Hyperbolic Discounting in Search Equilibrium

Espen R. Moen and Åsa Rosén

February 12, 2005

Abstract

We ananlyze general equilibrium effects of hyperbolic discounting among unemployed workers in search equilibrium. We show that hyperbolic discounting changes the workers' trade-off between high wages and a high exit rate from unemployment, thus influencing the behavior of firms. More specifically, firms 1) increase their number of vacancies, 2) reduce the quality of the vacancies (less capital per worker), and 3) reduce the wage attached to the vacancies. We discuss welfare consequences and derive policy implications. We find that unemployment benefits together with subsidized (or monitored) job search may increase welfare. If these policy measures are not availiable, minimum wages and subsidies to high-quality jobs may be warranted.

Key words: search equilibrium, unemployment, hyperbolic discounting, welfare

JEL codes: D60, J64, J68

1 Introduction

Hyperbolic discounting is present if a person's discount rate diminishes as the time horizon increases. At any given point in time, a consumer finds it more unattractive to delay consumption scheduled in the near future than to delay consumption scheduled in the more distant future. This leads to time-inconsistent behavior: The consumer will tend to consume more and save less than he or she would prefer from an *ex-ante* perspective (Frederick et al 2002).

Hyperbolic discounting may have consequences for the behavior of unemployed workers, who face several intertemporal choices. First, a worker's choice of search intensity influences future incomes. Second, when choosing whether to accept a job offer, the alternative is to continue searching for a better job. Hyperbolic discounting leads to a lower search intensity and a greater tendency to accept low wage offers than the person would have preferred from a prior perspective. This is discussed in a detail in DellaVigna and Paserman (2004) and Paserman (2004).

Paserman and DellaVigna analyses job search behavior in a partial equilibrium model of job search, with an exogenous distribution of wages. In this paper we explore the equilibrium effects of hyperbolic discounting in a two-sided search model with wage posting. We analyze the effects of hyperbolic discounting among workers on the behavior of (non-hyperbolic) firms. As in DellaVigna and Paserman (2004), we find that hyperbolic discounting reduces worker search intensity. However, we show that this has no effects on firm behavior. More importantly therefore, hyperbolic discounting also changes the workers' trade-off between high wages and a high exit rate from unemployment, and this influences firm behavior. Specifically, firms 1) increase their number of vacancies, 2) reduce the quality of the vacancies (less capital per worker), and 3) reduce the wage attached to the vacancies. We discuss welfare consequences and derive policy implications. We find that unemployment benefits together with subsidized (or monitored) job search may increase welfare. Absent these policy measures minimum wages and subsidies to high-quality jobs may be warranted.

2 The model

Our starting point is the standard Diamond-Mortensen-Pissarides model of labor market search. There exists a continuum of *ex ante* identical workers with measure normalized to one. Workers leave the market at an exogenous rate s. New workers enter the market as unemployed at the same rate. The unemployment rate is denoted by u. There exists a continuum of firms in the economy. A firm is either matched with a worker and producing or unmatched and searching for a worker. The number (measure) of searching firms is endogenous and denoted by v.

The number of matches between searching workers and firms is determined by a constant return to scale matching function $x(eu, v)$. This matching function maps a measure of workers u who search with an average intensity e for a measure of v vacancies into a flow x of new matches. Let p denote the probability rate that a worker finds a (new) job per unit of search intensity, and q the probability rate that a firm with a vacancy finds a worker. The arrival rates p and q are interrelated, as both depend on the labour market tightness θ defined as v/eu. Note that p and q only depend on e through θ. Due to constant returns to scale, the matching function can therefore be summarized as $q = q(p)$.[1] Matches only dissolve when the worker leaves the market, in which case the value of the firm is zero.

