denoted by w_p. At the production stage the worker produces without any moral hazard problems, and in the remuneration phase the worker receives a wage according to the contract.

At the search stage the specialist firms may advertise a vacancy at cost K. A firm that advertised a vacancy last period but did not receive any applicants has to pay the cost K over again if it wants to search for a worker. The employees in specialized firms leave the market at the end of the period and hence do not search.

Matching

Matching takes place between the periods. The number of matches between searching workers and specialized firms is determined by a constant return to scale matching function $x(su, v)$ up to the point where $x = \min[u, v]$, where u is the measure of searching workers, s their average search intensity, and v the measure of vacancies posted by specialized firms. We assume that the matching function is Cobb-Douglas, i.e., $x(su, v) = A(su)^{\beta} v^{1-\beta}$. Let p denote the probability of finding a job per unit of search intensity and q denote the probability that a vacancy is filled. It follows that

$$p(\theta) = A\theta^{1-\beta}, \tag{1}$$

$$q(\theta) = A\theta^{-\beta}, \tag{2}$$

where $\theta = v/su$ (The bound on the matching function ensures that the probabilities are less than one).[4] It follows from the definition of the matching function that the probability of

[4] We think of our matching process as a reduced form of a matching process set in continuous time. The probability of finding a job may then be interpreted more broadly as the fraction of the available time the worker is in the specialized firm.

finding a job for a worker with search intensity s is sp, $sp \leq 1$. [5]

If the worker is matched with a specialized firm and the bargaining game is successful, the worker quits and starts working in the specialized firm. Otherwise, the worker chooses between staying in the ordinary firm or to move to another ordinary firm.

Bargaining

We assume that $y_p - K$ is sufficiently large so that the market for specialized jobs is operating. The value of a vacancy is given by the value of hiring a worker times the probability of finding one. That is,

$$\begin{aligned} V &= q(y_p - w_p) \\ &= K, \end{aligned} \tag{3}$$

where the last equation follows from entry. When bargaining, the outside option of the worker is the contracted wage w_2 in the ordinary firm. In order to avoid uninteresting technicalities we assume that w_2 is unobservable to the specialized firm, which only knows the distribution of wages in the economy. As we only consider pure strategy equilibria, all workers in equilibrium have the same fallback wage w_2, and this equilibrium wage is thus the outside option of the workers.[6] Wages are determined by the Nash sharing rule. In the discussion section we derive the solution of the bargaining game as the equilibrium of an ultimate offer game.

We assume that the worker's bargaining power is given by β and thus equal to the weight on unemployment in the Cobb-Douglas matching function. It follows that the Hosios condition for constrained efficiency applies (Hosios, 1990). Hence, the matching process in

[5] Albrecht, Gautier and Vroman (2006) have pointed out that coordination externalities associated with multiple applications may arise in a discrete setting. The coordination externality will not arise if search effort relates to information gathering to find out which job to apply to and to the quality of the application rather then number of applications. Note also that the coordination externality disapears if the matching processes is set in continuos time or if firms may give a job offer to more applicants if the first applicant(s) turn down the offer (Kircher, 2008).

[6] If w_2 was observable, this would imply that the current employer could jack up the wage and thereby the value of search for her employees. As shown by Shimer (2006), this may lead to an untractable equilibrium distribution of wages.

itself does not lead to distortions. In Appendix 3 we show that the search equilibrium is efficient also in our setting.

It follows from the bargaining game that the wage in a specialized firm is given by (see the discussion section for details)

$$w_p = \beta y_p + (1-\beta) w_2. \tag{4}$$

From (1) and (2) it follows that $q = A^{\frac{1}{1-\beta}} p^{-\frac{\beta}{1-\beta}}$. Substituting $q = A^{\frac{1}{1-\beta}} p^{-\frac{\beta}{1-\beta}}$ and (4) into the zero profit condition (3) gives

$$p = A^{\frac{1}{\beta}} [\frac{(1-\beta)(y_p - w_2)}{K}]^{\frac{1-\beta}{\beta}}. \tag{5}$$

3 Equilibrium

Before characterizing the equilibrium of the model we make some observations. The zero profit requirement of ordinary firms implies that the wage for newly hired old workers in ordinary firms equals productivity; $w^o = y_2$. Thus, in order to retain an old worker if she does not obtain a job offer in a specialized firm, the contract must specify a wage $w_2(e) \geq w^o = y_2$. We refer to this as the worker's interim participation constraint. As we will see shortly, the interim participation constraint does not bind if the worker exerts effort. If the worker does not exert effort, the firm is indifferent between retaining the worker at wage y_2 and letting the worker go. We assume without loss of generality that the wage schedule satisfies the worker's interim participation constraint. Thus, the expected utility of a worker is,

$$u(\omega, e, s) = w_1 - ec + spw_p + (1 - sp)w_2(e) - \gamma s^2/2. \tag{6}$$

With some abuse of notation, let $u(\omega, e) = \max_s u(\omega, e, s)$. Let s^w denote the worker's choice of s, given by the first order condition

$$s^w = \frac{p(w_p - w_2(e))}{\gamma} \tag{7}$$

Let $\overline{u}$ denote the expected utility of a young worker that enters the market. The profit of an ordinary firm reads

$$\pi(\omega, e) = y_1 + e - w_1 + (1 - s(w_2(e))p)(y_2 - w_2(e)). \tag{8}$$

The optimal contract can now be defined as follows:

Definition 1 *The* optimal contract $(\widetilde{\omega}, \widetilde{e})$ *is a wage schedule* $\widetilde{\omega} = \{\widetilde{w}_1, \widetilde{w}_2(e)\}$ *and an effort level* $\widetilde{e}$ *that solves*

$$\max \pi(\omega, e) \qquad \textit{subject to}$$

1. Incentive compatibility:

$$u(\widetilde{\omega}, \widetilde{e}) = \max_{e \in \{0, \overline{e}\}} u(\widetilde{\omega}, e).$$

2. Interim participation:

$$\widetilde{w}_2(e) \geq y_2, \quad e \in \{0, \overline{e}\}.$$

3. Participation:

$$u(\widetilde{\omega}, \widetilde{e}) \geq \overline{u}.$$

We are now ready to define the equilibrium.

Definition 2 *The equilibrium is a contract* (ω^*, e^*)*, a search intensity* s^**, a job finding rate* p^**, a wage* w_p^* *and a utility* $\overline{u}^*$ *such that*

1. *The contract* (ω^*, e^*) *is an optimal contract.*
2. *Optimal search intensity:* $s^* = \arg\max_s u(\omega^*, e^*, s)$.
3. *Equilibrium in the search market:* w_p^* *and* p^* *solve (4) and (5).*
4. *Zero profit of ordinary firms:* $\pi(\omega^*, e^*) = 0$.

