Taking Competitive Search to Data

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1 Introduction

In search theory, an important distinction can be drawn between models with directed search and with random /undirected search. With directed search, wages are observed by workers prior to their search decision. With random search, workers learn the wages a firm posts after being matched with the firm. In the Burdett-Mortensen 1998 (BM) model the firms post wages. Other models of random search assume wage bargaining. The two modes of wage determination have different implications for the role of wages in allocating resources. With directed search, wages influence which firms workers direct their search. With random search, wages only influence the *ex post* decision of whether or not to accept the match. As a result, directed search gives rise to an efficient allocation of resources in a wide variety of environments, while random search typically does not.

In the present paper we first present a simple model of competitive on-the-job search, where firms' productivity differences emerge endogenously through the firms' investment choices. This gives a purified version of the competitive search equilibrium with on-the-job search relative to that of Garibaldi and Moen (2009,2010). We believe this model gives some new insights in its own right. Then we discuss testable differences between our model and the BM model. Finally we use Norwegian data to discriminate between the theories.

2 The model

The paper is related to Garibaldi and Moen (2010), but with some important differences, as the productivity of the firms in our model is endogenous.

- All agents are risk neutral, infinitely lived, and discount the future at rate r.
- Workers are identical, search equally well on and off the job, and receives y_0 when unemployed. Their number (measure) is exogenous and normalized to 1.
- Before they enter the market, firms choose a technology from a menu $y = f(K)$, where y is output, K is investment, and $f(K)$ is strictly increasing and concave.
- Firms constantly post one vacancy, and hence hire continuously (can easily be extended to v vacancies with convex maintenance cost $c(v)$).
- Firms die at rate δ. In addition, workers separates from firms at an exogenous rate s.
- No internal coordination problems in firms -workers search so as to maximize joint income. This may be because workers and firms can contract upon the worker's on-the job search behaviour directly. Alternatively, the worker may pay a quit fee to the firm if leaving or buy the job up-front after being matched. (see Moen and Rosen (2004) for more examples).

The assumption that firms contract efficiently implies that the remuneration of the worker does not influence her on-the-job search behaviour.

The search market endogenously separates into submarkets, consisting of a set of workers and firms with vacancies searching for each-other. In each submarket, the flow of matches is determined by a constant-returns-to scale matching function $x(u, v)$, where u and v are the measures of workers and firms in that submarket, respectively. Let $\theta = v/u$, and define $p(\theta) = x(u, v)/u = x(1, \theta)$ and $q(\theta) = x(u, v)/v = x(1/\theta, 1)$. Finally. let $\eta = |q'(\theta)\theta/q|$ denote the absolute value of the elasticity of η with respect to θ. It is convenient to assume that $\eta(\theta)$ is non-decreasing in θ.

Firms advertise contracts and workers search for the different contracts. Suppose a firm offers a wage and on-the-job search possibilities so that the workers' expected lifetime income is W. We then say that the firm advertises an NPV wage (or just wage) W. Suppose a vector $W_1, ..., W_k...$ of wages are advertised. The firms that advertise a given wage and the workers that apply to those firms form a submarket. Let $\theta_1, ..., \theta_k...$ denote the associated vector of labor market tightness.

As will be clear below, firms will diversify and choose different investment levels, and hence enter the market with different productivities. Suppose the different values of y forms a countable set $\{y_1, y_2, ...\}$, where $y_i < y_{i+1}$. Let $M_i = M(y_i)$ denote the joint expected discounted income flow of a worker and a job in a firm of type i, where the gains from on-the-job search is included. Since on-the-job search is efficient, it follows that M_i is given by

$$rM_i = y_i + (s+\delta)(M_0 - M_i) + \max_k p(\theta_k)[W_k - M_i] \tag{1}$$

The first term is the flow production value created on the job. The second term captures the expected capital loss due to job separation, which happens at rate $s+\delta$, and reduces the joint income to M_0 (since the firm then earns zero on this match). The last term shows the expected joint gain from on-the-job search. Since the current wage is a pure transfer from the employer to the worker, it does not appear in the expression. From (1) it follows that the optimal search behaviour of a worker depends on her current position, as this influences M_i.