Hyperbolic discounting means that the agent has a declining rate of time preference. In a continuous time, this can be obtained by assuming that the utility obtained after a certain (stochastic) period of time is discounted with a factor $\beta < 1$ relative to the utility obtained at present, where "the future" arrives at a constant probability rate γ. Let W^p and W^f denote the expected discounted income for an employed worker with constant wage w evaluated at present and in the future. It follows that

$$(r+s)W^p = w + \gamma(\beta W^f - W^p). \tag{1}$$

where r denotes the long-term discount rate. Since $W^f = \frac{w}{(r+s)}$ it follows that

$$\begin{aligned}(r+s)W^p &= \frac{r+s}{r+s+\gamma}w + \frac{\gamma}{r+s+\gamma}\beta w \\ &= kw, \end{aligned} \tag{2}$$

where

$$k = \frac{r+s+\gamma\beta}{r+s+\gamma}.$$

Note that $k < 1$ and that k decreases as β decreases (stronger hyperbolic discounting) and when γ increases. In the limit, as $\gamma \to \infty$, we find that $k = \beta$. In the literature, this is referred to as instantaneous gratification, (Harris and Laibson, 2004)

Let us now turn to unemployed workers. Let U^p and U^f denote the expected discounted income evalated at present and in the future of an unemployed worker, with job finding rate p, wage w, and income (utility flow)

[1] The probability rates p and q can be written as $p = x(eu,v)/eu = x(1,\theta) = \widetilde{p}(\theta)$ and $q = x(eu,v)/v = x(1/\theta,1) = \tilde{q}(\theta)$.. The matching technology can thus be summarised by a function $q = \tilde{q}(\theta) = \tilde{q}(\widetilde{p}^{-1}(p)) = q(p)$.

during unemployment z. Then [2]

$$(r+s)U^p = z + \gamma(\beta U^f - U^p) + ep(W^p - U^p) - c(e), \tag{3}$$

where $c(e)$ denotes the worker's search cost given search effort e. Similarly, U^f is given by

$$(r+s)U^f = z + e^f p^f (W^l - U^l) - c(e^f). \tag{4}$$

Note that e and p have topscript f. This is to indicate that the workers, when evaluating the future, may believe that they will behave differently than today. *Sophisticated* agents realize that their future self also will have an hyperbolic discount rate, while *naive* agents do not. In what follows, we assume that the workers are sophisticated unless stated otherwise.

With endogenous worker effort, the worker's choice of effort is given by

$$e^* = \arg\max_e ep(W^p - U^p) - c(e).$$

It follows that we can write U^p as a function of w and p, $U^p = U^p(w,p)$.

Firms are assumed to discount the future at a constant rate r. Thus, firms are not subject to hyperbolic discounting. The cost of creating a vacancy is endogenous and denoted by K. The corresponding productivity when the position is filled is given by $f(K)$, where $f(K)$ is increasing and concave in K. The value of a filled vacancy is thus

$$(r+s)J(w,K) = f(K) - w.$$

Let q denote the arrival rate of workers to the vacancy. The expected discounted value of a vacancy is thus

$$(r+s)V(w,p,K) = q(p)(J(w,K) - V(w,p,K)). \tag{5}$$

3 Equilibrium

A crucial assumption in our model is that workers can direct their search intensity towards certain firms based on the wages these firms pay. To capture

[2] Due to the law of rare events, a change in discount factor and a change in the status in the labour market never happens simultaneously

this we apply the competitive search equilibrium. This equilibrium concept can be given several interpretations. A core element of the competitive search equilibrium concept is the unique relationship between the advertised wage and the expected rate at which the vacancy will be filled (Acemoglu and Shimer 1999b). The relationship can be derived in several settings. Moen (1997) considers an economy in which a market maker creates submarkets, each characterized by a single wage. Workers and firms are free to choose which submarket to enter. As shown by Moen, wage advertisements by firms or reputation among workers about wages firms' pay may ensure that the same equilibrium wage prevails. Mortensen and Pissarides (1999, section 4.1) and Mortensen and Wright (2002) give a similar interpretation to the one of the market maker, by assuming that a "middle man" (like a job centre) sets the wage. Alternatively, the matching technology may be derived from the urn-ball process (Montgomery (1991), Peters (1991), Burdett et al. (2001)), and Shi (2001).

An intuitive explanation can be as follows. When a firm advertises a wage, it takes into account that the workers it attracts must get the same expected income as they get if applying to any other firm. Thus, the higher wage a firm offers, the more applicants it attracts (in expected terms). The firm thus faces a trade-off between q and w, and chooses w so as to maximize the value of V from equation (5) given this trade-off. In addition, free entry of firms ensures that the value of a vacancy is equal to its creation cost. As shown in Acemoglu and Shimer (1999a), the resulting equilibrium maximizes the expected income of a searching worker. In addition, we require that the firms choose K so as to maximize $V(K) - K$. The equilibrium values of p, w, and K are thus given by

