# Invisible markets

Espen R. Moen and Åsa Rosen

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**Abstract**

Efficiency in competitive search equilibrium requires that heterogeneous workers search in different search markets. In this paper we construct a competitive search model with a given number of search markets, where the workers only observe the wages offered in a limited number of submarkets. As a result, heterogeneous workers are not able to fully self-select into different submarkets, and all workers will end up searching in search markets in which there are also workers of other types. We show that the equilibrium of the model is a "Maximum segmentation allocation" (MSA), where workers are segmented to the largest degree possible given the information constraints. We show that the expected income of a given worker depends positively on the fraction of workers of his type in the economy. This gives rise to feedback effects. For instance, the return from investments in human capital is an increasing function of the fraction of workers that do invest, and this may lead to multiple equilibria.

# 1 Introduction

Search frictions by definition create allocative costs. An important issue is to what extent search frictions create additional costs by distorting the price mechanism. Diamond (1971) showed that with sequential search, arbitrarily small frictions may give rise to huge welfare losses. This happens if firms have all the bargaining power and cannot communicate their wage offers to workers prior to search. Mortensen (1986) and Pissarides (2000) extended Diamonds analysis by assuming that wages are determined by Nash bargaining. It can be shown (Mortensen 1982, Hosios 1990, Julien et.al. 2005) that with identical agents, efficiency will prevail (the correct number of firms will enter the market) if the so-called Hosios condition is satisfied. However, efficiency will typically not prevail if the agents are heterogeneous, or if investments take place before market entry, as long as workers with different characteristics cannot separate themselves into different submarkets.

An unsatisfactory feature of the Diamond-Mortensen-Pissarides model is the limited role wages play for the allocation of resources in the economy. Firms cannot speed up their hiring process by increasing the wage they offer. This is obviously an unrealistic and unsatisfactory property of the model. Partly in response to this the competitive search equilibrium where developed (Moen 1997, Shimer 1996). In the competitive search equilibrium, firms can advertise their wages prior to the workers' search decision, and the wage offered thus determine the queue length of workers in any firm. When setting the wage, a firm thus trade off wages and search costs, and it is easy to show that efficiency prevails.

The efficiency results of competitive search equilibrium are strong and robust. If workers are heterogeneous along some dimensions, (say productivity), the competitive search equilibrium has the property that new submarkets will open up. As a result, all workers in any submarket are identical, and the wage-tightness trade-off will be balanced so as to maximize the expected utility of workers in that submarket. As there is no interaction between the submarkets, heterogeneity both on the firm-and the worker side has no spillover to other agents in the market.

In this paper we assume that the market cannot so easily sort heterogeneous agents into different submarkets. We do this by assuming that there exist a large number of different submarkets. However, a given worker only observes a limited number of markets (with replacement). Thus, although efficiency requires that different workers search in different submarkets, workers may end up in markets with predominantly workers of the other type. We show that the equilibrium of the model is what we refer to as a "Maximum segmentation allocation" (MSA), where the workers are allocated on submarket such that they to the largest extent possible are allocated to search markets dominated by workers of the same type. Still, there will always be overlap in the equilibrium, in the sense that many (almost all) the submarkets contain workers of both types. Hence, the market is small in the sense that no workers find a market that is "designed" exactly for them. Furthermore, (productivity contingent) wages for a given worker type are an increasing function of the fraction of that worker type in the submarket in question.

This equilibrium has many interesting properties. First, although the equilibrium is ex post efficient, the expected income of a given worker depends on the fraction of workers of his type in the economy. The reason is that the more workers there are of a given type, the more are the search markets designed in accordance with their needs. Hence, if the fraction of (say) high-type workers increases, this increases the expected income of high-type workers and reduces the expected income of low-type workers. This runs contrary to the findings in standard search models with bargaining. Furthermore, it gives rise to interesting feedback effects in the market, which again may give rise to multiple equilibria. Suppose for instance that the workers can upgrade their skills before they enter the market at a given cost. If few workers upgrade their skills, most of the workers in the economy are unskilled, and the market gives a relatively high return to unskilled workers and a relatively low return to low-skill workers. However, if more workers upgrade their skills, more workers will be of the high type and will constitute a larger share of the workers in each submarket. As a result, the markets will to a larger degree be tailor maid to suit their type, and thus give high-type workers a higher pay-off and low-type workers a lower pay-off. Thus, the pay-off from upgrading skills is an increasing function of the fraction of workers that do upgrade their skills, and multiple equilibria may arise.

