Incentives in Competitive Search Equilibrium*

Espen R. Moen† and Åsa Rosén‡

June 25, 2008

Abstract

This paper analyzes the interaction between internal agency problems within firms and external search frictions when workers have private information. Firms use wage contracts to motivate workers. In addition, wages are also used to attract workers. The resulting allocation of resources satisfies a modified Hosios rule. Then, we use the model to analyze the effect of changes in the macroeconomic variables on the wage contract and the unemployment rate. We find that private information may increase the responsiveness of the unemployment rate to changes in productivity. The incentive power of the wage contracts is positively related to high productivity, low unemployment benefits and high search frictions.

Key words: Private information, incentives, search, unemployment, wage rigidity

JEL classification: E30, J30, J60

1 Introduction

There exists a large literature analyzing the effects of search frictions in the labor market. In this literature, firms are typically modeled in a parsimonious way, with exogenous output per worker. In particular, agency problems between workers and firms are ignored. The focus is thus solely on the effects of search frictions on the flows into and out of employment.

*This is a revised version of an earlier paper entitled "Incentives in Competitive Search Equilibrium and Wage Rigidity" CEPR Discussion Paper No 5554, 2006. We would like to thank Per Engstrom, Matthew Lindquist, Marta Lochowska, Robert Shimer, and seminar participants at Northwestern University, University of Pennsylvania, University of Chicago, University of Albany, Federal Reserve Bank of Richmond, ESSLE, Stockholm University, Göteborg University, Humboldt University in Berlin, University of Bonn, the Research Institute of Industrial Economics and the Swedish Institute for Social Research for valuable comments. Financial support from the Norwegian Research Council and the Swedish Research Council is gratefully acknowledged.

†Norwegian School of Management and CEPR. e-mail: espen.moen@bi.no

‡Swedish Institute for Social Research, Stockholm University. e-mail: asa.rosen@sofi.su.se

In the present paper, we allow a firm's output to depend on the wage contracts firms offer their workers. A worker's output depends on both her effort and a match-specific component. The firm observes total output, but cannot disentangle output into its different components. It acts as a principal and chooses a wage contract that maximizes profits given the information constraints. Our aim is to analyze the interplay between search frictions in the market place and agency problems created by workers' private information.

In competitive search equilibrium, search frictions and agency problems interact because of the amount of "rents" that accrues to the worker. A worker's private information gives her an information rent, which is larger the closer the wages are linked to workers' output. When the firm sets the wage contract, it trades off incentives for the worker to provide effort and rent extraction from the worker. However, when there are search frictions in the labor market and firms compete for workers, more rents to the worker also benefit the firm as it speeds up the hiring process. Hence, it is less costly for a firm to provide workers with incentives when operating in a competitive, frictional market rather than in a non-competitive or frictionless market.

We show that the resulting search equilibrium, which we refer to as generalized competitive search equilibrium, has a simple form. The agency problem and the wage posting problem can be disentangled into two separate maximization problems. The solution to the firms' problem satisfies a modified Hosios rule that determines the constrained efficient resource allocation. When the information constraints are tight in a well-defined sense, the optimal wage contract prescribes that a large share of the match surplus is allocated to the employees. As a result, the profit will be lower, and fewer resources are used to create new jobs as compared to the equilibrium without agency problems.

We then analyze the effects of changes in the macroeconomic environment on the wage contract and the unemployment rate. First, we analyze the effects of negative productivity changes where all firms are hit equally hard. Such a negative shock tightens the constraints imposed by the workers' private information, and the worker's share of the match surplus *increases*. Therefore, the unemployment rate becomes more responsive to such shocks than in the standard search model. If the recession is caused by changes in the information structure, or if worker effort is more crucial after a negative change, the responsiveness of the unemployment rate to negative shocks is further increased and may be arbitrarily large.

Our model can thus contribute to the debate following Shimer (2005) and Hall (2004) who document that fluctuations in the unemployment rate predicted by the model in response to observed productivity shocks are much smaller than actual fluctuations in the unemployment rate, as wages in the model absorb much of the shock. Note also that our analysis indicates that a counter-cyclical sharing rule, where workers receive a larger share of the surplus after a negative shock, may be an *optimal* response to information problems between employers and employees.

We also find that a positive productivity shock or a fall in unemployment benefits both tend to increase the incentive power of the wage contract. In both cases, the shift increases the average match surplus. As a result, there are more rents available in the relationship and therefore also room for more high-powered incentive contracts. Similarly, an increase in search frictions also tends to increase the incentive power of the wage contract.

Our private information model builds on the procurement model by Laffont and Tirole (1993) and its adoption to a frictionless labor market by Moen and Rosén (2006a). As the emphasis in the present paper is on the interplay between search frictions and wage contracts, its analysis differs radically from that of Moen and Rosén (2006a).

In a related model, Faig and Jerez (2005) analyze a retail market with search frictions when buyers have private information about their willingness-to-pay. Although their paper studies private information in a competitive search environment, their model and its emphasis differ from ours. They focus on welfare analysis and abstract from moral hazard problems. They do not derive the modified Hosios condition, nor analyze the impact of macroeconomic variables on sharing rules and incentives.

Shimer and Wright (2004) consider a competitive search model where firms (not workers) have private information about productivity and workers have private information about effort. They show how private information may distort trade, thereby increasing unemployment. However, the mechanism in their paper differs from ours. We focus on the division of the match surplus between workers and firms as an instrument to mitigate the inefficiencies caused by private information, summarized in the modified Hosios condition. This is absent in Shimer and Wright which instead focuses on the direct effect of the inefficiencies created by two-sided private information on unemployment and vacancy rates. Guerrieri (2007) studies the welfare effects of including non-pecuniary aspects of a match which are private information to workers. She finds that the resulting allocation is inefficient out of steady state.

Several recent studies seek to make the search model consistent with Shimer and Hall's empirical findings. In Kennan (2007), workers and firms bargain over wages once they meet. Firms have private information in booms, but not in recessions, and thus earn information rents in booms. This increases the profits in booms and thus also unemployment volatility. Nagypál (2006) and Krause and Lubik (2007) show that on-the-job search in a matching model may amplify the effects of productivity shocks on the unemployment rate. Menzio (2005) illustrates that firms with private information may find it optimal to keep wages fixed if hit by high-frequency shocks. In Rudanko (2008), the effect of risk averse workers and contractual incompleteness on volatility is explored. Reiter (2007) shows that the responsiveness of the unemployment rate to productivity shocks may be increased if one allows for technological change that is embodied into the match. Gertler, Sala and Trigari (2007) explain wage rigidity by staggered wage contracts, among other things. For an extended survey of this literature, see Mortensen and Nagypál (2006).

Our model is also related to the literature on efficiency wage models (e.g. Weiss, 1980; Shapiro and Stiglitz, 1984). Some of these papers examine the comparative static properties of efficiency wage models (Strand, 1992; Danthine and Donaldson, 1990; Ramey and Watson, 1997; MacLeod, Malcomson and Gomme, 1994; MacLeod and Malcomson, 1998). In a static model, Rocheteau (2001) introduces shirking in a search model and shows that the non-shirking constraint forms a lower bound on wages.

The paper is organized as follows: Section 2 presents the model. In section 3, we study the full-information benchmark. In section 4, we introduce and characterize the generalized competitive search equilibrium. In section 5 we apply the model and analyze the effects of macroeconomic variables on the wage contract and the unemployment rate. Section 6 offers final comments.

2 The model

The matching of unemployed workers and vacancies is modeled using the Diamond-Mortensen-Pissarides framework (Diamond, 1982; Mortensen, 1986; Pissarides, 1985) with competitive wage setting. The economy consists of a continuum of *ex ante* identical workers and firms. All agents are risk neutral and have the same discount factor r. The measure of workers is normalized to one. Workers leave the market at an exogenous rate s and new workers enter the market as unemployed at the same rate. Abandoned firms have no value.