$$(w^*, p^*; K) \text{ solve } \max_{w,p} U^p(w,p) \quad \text{S.T. } V(w,p,K) = K \tag{6}$$

and

$$K^* = \arg\max_K V(w,p,K).$$

We first derive the equilibrium in the search market for a given K, and then derive the optimal K. Let $w(p; U^p)$ denote the indifference curve of a worker with expected income U^p. From equation (2) and (3) it follows that

$$\begin{aligned}(r+s+\gamma+ep)\frac{\partial U^p}{\partial w} &= \frac{epk}{r+s} \\ (r+s+\gamma+ep)\frac{\partial U^p}{\partial p} &= e[wk/(r+s)-U^p]\end{aligned}$$

(from the envelope theorem we know that changes in e can be ignored). It thus follows that the marginal rate of substitution between p and w is given by

$$|\frac{dw(p)}{dp}| = \frac{e[wk/(r+s)-U^p]}{epk/(r+s)} = \frac{1}{p}\left(w - \frac{(r+s)U^p}{k}\right).$$

Note that the marginal substitution between p and w is independent of e. With hyperbolic discounting, $k < 1$, while with exponential discounting, $k = 1$.. The next lemma is central to our analysis:

Lemma 1 *Hyperbolic discounting increases the marginal rate of substitution* $|dw/dp|$ *for given values of* (w, p).

Proof: First note that $\frac{(r+s)U^p}{k} = wU^p/W^p$. It is sufficient to show that for any $\gamma > 0$, U^p/W^p is decreasing in β. However, since U^p is the net present value of an income flow that is backlogged, while W^p denotes the net present value of an income flow that is constant, it follows that the relative decrease in U^p is larger than the relative decrease in W^p when β increases. The lemma thus follows.

To gain more intuition, note that a person with a hyperbolic discount rate puts less weight on the future than a person with exponential discounting (and the same underlying discount rate). A person with hyperbolic discounting is therefore less inclined to wait longer in order to get a higher wage than is a person with exponential discounting. The next proposition follows immediately:

Proposition 2 *With hyperbolic discounting, wages are lower and the job finding rate* p *is higher than with exponential discounting (given* r *and* K*).*

From the lemma above, it follows that at any point (p, w) at the possibility frontier defined by $V(w, p; K) = K$, workers with hyperbolic discounting are more inclined to prefer a higher value of p in return for a lower wage. The

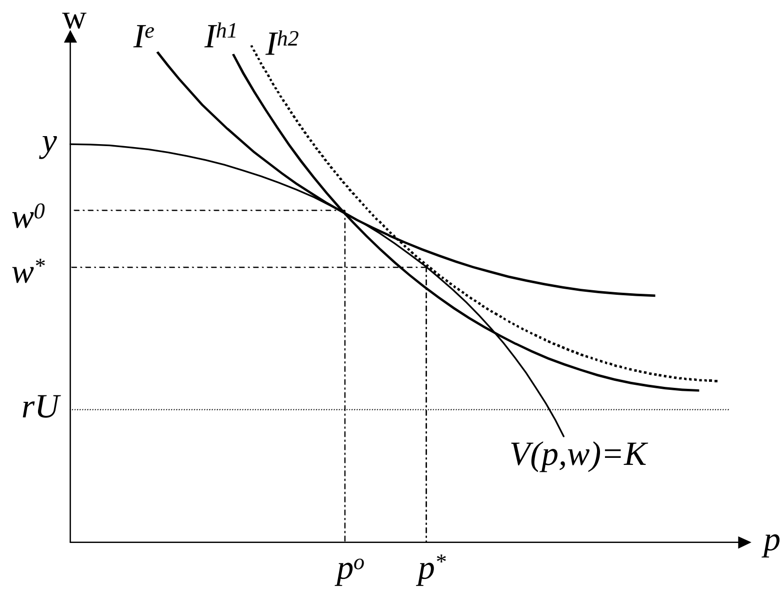

Figure 1. *V(p,w)=K* is the possibility set, I^e the indifference curve for a worker with exponential discounting, and I^{h1} and I^{h2} two indifference curves for a worker with hyperbolic discounting. Finally (w^*,p^*) and (w^o,p^o) denotes equilibrium with and without hyperbolidc discounting.

solution to the maximization problem (6) thus implies a higher p and a lower w with hyperbolic discounting than with exponential discounting. This is illustrated in the figure below.