The indifference curve of a worker of type i shows combinations of θ and W that gives a joint income equal to M_i. We can represent this as $\theta_i = f_i(W; M)$.[1] It follows that f_i is defined implicitly by the equation

$$rM_i = y_i + (s+\delta)(M_0 - M_i) + p(g_i(W, M))[W - M_i] \tag{2}$$

where M_i is the equilibrium joint income in firm i. It follows that for $M_i < W_i$

$$g_i(W; M) = p^{-1}\left(\frac{(r+s+\delta)M_i - y_i - (s+\delta)M_0}{W - M_i}\right) \tag{3}$$

The indifference curve is defined for all W, not only the values advertised in equilibrium. Define

$$g(W; M) = \min_{i\in\{0,1,\dots,n\}} g_i(W; M) \tag{4}$$

The function $g(W; M)$ is thus the lower envelope of the set of functions $g_i(W; M)$. In equilibrium, $g(W; M)$ shows the relationship between the wage advertised and the labor market tightness in a submarket. Suppose that for a given W, the minimum in (4) is obtained for worker type i'. This worker type will then flow into the market up to the point where $\theta = g_{i'}(W; M)$. At this low labor market tightness, no other worker

[1] Strictly speaking, f_i only depends on M_i and M_0, but we write it as a function of the vector M for convenience

types want to enter this submarket. The labor market tightness is thus given by $g_{i'}(W;M)$, and only workers of type i' enter the market.

Consider then a firm with productivity y_j. At any point in time, a firm of type j maximizes the value of search given by[2]

$$\pi(y_j.W) = -c + q(\theta)[M_j - W]. \tag{5}$$

where W_j is the NPV wages paid by the firm. The first part is the flow cost of posting vacancies, while the second part is the gain from search. The firm maximizes profit with respect to W given that $\theta = g(W;M)$. Denote the associated maximum profit flow by $\pi_j^*(y_j)$. The expected profit of a firm entering the market as a type j firm is thus

$$\Pi_j = \frac{\pi_j^*(y_j)}{r+\delta} \tag{6}$$

At the entry stage, the firm chooses the investment level K, hence we can write $\Pi(K) = \frac{\pi^*(y(K))}{r+\delta}$. Firms choose K so as to maximize $\Pi(K) - K$.

Let θ_{ij} denote the labor market tightness in a labor market where workers in firms of type i searches for firms of type j.

Definition 1 *A competitive search equilibrium is a vector $K_1, K_2....$ of investment levels, a vector of joint incomes M_0,M_1, ..., a matrix of wages W_{ij}, $i,j \in \{1,2,...\}$, and a matrix θ_{ij}, $i,j \in \{1,2,...\}$ such that*

1) M_i is given by (1) with $y_i = f(K_i)$

2) W_{ij} maximizes $\pi_j(y(K_j), W)$ for some j

3) $\Pi(K) - K \leq 0$ for all K with equality for all K chosen in equilibrium

4) Aggregate consistency: inflow of workers equal to outflows in all submarkets

3 Characterizing equilibrium

We will now show that the equilibrium of the model can be characterized in a simple way. Our first observation is that the vector of investment levels $K_1, K_2, ...$ cannot be finite. Suppose there is a highest value K_n. Consider a firm that is investing $K_n + 1$. Since $f'(K) > 0$ it follows that this firm can offer a wage slightly above $y(K_n)$, fill its vacancies infinitely quickly, and hence obtain an infinite profit.

The next result is key for characterizing equilibrium

[2] At any point in time, the firm decides on the number of vacancies to be posted and the wages attached to them. This only influences profits through future hirings, and is independent of the stock of existing workers.

Proposition 2 *The equilibrium is a pure job ladder, where workers working in firms of type j with capital K_j search for jobs in firms of type $j+1$ with capital K_{j+1} only.*

Consider a firm of type j. To see why, consider a firm that attracts workers employed in firms of type j. The investment level and wage must solve the dual problem

$$\max_{W,\theta,K} M_j = \max_{W,\theta,K} y_j + p(\theta)(W - M_j) \qquad \text{S.T. } \Pi(K) = K \tag{7}$$

The solution of this problem depends on j, hence a worker of type j will never want to search for other firms than those solving (7) of worker type j. In addition it is easy to show that the value of K that solves (7) is increasing in worker type.