Let u denote the unemployment rate and v the vacancy rate in the economy. Firms are free to open vacancies at no cost, but maintaining a vacancy entails a flow cost, c. The number of matches is determined by a concave, constant return to scale matching function $x(u, v)$. Let p denote the matching rate of workers and q the matching rate of firms. The probability rates p and q can be written as $p = x(u, v)/u = x(1, \theta) = \widetilde{p}(\theta)$ and $q = x(u, v)/v = x(1/\theta, 1) = \tilde{q}(\theta)$, where $\theta = v/u$. We assume that $\lim_{\theta \to 0} p(\theta) = 0$ and $\lim_{\theta \to 0} q(\theta) = \infty$. The matching technology can thus be summarized by a function $q = \tilde{q}(\theta) = \tilde{q}(\widetilde{p}^{-1}(p)) = q(p)$. Since the matching function has constant return to scale, we can write $q = q(p)$, with $q'(p) < 0$.

Our equilibrium concept is the competitive search equilibrium (Moen 1997), which combines competitive price determination and search frictions. One of its core element is the unique relationship between the attractiveness of the offered wage contract and the expected rate at which the vacancy is filled. This relationship can be derived in several alternative settings.[1] In the present paper, we choose the interpretation that firms advertise wage contracts.

As mentioned, we study a segment of the labor market where workers are ex ante identical.[2] Compared with the simplest bench-mark search model, we bring in two new elements, both of which are common in labor economics. First we assume that the output of a match depends on worker effort, e. Second, we include stochastic job matching. As in Jovanovic

[1] Moen (1997) assumes that a market maker creates submarkets and shows that the same equilibrium can be obtained if firms advertise wages. This interpretation is further developed in Mortensen and Wright (2002). Mortensen and Pissarides (1999, section 4.1) interpret the market maker as a "middle man" (like a job center) that sets the wage. In Acemoglu and Shimer (1999a and 1999b), the labor market is divided into regional or industrial submarkets offering potentially different wages.

[2] Workers with different observable (and contractible) characteristics would be offered different wage contracts. Furthermore, in competitive search equilibrium, they search in separated search markets and hence do not create search externalities towards each other.

(1979) and many subsequent papers (see Pissarides 2000, ch. 6 for an overview), we allow the productivity of a given worker-firm pair to be match-specific. The output y of a worker-firm pair is given by

$$y(\varepsilon, e) = \overline{y} + \varepsilon + \gamma e, \tag{1}$$

where $\overline{y}$ is a constant, ε the match-specific term (or stochastic matching term) and e is worker effort.

Output y is observable and contractible. However, as in most models of optimal wage contracts, we assume that the worker (the agent) has an information advantage over the firm (the principal) – we assume that only the worker can decompose output y into effort e and the stochastic matching term ε. As a firm cannot tell whether a high output level y is due to high effort or a high match-specific term, this will give rise to a moral hazard problem between workers and firms. Note that the information problem facing firms is one-dimensional as firms observe the sum of the two unobserved variables ε and e. If the firm could observe say e, it would be able to back out ε. Thus, with observable output, we need both components to be private information for the worker in order to get an interesting agency problem.

In equation (1) the parameter γ is a measure of the relative importance of worker effort. As is common in the stochastic matching literature, we assume that ε is i.i.d across all worker-firm matches. In footnote 5, we argue that our results also hold when allowing for some correlation of the stochastic match component. For any given match, ε is constant over time and continuously distributed on an interval $[\underline{\varepsilon}, \overline{\varepsilon}]$ with the cumulative distribution function H and density function h. We assume that H has an increasing hazard rate.

The wage contracts advertised by firms can be described as direct revelation mechanisms designed so that workers truthfully reveal their match-specific term, ε. When a worker and a firm meet, the worker learns ε and reports it to the firm. If the contract prescribes that a match should not be formed for the reported ε, workers and firms continue to search. Formally, a contract is given by a triple $\phi = (w(\varepsilon), e(\varepsilon), \varepsilon_c)$, where $\varepsilon_c \geq \underline{\varepsilon}$ denote the threshold value of ε below which a match is not formed. We do not consider tenure-dependent contracts. This is without loss of generality, as we show later that the optimal contract is tenure-independent.

Before we continue, we would like to make two comments regarding the set-up, both related to the match-specific term ε. The first comment regards the exact timing of when a worker learns the match-specific productivity term ε. We assume that a worker learns ε before the contract is signed. This sequence rules out up-front payments from the worker to the firm before the worker learns ε.[3]

The second comment regards our assumption that the match-specific productivity term is unobservable to the firm. An alternative interpretation is that, although able to observe ε, firms are unwilling or unable to differentiate output-contingent wage contracts between workers with the same observable characteristics but different stochastic matching terms. Different wage contracts would here mean offering less attractive contracts to workers with a high stochastic match term. Evidence of workers with different productivities working under the same bonus scheme is given in e.g., Lazear (2000).

Asset value equations

The asset value equations define the parties' payoffs for a given contract, $\phi = (w(\varepsilon), e(\varepsilon), \varepsilon_c)$. Let U denote the expected discounted utility of an unemployed worker and $\widetilde{W}(\varepsilon)$ the expected discounted utility of an employed worker with a match-specific productivity term ε, hereafter somewhat imprecisely referred to as her type. Then, $\widetilde{W}(\varepsilon)$ is defined as

$$\begin{aligned} (r+s)\widetilde{W}(\varepsilon) &= w(\varepsilon) - \psi(e(\varepsilon)) \\ &\equiv \omega(\varepsilon), \end{aligned} \tag{2}$$

where w denotes the wage, $\psi(e)$ the cost of effort and $\omega(\varepsilon)$ the wage net of effort costs. The function $\psi(e)$ is increasing and its derivative $\psi'(e)$ is increasing and convex in e. The expected discounted value of a worker being matched is

$$W = \int_{\varepsilon_c}^{\overline{\varepsilon}} \widetilde{W}(\varepsilon) dH + H(\varepsilon_c)U.$$

[3]If up-front payments are not admitted, it is sufficient that the worker learns ε after exerting effort and observing y.

The expected discounted utility of an unemployed worker is given by

$$(r+s)U = z + p(W-U),$$

where z is the utility flow when unemployed.

Let V denote the expected discounted value of a firm with a vacancy and $\widetilde{J}(\varepsilon)$ the expected discounted value of a filled job with a worker of type ε, where $\widetilde{J}(\varepsilon)$ is defined as

$$(r+s)\widetilde{J}(\varepsilon) = y(e(\varepsilon),\varepsilon) - w(\varepsilon).$$

The expected value of a firm being matched is

$$\begin{aligned} J &= \int_{\varepsilon_c}^{\overline{\varepsilon}} \widetilde{J}(\varepsilon)dH + H(\varepsilon_c)V \\ &= \int_{\varepsilon_c}^{\overline{\varepsilon}} \frac{y(e(\varepsilon),\varepsilon) - w(\varepsilon)}{r+s} dH + H(\varepsilon_c)V. \end{aligned} \tag{3}$$

The value of a vacancy can be written as

$$rV = -c + q(J-V).$$

For our subsequent analysis, it is convenient to use the concept of worker rents associated with a match. The rents from a match reflect the workers' expected "capital gain", or expected income (net of effort costs) in excess of U, of being matched to a vacancy. Note that the expected rent associated with a match may be lower than the expected rent associated with employment, because not all matches need to end up in employment. The expected worker rents of a match can be expressed as

$$\begin{aligned} R &\equiv W - U \\ &= \int_{\varepsilon_c}^{\overline{\varepsilon}} [\frac{\omega(\varepsilon)}{r+s} - U] dH. \end{aligned} \tag{4}$$

Using the definition of worker rents, the expected utility of an unemployed worker takes a particularly simple form

$$(r+s)U = z + pR. \tag{5}$$

That is, the flow value of an unemployed worker is equal to the utility flow when unemployed plus the expected gain from search, which is equal to the matching rate times the expected rent associated with a match. The total expected surplus of a match is $S \equiv J - V + R$, or (using equations (3) and (4))

$$(r+s)S = \int_{\varepsilon_c}^{\overline{\varepsilon}} [y(e(\varepsilon), \varepsilon) - \psi(e(\varepsilon)) - (r+s)U - (r+s)V] dH. \tag{6}$$

Finally, the unemployment rate is given by

$$u = \frac{s}{s + p(1 - H(\varepsilon_c))}. \tag{7}$$

3 Equilibrium with full information

In this section, we characterize the equilibrium outcome in the special case where ε and e are observable and contractible. This will serve as a benchmark for later analysis. When defining the equilibrium, we do not require the participation constraint to be satisfied for hired workers. However, as we will see, the workers' participation constraint can be satisfied in equilibrium without affecting the equilibrium allocation.