4 Characterizing equilibrium

In this section we characterize equilibrium. First we study on-the-job search and deferred compensation in some detail. The sum of u and π is given by (from 6 and 8)

$$\begin{aligned} u+\pi &= y_1+y_2+e(1-c)+sp(w_p-y_2)-\gamma s^2/2 \\ & y_1+y_2+e(1-c)+\Omega(s), \end{aligned} \tag{9}$$

where

$$\Omega(s)=sp(w_p-y_2)-\gamma s^2/2, \tag{10}$$

is the *joint gain from search* (the functional dependence on p and w_p is suppressed). Define $\Omega^{\max}=\max_s \Omega(s)$ and let $s^{\max}$ denote the corresponding value of s. Then

$$s^{\max} = \frac{p(w_p-y_2)}{\gamma}, \tag{11}$$

$$\Omega^{\max} = \frac{p^2(w_p-y_2)^2}{2\gamma}. \tag{12}$$

(where the functional dependences on w_p and p are suppressed). The worker, by contrast, chooses s^w defined by (7), which inserted into (10) gives (with $\Omega^w=\Omega(s^w)$)

$$\Omega^w=\frac{p^2(w_p-w_2)((w_p-y_2)-(y_2-w_2))}{2\gamma}, \tag{13}$$

with $w_2=w_2(e)$. By comparing (11) and (7) it follows that the worker maximizes the joint gain from search Ω if and only if $w_2(e)=y_2$, in which case there is no externality on the firm from the worker's search behavior. Define

$$L=\Omega^{\max}-\Omega^w. \tag{14}$$

We refer to L as the loss associated with too low search intensity when the worker receives deferred compensation. Finally, inserting $u(\omega,e)=\overline{u}$ and (14), into (9) gives

$$\pi=y_1+y_2+e(1-c)+\Omega^{\max}(w_p,p)-L(D,p)-\overline{u}. \tag{15}$$

Let $D=w_2-y_2$ denote the *amount of deferred compensation* the wage schedule gives to the worker.

Lemma 1 *The loss L is a function of p and D, and reads*

$$L(D,p)=\frac{D^2p^2}{2\gamma}. \tag{16}$$

The loss is increasing in the amount of deferred payment D and tightness p in the search market. The higher D is, the further away is the worker's search intensity from the search intensity that maximizes the joint gain from search. The higher p is, the more it matters that the search intensity is too low. Hence deferred compensation is less attractive when the overall turnover rate in the market is high. Note that the loss is independent of w_p.

If the firm wants to implement high effort, incentive compatibility clearly implies that $w_2(\overline{e})>w_2(0)$. It is trivial to show that the interim participation constraint binds; $w_2(0)=y_2$ (the "shirker" is punished as hard as possible). Let $\overline{D}=w_2(\overline{e})-y_2$ denote the lowest amount of deferred compensation consistent with the incentive compatibility constraint $u(\overline{e})\geq u(0)$. The following then holds:

Lemma 2 $\overline{D}=\overline{D}(p,w_p)$ *is increasing in both p and w_p, and is implicitly defined by the expression*

$$\overline{D}p^2[\overline{D}-2(w_p-y_2)]+2\gamma\overline{D}=2\gamma c\overline{e}. \tag{17}$$

Furthermore, $\overline{D}$ is strictly increasing in c.

The value of $\overline{D}$ has to be higher when the probability that the worker is there to pick it up is low (p high) and when the wage in the specialized firm, w_p, is high. This explains why $\overline{D}$ is increasing in p and w_p.

We distinguish between two types of equilibrium, the effort equilibrium $(\omega^e, s^e, p^e, w_p^e, \overline{u}^e)$ and the no-effort equilibrium $(\omega^n, s^n, p^n, w_p^n, \overline{u}^n)$, where the asterix is skipped for convenience.

The no-effort equilibrium

In this case the incentive compatibility constraint is trivial to satisfy and with slight abuse of notation we set $w_2(\overline{e})=w_2(0)=w_2^n$. From (15) it follows that the firm will set $D=0$, and thus that $w_2^n=y_2$. Equations (4) and (5) then gives

$$\begin{aligned} w_p^n &= \beta y_p+(1-\beta)y_2, \\ p^n &= A^{\frac{1}{\beta}}[\frac{(1-\beta)(y_p-y_2)}{K}]^{\frac{1-\beta}{\beta}}. \end{aligned}$$

From (15) it follows that the profit in the ordinary firm is given by

$$\pi^n = y_1 + y_2 + \Omega^{\max}(p^n, w_p^n) - \overline{u}^n. \tag{18}$$

The zero profit constraint implies that

$$\overline{u}^n = y_1 + y_2 + \Omega^{\max}(p^n, w_p^n). \tag{19}$$

Since $w_2^n = y_2$ the zero profit condition implies that $w_1^n = y_1$. Finally, for this to be an equilibrium, it must be true that $\max_\omega \pi(\omega, 0) \geq \max_\omega \pi(\omega, \overline{e})$. If the firm implements effort it sets $w_2^n(0) = y_2$ and $w_2^n(\overline{e}) = y_2 + \overline{D}(p^n, w_p^n)$. From (15) it follows that the no-effort equilibrium exists, if and only if

$$L(\overline{D}(p^n, w_p^n), p^n) \geq \overline{e}(1-c). \tag{20}$$

Thus, the no-effort equilibrium exists if the loss of implementing effort due to distortions in the search effort, given the equilibrium values p^n and w_p^n, is greater than the gain from implementing a high effort level. The no-effort equilibrium obviously exists for $c = 1$, and by a continuity argument it follows that the no-effort equilibrium exists if c is sufficiently close to 1.

The effort equilibrium

In the effort equilibrium the firm sets $w_2^e(0) = y_2$ and $w_2^e(\overline{e}) = y_2 + \overline{D}(p^e, w_p^e)$. From (4) and (5) it then follows that

$$\begin{aligned} w_p^e &= \beta y_p + (1-\beta)(y_2 + \overline{D}(p^e, w_p^e)), \\ p^e &= A^{\frac{1}{\beta}}[\frac{(1-\beta)(y_p - y_2 - \overline{D}(p^e, w_p^e))}{K}]^{\frac{1-\beta}{\beta}}. \end{aligned} \tag{21}$$

Using (15) the profit of the firm reads

$$\pi^e(\overline{e}) = y_1 + y_2 + \overline{e}(1-c) + \Omega^{\max} - L - \overline{u}^e. \tag{22}$$

Zero profits gives that

$$\overline{u}^e = y_1 + y_2 + \overline{e}(1-c) + \Omega^{\max} - L. \tag{23}$$

In period 2, the firm pays an expected deferred compensation of $s^e p^e \overline{D}$. Thus, $w_1^e = y_1 - s^e p^e \overline{D}$. Finally, for high effort to be an equilibrium, it must be true that $\max_\omega \pi(\omega, \overline{e}) \geq \max_\omega \pi(\omega, 0)$. Analogous with (20), this is true if and only if

$$L(\overline{D}(p^e, w_p^e), p^e) \leq (1-c)\overline{e}. \tag{24}$$

Thus, the effort equilibrium exists if the loss of implementing effort due to distortions in search effort, given the equilibrium values p^e and w_p^e, is smaller than the gain from implementing a high effort level. The effort equilibrium obviously exists for $c = 0$, and by a continuity argument it follows that the effort equilibrium exists if c is sufficiently close to 0. We summarize our findings so far:

Proposition 1 *The no-effort equilibrium exists for values of* c *sufficiently close to one. The effort-equilibrium exists for values of* c *sufficiently close to* 0*.*

We want to illustrate the different equilibria in a figure. Define $\overline{D}^W(p) \equiv \overline{D}(p, w_p(p))$, where $w_p(p)$ is defined by (21). In the appendix we show that

$$\frac{d\overline{D}^W(p)}{dp} > 0 \tag{25}$$

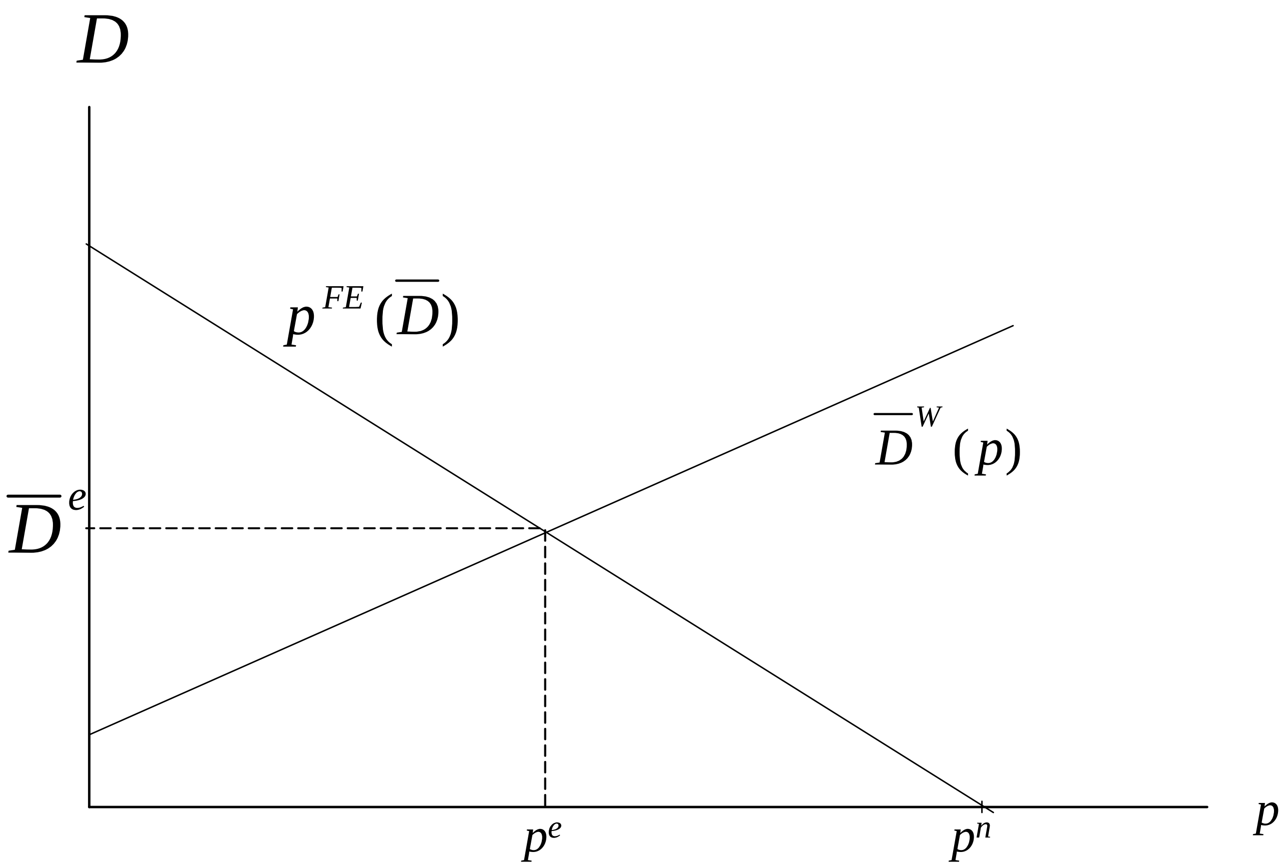
The curve is upward sloping, reflecting that the higher is the job finding rate, the larger amount of deferred compensation is necessary to induce effort.

Second, from the zero profit condition (5) we can a write p as a function of $\overline{D}$,

$$p = p^{FE}(\overline{D}), \quad \frac{dp^{FE}(\overline{D})}{d\overline{D}} < 0. \tag{26}$$

The curve shows the job finding rate p that is consistent with the zero profit condition for specialized firms (or free entry condition) as a function of $\overline{D}$. The higher is the deferred compensation, the higher is w_p, and hence the lower is the number of firms that enter the market.

In the $p - \overline{D}$ space the effort equilibrium is is obtained at the intersection of the two curves. The no-effort equilibrium is defined by $\overline{D} = 0$ and $p^n = p^{FE}(0)$. The two equilibria are shown in figure 1.



Effort and no-effort equilibrium

5 Multiple equilibria

In the previous section we derived conditions under which the effort and the no effort equilibrium may exist. An interesting issue is whether they may exist simultaneously.

Inspecting the loss function gives us an indication that this may actually be the case. As $w_2^e > w_2^n$, it follows immediately that $p^e < p^n$. With some abuse of notation, write $L(p) \equiv L(p, \overline{D}^W(p))$. From lemma 1 and (25) it then follows that $L'(p) > 0$, and thus in particular that

$$L(p^e) < L(p^n).$$

Thus, the cost of implementing high effort is higher in the no-effort equilibrium than in the effort equilibrium.

Let c^e denote the highest value of c such that (24) is satisfied for all $c \leq c^e$ and let c^n denote the lowest values of c such that (20) is satisfied for all $c \geq c^n$. It follows that c^e and c^n satisfy the equations

$$L(p^e) = \bar{e}(1 - c^e) \tag{27}$$

$$L(p^n) = \bar{e}(1 - c^n) \tag{28}$$

Furthermore, define turnover in the economy by sp.

Proposition 2 *There exists a non-empty interval $[c^n, c^e]$ such that the model exhibits multiple equilibria whenever $c \in [c^n, c^e]$. One equilibrium is characterized by high effort, low turnover and deferred compensation, while the other is characterized by no effort, high turnover and no deferred compensation.*

The intuition for this result is as follows. Suppose we are in the no-effort equilibrium. Then w_2 is relatively low, as no firms defer compensation. Therefore many specialized firms enter the market and on-the-job search is valuable. If a firm deviates and implements high effort, it has to defer compensation and distort workers' search effort. Since search is so valuable this comes at a high cost (L is high) and hence, the deviation is not profitable.