Taking the derivative of (1) with respect to y gives (due to the envelope theorem)

$$M'(y) = \frac{1}{r + s + \delta + p_{j+1}}$$

First order condition for K requires that $\Pi'(K) = 1$, or

$$q_j \frac{f'(K_j)}{r + s + \delta + p_{j+1}} = r + \delta$$

The first order condition for wages is given by

$$W_j = M_{j-1} + \eta(M_j - M_{j-1}) \tag{8}$$

which uniquely defines W_j (assuming that η is non-decreasing in θ). It follows that the zero profit constraint writes

$$q(\theta_j)(1 - \eta)M(y(K_j) = r + \delta$$

For given $M()$, this uniquely pins down θ_j.

Proposition 3 *1. The equilibrium can be characterized as a vector of investments $K_1^*, K_{2,}^*...$,joint incomes $M_0^*, M_1^*, ...$, and labor market tightness $\theta_0^*, \theta_1^*, ...$ satisfying the following conditions*

a) Optimal investments

$$q_j(\theta_i^*) \frac{f'(K_j)}{r + s + \delta + p_{j+1}} = r + \delta$$

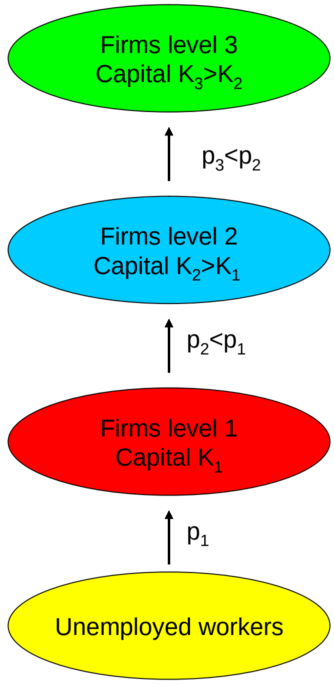
b) Joint incomes

$$M(y(K_j^*)) = \frac{y(K_j^*) + \eta(M(y_{j+1}) - M(y_j)) + (s + \delta)M^0}{r + \delta + s}$$

c) Zero profits

$$q(\theta_j)(1 - \eta)M(y(K_j)) = r + \delta$$

2. $K_1^* < K_2^* < K_3^*....$ and $\theta_1 > \theta_2 > \theta_3......$



4 Empirical analysis

In this section we will briefly discuss testable differences in predictions between our model and some other important models of on-the-job search.

The Burdett-Mortensen (BM) model. In the BM model, a firm's output is proportional to the labor force,and firms post wages prior to being matched with workers. Firms and workers match randomly, hence the distribution of wages after successful on-the-job search is equal to the wage distribution over vacancies truncated at previous wage w^o ($w \geq w_o$). Hence, if the wage distribution over vacancies is denoted by $H^v(w)$, it follows that

$$H(w|w^o) = \frac{H^v(w) - H^v(w^0)}{1 - H^w(w^0)}$$

The support of the distribution is $[w^o, w^s]$, where w^s is the supremum of the support of advertised wages. Define $H(w|w \geq \overline{w}, w^0)$ as the distribution function of new wages w, contingent on $w \geq \overline{w}$, as a function of the old wage w^0. Then for any $w \geq \overline{w} \geq w^0$,

$$\begin{aligned} H(w|w \geq \overline{w}, w^0) &= \frac{H(w|w^0) - H(\overline{w}|w^0)}{1 - H(\overline{w}|w^0)} \\ &= \frac{H^w(w) - H(\overline{w})}{1 - H^w(\overline{w})} \end{aligned}$$

independently of w_i^o.

Similarly, the distribution of prior wages $H^{old}(w^0|w^1)$ (where w^1 is still the new wage) is equal to the distribution of wages over employees (including unemployment benefit) truncated at w^1. By using the same argument as above it follows that $H^{old}(w|w^1, w \leq \overline{w})$ is independent of w^1 for all $w^1 > w$.

The Postel-Vinay and Robin (PR) wage setting procedure. Postel-Vinay and Robin (2002) assume that after successful on-the-job search, the incumbent firm and the new firm compete for the worker in a Bertrand fashion. Furthermore, firms compete in NPV wages, hence a worker takes into account that expected future wages (after encountering another job offer) will be higher the higher is the productivity of the employer. The latter is referred to as the option value of the job.