The paper that comes closest to the present one is Acemoglu and Shimer (2000). In an urn-ball model they assume that workers only observe a limited number of wage offers before they make their search decision, and they show that it is sufficient that the workers observe two independent wage offers to obtain efficiency. However, this very interesting result is derived under the assumption that workers are identical. The main focus in the present paper is on worker heterogeneity and the workers' ability to self-select into different submarkets.

# 2 The model

The model is defined as follows:

- The model is set as a one-shot game with risk neutral agents.
- There is a continuum of workers in the economy with measure equal to 1 (exogenous).
- A fraction $\alpha_l$ of the workers is of low type with productivity $y^l$, and a fraction $\alpha_h$ of the workers are high-type with productivity $y^h$. An unemployed worker receives an income $y^0$.
- There is a market maker that may operate $n$ different submarkets.
- Each worker observes exactly $m$ markets with replacement.
- A submarket consists of a pair of wages $(w_i^l, w_i^h)$ for low- and high-type workers, respectively.
- A firm consists of one job. The cost of posting a vacancy is $c$.
- The number of matches in submarket $i$ is given by a c.r.s. matching function $\min[x(u_i, v_i), u_i, v_i]$ where $u_i$ is the number of searching workers and $v_i$ the number of firms with vacancies in that market.

We assume that number of matches is given by $x(u_{i,}v_i)$. Let $p = x(u_{i,}v_i)/u_i = p(\theta)$ and $q = x(u_{i,}v_i)/v_i = q(\theta)$. Finally let $\eta = -q'(\theta)\theta/q(\theta)$. To simplify the analysis we assume that $\eta(\theta)$ is non-decreasing in $\theta$.

# 3 Equilibrium

Let $U_i(w_i^l, w_i^h, \theta)$ denote the expected income of a worker of type $j \in \{l, h\}$ that enters a submarket $(w_i^l, w_i^h)$ and where the labor market tightness is $\theta$. It follows that

$$U(w_i^l, w_i^h, \theta) = y^0 + p(\theta(w_i^l, w_i^h))(w_i^j - y^0) \tag{1}$$

The profit of a firm in a submarket $(w_i^l, w_i^h)$ is given by

$$V = -c + q(\theta(w_i^l, w_i^h))[(1-\kappa_i)(y^l - w_i^l) + \kappa_i(y^h - w_i^h)] \tag{2}$$

where $\kappa_i$ is the ratio of high-to low type workers in this market. In any submarket that attracts workers, firms will flow into the submarket, and $\theta$ adjusts so that $V = 0$. It follows that we can write $\theta = \widetilde{\theta}(w_i^l, w_i^h, \kappa_i)$. Define $\widetilde{U}^j(w_i^l, w_i^h, \kappa_i) \equiv U^j(w_i^l, w_i^h, \widetilde{\theta}(w_i^l, w_i^h, \kappa_i))$.

A special feature of our model is that the pure existence of a particular submarket reduces the probability that a worker will observe the other submarkets, as workers only observe a given number of submarkets. To illustrate, suppose a submarket $(w_1^l, w_1^h)$ gives both worker types a reasonably high utility. Suppose now that we add two submarkets $(w_2^l, w_2^h)$ and $(w_3^l, w_3^h)$ where wages for both types are only slightly higher than the unemployment income $y^0$ and hence give an expected income close to $y^0$ for both types. Suppose also that market 2 gives a slightly higher utility for the low type than market 3, while the opposite is true for market 3. With only market 1 operating, both worker types get a reasonably high expected income. If all three markets operate, 4/9 of the workers do not observe market 1, and end up in a lousy market. The expected income of workers thus fall. Furthermore, among these workers, the low-type workers chose market 2 while the high-type workers chose market 3. Hence all markets attract workers.

In order to avoid an excessive number of markets that blur the workers' choice of submarkets, we make the following requirement on superfluous markets

**Definition 1** *Consider a submarket* $(w^l, w^h)$ *in which a fraction* $\kappa$ *of the workers is high-type. Then the market is superfluous if there exists another market* $(w^l_x, w^h_x)$ *(active or inactive) such that*

$$\widetilde{U}^j(w^l, w^h, \kappa) < \widetilde{U}^j(w^l_x, w^h_x, \kappa)$$

*for both* $j = l$ *and* $j = h$.