Suppose then instead that all firms in the economy implement high effort and thus defer wages. In this case, few specialized firms enter the market, and the return from search is

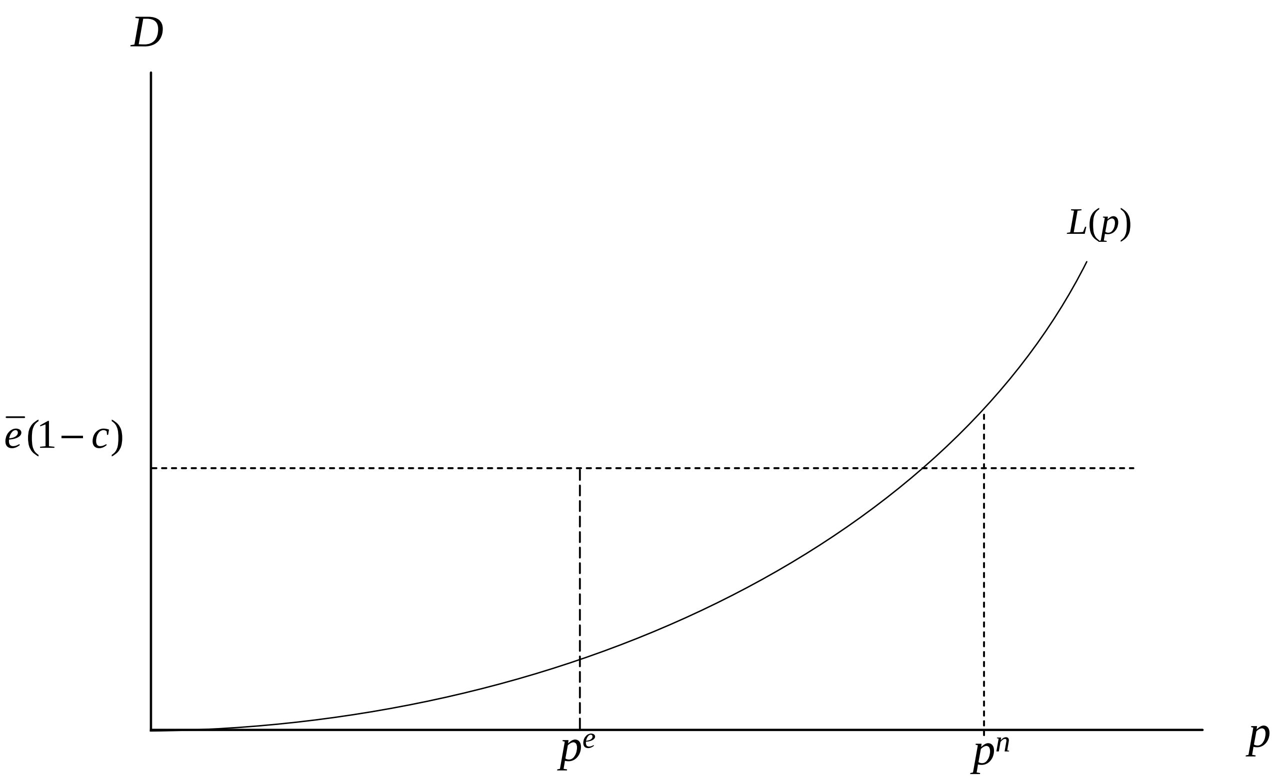


Figure 2. Multiple equilibria

lower. Consider then a deviating firm, that does not implement a high effort and thus does not defer compensation. Although this increases the joint gain from search, the increase is moderate since there are relatively few firms to search for anyway. Multiple equilibria are illustrated in figure 2.

More generally, when all the other firms use deferred compensation, the search market is "designed" for workers with a high period-two wage, in the sense that the equilibrium maximizes the value of search for workers with a high period 2 wage. The gain for a worker-firm pair of improving the incentives for the worker to do on-the-job search is lower in this situation than in the situation where the equilibrium of the search market is designed for workers with a low period two wage.

Put differently, the outcome in the search market depends on the behavior of the agents on the other side of the market, and that will again depend on the agents on the same side of the market. Thus, there exists a feedback effect from the search behavior of the average worker in the market to the gain from search for any individual worker. Since the search behavior depends on the wage contract in question, it follows that the gain from implementing high effort and defer payment depends on the extent to which the other firms in the market defer compensation.

We want to analyze how the equilibrium configurations depend on search frictions, measured by A (the efficiency of the matching technology), or equivalently by K. We say that the effort equilibrium is more likely if c^e increases, and that the no-effort equilibrium is more likely if c^n decreases.

Proposition 3 *An increase in the search frictions (reduced A or increased K) makes the effort equilibrium more likely and the no-effort equilibrium less likely.*

Welfare As the workers receive the entire economic surplus, the relevant welfare measure is the utility of workers entering the market, $\overline{u}$. From (19) and (23) it follows that the utility in the no-effort and effort equilibrium can be written as.

$$\overline{u}^n = y_1 + y_2 + \Omega^{\max}(p^n, w_p)$$

$$\overline{u}^e = y_1 + y_2 + \Omega^{\max}(p^e, w_p) + \overline{e}(1-c) - L(p^e, \overline{D}^e)$$

From the proof in Appendix 3 it follows that the gain from search $\Omega^{\max}$ is maximized (given the zero profit constraint of specialized firms) when $w_2 = y_2$. The unconstrained efficient allocation thus requires that $e = \overline{e}$, $p = p^n$, and $s = s^n$. It follows that both equilibria are inefficient. The no-effort equilibrium because there is no effort. The effort equilibrium is inefficient for two reasons. First, due to deferred compensation workers search too little, and this is captured in the dead weight loss L. Second, there are too few firms entering the market, and for that reason $\Omega^{\max}(p^n, w_p) > \Omega^{\max}(p^e, w_p)$ (see 12).

In general one can not show that one equilibrium welfare dominates the other. The exception is if the effort cost, c, is close to the upper boundary c^n for when the high-effort equilibrium exists. In this case the low-effort equilibrium dominates:

Proposition 4 *Given c is close to c^n, social welfare is higher in the no-effort equilibrium than in effort equilibrium.*

Proof. When c is smaller but close to c^n then $\overline{e}(1-c) - L(p^e, \overline{D}^e) \approx 0$. Since $\Omega(p^n, w^n) > \Omega(p^e, w^e)$ it follows that $\overline{u}^n > \overline{u}^e$. ■

The opposite does not hold. It may be that if c is close to c^e, the no-effort equilibrium still dominates the effort equilibrium.

6 Implications

In this section we study contractible effort, investments in firm specific human capital and supply of entrepreneurs.

6.1 Piece rate payment or deferred compensation?

Suppose young workers undertake two types of effort, unobservable effort $e \in \{0, \overline{e}\}$ as in the last section and observable and contractible effort d, where d is a continuos variable. The total cost of effort is $\kappa(e, d)$, where κ is convex in (e, d). We define the cost of unobservable effort as $c(d) \equiv \kappa(\overline{e}, d) - \kappa(0, d)$. It follows that $c'(d) > 0$.