Key for characterizing the competitive search equilibrium is the relationship between the wages a firm offers and the speed at which the vacancy is filled. Let U^* denote the equilibrium utility of a searching worker. Then, for any expected rent R a firm offers, the queue length of workers adjusts so that the applicants get the equilibrium expected utility equal to U^*. That is,

$$z + pR = (r+s)U^*, \tag{8}$$

which defines p as a decreasing function of R, $p = p(R)$ (the dependence of U^* is suppressed).

In equilibrium, firms choose wage contracts so as to maximize profits. In addition, free entry of firms implies that the value V of a vacancy is zero.

Definition 1 *The competitive search equilibrium under full information is a contract* $\phi^F = (w^F(\varepsilon), e^F(\varepsilon), \varepsilon_c^F)$, *a vector of asset values* (S^F, R^F, U^F), *and a job finding rate* p^F *such that the following holds:*

1. *Profit maximization. The equilibrium solves the program P1 given by*

$$rV^{\max}(U) = \max_{w(\varepsilon),e(\varepsilon),\varepsilon_c,S,R,p} -c + q(p)(S-R)$$

s.t.

$$(r+s)U = z + pR \quad \text{(C1)}$$

$$(r+s)R = \int_{\varepsilon_c}^{\overline{\varepsilon}} [w(\varepsilon) - \psi(e(\varepsilon)) - (r+s)U] dH(\varepsilon) \quad \text{(C2)}$$

$$(r+s)S = \int_{\varepsilon_c}^{\overline{\varepsilon}} [y(e(\varepsilon),\varepsilon) - \psi(e(\varepsilon)) - (r+s)U - (r+s)V] dH(\varepsilon). \quad \text{(C3)}$$

2. *Free entry:*

$$V^{\max}(U^F) = 0. \quad (9)$$

We solve the profit maximization program P1 in two steps.

1. For a given U, find $S^{\max}(U)$ such that

$$(r+s)S^{\max}(U) = \max_{e(\varepsilon),\varepsilon_c} \int_{\varepsilon_c}^{\overline{\varepsilon}} [y(e(\varepsilon),\varepsilon) - \psi(e(\varepsilon)) - (r+s)U - (r+s)V] dH(\varepsilon). \quad (10)$$

2. For a given U and $S^{\max}(U)$, find $V^{\max}(U)$ such that

$$rV^{\max}(U) = \max_{R} -c + q(p(R))[S^{\max}(U) - R], \quad (11)$$

   where $p(R)$ is defined by (8).

The first-order condition for the optimal effort levels reads:

$$\psi'(e(\varepsilon)) = \gamma \text{ for all } \varepsilon. \quad (12)$$

The optimal cut-off level is given by either $\varepsilon_c = \underline{\varepsilon}$ or (with $V = 0$)

$$\overline{y} + \varepsilon_c + \gamma e(\varepsilon_c) - \psi(e(\varepsilon_c)) = (r+s)U. \quad (13)$$

The equation defines the efficient cut-off level, which equalizes the worker's net productivity with the outside options. Note that the solution is independent of R.

Then, we turn to the second step. In Appendix 1, we show that the solution to (11) satisfies the Hosios condition (Hosios, 1990):

$$\frac{R^F}{S^F - R^F} = \frac{\eta}{1-\eta}, \tag{14}$$

where η denotes the absolute value of the elasticity of q with respect to $\theta = v/u$. Finally, the free-entry condition (9) pins down U^F.

Given U^F, equations (12) and (13) define e^F (which is independent of both ε and U) and ε_c^F, while equation (14) determines R^F and indirectly p^F. (We assume the matching function to be well behaved so that equation (14), for a given S^F, uniquely determines R^F).

The equilibrium does not pin down a unique wage schedule, $w^F(\varepsilon)$. The equilibrium wage schedule may, for instance, be an appropriately chosen constant wage, independent of ε for all $\varepsilon \geq \varepsilon_c$. The workers' participation constraint is then satisfied for all types.

4 Generalized competitive search equilibrium

When e and ε are private information, the contracts must satisfy the workers' incentive compatibility and participation constraints, which read

$$\begin{aligned} w(\varepsilon) - \psi(e(\varepsilon)) &\geq w(\widetilde{\varepsilon}) - \psi(e(\widetilde{\varepsilon}) - \frac{\varepsilon - \widetilde{\varepsilon}}{\gamma}) && \text{for all } \varepsilon, \widetilde{\varepsilon} && \text{(C4)} \\ w(\varepsilon) - \psi(e(\varepsilon)) &\geq (r+s)U && \text{for all } \varepsilon \geq \varepsilon_c. && \text{(C5)} \end{aligned}$$

The left-hand side of constraint (C4) shows the utility flow of a worker of type ε who reports her type truthfully, while the right-hand side shows the utility flow if she instead reports $\widetilde{\varepsilon}$. The last equation ensures that all workers at or above the cut-off level ε_c accept the contract.

Definition 2 *The generalized competitive search equilibrium (GCS-equilibrium) is a contract* $\phi^* = (w^*(\varepsilon), e^*(\varepsilon), \varepsilon_c^*)$, *a vector of asset values* (S^*, R^*, U^*), *and a job finding rate* p^* *such that the following holds:*

1. *The equilibrium solves program P1 with (C4) and (C5) as additional constraints. We refer to this as program P2.*

2. *Free entry:*

$$V^{\max}(U^*) = 0. \tag{15}$$

Let $\widetilde{\omega}(\varepsilon, \widetilde{\varepsilon})$ denote the utility flow of a worker of type ε who reports type $\widetilde{\varepsilon}$ (the right-hand side of constraint (C4)). The utility flow of a worker of type ε is $\omega(\varepsilon) \equiv \arg\max_{\widetilde{\varepsilon}} \widetilde{\omega}(\varepsilon, \widetilde{\varepsilon})$. From the envelope theorem, it follows that

$$\omega'(\varepsilon) = \frac{\partial \widetilde{\omega}(\varepsilon, \widetilde{\varepsilon})}{\partial \varepsilon}.$$

Truth-telling requires that $\widetilde{\varepsilon} = \varepsilon$ and hence, that

$$\omega'(\varepsilon) = \psi'(e(\varepsilon))/\gamma. \tag{16}$$

Note that with truth-telling, $\omega(\varepsilon) = w(\varepsilon) - \psi(e(\varepsilon))$ denotes the utility flow of a worker of type ε. If a worker's type increases by one unit, she can reduce her effort by $1/\gamma$ units and still obtain the same output, thereby increasing her utility by $\psi'(e(\varepsilon))/\gamma$ units. Incentive compatibility requires that the worker obtains the same gain by reporting her type truthfully. Using equation (16), the utility flow to a worker of type $\varepsilon \geq \varepsilon_c$ can be written as

$$\omega(\varepsilon) = \omega(\varepsilon_c) + \int_{\varepsilon_c}^{\varepsilon} \frac{\psi'(e(x))}{\gamma} dx. \tag{17}$$

Note that contracts that prescribe more effort from low-type workers must give larger rents to high-types to keep the incentive compatibility constraint satisfied.

A first question that arises is whether the full information equilibrium $(S^F, R^F, U^F, p^F, \phi^F)$ is still feasible. The next lemma addresses that question.