If we we consider productivity instead of wages, the results for the BM model carries over to this model. If we let D denote the distribution of productivities of the new firm after successful on-the-job search, it follows that D is equal to the productivity distribution of the vacancies truncated at the productivity level of the previous employer. Hence, if the productivity distribution over vacancies is given by $D^p(y)$, it follows that

$$D_p(y^1|y^0) = \frac{D^p(y) - D^p(y^0)}{1 - D^p(y^0)}$$

Consider then the distribution of wages $D_w(w|w_o)$. There is no one-to-one correspondence between wages and productivity in a given job. However, wages and productivity are positively correlated. Hence there will be a positive correspondence between wages in a previous job and wages in the new job, even though the distribution of underlying productivities does not show such a correspondence.

Due to the fact that the option value is increasing in productivity, the model predicts that, contingent the productivity of the employer, there is a *negative* relationship between wages in the new job and the productivity of the new employer.

Consider then the distribution of wages in a given firm. On average, a high-productivity firm will pay higher NPV wages when attracting workers, since they will be willing to bid higher and be able to attract workers previously employed in more productive firms. On the other hand, since the option value of staying in a high-productivity firm is higher than the option value of staying in a low-productivty firm, hence contingent on the productivity of the previous employer, the more productive workers pay less.

Competitive on-the-job search. Our model is not a model of wages, but rather of NPV wages. However, assume that the wage that the worker obtains in a firm is constant, and that the workers' search behaviour is contracted upon directly. As the value of job search is lower the higher is the worker in the hierarchy, it follows easily that $w = w(W)$, $w\prime(W) > 0$.In the competitive search equilibrium it follows that there is a one-to-one correspondence between wages and productivity and between wages before and after a job change.

To sum up,

To incorporate observable heterogeneity, we assume that wages of individual i in job number k can be written as a function of observables X_{ik} plus some residual u_{ik}.

$$\begin{aligned} w_{ik} &= X_{ik}\beta + u_{ik} \\ X_{ik} &= \{Year_{ik}, Age_{ik}, Industry_{ik}.Educ_i\} \end{aligned}$$

Where X_{ik} is a vector of indicator dummies for calendar year, age, industry and education. The residual u_{ik} captures the variation in wages not explained by observable characteristics. This is the component of wages that can be interpreted as an outcome of job search behavior/endogenous wage dynamics. To make this point more explicit, consider u_{ik} as a composite error term, which is in part a function of the wage in the last job, controlling for observables:

$$\begin{aligned} u_{ik} &= g(u_{ik-1}) + \varepsilon_{ik} \\ &= \gamma u_{ik-1} + \varepsilon_{ik} \end{aligned}$$

Assuming g is linear. Consider a sample of workers where $w^0 < \overline{w} < w^1$ for some $\overline{w}$. In this sample, BM predicts $\gamma = 0$, while our model predicts $\gamma > 0$

From this, it follows that

$$\begin{aligned} w_{ik} &= X_{ik}\beta + u_{ik} \\ &= X_{ik}\beta + \gamma u_{ik-1} + \varepsilon_{ik} \\ &= X_{ik}\beta + \gamma(w_{ik-1} - X_{ik-1}\beta) + \varepsilon_{ik} \\ &= X_{ik}\beta + \gamma w_{ik-1} + X_{ik-1}\beta_1 + \varepsilon_{ik} \end{aligned}$$

where $\beta_1 = -\gamma\beta$

Table 1: Descriptive statistics

	Mean	Std. dev.
Wage in job 0	144.7	62.50
Wage in job 1	190.9	84.73
Wage increase	46.17	54.18
Age job 0	35.56	11.11
Age job 1	38.70	11.38
Changed industries	38.9%	
Male	62.9%	
Basic education only or unknown	11.9%	
Secondary school degree	61.3%	
Higher education degree	26.8%	
Observations	477941	

5 Data and results

The principal source of data on wages is Statistics Norway's "Wage Statistics". This dataset contains information on individual wages and hours worked. Data is collected yearly by Statistics Norway.