In a no-effort ($e = 0$) equilibrium, the firm maximizes profits given by

$$\pi^n = y_1 + y_2 + d - \kappa(0, d) + \Omega^{\max} - \bar{u}^n.$$

It follows trivially that the firm wants to implement the first best level of d, given by $\frac{\partial \kappa(0,d)}{\partial d} = 1$. With a linear incentive scheme $w_1 = a + b(y_1 + d)$ this can be implemented by setting $b = 1$. Since $w_2 = y_2$ the zero profit condition then implies that $a = y_1$.

Consider then the effort-equilibrium ($e = \bar{e}$). From lemma 2 we know that $\overline{D}$ is increasing in c. The profit in this case reads

$$\pi^e = y_1 + y_2 + \bar{e} + d - \kappa(\bar{e}, d) + \Omega^{\max} - L(p, \overline{D}) - \bar{u}^e.$$

The first order condition for d reads

$$\frac{\partial \pi^e}{\partial d} = 1 - \frac{\partial \kappa(\bar{e}, d)}{\partial d} - L_{\overline{D}} \frac{d\overline{D}}{dc} c'(d) = 0,$$

or

$$\frac{\partial \kappa(\bar{e}, d)}{\partial d} = 1 - L_{\overline{D}} \frac{d\overline{D}}{dc} c'(d) < 1.$$

Thus, the marginal effort cost of d is less than one. The reason is that increasing contractible effort (short-term bonuses), d, means increasing the costs of unobservable effort (which in itself is captured in κ). This increases the amount of deferred compensation and thus the implementation cost L of high effort. This effect is a direct response to the deadweight loss associated with deferred compensation.

Suppose again that the firm implements the optimal d, d^e, by a linear wage schedule of the form $a + b(y_1 + d)$, with an upper bound at $y_1 + d^e$. Up to the bound, the worker chooses d such that $\kappa_d = b$, thus the optimal effort level is obtained by setting

$$b = 1 - L_{\overline{D}} \frac{d\overline{D}}{dc} c'(d^e)$$

Proposition 5 *The incentive power of the short-term wage contract is lower in the effort-equilibrium than in the no-effort equilibrium*

As mentioned in the introduction, the prediction that firms which use deferred compensation to a lesser extent than other firms are likely to use short-term bonuses is supported by Bayo-Moriones et al, (2004).

6.2 Firm-specific human capital

We now turn to firm-specific human capital h. The firm can invest in human capital in young workers, and the gain is reaped when the workers are senior. The cost of investing h efficiency units of firm-specific human capital is denoted by $g(h)$, where g is increasing and convex in h. The investment increases second-period productivity of the worker in that firm from y_2 to $y_2 + h$.

Consider a firm that does not implement effort. In order to obtain optimal job search, the ordinary firm sets the period 2 wage equal to worker productivity, $w_2 = y_2 + h$. The profit of the firm reads (analogue to equation 18)

$$\pi^n = y_1 + y_2 + h - g(h) + \max_s [sp^n(w_p - y_2 - h) - \frac{s^2}{2\gamma}].,$$

The first order conditions for h is given by (due to the envelope theorem, the effect through s^n can be ignored)

$$g'(h) = 1 - s^n p^n. \tag{29}$$

Consider then a firm that does not implement effort. As before we define $w_2 = y_2 + \overline{D}$. As the interim participation constraint and the incentive compatibility constraint is unchanged, so is $\overline{D}$. From (22) it follows that

$$\pi^e(\overline{e}) = y_1 + y_2 + h + (1 - c)\overline{e} + s^e p^e(w_p^e - y_2 - h),$$

where s^e is defined in (7). Since $\overline{D}$, and thus s^e is independent of h it follows that

$$g'(h) = 1 - s^e p^e. \tag{30}$$

Since $s^n > s^e$ and $p^n > p^e$ it follows that the investments in firm-specific human capital is higher in the effort equilibrium than in the no-effort equilibrium.

The loss associated with deferred compensation also falls in the presence of human capital investments. The amount of deferred compensation is given by $\overline{D} - h$, and the loss function (16) can thus be written as

$$L(\overline{D}, p, h) = \frac{(\overline{D} - h)p^2}{2\gamma}.$$

Above we said that the effort equilibrium becomes more (less) likely if c^e increases (decreases), and the no-effort equilibrium becomes more (less) likely if c^n decreases. As investments in firm-specific human reduces the loss L, it follows that the introduction of firm-specific human capital makes the effort equilibrium more likely and the no-effort equilibrium less likely.

Proposition 6 *a) Firms in effort equilibrium invest more in firm-specific human capital than firms in no-effort equilibrium.*

b) The presence of firm-specific human capital makes the effort equilibrium more likely and the no-effort equilibrium less likely.

The presence of firm-specific human capital creates a wedge between the productivity of senior workers and newly hired old workers in ordinary firms, and this increases the scope of punishing the worker without deferring compensation. In addition, firm-specific human capital reduces the optimal turnover rate in the economy. As a result, the option implementing effort becomes relatively more profitable.

6.3 Entrepreneurship and venture capital

Entrepreneurs are often former employees of firms in the same industry. Furthermore, entrepreneurs often need access to particular kinds of funding, (e.g., venture capital), for which the market may be thin. The matching process between venture capitalists and entrepreneurs may be similar to the search market described above.

For a potential entrepreneur, the shadow price of becoming an entrepreneur is continued employment. This shadow price is higher with deferred compensation than without. Furthermore, when bargaining over terms of trade with a venture capitalist, the economic compensation of continued employment is likely to influence a potential entrepreneur's bargaining position. Thus, in low-turnover equilibrium with deferred compensation, entrepreneurship is less attractive, because the shadow price in terms of foregone wages is high. Just as with specialized firms, this may also reduce the number of entrepreneurs entering the market. The mechanism creating multiple equilibria may then again be at work: in low-turnover equilibrium, few venture capitalists enter the market, hence the loss of deferred compensation caused by reduced entrepreneurship is small. In high-turnover equilibrium, by contrast, a large number of venture capitalists enter the market, and distortions associated with low entrepreneurial activity are large.

7 Discussion

In this section we first analyze alternative ways to model worker effort. Then we study an extensive form bargaining game that leads to the wage equation (4).

7.1 Main assumptions

Modelling of effort

In this paper we assume that effort is observable with a time lag, and this forces firms to use deferred compensation in order to motivate the worker. However, this is only one reason why deferred compensation may be warranted. Another reason may be that effort is observed in the same period but with noise. We want to demonstrate the need for deferred compensation under this alternative assumption more precisely, and show that also in this case the trade-off between effort provision and efficient turnover arise. We make the following additional assumptions:

1. A worker exerts effort in both periods, so that output in period i is $y_i = y + e_i$, where e_i is effort in period i, $i = 1, 2$
2. There is a lower bound on the wage a worker can offer in any period. To simplify the exposition we set the lower bound equal to y.