Lemma 1 *a) For $\varepsilon_c^F > \underline{\varepsilon}$, the GCS-equilibrium with full information is not feasible when ε and e are private information to the worker.*

b) For $\varepsilon_c^F = \underline{\varepsilon}$, the GCS-equilibrium with full information is feasible with private information if and only if $R^F > \overline{R}$, where

$$\overline{R} = \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \frac{\varepsilon - \underline{\varepsilon}}{r+s} dH(\varepsilon). \tag{18}$$

Proof. a) Suppose the full information equilibrium is feasible. Denote the full information output level by $y^F(\varepsilon)$. The participation constraint (C5) then binds, and together with equation (13), it follows that $w(\varepsilon_c) = y^F(\varepsilon_c)$. Recall that $w(\varepsilon) \equiv \omega(\varepsilon) + \psi(e^F)$. Inserting (12) into (17) thus gives

$$\begin{aligned} w(\varepsilon) &= y^F(\varepsilon_c) + \varepsilon - \varepsilon_c \\ &= y^F(\varepsilon). \end{aligned}$$

Hence, the profit is zero for all worker types and no firm enters the market. This is inconsistent with the equilibrium.

b) For $\varepsilon_c = \underline{\varepsilon}$ we may have that $(r+s)U < y^F(\underline{\varepsilon})$ and hence $\omega(\varepsilon_c) < y^F(\varepsilon_c) - \psi(e^F)$. Set $\omega(\varepsilon_c)$ at its lowest possible value that satisfies the participation constraint, $\omega(\varepsilon_c) = (r+s)U$. Inserting $\psi'(e(\varepsilon)) = \gamma$ into (17) then gives $\omega(\varepsilon) = (r+s)U + \varepsilon - \underline{\varepsilon}$. The lowest possible rent R that implements the full information allocation is thus given by (18). ■

If $R^F > \overline{R}$, the full information equilibrium can be implemented by setting $\widetilde{W}(\underline{\varepsilon}) = U + R^F - \overline{R}$. In the following, we consider the case where $R^F < \overline{R}$. To solve the firms' maximization program P2, we use the standard method of integrating up the incentive compatibility constraint using integration by parts. As rent is valuable, firms do no not leave rents to the marginal worker, $\omega(\varepsilon_c) = (r+s)U$. From equation (17), we then get

$$\begin{aligned} \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \omega(\varepsilon) dH(\varepsilon) &= (r+s)U + \int_{\varepsilon_c}^{\overline{\varepsilon}} \int_{\varepsilon_c}^{\varepsilon} \frac{\psi'(e(x))}{\gamma} dx dH(\varepsilon) \\ &= (r+s)U + \int_{\varepsilon_c}^{\overline{\varepsilon}} \frac{\psi'(e(\varepsilon))}{\gamma} \frac{1-H(\varepsilon)}{h(\varepsilon)} dH(\varepsilon). \end{aligned}$$

Using (4) thus gives

$$(r+s)R = \int_{\varepsilon_c}^{\overline{\varepsilon}} \frac{\psi'(e(\varepsilon))}{\gamma} \frac{1-H(\varepsilon)}{h(\varepsilon)} dH(\varepsilon). \tag{C6}$$

As for the full-information equilibrium, program P2 is solved in two steps:

1. (Optimal contracts) For a given U and R, find $S^{\max}(U,R)$ defined as

$$\begin{aligned}(r+s)S^{\max}(R,U) &= \max_{e(\varepsilon),\varepsilon_c}\int_{\varepsilon_c}^{\overline{\varepsilon}}[y(e(\varepsilon),\varepsilon)-\psi(e(\varepsilon))-(r+s)U-(r+s)V]dH(\varepsilon)\\ &\quad \text{s.t.}\\ (r+s)R &= \int_{\varepsilon_c}^{\overline{\varepsilon}}\frac{\psi'(e(\varepsilon))}{\gamma}\frac{1-H(\varepsilon)}{h(\varepsilon)}dH(\varepsilon).\end{aligned}$$

Denote the associated contract by $\phi^{\max}(R,U)$.

2. (Optimal sharing rule) For a given U and $S^{\max}(R,U)$, find $V^{\max}(U)$ such that

$$rV^{\max}(U)=\max_{R}-c+q(p(R))[S^{\max}(R,U)-R], \tag{19}$$

where $p(R)$ is defined by (8).

Finally $S^{*}=S^{\max}(R^{*},U^{*})$ and $\phi^{*}=\phi^{\max}(R^{*},U^{*})$.

Step 1: Optimal contracts Denote the Lagrangian parameter associated with the rent constraint (C6) by α. The Lagrangian is given by

$$\begin{aligned}L &= \int_{\varepsilon_c}^{\overline{\varepsilon}}[\overline{y}+\varepsilon+\gamma e(\varepsilon)-\psi(e(\varepsilon))-(r+s)U-(r+s)V]dH(\varepsilon)\\ &\quad -\alpha[\int_{\varepsilon_c}^{\overline{\varepsilon}}\frac{\psi'(e(\varepsilon))}{\gamma}\frac{1-H(\varepsilon)}{h(\varepsilon)}dH(\varepsilon)-(r+s)R].\end{aligned}$$

The first-order conditions for maximum are now immediate:

Proposition 1 *(Solution to the first step). For given R and U, the optimal contract satisfies the following conditions*

a) The first-order condition for the effort level:

$$\gamma-\psi'(e(\varepsilon))=\alpha\frac{1-H(\varepsilon)}{h(\varepsilon)}\psi''(e(\varepsilon))/\gamma. \tag{20}$$

b) The first-order condition for the optimal cut-off level, given by either $\varepsilon_c=\underline{\varepsilon}$ or (with $V=0$)

$$[\overline{y}+\varepsilon_c+\gamma e(\varepsilon_c)-\psi(e(\varepsilon_c))-(r+s)U]h(\varepsilon_c)=\alpha(1-H(\varepsilon_c))\frac{\psi'(e(\varepsilon_c))}{\gamma}. \tag{21}$$

c) The rent-constraint defined by equation (C6).

Before we explain the first-order conditions in some detail, note that α is the shadow flow value of worker rents for the match surplus $S^{\max}(R;U)$. More precisely,

$$(r+s)S_R^{\max} = \alpha, \tag{22}$$

where subscript R denotes the derivative with respect to R.

Equation (22) captures a fundamental role of private information. In the full-information equilibrium derived in the previous section, the match surplus $S^{\max}$ was independent of R, the division of surplus between the worker and the firm did not influence the match surplus (i.e., output). In contrast, with private information, the amount of surplus and the division of the surplus are interrelated (as long as $R^F \leq \overline{R}$). The higher is the share of the surplus that is allocated to the worker, the higher is the match surplus.

The two first-order conditions generalize optimal contracts with private information (as in e.g. Laffont and Tirole, 1993) to a setting with search frictions. Without frictions, the shadow value of rents α would be equal to 1. For $\alpha = 0$, the first-order conditions coincide with those of the full-information case. As shown above, this is only feasible when $R^F \geq \overline{R}$.

Consider the optimal effort equation (20) and suppose that the effort level of a type $\hat{\varepsilon}$ worker increases by one unit. The left-hand side of equation (20) captures the resulting efficiency gain $\gamma - \psi'(e(\hat{\varepsilon}))$. The right-hand side captures the costs associated with an increase in effort. A one unit increase in effort of a type $\hat{\varepsilon}$ worker increases the rents of all workers above $\hat{\varepsilon}$ by $\psi''(e(\hat{\varepsilon}))/\gamma$ units (from equation 17) and the shadow value of this rent is α. The likelihood of obtaining a worker of type $\hat{\varepsilon}$ is reflected in $h(\hat{\varepsilon})$, while the measure of workers with higher match-specific productivity is $1 - H(\hat{\varepsilon})$. This explains the factor $(1 - H(\hat{\varepsilon}))/h(\hat{\varepsilon})$. Note that $e^*(\overline{\varepsilon}) = e^F$ (no distortion at the top). Since h has increasing hazard rate and $\psi'''(e) \geq 0$, $e(\varepsilon)$ is increasing in ε (hence, the second-order condition is satisfied).

The left-hand side of the cut-off equation (21) shows the net productivity loss of increasing ε_c. The right-hand side represents the gain in terms of reduced rents, which have a shadow flow value α. In Appendix 2, we show that the cut-off level is unique for a given α.