For public sector employees, data on wages and hours are collected using existing register data. In later years, this covers all public sectors employees. For private sector workers, the data source is questionnaires on individual workers filled out by all firms included in the annual sampling. All large firms are included in the sample, as well as a sample of smaller firms. In total, the survey covers 50-65% of all workers in each industry. Wage data is available from 1997 to 2006. Education data is collected from the registry of the population's highest education.

In our sample, we include persons who at some point in time during the years 1997 - 2005 were registered in a job in the wage statistics, and who were registered as working in some other job at a later time during 1998 - 2006. We include only observations where data is available on contracted hours and monthly wages for both jobs.

In order to keep only job-to-job transition, we exclude persons that were registered unemployed between the two jobs. We also exclude persons who experienced a wage drop in the new job, as some of these observations are likely involuntary job loss even though there was no period of registered unemployment. Persons who changed their education level between jobs are also excluded from the sample.

The final sample covers 477941 job transitions. Tables 1, 2 and 3 present some descriptive statistics.

Table 2: Grouped industry shares in sample

	Industry share
Wholesale and retail trade, repairs	22.6%
Manufacturing	21.8%
Other business activities	13.5%
Public sector	10.1%
Transport and communication	9.45%
Construction	8.44%
Financial intermediation	6.55%
Electricity, gas and water supply	2.50%
Hotels and restaurants	2.47%
Mining and quarrying	2.44%
Fishing	0.101%
Observations	955882

Table 3: Grouped industry shares, first and second job

	Share job 0	Share job 1
Wholesale and retail trade, repairs	24.1%	21.0%
Manufacturing	22.5%	21.2%
Other business activities	13.0%	13.9%
Public sector	9.5%	10.7%
Transport and communication	9.0%	9.9%
Construction	8.2%	8.7%
Financial intermediation	6.3%	6.8%
Electricity, gas and water supply	2.6%	2.4%
Hotels and restaurants	2.4%	2.6%
Mining and quarrying	2.1%	2.8%
Fishing	0.1%	0.1%
Observations	477941	477941

Table 4: Estimated wage equations

$\bar{w}_1$ is percentile	20th	40th	60th	80th
γ	0.101***	0.105***	0.0808***	0.0957***
	(14.78)	(16.75)	(12.52)	(12.30)
Controls	Dummies for Age1, Age0, Ind1, Ind0, Educ, Year1, Year0			
Observations	185040	163545	118986	66630
Adjusted R^2	0.183	0.128	0.095	0.072

t statistics in parentheses

$^{*}p < 0.05$, $^{**}p < 0.01$, $^{***}p < 0.001$

Estimation of

$$w_{i1} = X_{i1}\beta + \gamma w_{i0} - X_{i0}\beta_1 + \varepsilon_{ik}$$

is carried out choosing $\bar{w}_1$ equal to some value (e.g. the 20th percentile), and estimating the equation on the subsample with $w_1 > \bar{w}_1$ and $w_0 < \bar{w}_1$. Results are shown in table 4

6 Conclusion

In the theory paper of this model, we have derived a model of on-the-job search where productivity differences between firms emerge endogenously as a response to search frictions and directed search. Furthermore, the equilibrium of the model can be described as a pure job ladder, where workers advance the ladder one step at the time.

We then show that the competitive search model implies that wages before on-the-job search influences wages after successful on-the-job search. Furthermore, we also argue that this is not the case for the (properly conditioned) wage distributions after on-the-job search predicted by the Burdett-Mortensen model. Hence competitive search models and random search models give different predictions at this point. We have used Norwegian data to discriminate between the two models. Our preliminary findings suggest that there is a positive relationship between wages before and after the job shift, which tend to favour competitive search. However, we want to point out that this may also be due to unobservable fixed worker effects.

7 References

Garibaldi, P., and Moen, E.R., “Job to Job Movements in a Simple Search Model”. *American Economic Review* (p&p) Vol 100, no 2, May 2010, p 343-347.

Garibaldi, P., and Moen, E.R., (2010) “Competitive on-the-job search”. Miemo.

Burdett, Kenneth, and Dale T. Mortensen, (1998) Wage Differentials, Employer Size, and Unemployment, *International Economic Review*, vol. 39(2), pages 257-73, May.

Postel-Vinay, R., and Robin, J.M. (2002), "Equilibrium Wage Dispersion with Worker and Firm Hetrogeneity". *Econometrica* vol 70 no 6, 2295-2350.