The rest of the model is as before. Let δ denote the probability that the firm observes that the worker provides no effort, $e = 0$. Suppose the contract specifies that $e = \overline{e}$. Suppose this is to be implemented in a period by period basis, without deferred compensation. The highest wage the firm can profitably pay is $y + \overline{e}$. The cost of effort is $\overline{e}c$. The non-shirking condition reads $y + \overline{e}(1-c) \geq y + \overline{e}(1-\delta)$. High effort can thus only be implemented if

$$c/\delta \leq 1$$

We assume that this is not the case.

Consider deferred compensation. If a worker that is detected "shirking" in period 1, she cannot profitable be incentivized in period 2 as $c/\delta \leq 1$. She thus obtains obtains y in period 2. Consider a contract with deferred compensation, where a worker who is not detected shirking in any period gets $w_1 = y$ in period 1 and $w_2 = y + 2\overline{e}$ in period 2. The worker will not shirk in period 2 if $c/\delta \leq 2$. The period 2 utility of a worker is thus $2\overline{e}+y-c\overline{e}$, and independent of wether the worker provided effort in period 1 or not.

In period 1, the lifetime utility of a shirker is $2y\delta + (1-\delta)(2y + 2e - c\overline{e})$. The lifetime utility of a non-shirker is $2y + 2\overline{e} - 2c\overline{e}$. The non-shirking condition in period 1 thus reads

$$2y + 2\overline{e} - 2c\overline{e} \geq 2y\delta + (1-\delta)(2y + 2\overline{e} - c\overline{e}),$$

or

$$c/\delta \leq 2 - c.$$

Thus, if the parameters satisfy

$$1 < c/\delta \leq 2 - c,$$

high effort can be implemented if and only if the firm uses deferred compensation.

The point is that even if the period-by period bonus available is insufficient to motivate the worker, the aggregate surplus over the workers' career is. Deferring the compensation to the end of the second period allows the firm to use the bonuses in both periods to motivate the worker. This doubles the incentives to exert effort in the second period. Furthermore, since the effort cost in period 2 is less than the bonus available in that period ($c < 1$), it also increases the incentives to exert effort in period 1. Put differently, with deferred compensation the firm makes the decision based on two observations instead of one. The increased information increases the scope for implementing a high effort level.

7.2 Bargaining and stability of equilibrium

When deriving wages we used the Nash bargaining solution. At the same time we assumed that firms cannot observe individual wages, only the distribution of wages in the economy. As all workers have the same fallback wage w_2 in equilibrium, applying the Nash bargaining solution is well defined. However, in order to analyze the stability of the equilibrium we set up and solve a strategic bargaining game between individual workers and firms. We then show that our equilibrium is stable in a well defined sense.

The bargaining game is modelled as an ultimate offer game with the following timing:

1. With probability β and $1-\beta$, respectively, nature chooses the worker or the firm as *proposer*.
2. The proposer offers a wage offer w_p^i, where $i = w$ (the worker proposes) or $i = f$ (the firm proposes).
3. The other agent accepts or rejects the offer. If the offer is accepted, the game ends, and production starts immediately at the agreed terms. If the wage offer is not accepted, the match ends. In this case the worker continues in her current job, while the firm remains unmatched and obtains zero profit.

If the worker proposes, she proposes a wage equal to her productivity, $w_p^w = y_p$ and the wage claim is accepted. Suppose then that the firms assign probability 1 to the event that the workers' wage is w_2. If the firm proposes, it proposes a wage $w_p^f = w_2$. The expected wage is thus given by $w_p = \beta y_p + (1-\beta)w_2$, identical to the Nash bargaining solution.

Consider then an effort equilibrium, and suppose a fraction Δ of the firms deviates, implement no effort, and set $w_2 = y_2$. (The other firms set $w_2 = w_2^e$) In the bargaining game, the probability that the firms' opponent has a fallback wage $w_2 = y_2$ is given by

$$\rho = \frac{\Delta s^n}{\Delta s^n + (1-\Delta)s^e} \tag{31}$$

The pay-off to a proposing firm is thus

$$
\begin{aligned}
\pi &= y_p - w_p^f && \text{if } w_p^f \geq w_2^e \\
\pi &= \rho(y_p - w_p^f) && \text{if } y_2 > w_p^f \geq w_2^e \\
\pi &= 0 && \text{if } w_2^e > w_p^f
\end{aligned}
$$

Clearly, if Δ is sufficiently small and ρ sufficiently close to 1 the firm proposes w_2^e, and the expected wage is equal to the Nash wage when the outside option of the worker is w_2^e. It follows that the effort equilibrium still exists, and one can show that as $\Delta \to 0$ the equilibrium converges to the effort equilibrium without deviators.

Analogously, consider a no-effort equilibrium, and suppose a fraction Δ of the ordinary firms deviate and implement effort. Denote the corresponding period 2 wage by w_2^d. By the same argument it follows that the firm will offer a wage y_2, which a worker in a deviating firm will reject. Again the Nash wage $w_p = \beta y_p + (1-\beta)y_2$ prevails, and as $\Delta \to 0$ the equilibrium converges to the no effort equilibrium without deviators.

8 Conclusion

This paper analyses moral hazard in a model of on-the-job search. As worker effort is observed with a time lag, the optimal incentive contract includes deferred compensation. However, deferred compensation distorts the workers' on-the-job search decisions, as it gives the workers too weak incentives to search on the job. Due to feedback effects between firms' choice of wage contracts and entry in the on-the-job search market, multiple equilibria may emerge. In one equilibrium, firms offer incentive contracts with deferred compensation, which lead to high effort and low turnover rates. In the other equilibrium firms do not offer deferred compensation, and this lead to low effort and high turnover. Our model contributes to a growing literature that incorporates private information into matching models of the labor market. Our model also sheds light on the observed differences in turnover rates between countries (U.S. and Europe/Japan) and regions (Silicon valley and Massachusetts' route 128).

Our model has several empirical implications. The equilibrium with deferred compensation is more likely to prevail in markets with large search frictions, inclined to give weaker incentives to contractible performance (less use of short-term bonuses) and lead to higher investments in firm-specific human capital than in the equilibrium without deferred compensation. Furthermore, entrepreneurship and venture capital may be more frequent in high-turnover equilibrium than in low-turnover equilibrium. These implications are in line with popular perceptions of the differences between e.g. the US and Japan or between Silicon Valley and Massachusetts.

Appendix

Appendix 1. Proof of Lemma 1

We have that

$$\begin{aligned}
L(D,p) &= \Omega^{\max}(p) - \Omega(p, s^w(p,D)) \\
&= \frac{p^2(w_p - y_2)^2}{2\gamma} - s^w p(p,D)(w_p - y_2) + \frac{\gamma s^{w2}(p,D)}{2} \\
&= \frac{p^2(w_p - y_2)^2}{2\gamma} - \frac{p^2(w_p - w_2)(w_p - y_2}{\gamma} + \frac{p^2(w_p - w_2)^2}{2\gamma} \\
&= \frac{p^2}{2\gamma}((w_p - y_2)^2 - 2(w_p - w_2)(w_p - y_2) + (w_p - w_2)^2) \\
&= \frac{p^2}{2\gamma}((w_p - y_2) - (w_p - w_2))^2 \\
&= \frac{p^2}{2\gamma}(w_2 - y_2)^2 \\
&= \frac{p^2 D^2}{2\gamma}
\end{aligned}$$

which completes the proof.