Let (a, b) denote a linear contract of the form $w = a + by$. It is well known that the optimal non-linear contract can be represented by a menu $(a(\varepsilon), b(\varepsilon))$ of linear contracts.[4] For any b, the worker chooses the effort level such that $\psi'(e) = b\gamma$. Inserting this condition into equation (20), we obtain

$$b(\varepsilon) = 1 - \alpha \frac{1 - H(\varepsilon)}{h(\varepsilon)} \frac{\psi''(e)}{\gamma^2}. \tag{23}$$

We refer to $b(\varepsilon)$ as the incentive power of the optimal contract. The value of $a(\varepsilon)$ is set so that (C6) is satisfied. For later reference, we also express the expected rent in terms of $b(\varepsilon)$. Inserting $\psi'(e) = b\gamma$ into equation (C6) gives

$$(r + s)R = \int_{\varepsilon_c}^{\overline{\varepsilon}} b(\varepsilon) \frac{1 - H(\varepsilon)}{h(\varepsilon)} dH(\varepsilon). \tag{24}$$

Proposition 2 *The optimal contract $\phi^{\max}(R, U)$ and match surplus $S^{\max}(R, U)$ have the following properties:*

a) The effort level $e(\varepsilon)$ is strictly increasing in R for all $\varepsilon < \overline{\varepsilon}$ and the cut-off level ε_c is decreasing in R.

b)The match surplus $S^{\max}(R, U)$ is strictly increasing and concave in R.

c)If all types are hired ($\varepsilon_c = \underline{\varepsilon}$), then

i) a shift in U shifts $a(\varepsilon)$ but leaves $b(\varepsilon)$ unchanged for all ε.

ii) a shift in U does not influence the marginal value of rents, i.e., $S^{\max}_{RU} = 0$.

Proof. See Appendix 3. ■

First consider result a). When the principal has more rents to dole out, she can afford to give stronger incentives to all workers. Furthermore, as the expected rent is decreasing in the cut-off level, a higher R also implies that the principal can afford to hire workers of a lower type, by reducing ε_c. The proposition states that the principal does both.

The first part of b), that the match surplus, $S^{\max}$, increases in R, directly follows from the fact that the rent constraint is binding. The second part of b), that $S^{\max}$ is concave in R, follows from the convexity of the maximization problem, i.e. that the marginal return from higher effort or a lower cut-off level is decreasing.

[4] See, e.g., Laffont and Tirole, 1993.

Result c) states that if all workers are hired, the workers' outside option U neither influences the incentive power of the contract nor the shadow value of rents. Intuitively, for a given cut-off, a change in U (for a given R) only implies that more income is transferred to the worker and the effort level remains constant for all types. This property of the optimal contract will be extensively used below.[5]

Above, we have derived the optimal static (tenure independent) contract.

Lemma 2 *The optimal dynamic contract repeats the static contract, provided that the firm can commit not to renegotiate the contract.*

Proof. See Appendix 4. ■

Providing incentives is costly for firms, as it yields information rents to the inframarginal workers. Deferred compensation or other time dependent wage contracts do not reduce this information rent, as they do not reduce the rent high types can obtain by pretending to be low types. Furthermore, deferred compensation does not influence the participation constraint at the hiring stage. It may loosen the participation constraint for tenured workers, but this has no value to the firm as the worker's outside option is time independent.

Step 2: Optimal sharing rules In Appendix 5, we derive the first-order condition for the maximization problem (19). With the equilibrium value of U^* inserted, the first-order condition reads

$$[1 - S_R^{\max}(R^*, U^*)] \frac{R^*}{S^* - R^*} = \frac{\eta}{1-\eta}, \tag{25}$$

where, as before, η denotes the absolute value of the elasticity of q with respect to $\theta = v/u$. We refer to this equation as the modified Hosios condition. The modified Hosios condition states that the workers' share of the match surplus, $R^*/(S^* - R^*)$, increases with the marginal

[5] If the match productivities ε were correlated between firms, a worker's outside option would increase with ε. However, $U'(\varepsilon) < 1/(r+s)$ would still hold and $U'(\varepsilon)$ would be smaller when the correlation is weaker. The incentive compatibility constraint would be unaltered. Furthermore, the participation constraint would still only bind for the lowest type, provided that the correlation is not too high. Hence, our main argument would still hold. However, the rents associated with a given contract and thus also $\overline{R}$ would be lower.

value of worker rents, $S_R^{\max}$. Thus, a smaller fraction of the match surplus is allocated to job creation. When $S_R^{\max} = 0$, equation (25) is identical to the Hosios condition with full information given by equation (14). With full information, a wage increase is purely redistributional. It reduces the value of a match for the firm by exactly the same amount as it increases its value for the worker. With private information, this no longer holds. A one-unit increase in R increases the match surplus $S^{\max}$ by $S_R^{\max}$ units, thereby reducing the firm's profit J by $1 - S_R^{\max}$ units.

Proposition 3 *Suppose $z < \overline{y} + \gamma e^F + \overline{\varepsilon}$. Then the generalized competitive search equilibrium exists. If $\eta = -q'(\theta)\theta/q(\theta)$ is non-decreasing in θ, the equilibrium is unique.*

Proof. By definition, the value function $V^{\max}(U)$ is unique, and it is trivial to show that it is continuous and strictly decreasing in U. If $U \to z^+$, it follows from equation (8) that $p(R)$ converges to zero and thus, that $q(p(R))$ converges to ∞. Hence, $R = 0$ and $(r + s)V = \overline{y} + \gamma e^F + \overline{\varepsilon} - U > 0$. Similarly it follows that $\lim_{U \to \overline{y} + \gamma e^F + \overline{\varepsilon}} V(U) < 0$. Thus, there exists a unique $U^* \in (z, \overline{y} + \gamma e^F + \overline{\varepsilon})$ such that $V^{\max}(U^*) = 0$.

Then we turn to uniqueness. The left-hand side in equation (25) increases in R on the relevant intervals of R. An increase in R for a given U implies a fall in θ. Given that $\eta'(\theta) \geq 0$, it thus follows that the right-hand side of equation (25) is decreasing in R and this ensures uniqueness. ∎

With the Cobb-Douglas matching function $x(u, v) = Au^{\beta}v^{1-\beta}$, it follows that $\eta = \beta$. The modified Hosios condition then reads

$$[1 - S_R(R^*, U^*)]\frac{R^*}{S^* - R^*} = \frac{\beta}{1 - \beta}. \qquad (26)$$

The competitive search equilibrium with full information maximizes the asset value of unemployed workers, given that firms break even (Acemoglu and Shimer, 1999b). This property also holds for the GCS-equilibrium:

Lemma 3 *The generalized competitive search equilibrium maximizes U given the free entry constraint $V = 0$ and the relevant information constraints.*

Proof. Suppose the contrary, i.e. that there exists a wage contract $\tilde{\phi}$ such that $U(\tilde{\phi}) = \tilde{U} > U^*$ and $V = 0$. By definition, a firm offering $\tilde{\phi}$ breaks even at $U = \tilde{U}$. Thus, the firm makes a strictly positive profit if it advertises this contract when $U = U^* < \tilde{U}$ (recall that V only depends on U). But then ϕ^* cannot be a profit-maximizing contract, which is a contradiction. ■

5 Applications

In this section, we address the effects of aggregate shocks on sharing rules (wages), incentives and unemployment. Ideally, this should be done by specifying a stochastic process for the variable in question (for instance $\overline{y}$), and then solving the model assuming that the agents have rational expectations. We have chosen a simpler alternative and instead do comparative statics analysis. Note, however, that all equilibrium variables but the unemployment rate are jump variables, and the unemployment rate converges much more quickly to its steady steady state level than the average duration of the cycle. Hence, the transition path towards steady expectations about future changes in macroeconomic conditions may not be very important. Furthermore, both Shimer (2005) and Mortensen and Nagypal (2006) argue that the analysis of productivity shocks can be carried out without explicitly modeling the dynamics. An adequate approximation is to do comparative statics with respect to the productivity variable.[6] To obtain simple expressions, we assume that the matching function is Cobb-Douglas, $x(u, v) = Au^{\beta}v^{1-\beta}$, so that the modified Hosios condition is given by (26).