We can now define competitive search equilibrium as follows[1 2]

**Definition 2** *An* $n$*-market competitive search equilibrium is a set of* $n$ *wage pairs* $(w^l_i, w^h_i)$*, with associated labor market tightness* $\theta_i$ *and inflow of low-types* $N^l_i$ *and of high-type workers* $N^h_i$*, satisfying the following criteria:*

*1) Free entry of firms. For all* $(w^l_i, w^h_i)$, $\theta_i = \widetilde{\theta}(w^l_i, w^h_i, \kappa_i)$*, where* $\kappa_i = N^h_i/N^l_i$

*2) Optimal choice of submarkets. Among the* $m$ *observed submarkets, workers chose the submarket that gives the highest expected income* $\widetilde{U}^j(w^l_i, w^h_i, \kappa_i)$.

*3) None of the markets are superfluous*

# 4 Characterizing equilibrium

In order to characterize the equilibrium, we first derive conditions for markets not being superfluous. Consider a submarket with $N_l$ low-type and $N_h$ high-type workers, and with $\kappa = N_h/N_l$. Aggregate income for the workers in this

[1] An alterantive is to assume that there is a smallest unit of account for wages, so that the wage space is discretized. More specifically, to assume that the the smallest unit of account is $1/z$, so that a unit $[n, n+1)$ is devided into $z$ units.

[2] This definition is not sufficient to ensure uniqueness. In order to obtain this, we have to require that the utility of joining a market is higher the higher is the social surplus associated with the worker joining the market.

submarket is proportional to

$$S(\kappa) = (N_l + N_h)[y^0 + p(\theta)[(1-\kappa)w^l + \kappa w^h - y^0] \tag{3}$$

Let $J$ be the expected profit of a firm that finds a worker, given by

$$\begin{aligned} J &= \kappa(y^h - w^h) + (1-\kappa)(y^l - w^l) \\ &= \frac{c}{q(\theta)} \end{aligned} \tag{4}$$

where the last equation follows from the free entry condition. It follows that

$$S(\kappa) = (N_l + N_h)[y^0 + p(\theta)[(1-\kappa)y^l + \kappa y^h - J - y^0]]$$

Non-superfluos markets solves

$$\begin{aligned} &\max(N_l + N_h)[y^0 + p(\theta)[(1-\kappa)y^l + \kappa y^h - J - y^0]] \\ &\text{S.T} \\ J &= \frac{c}{q(\theta)} \end{aligned}$$

By substituting $J = c/q$ into the expression of $S$, to get that

$$S(\kappa) = (N_l + N_h)[y^0 + p(\theta)[(1-\kappa)y^l + \kappa y^h - \frac{c}{q(\theta)} - y^0]]$$

Maximizing this expression wrt $\theta$, we obtain the following first-order conditions (where $\eta = -q'(\theta)\theta/q$):

$$(1-\eta)[(1-\kappa)y^l + \kappa y^h - J - y^0] = \eta J \tag{5}$$

This "sharing rule" can be interpreted as the Hosios condition in this setting. By slightly rearranging the equation it follows that the worker's expected share of the match surplus is equal to $\eta$.

For any $\kappa$, (5) uniquely defines the labor market tightness $\theta^*$. It follows that we can write $\theta^* = \theta^S(\kappa)$. It follows that $\theta^*$ is strictly increasing in $\kappa$ (this is not difficult to show).

Let us calculate the social value of an additional low-type worker, $U^{sl}(\kappa)$ entering the market. By the envelope theorem it follows that

$$\begin{aligned} U^{sl} &= \frac{d}{dN_l} S \\ &= y^0 + p(\theta^S(\kappa))[y^l - J - y^0] \end{aligned} \tag{6}$$

Note that $U^{sl}$ is proportional to $S(0)$. It follows that $U^{sl}$ is maximized at $\kappa = 0$ and is strictly decreasing in $\kappa$ (as $J$ and $\theta$ increases).

**Lemma 3** *The social value of a low-type worker, $U^{sl}(\kappa_i)$, is strictly decreasing in $\kappa_i$. The social value of a high-type worker, $U^{sh}(\kappa_i)$, is strictly increasing in $\kappa_i$.*

To gain intuition, note that the social value of one more low-type worker in the market can be written as (keeping $\theta$ constant due to the envelope theorem) $y^0 + p(\theta)(y^l - J - y^0)$. By definition this is maximized in a submarket where $\kappa_i = 0$, i.e., where all workers are of low-type. The higher is $\kappa_i$, the higher is $\theta^*$, and the longer is the difference between the labor market tightness in this market and the optimal labor market tightness.