Appendix 2. Proof of Lemma 2

Incentive compatibility requires that $u(\widetilde{\omega}, \overline{e}) \geq u(\widetilde{\omega}, 0)$. From (6) and (7) it follows that the incentive compatibility constraint thus reads

$$\frac{p^2(w_p - w_2)^2}{2\gamma} + w_2 - c\overline{e} \geq \frac{p^2(w_p - y_2)^2}{2\gamma} + y_2 \Leftrightarrow$$

$$\frac{p^2(w_p - y_2 - D)^2}{2\gamma} + y_2 + D - c\overline{e} \geq \frac{p^2(w_p - y_2)^2}{2\gamma} + y_2 \Leftrightarrow$$

$$p^2(w_p - y_2 - D)^2 + 2\gamma D - 2\gamma c\overline{e} \geq p^2(w_p - y_2)^2 \Leftrightarrow$$

$$p^2\left[(w_p - y_2 - D)^2 - (w_p - y_2)^2\right] + 2\gamma D \geq 2\gamma c\overline{e} \Leftrightarrow$$

$$p^2\left[(w_p - y_2)^2 - 2D(w_p - y_2))^2 + D^2 - (w_p - y_2)^2\right] + 2\gamma D \geq 2\gamma c\overline{e} \Leftrightarrow$$

$$p^2 D\left[D - 2(w_p - y_2)\right] + 2\gamma D \geq 2\gamma c\overline{e}$$

Thus $\overline{D}$ is defined by

$$p^2\overline{D}\left[\overline{D} - 2(w_p - y_2)\right] + 2\gamma\overline{D} = 2\gamma c\overline{e} \tag{32}$$

(ii) Differentiating (32) w.r.t $\overline{D}$ and p gives

$$\left(2p^2\overline{D} - 2p^2(w_p - y_2) + 2\gamma\right)d\overline{D} + 2p\overline{D}\left[\overline{D} - 2(w_p - y_2)\right]dp = 0$$

which gives

$$\frac{d\overline{D}}{dp} = \frac{-2p\overline{D}\left[\overline{D} - 2(w_p - y_2)\right]}{2p^2\overline{D} - 2p^2(w_p - y_2) + 2\gamma}$$

Using (32) we have that $-2p\overline{D}\left[\overline{D} - 2(w_p - y_2)\right] = \frac{2\gamma(\overline{D} - c\overline{e})}{p} > 0$ and hence the numerator is positive. Since $2p^2\overline{D} - 2p^2(w_p - y_2) + 2\gamma > \frac{p^2\overline{D}\left[\overline{D} - 2(w_p - y_2)\right] + 2\gamma\overline{D}}{\overline{D}}$ and (using (32)) $\frac{p^2\overline{D}\left[\overline{D} - 2(w_p - y_2)\right] + 2\gamma\overline{D}}{\overline{D}} = \frac{2\gamma c\overline{e}}{\overline{D}} > 0$ the denominator is also positive, hence $\frac{d\overline{D}}{dp} > 0$.

Differentiating (32) w.r.t to $\overline{D}$ and w_p.

$$\left(2p^2\overline{D} - 2p^2(w_p - y_2) + 2\gamma\right)d\overline{D} - 2p^2\overline{D}dw_p = 0$$

or

$$\frac{d\overline{D}}{dw_p} = \frac{2p^2\overline{D}}{2p^2\overline{D} - 2p^2(w_p - y_2) + 2\gamma}$$

We have already shown that the denominator is positive. Hence $\frac{d\overline{D}}{dw_p} > 0$.

The claim that $\overline{D}$ is increasing in c follows directly from that the left-hand side of (17) is increasing in $\overline{D}$ and the right-hand side increasing in c.

Appendix 3. Efficiency

The problem of maximizing worker utility given the zero profit constraint writes

$$\max_{s,p} sp(w_p - w_2) - \frac{\gamma s^2}{2} + w_2 \quad \text{s.t.} \quad A^{\frac{1}{1-\beta}} p^{-\frac{\beta}{1-\beta}} (y_p - w_p) = K.$$

The optimal search effort reads $s = \frac{p(w_p - w_2)}{\gamma}$, which inserted gives

$$\max_{p} \frac{p^2(w_p - w_2)^2}{2\gamma} + w_2 \quad \text{s.t.} \quad A^{\frac{1}{1-\beta}} p^{-\frac{\beta}{1-\beta}} (y_p - w_p) = K.$$

The associated Lagrangian is

$$L = \frac{p^2(w_p - w_2)^2}{2\gamma} + w_2 - \lambda[A^{\frac{1}{1-\beta}} p^{-\frac{\beta}{1-\beta}} (y_p - w_p) - K],$$

with first order conditions

$$\begin{aligned}
\frac{\partial L}{\partial w_p} &= 0 \Leftrightarrow \frac{p^2(w_p - w_2)}{\gamma} + \lambda A^{\frac{1}{1-\beta}} p^{-\frac{\beta}{1-\beta}} = 0 \\
\frac{\partial L}{\partial p} &= 0 \Leftrightarrow \frac{p(w_p - w_2)^2}{\gamma} + \lambda A^{\frac{1}{1-\beta}} \frac{\beta}{1-\beta} p^{-\frac{\beta}{1-\beta}-1} (y_p - w_p).
\end{aligned}$$

Solving out gives

$$w_p = \beta y_p + (1-\beta) w_2 = \beta y_p + (1-\beta)(y_2 + \overline{D}).$$

Appendix 4. Proof of equation 25

Differentiating $\overline{D}(p, w_p)$ gives

$$\frac{d\overline{D}(p, w_p)}{dp} = \frac{\partial \overline{D}(p, w_p)}{\partial p} + \frac{\partial \overline{D}(p, w_p)}{\partial w_p} \frac{dw_p}{dp}$$

Using (21) gives

$$\frac{dw_p}{dp} = (1-\beta) \frac{d\overline{D}}{dp}$$

or

$$\frac{d\overline{D}(p,w_p)}{dp}(1-(1-\beta)\frac{\partial\overline{D}(p,w_p)}{\partial w_p}) = \frac{\partial\overline{D}(p,w_p)}{\partial p}$$

From lemma 2 we know that the right-hand side is strictly positive, ($\frac{\partial\overline{D}(p,w_p)}{\partial p} > 0$). It is thus sufficient to show that $\frac{\partial\overline{D}(p,w_p)}{\partial w_p} < 1$.