Some shifts in parameter values change the optimal contracting problem (stage 1). We refer to such shifts as information-changing shifts. However, when $\varepsilon_c = \underline{\varepsilon}$, some shifts do not affect the stage-one maximization problem. We refer to such shifts as information-neutral shifts. They are changes in general productivity $\overline{y}$, unemployment benefit / value of leisure z, vacancy cost c and matching efficiency, A.

As we will see, information-neutral shifts in productivity $\overline{y}$ with an exogenous cut-off at most lead to rent rigidity. As shown in Brugemann and Moscarini (2007), this is not sufficient

[6] The cut-off level ε_c may increase after a negative shock. Thus, a measureof workers employed initially will leave their jobs immediately after a negative shock. This effect is not captured in our comparative statics analysis.

to fully explain the Shimer puzzle discussed in the introduction. However, information changing shifts may cause output and worker rents to move in different directions, thereby violating assumption 1a) in Brugemann and Moscarini. We show that in this case, there are no bounds on how large the effects may be. In addition, effects through endogenous cut-off levels may also increase the responsiveness to unemployment after aggregate shocks.

5.1 Information-neutral shifts

We first assume all worker types to be hired, i.e. $\varepsilon_c = \underline{\varepsilon}$. Later on, we analyze the case with an interior cut-off level.[7]

The effect of information-neutral shifts depends on their effect on the match surplus, and the core equation for evaluating the effects of the shifts is equation (26). Since $S_{RU} = 0$, the equation is independent of U^*. Differentiating (26) gives

$$\begin{aligned} dS^* &= \frac{1-\beta}{\beta}[-S_{RR}(R^*)R^* + (1 - S_R(R^*)) + \frac{\beta}{1-\beta}]dR^* \\ &= \Gamma dR^*. \end{aligned} \tag{27}$$

Since $S_R(R^*) < 1$ and $S_{RR}(R^*) < 0$ (from Proposition 2b), it follows that $\Gamma > 0$. Slightly rewriting (26) gives

$$\frac{R^*}{S^* - R^*} = \frac{1}{1 - S_R(R^*)} \frac{\beta}{1-\beta}. \tag{28}$$

The workers' share of the surplus thus increases if and only if R^* decreases. Furthermore, since $\alpha = S_R(R^*)$, it follows from equation (20) that an increase in R^* increases the incentive power of the wage contracts and hence, also the effort level for all ε.

From (27) and (28), it follows that if an information-neutral shift increases S^*, it also increases R^*, reduces the workers' share of the surplus and increases the incentive power of the wage contracts.

Shifts in $\overline{y}$. A shift in $\overline{y}$ has two effects on S^*. On the one hand, it increases S^* for a given U^*. On the other hand, it also increases U^*, which tends to reduce S^*. However, since

[7] In Appendix 6, we show that $\varepsilon_c = \underline{\varepsilon}$ and simultaneously, $R^F < \overline{R}$ is indeed possible.

there is a time delay before an unemployed worker finds a job, the former effect dominates and S^* increases (See Appendix 7 for a formal proof).

Thus, from equations (27) and (28) and the following discussion, a drop in $\overline{y}$ increases the worker's share of the surplus. This is an interesting observation and it is relevant for the discussion about rigid wages following the findings in Shimer (2005). As discussed in the introduction, Shimer documents empirical regularities of the business cycle that the standard matching model of the labor market can hardly account for. With private information, the workers' share of the surplus is counter-cyclical. After a negative shock to $\overline{y}$, there is a fall in the match surplus. Hence, for a given sharing rule, the shadow value of worker rents increases. As a result, firms find it optimal to increase the worker's share of the surplus. Thus, wages are more rigid and the unemployment rate is more volatile than in the standard model without private information.

Note in particular that our result relates to the discussion in Hall (2005a). Hall argues that due to social norms, the worker's share of the match surplus is counter-cyclical.[8] Our model generates a counter-cyclical sharing rule as an optimal response to changes in aggregate variables in the presence of private information.

As $\overline{y}$ reduces S_R, it follows from the discussion below (28) that the incentive power of the equilibrium wage contracts measured by $b(\varepsilon)$ falls for all ε. Loosely interpreted, the model thus predicts that pay should be less variable when aggregate productivity is low than when it is high.

Shifts in unemployment benefits. An increase in z, unemployment benefits or value of leisure shifts U^* upwards and S^* downwards (see Appendix 7). Higher unemployment benefits increase the workers' outside options and reduce the available match surplus. As a result, there is an increase in the workers' share of the surplus.

An increase in unemployment benefits thus both has a direct and an indirect effect on the unemployment rate. The direct or standard effect is that it reduces the match surplus, leading to less entry for a given sharing rule. Our new, indirect effect is that the share of the

[8] Hall (2005b) also shows that wage rigidity may be the result of alternative specifications of the bargaining procedure or self-selection among workers.

match surplus allocated to the worker increases, which further increases the unemployment rate.

Note that there is a link between unemployment benefits and the optimal wage contract. As z increases and S^* falls, there are less rents to the workers and the incentive power of the wage contract falls. Thus, our model predicts that higher unemployment benefits are associated with less incentive pay and lower effort provision.

Shifts in the search cost c and the matching technology parameter A. An increase in search cost c reduces the equilibrium value U^* and increases S^* (see Appendix 7). As a result, the worker's share of the match surplus decreases. Once more, the change has a direct and an indirect effect on the unemployment rate, but they now go in opposite directions. The direct (standard) effect of an increase in c is higher unemployment. For a given sharing rule, fewer firms enter the market and the unemployment rate increases. The indirect, countervailing effect is that the workers' share of the surplus falls. As a result, private information tends to dampen the effects of higher search costs on the unemployment rate.

Since there is an increase in the rents that are allocated to worker's in equilibrium, a higher value of c implies that the wage contracts become more incentive-powered. If the search costs are sufficiently large, it follows that $R^F > \overline{R}$, and all workers are given first-best incentives.

A decrease in A has the same effect as an increase in c. As the match surplus increases, the worker's share of the surplus decreases and the incentive power of the contract increases for all ε.

When search frictions are high, it is more important for firms to speed up the hiring process by offering workers more rents. Thus, the cost of providing incentives in terms of higher worker rents falls, and firms increase the incentive power of the contract.

We summarize our findings in a proposition:

Proposition 4 *For a given cut-off $\varepsilon_c = \underline{\varepsilon}$, the following is true*

a) A positive shift in $\overline{y}$ decreases the workers' share of the surplus and shifts $b(\varepsilon)$ upwards.

b) An increase in the unemployment benefit z increases the workers' share of the surplus and shifts $b(\varepsilon)$ downwards.

c) An increase in search frictions (an increase in c or a reduction in A) decreases the workers' share of the match surplus and shifts $b(\varepsilon)$ upwards.

5.1.1 Interior cut-off level

We now turn to the case with an interior cut-off. To facilitate the reading, we repeat the first-order condition for the optimal cut-off level, ε_c.

$$\overline{y} + \varepsilon_c + \gamma e(\varepsilon_c) - \psi(e(\varepsilon_c)) - (r+s)U = \alpha \frac{(1 - H(\varepsilon_c))}{h(\varepsilon_c)} b(\varepsilon_c) \tag{29}$$

(where we have used that $\psi'(e(\varepsilon_c)) = \gamma b(\varepsilon_c)$). The left-hand side is the match surplus associated with the marginal worker. It reflects the cost of increasing ε_c and thereby not realizing matches with a positive match surplus. The right-hand side reflects the gain of increasing ε_c in terms of lower rents for higher types that are hired.

Proposition 5 *The cut-off level ε_c decreases in $\overline{y}$ and c and increases in z and A.*

Proof. See Appendix 8. ∎

A fall in $\overline{y}$ implies that the left-hand side of equation (29) falls (since $\overline{y}$ falls more than $(r+s)U$), which tends to increase the cut-off level ε_c. Furthermore, we know that an increase in α also increases ε_c (Appendix 3). A similar argument holds for shifts in z, A and c.