In the figure, (w^o, p^o) is the equilibrium without hyperbolic discounting. At this point, the indifference curve I^e for a worker with exponential discounting is tangent to the zero profit curve $V(w,p) = K$. However, at this point the indifference curve I^{h1} for a worker with hyperbolic discount rate is steeper than the zero profit curve. In the equilibrium point (w^*, p^*) it follows that the wage w^* is lower and the job finding rate p^* is higher than in the exponential-discounting equilibrium.

Let us then turn to job quality K. The first order condition for K is given by [3]

$$\frac{f'(K)}{r+s} = \frac{r+s+q}{q}$$

independently of the workers' discounting. Since q is lower when workers have hyperbolic discounting, it follows that K is lower as well.

Proposition 3 *The equilibrium job quality K^* is lower with hyperbolic discounting than with exponential discounting (for a given r).*

As $W^p - U^p$ is lower with hyperbolic discounting than without, it follows that the search intensity falls as well:

Proposition 4 *Search intensity is lower when workers have hyperbolic discounting than when they have exponential discounting (for a given r).*

Up to now we have assumed that workers are sophisticated. Now, we assume that the workers where naive, i.e., workers believe they will act as if they had exponential discount rate in the future. We will not do the analysis in detail, only sketch the main argument. Naive workers believe they will maximize the preference of the "long-run-self" in the time intervals for which they (today) discount exponentially. This will increase the future expected discounted income of being unemployed relative to the one for sophisticated workers. This in turn leads to a lower search intensity, but higher wages and lower job finding rates than when the agents are sophisticated:

Conjecture 5 *If the workers are naive, this reduces the effects of hyperbolic discounting on wages and job-finding rates, but increases the effect of hyperbolic discounting on search effort.*

[3] Due to the envelope theorem, changes in p and w do not influence the maximand and can therefore be ignored.

4 Welfare and Policy

The proper definition of an optimal allocation of resources is not as clear with hyperbolic discounting as it is with exponential discounting, and there exists a literature discussing the appropriate welfare measure, see e.g. Bhattacharya and Lakdawalla (2004), Harris and Laibson (2004) and O'Donoghue and Rabin (1999). In our setting, all welfare measures will imply that the equilibrium value of p, K, and e are below their social values provided that z does not include transfers (unemployment benefits). In order to simplify the discussion we use as welfare measure the welfare measure the utility is the "long run self." Thus, the optimal allocation of resources is equivalent to the competitive search equilibrium with exponential discounting.

Let b denote unemployment benefits (transfers). With positive unemployment benefits the value to an unemployed worker is

$$(r+s)U^p = b + z + \gamma(\beta U^f - U^p) + ep(W^p - U^p) - c(e),$$

We then have the following result:

Proposition 6 *First best can be obtained by a combination of unemployment benefit and subsidized search.*

Proof: Denote the socially optimal values of e, p, and K by e^o, p^o and K^o. Fix $e = e^o$. Then we can write $p = p(b)$, where b is the unemployment benefit. Obviously, $p = 0$ as $b \to \infty$. Thus, due to continuity it follows that there exists a $b = b^o$ such that $p = p^o$ given $e = e^o$. Let a denote the subsidy rate for search, such that the private cost to the worker is $(1-a)c(e)$. Choose a such that $e^* = e^o$ given that $p = p^o$, and denote the corresponding a by a^o. Finally, note that $K = K^o$ given that $p = p^o$. It follows that first best will be achieved by implementing $b = b^o$, $a = a^o$.

The result may be surprising, as there are three variables K, p, and w to be manipulated and only two instrument. The reason why we still get efficiency is the unique relationship between w and p.

What if the only remedy is to subsidize search? Then the following still holds.

Corollary 7 *It is possible to improve welfare by subsidizing search.*

Without parametrizing the model it is not possible to determine the effects of an isolated increase in the unemployment benefit b, as an increase in b even from $b = 0$ has first order effects on both e and p. An increase in b decreases e, which has a negative welfare effect, and decreases p which has a positive welfare effect. For the same reason, one cannot predict the welfare outcome of an entry tax, as this will decrease p (a positive first-order welfare effect) and decrease e (a negative fist-order welfare effect).