To this end, note that incentive compatibility requires that $u(\widetilde{\omega},\overline{e}) \geq u(\widetilde{\omega},0)$, which is satisfied with equality. Written out, this implies that

$$s^n p(w_p - y_2) + y_2 = s^e p(w_p - y_2 - \overline{D}) + y_2 + \overline{D} - c\overline{e}$$

where s^n is the search intensity with no effort and s^e is the search intensity with effort. Differentiating with respect to $\overline{D}$ and w_p gives (due to the envelope theorem we can ignore changes in s^n and s^e)

$$s^n p dw_p = s^e p dw_p + (1 - s^e p)d\overline{D}$$

or that

$$\frac{d\overline{D}}{dw_p} = \frac{s^n p - s^e p}{1 - s^e p} < 1$$

provided that $s^n p < 1$, which is true by assumption. This completes the proof.

Appendix 5. Proof of proposition 2

First we want to show that c^n is unique. To this end, note that p_n is independent of c. From lemma 2 we know $\overline{D}$ is increasing in c, and hence that an increase in c increases $L(p^n)$. The right-hand side of (28) is strictly decreasing in c. The left-hand side of (28) is zero for $c = 0$ and strictly positive for $c > 0$. The right-hand side is strictly positive for $c = 0$ and zero for $c = 1$. It follows that (28) uniquely defines c^n.

Then consider c^e defined by equation (27). Since $p^e < p^n$, it follows trivially that $L(p^e) < L(p^n)$ at $c = c^n$. Hence $L(p^n) > \overline{e}(1 - c^n)$. Since (28) defines c^n uniquely it follows that $c^n < c^e$. The result thus follows.

From (4) we have that $w_p - w_2$ is decreasing in w_2. From (7) it thus follows that $s^n > s^e$, and hence that $s^n p^n > s^e p^e$.

Appendix 6. Proof of proposition 3

We have to show that c^e and c^n both increases. Consider figure 1. A decrease in A shifts $p^{FE}(\overline{D})$ down (from equation (5)) but does not influence $\overline{D}^W(p)$. It follows that both p_n and p_e shifts down, while $L(p)$ does not shift. From equation (27) and (28) it follows that c^e and c^n increases. Exactly the same argument applies for K.

References

Acemoglu, D. and Pischke, J-S. (1998), "Why Do Firms Train?, Theory and Evidence", *Quarterly Journal of Economics*, 113, 79-119.

Albrecht, J., Gautier, P. and Vroman, S. (2006), "Equilibrium Directed Search with Multiple Applications," *Review of Economic Studies*, October, 869-891.

Barth, E. (1997), "Firm-Specific Seniority and Wages", *Journal of Labor Economics*, 15, 495-506.

Bayo-Moriones, A., Galdon-Sanchez, J.E. and M. Guell (2004), "Is Seniority-Based Pay Used as a Motivation Device? Evidence from Plant Level Data", IZA Discussion Paper no 1321.

Burdett, K. and Coles, M. (2003), "Equilibrium Wage Tenure Contracts", *Econometrica*, 71, 1377-1404.

Burdett, K. and Mortensen, D.T. (1998), "Wage Differentials, Employer Size, and Unemployment", *International Economic Review*, 39, 257-273.

Carmichael, L. (1983), "Firm specific human capital and promotion ladders", *Rand Journal of Economics*, 14, 251-258.

Chang, C. and Wang, Y (1995), "A Framework for Understanding Differences in Labor Turnover and Human Capital Investments", *Journal of Economic Behaviour and Organization*, 28, 91-105.

Guerrieri, V. (2008), "Inefficient Unemployment Dynamics under Asymmetric Information“, *Journal of Political Economy*, **116**, 667-708.

Guerrieri, V., Shimer, R., and Wright, R. (2009), "Adverse Selection in Competitive Search", Working Paper.

Hosios, A.J. (1990), "On The Efficiency of Matching and Related Models of Search and Unemployment", *Review of Economic Studies*, 57, 279-298.

Johnson, W.R. (1978), "A Theory of Job Shopping", *Quarterly Journal of Economics*, 92, 261-277.

Jovanovic, B. (1979), "Job Matching and the Theory of Turnover", *Journal of Political Economy*, 87, 972-990.

Kircher, P. (2008), "Efficiency of Simultaneous Search", PIER Working Paper No 08-004.

Kotlikoff, L.J. and Gokhale, J (1992). "Estimating a Firm's Age-Productivity Profile Using the Present Value of Workers' Earnings", *The Quarterly Journal of Economics* 107, 1215-42.

Lazear, E.P (1979). "Why is there Mandatory Retirement?", *Journal of Political Economy*, 87, 1261-1264.

Lazear, E.P. (1981), "Agency, Earnings Profiles, Productivity, and Hours Restrictions", *American Economic Review*, 71, 606-620.

Lazear, E.P. and Moore, R.L. (1984). "Incentives, Productivity, and Labor Contracts", *The Quarterly Journal of Economics*, 99, 275-296.

Medoff, J.L. and Abraham, K.G. (1980), "Experience, Performance, and Earnings", *Quarterly Journal of Economics*, 95, 186-216.

Menzio, G. and Moen, E.R. (2009), "Worker Replacement", Working Paper.

Moen, E. R., and Rosén, Å (2004), "Does Poaching Distort Training?", *Review of Economic Studies*, 71, 1143-1162.

Moen, E.R. and Rosén, Å (2009), "Incentives in Competitive Search", Working paper.

Moene, K.O., and Wallerstein, M. (1997), "Full Employment as a Worker Discipline Device.", in *Property Relations, Incentives and Welfare.* Ed J. Roemer, London: MacMillan Press.

Morita, H. (2001), "Choice of Technology and Labour Market Consequences", *Economic Journal*, 111, 29-50.

OECD (1993), *Employment Outlook.* ch 4. Paris

Owan, H. (2004), “Promotion, Turnover, Earnings, and Firm-Sponsored Training”, *Journal of Labor Economics*, 22, 995-978.

Pries, M., and Rogerson, R. (2005), “Hiring Policies, labor market institutions, and Labor Market Flows”, *Journal of Political Economy*, 113(4), 811-839.

Rudanko, L. (2009), “Labor Market Dynamics under Long Term Wage Contracting and Incomplete Markets”. Boston University, Mimeo.

Saint-Paul, G. (1995), “The High Unemployment Trap”, *Quarterly Journal of Economics*, 110, 527-550.

Saxenian, A.L. (1994), *Regional advantage: culture and competition in Silicon Valley and Route 128.* Cambridge, Mass.: Harvard University Press.

Shapiro, C. and Stiglitz, J.E. (1984), “Equilibrium Unemployment as a Worker Discipline Device”, *American Economic Review*, 74, 433-444.

Stevens, M. (2004). “Wage-Tenure Contracts in a Frictional Labour Market: Firms’ Strategies for Recruitment and Retention”, *Review of Economic Studies*, 71, 535-551.

Shimer, R. (2006), “On-the-Job Search and Strategic Bargaining”, *European Economic Review*, 50, (4), 811-830.