Thus, in all cases, the effects through the cut-off level seem to exacerbate our previous findings regarding the responsiveness of the unemployment rate to shocks. In particular, a negative shift in $\overline{y}$ increases the cut-off level, thereby leading to a further increase in the unemployment rate. However, there is a caveat here: As ε_c shifts upwards after a fall in $\overline{y}$, this tends to dampen the increase in α, which may even fall. However, this typically happens when the increase in ε_c (and thus, its adverse effect on the unemployment rate) is large.

Moreover, also with endogenous cut-off levels, a fall in $\overline{y}$ leads to a fall in R^*; hence, we do not go beyond rent rigidity as defined in Brugemann and Moscarini (2007) and Brugemann (2008). Thus, the effects on the unemployment rate through labor market tightness are still insufficient for explaining the Shimer puzzle. However, the effects through the cut-off level come in addition and are not limited by the Brugemann-Moscarini bound.

5.2 Information-changing shifts

In this subsection, we analyze the effects of shifts that have a direct influence on the marginal value of rents $S_R^{\max}(R,U)$. Once more, we first assume that all worker types are hired, i.e. $\varepsilon_c = \underline{\varepsilon}$, and return to the case with an interior cut-off level below.

Consider first a shift in the distribution of ε. To this end, write the match-specific productivity term as $\varepsilon = k\mu$, where μ is symmetrically distributed on $[-1,1]$ and k is a scalar. Let $\widetilde{H}(\mu)$ denote the cumulative distribution function of μ. Let $\overline{k}$ denote the value of k such that $R^* = \overline{R}$ ($\overline{k}$ is thus the highest possible k for which the full-information equilibrium is feasible). We study the effects of an increase in k for $k \geq \overline{k}$. On the one hand, an increase in k increases the amount of private information workers possess. For a given R, the incentive power of the wage contract thus decreases, which tends to increase the marginal value of effort and thus, $S_R^{\max}$. On the other hand, an increase in k implies that more rents are needed to increase workers' incentives, which tends to reduce the value of $S_R^{\max}$. It turns out that if the private information problems are moderate (k relatively close to $\overline{k}$), the first effect dominates, and an increase in k increases $S_R^{\max}$. Define the "average" incentive power as[9]

$$\overline{b} = \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \frac{b(\varepsilon)}{\overline{\varepsilon} - \underline{\varepsilon}} d\varepsilon.$$

In Appendix 9, we show that a sufficient condition ensuring that an increase in k increases $S_R^{\max}$ is that $\overline{b} \geq 1/2$. Note that if R is close to $\overline{R}$, then b is close to 1 for all ε and this condition is certainly satisfied

As long as $\varepsilon_c = \underline{\varepsilon}$, an increase in k reduces the expected output: for a given R, an increase in k implies that the optimal contract cuts back on worker effort, and output falls. The firm may compensate by increasing R, but this only has a second-order effect on U^* due to the

[9]Note that $\overline{b}$ is generally not equal to the expected value of b, unless ε is uniformly distributed.

envelope theorem. Hence, an increase in k decreases U^* and can thus be considered as a recession.

We want to illustrate with an example that the effects of changes in the information structure may lead to large changes in the unemployment rate, relative to the change in output per worker (net of effort costs). To this end, suppose that initially $k = \overline{k}$, such that the full information outcome is achievable but with no slack. Let $\widetilde{y}(k)$ denote expected output net of effort cost and let $u(k)$ denote the equilibrium unemployment rate given by equation (7) and p^* as a function of k.

Proposition 6 *Consider an increase in k and suppose that initially, $k = \overline{k}$. Then,*

$$\lim_{k \to \overline{k}^+} \frac{du(k)}{d\widetilde{y}(k)} = \infty.$$

Proof. See Appendix 10. ■

An increase in k from $\overline{k}$ only has a second-order effect on the match surplus. However, it has a first-order effect on α, the shadow value of rents and thus, also on R^*. Furthermore, an increase in R^* implies that firms offer fewer vacancies and hence, that p falls and u increases (from 7). Thus, although the change has negligible effects on net output, it may have a substantial effect on the sharing rule and thus also on the unemployment rate.

Let us next consider the effects of shifts in the importance of unobservable effort, γ. An increase in γ implies that effort is more more productive. Consider equation (23). For any given $b(\varepsilon)$, an increase in γ increases the effort level (recall that $\psi'(e(\varepsilon)) = b(\varepsilon)\gamma$). Furthermore, the rent-constraint (24) is unaffected. Therefore, the issue is how an increase in γ influences the marginal gain from increasing the incentives $b(\varepsilon)$ and thereby the marginal value of rents. The next lemma shows that under reasonable assumptions on $\psi(e)$, an increase in γ increases the marginal value of worker rents:

Lemma 4 *Given that $\varepsilon_c = \underline{\varepsilon}$, $S_R^{\max}(R, U)$ is increasing in γ provided that $\psi''/(\psi')^2$ is non-increasing in e.*[10]

[10] This restriction is rather mild and is satisfied for most convex functions. For instance, any polynomial of the form $\psi(e) = e^n$ $(n > 1)$ satisfies this condition, as well as the exponential function $\exp e$.

Proof. See Appendix 11. ■

Thus, for given values of R and U, an increase in γ increases the workers' share of the surplus. An increase in γ tends to increase output and is in that sense a positive shock. However, a shock may influence both $\overline{y}$ and γ. For instance, if a fall in $\overline{y}$ is caused by an increase in input prices (e.g. energy prices) and effort and other inputs are substitutes, a fall in $\overline{y}$ goes hand in hand with an increase in γ. Furthermore, the elasticity of the unemployment rate to average productivity may be arbitrarily high if the fall in $\overline{y}$ and the increase in γ imply that average productivity barely falls while the change in γ is substantial.

More generally, if correlated with the business cycle, information-changing shifts may increase the volatility of the unemployment rate. If workers have more private information during a downturn, or if unobservable effort is more important during a downturn, this will further increase the negative effect on the unemployment rate.

Note also that for information-changing shifts, the conditions in Brugemann and Moscarini (2007) are not satisfied. For instance, the effect of an increase in k, for k close to $\overline{k}$, is a decrease in average productivity together with an increase in the expected rent. Hence, their assumption 1a is violated and their volatility bound does not apply. The same may be true if a reduction in $\overline{y}$ goes hand in hand with an increase in γ.

5.2.1 Interior cut-off level

We now turn to the case with an interior cut-off level.

Lemma 5 *a) Suppose that $b(\varepsilon_c) \geq 1/2$ and $\varepsilon_c < 0$. Then an increase in k increases ε_c.*

b) Suppose $\psi''/(\psi')^2$ is non-increasing in e. Consider an increase in γ combined with a reduction in $\overline{y}$ such that U^* is unchanged. Then ε_c increases.

Proof. See Appendix 12. ■

Thus, the effects through ε_c tend to exacerbate our previous findings, although the caveat regarding α pointed out in section 5.1.1 still applies. Note that when k is close to $\overline{k}$, the qualifiers under a) are certainly satisfied; hence the effects through the cut-off level will increase the responsiveness of the unemployment rate to shocks to k. The qualifier that U^*

is unchanged in b) is needed because, in addition to changing the information structure, an increase in γ also increases average productivity, and the latter effect tends to reduce ε_c.

6 Final comments

In this paper, we define and characterize what we refer to as the generalized competitive search equilibrium, in which workers have private information regarding their effort and "type". In our model, the firms face a trade-off between extracting rents from workers and providing incentives to exert effort. Search frictions with competitive wage setting imply that the cost of leaving rents to the worker are lower than in the standard frictionless model, as worker rents save on search costs for the firms. We show that the resulting equilibrium satisfies what we refer to as the modified Hosios condition. We also analyze the equilibrium effects of changes in macroeconomic variables. Private information may increase the responsiveness of the unemployment rate to productivity changes. Furthermore, the incentive power of the wage contracts is positively related to high productivity, low unemployment benefits and high search frictions.