Suppose the government can set minimum wages, but nothing else? The following holds:

Proposition 8 *There exists a minimum wage $w^{Min} > w^*$ which improves welfare.*

Increasing w marginally above its equilibrium value has a first-order effect on p, but only second order effects on K and e. The result thus follows.

Finally, note that workers actually may benefit from being naive. Suppose e is optimally monitored by the government. Naive workers will then tend to search for jobs with a lower job-finding rate and higher wages than sophisticated workers, thus obtaining a higher long-run utility.

5 More on good and bad jobs

It is straightforward to extend the model to two sectors. In each sector, each firm employs at most one worker. Firms are price takers. Aggregate output in the economy is given by

$$Y = f(N_1, N_2),$$

where f is homogeneous of degree one. Let $y_i = f_i(N_1, N_2)$, where N_1 and N_2 denote employment in sector 1 and 2, respectively It follows that

$$Y = y_1 N_1 + y_2 N_2,$$

where y_1 and y_2 is the (real value of) output in each sector.

The vacancy costs in the two sectors are K_1 and K_2. We assume that $K_1 < K_2$. We refer to sector 2 jobs as good jobs, as they will pay higher wages in equilibrium (although workers will be indifferent as to which sector to enter).

Without hyperbolic discounting, it is straightforward to show that competitive search equilibrium is efficient. In Moen and Rosen (2004) we show that the following holds:

Proposition 9 *With hyperbolic discounting, a larger fraction of the work force is allocated to the low-wage sector (sector* 1*) than with exponential discounting. Furthermore, in both sectors the job finding rates* p_i *are higher with hyperbolic discounting than with exponential discounting.*

6 Conclusion

We have analyzed the effects of hyperbolic discounting in competitive search equilibrium where wages are determined in a competitive fashion. We find that hyperbolic discounting tends to reduce wages, increase the number of jobs, decrease the quality of jobs, and decrease job search intensity by workers. The allocation of resources in the economy can be improved by introducing unemployment benefits and subsidizing (or monitoring) search.

7 References

Acemoglu, D. and Shimer, R. (1999a), "Efficient Unemployment Insurance", *Journal of Political Economy,* **107**, October, 893-928.

Acemoglu, D. and Shimer, R. (1999b), "Holdups and Efficiency with Search Frictions" *International Economic Review*, **40**, November, 827-849.

Bhattacharya, J. and Lakdawalla, D. (2004), "Time-Inconsistency and Welfare", NBER Working Paper Series No. 10345.

Burdett, K., Shi, S. and Wright, R. (2001), "Pricing and Matching with Frictions", *Journal of Political Economy*, **109**, October, 1060-1085.

DellaVigna, S. and Paserman, M.D. (2004), "Job Search and Impatience", NBER Working Paper Series 10837.

Frederick, S., Loewenstein, G., and O'Donoghue, T. (2002), "Time Discounting and Time Preference: A Critical Review", *Journal of Economic Literature*, **40**, 351-401.

Harris, C. and Laibson, D. (2004), "Instantaneous Gratification", Working Paper.

Moen, E. R. (1997), "Competitive Search Equilibrium", *Journal of Political Economy*, **105**, 385-411.

Moen, E.R., and Rosen, A. (2004). "Hyperbolic Discounting in Competitive Search Equilibrium". Working paper

Montgomery, J.D. (1991), "Equilibrium Wage Dispersion and Interindustry Wage Differentials", *Quarterly Journal of Economics*, **106**, February, 163-179.

Mortensen, D.T, and Pissarides, C.A. (1999), "New Developments in Models of Search in the Labor Market", in O. Ashenfelter and D. Card (eds), *Handbook of Labor Economics*, vol 3b, North-Holland, pp 2567-2627.

Mortensen, D.T, and Wright, R. (2002), "Competitive Pricing and Efficiency in Search Equilibrium", *International Economic Review*, **43**, 1-20.

O'Donoghue, T., and Rabin, M. (1999), "Doing It Now or Later", *American Economic Review*, **89**, 103-124.

Paserman, M. D. (2004) "Job Search and Hyperbolic Discounting: Structural Estimation and Policy Evaluation", IZA Discussion Paper No. 997.

Peters, M. (1991), "Ex Ante Price Offers in Matching Games: Non-Steady States", *Econometrica*, **59**, September, 1425-1454.

Shi, S. (2001), "Frictional Assignment I, Efficiency", *Journal of Economic Theory*, **98**, 232-260.