By combining (1) and (6) we get the following expression for wages

$$w^{js}(\kappa) = y^j - \frac{c}{q(\theta^S(\kappa))} \tag{7}$$

The wage $w^{js}(\kappa)$ shows the wage that equalizes the social and the private value of entering the market, provided that $\theta^* = \theta^S(\kappa)$. Note that the wage is decreasing in $\kappa$ for both types, as $\theta^S(\kappa)$ is increasing in $\kappa$. . The more workers that are of the high type, the higher is the labor market tightness, and the lower is the wage. For low types, the negative effect of lower wages dominates the positive effect of higher $\theta^*$ as $\kappa$ increases, hence $U^{lS}$ is decreasing in $\kappa$. The opposite is true for the high-type.

The next step is to analyze the values of $\kappa$ in the different submarkets. To this end we make the following definition

**Definition 4** *A maximum segmentation allocation (msa) obtains if all agent, when choosing between submarkets, choose the submarket with the highest proportion of agents of the same type.*

Our next step is to characterize the msa-equilibrium. Number the submarkets in such a way that the share of high type workers $\kappa_i$ is increasing with the index $i$. The measure of workers observing any given submarket is given by

$$
\begin{aligned}
N_i^{obs} &= (1-(1-\frac{1}{n})^m)N \\
&\approx \frac{1}{n}Nm
\end{aligned}
$$

if the number of markets $n$ is sufficiently large. In the limit, with infinitely many markets, the relationship is exact. In an msa-allocation a high-type worker chooses this submarket if and only if all the other submarkets she observes has a lower $\kappa_i$, i.e. a lower index. The probability that a high-type workers does not observe a submarket with higher index than $i$ is $(i/n)^m$. The fraction of high-type workers that observe submarket $i$, given that they do not observe a submarket with a higher $i$, is given by $1-(\frac{i-1}{i})^m$. The measure of workers that apply to job $i$ is thus

$$N_i^h = \alpha_h(\frac{i}{n})^m[1-(\frac{i-1}{i})^m]. \tag{8}$$

Analogously, a low type worker in an msa-allocation chooses market $i$ if and only if she does not observe a market with a lower index $j$. The fraction of low-type workers that does not observe a market $j < i$ is given by $(1-\frac{i-1}{n})^m$. The probability that a worker observes $i$, given that she does not observe a lower $i$, is given by $1-(\frac{n-i}{n-i+1})^m$

$$N_i^l = \alpha_l(1-\frac{i-1}{n})^m[1-(\frac{n-i}{n-i+1})^m]. \tag{9}$$

It follows that the fraction of low-type workers in market $i$ is given by

$$\begin{aligned}\kappa_i &= \frac{N_i^h}{N_i^l+N_i^h} \\ &= \frac{\alpha_h(\frac{i}{n})^m[1-(\frac{i-1}{i})^m]}{\alpha_l(1-\frac{i-1}{n})^m[1-(\frac{n-i}{n-i+1})^m]+\alpha_h(\frac{i}{n})^m[1-(\frac{i-1}{i})^m]}\end{aligned} \tag{10}$$

**Proposition 5** *a) The following constitutes an n-market competitive search equilibrium:*

*1) The allocation of workers on submarkets is a maximum separation allocation. For convenience we assume that market number* 1 *has the lowest value of* $\kappa_i$*. It follows that* $N_i^h$ *and* $N_i^l$ *are determined by (8) and (9), respectively. It follows that* $\kappa_i$ *is given by (10).*

*2)* $(w_i^l, w_i^h, \theta_i) = (w^{ls}(\kappa_i), w^{hs}(\kappa_i), \theta^S(\kappa_i))$

The proof is by construction. By definition, none of the markets are superfluous. From lemma 3 it follows that the low-type workers prefer the submarkets with a low $\kappa$, while the opposite is true for the high-type workers. Hence the resulting equilibrium is a maximum separation equilibrium, with values of $\kappa_i$ given by (10).

Note the following: From lemma (3) it follows that the social value of a low-type worker is decreasing in $\kappa_i$. Furthermore, by construction the wages in each submarket is set so that workers receive their social contribution. Hence, a low-type worker is always better off by choosing the submarket among the ones that she observes that has the lowest fraction of high-type workers. For high-type workers the opposite holds, for exactly the same reasons. Hence, workers will self-select on submarkets in such a way that the maximum-segmentation equilibrium occurs.

It follows by construction that the equilibrium is constrained efficient, in the sense that it maximizes output given the constraint that workers only observe $m$ submarkets. To understand why the maximum separation allocation is efficient, let $\kappa_i$ and $\kappa_j$ denote the fraction of high-type workers in submarket $i$ and $j$, respectively. Suppose $\kappa_i > \kappa_j$. Then it is sufficient to show that welfare increases by exchanging a small measure $a$ of low-type workers from market $i$ with high-type workers from market $j$. The measure of high-type workers thus increases and low-type workers decreases in market $i$, while the opposite holds in market $j$. From the envelope theorem we know that we can ignore effects from changes in the labor market tightness, as they are set optimally initially. Clearly, for a given tightness, the total number of hirings in the two markets stay constant. However, as the labor market tightness is higher in market $i$ than in market $j$ it follows that the proportion of high-type workers hired increases, and hence also welfare.

It is trivial to show the following:

$$\lim_{m\to\infty} \kappa_1 = 0$$
$$\lim_{m\to\infty} \kappa_n = 1$$

Hence, as the workers observe a large number of markets (with replacement), the equilibrium allocation converges to the equilibrium allocation with completely separated markets.

It is interesting to analyze the market in equilibrium as $n \to \infty$. Let $I = i/n$. As $n$ goes to infinity, the measure of workers in each market goes to zero. It follows from (10) that

$$\begin{aligned}\kappa(I) &= \lim_{i,n\to\infty} \frac{\alpha_h(\frac{i}{n})^m[1-(\frac{i-1}{i})^m]}{\alpha_l(1-\frac{i-1}{n})^m[1-(\frac{n-i}{n-i+1})^m]} \\ &= \frac{\alpha_h I^m}{\alpha_l(1-I)^m}\end{aligned}$$

where we have used L'Hopital's rule. Define the density of workers in market $I$, $f(I)$, as

$$\begin{aligned}f(I) &= \lim_{n\to\infty} \frac{N^j_{nI} n}{\alpha} \\ &= \lim_{n\to\infty} (1-I)^m[1-(\frac{n-In}{n-In+1})^m]\frac{1}{n^{-1}} \\ &= (1-I)^m \lim_{n\to\infty} \frac{m(\frac{n-In}{n-In+1})^{m-1}\frac{1-I}{(n-In+1)^2}}{\frac{1}{n^2}} \\ &= m(1-I)^{m-1}\end{aligned}$$

It follows that

$$\begin{aligned}1 &= \int_0^1 f(I)dI \\ &= \int_0^1 m(1-I)^{m-1}dI \\ &= -|_0^1(1-I)^m \\ &= 1\end{aligned}$$

## 5 Market feedback

The expected income of a worker of type $j$ can be written as

$$EU^j = \sum_{i=1}^{n} f(\frac{i}{n})U^{Sj}(\kappa_i)$$

Suppose $\alpha^h/\alpha^l$ increases. From equation (10) it follows that $\kappa_i$ increases in all submarkets. The next proposition follows directly

**Proposition 6** *An increase in the fraction of high type workers in the economy increases the expected pay-off for high-type workers and decreases the pay-off for low-type workers.*

There is thus a feed-back effect present in the model. The more workers of a given type there is in the market, the better off is this type, while the other type is worse off. This contrasts earlier findings in standard sequential search models with bargaining. In these models an increase in the fraction of high type workers typically makes *both* high type and low type workers better off, as the labor market tightness increases. (See e.g. Acemoglu 1996 and Masters 1998).[3]

The feedback effects may clearly give rise to multiple equilibria. Suppose workers prior to joining the market may choose to upgrade their skills at cost $K$. If no workers upgrade, $\kappa = 0$. Hence $\kappa_i = 0$ in all submarkets. hence the value of upgrading is low and may not cover the costs of upgrading. If

[3] Exceptions can be found Acemoglu (1999) and Albrecht and Wroman (2001). However, in these models the effects do not get through the labor market tightness. For instance, in Acemoglu firms don't know what worker type they will meet at the stage when they chose technology, and hence base their choice of the expected ratio of good to bad workers.

on the contrary all workers upgrade their skills, $\kappa = 1$. Now the gain from upgrading is larger, since from proposition 6 the value of being of the high-type is increasing while the value of being low-type has decreased.

Finally, note that as the value of being low-skilled decreases in the number of skilled workers, we do not know a priori which of the equilibria that is superior from a welfare point of view.

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