We want to point out that our definition of the generalized competitive search equilibrium is flexible and can easily accommodate other forms of incentive problems. In a working paper version of this paper (Moen and Rosén 2006b), we both analyze a model with shirking as in Shapiro and Stiglitz (1984) and a model with non-pecuniary aspects of employment. For instance, in the shirking model, workers are identical, but both worker effort and output are private information to the worker. Effort is either 0 or 1 and the effort cost is ψ. Let g denote the probability rate of a shirking worker being detected, in which case she is fired. The non-shirking condition is then given by

$$\psi \leq gR.$$

If we are in a region where the non-shirking constraint binds, the equilibrium rent is determined by $R^* = \psi/g$. A fall in $\overline{y}$ then has no impact on R^* and we get complete rent rigidity.

It is our belief that developing search models with a richer structure than the standard

Diamond-Mortensen-Pissarides model may add new insights, both within macroeconomics and different subfields of labor economics. In previous studies, the inclusion of human capital in search models has improved our understanding of human capital formation. The present paper addresses questions that are relevant for both macroeconomic fluctuations and personnel economics within a search framework. Adding more structure to search models may therefore be a fruitful avenue for future research.

Appendix

Appendix 1: Equation (14)

Taking the derivative of equation (11) with respect to R and utilizing that $V^{\max\prime}(R^*) = 0$ gives the first-order condition

$$q'(p)p'(R)(S^F - R) - q = 0 \tag{30}$$

or, by simple manipulation,

$$el_p q(p) el_R p(R) = \frac{R}{S^F(R) - R}. \tag{31}$$

From equation (8) it follows that $el_R p(R) = -1$. We want to show that $el_p q(p) = -\frac{\eta}{1-\eta}$. To see this, let $p = \widetilde{p}(\theta)$ and $q = \widetilde{q}(\theta)$. Then

$$\begin{aligned} el_p q(p) &= el_p \widetilde{q}(\widetilde{p}^{-1}(p)) \\ &= \frac{el_\theta \widetilde{q}(\theta)}{el_\theta \widetilde{p}(\theta)}. \end{aligned}$$

Since $el_\theta \widetilde{q}(\theta) = -\eta$ and $el_\theta \widetilde{p}(\theta) = el_\theta[\theta \widetilde{q}(\theta)] = 1 - \eta$, it follows that $el_p q(p) = -\frac{\eta}{1-\eta}$. The result thus follows.

Appendix 2: Unique cut-off level

Define

$$\Psi(\varepsilon_c) = \overline{y} + \varepsilon_c + \gamma e(\varepsilon_c) - \psi(e(\varepsilon_c)) - (r+s)U - \alpha \frac{1 - H(\varepsilon_c)}{h(\varepsilon_c)} \psi'(e(\varepsilon_c))/\gamma. \tag{32}$$

Equation (32) determines a unique ε_c iff $\Psi(\varepsilon_c) = 0$ is uniquely defined.

$$\frac{d\Psi(\varepsilon_c)}{d\varepsilon_c} = 1 + \gamma \frac{de}{d\varepsilon_c} - \psi'(e(\varepsilon_c)) \frac{de}{d\varepsilon_c} - \alpha \frac{\psi'(e(\varepsilon_c))}{\gamma} \frac{d\frac{1-H(\varepsilon_c)}{h(\varepsilon_c)}}{d\varepsilon_c} - \alpha \frac{1-H(\varepsilon_c)}{h(\varepsilon_c)} \frac{\psi''(e(\varepsilon_c))}{\gamma} \frac{de}{d\varepsilon_c}.$$

Inserting $-\alpha\frac{1-H(\varepsilon_c)}{h(\varepsilon_c)}\psi''(e(\varepsilon_c))/\gamma = \psi'(e(\varepsilon_c)) - \gamma$ (equation 20) and using that h has an increasing hazard rate yields:

$$\frac{d\Psi(\varepsilon_c)}{d\varepsilon_c} = 1 - \alpha \frac{\psi'(e(\varepsilon_c))}{\gamma} \frac{d\frac{1-H(\varepsilon_c)}{h(\varepsilon_c)}}{d\varepsilon_c} > 0.$$

Thus, $\Psi(\varepsilon_c)$ is strictly increasing on $[\underline{\varepsilon}, \overline{\varepsilon}]$. Hence, $\Psi(\varepsilon_c) = 0$ is uniquely defined.

Appendix 3: Proof of Proposition 2

We first show the following property:

Property 1: *The cut-off level ε_c is increasing in α (for a given U).*

Proof. Taking the partial derivative of (32) with respect to α gives

$$\begin{aligned} \frac{d\Psi}{d\alpha} &= (\gamma - \psi'(e(\varepsilon_c)))\frac{de}{d\alpha} - \alpha \frac{1-H(\varepsilon_c)}{h(\varepsilon_c)} \frac{\psi''(e(\varepsilon_c))}{\gamma} \frac{de}{d\alpha} - \frac{1-H(\varepsilon_c)}{h(\varepsilon_c)} \frac{\psi'(e(\varepsilon_c))}{\gamma} \\ &= -\frac{1-H(\varepsilon_c)}{h(\varepsilon_c)} \frac{\psi'(e(\varepsilon_c))}{\gamma} < 0 \end{aligned}$$

(where we have used the first-order condition 20). Differentiating the cut-off equation $\Psi = 0$ and using that $\frac{\partial \Psi}{\partial \varepsilon_c} > 0$ gives

$$\frac{d\varepsilon_c}{d\alpha} = -\frac{\frac{\partial \Psi}{\partial \alpha}}{\frac{\partial \Psi}{\partial \varepsilon_c}} > 0.$$

Property 1 thus follows. ■

Proof of Proposition 2b). Since the rent-constraint is binding by definition, it directly follows that $S^{\max}(R, U)$ is strictly increasing in R. To show that $S^{\max}(R, U)$ is concave in R, it is sufficient to show that $\alpha(R, U)$ is decreasing in R. Consider an increase in R and suppose instead that α increases. From Property 1, we know that ε_c is increasing in α. From (20) and the assumptions on ψ, it follows that $e(\varepsilon)$ is strictly decreasing in α for all $\varepsilon < \overline{\varepsilon}$. From (C6), it then follows that the expected rent is strictly decreasing, which is a contradiction.

Proof of Proposition 2a). We have just seen from the proof of 2b) that α is strictly decreasing in R and that $e(\varepsilon)$ is strictly decreasing in α for all $\varepsilon < \overline{\varepsilon}$. It follows that $e(\varepsilon)$ is strictly increasing in R for all $\varepsilon < \overline{\varepsilon}$. Since α is decreasing in R, it follows from Property 1 that ε_c is decreasing in R.

Proof of Proposition 2c). The results in part c) directly follows from the fact that when $\varepsilon_c = \underline{\varepsilon}$, U only influences the maximization problem through the participation constraint $\omega(\underline{\varepsilon}) = (r+s)U$. The first-order condition for optimal effort as well as α is independent of U.

Appendix 4: Proof of Lemma 2

We want to show that the optimal time-independent contract is also optimal within the larger class of time-dependent contracts. A similar proof, based on Baron and Besanko (1984), can be found in Fudenberg and Tirole (1991, p. 299). To simplify the proof and avoid uninteresting technicalities, we assume time to be discrete. We first consider the case where the cut-off level is $\underline{\varepsilon}$. This will be modified at the end.

The revelation principle still holds. Hence, it is sufficient to study the set of contracts that maps the worker's (reported) type into a sequence of wages and effort levels $\{w_t(\varepsilon), e_t(\varepsilon)\}_{t=0}^{\infty}$, where t denotes the tenure of the worker in question.

Let $\pi_t(\varepsilon, e_t) = \overline{y} + \varepsilon + \gamma e_t(\varepsilon) - w_t(\varepsilon)$. The expected discounted profit to the firm is given by

$$\Pi = E^{\varepsilon} \Sigma_{t=0}^{\infty} \pi_t(\varepsilon, e_t)\delta^t,$$

where $\delta = \frac{1-s}{1+r}$ is the discount factor, including the exit rate of the worker. The expected discounted utility of a worker of type ε who announce type $\widetilde{\varepsilon}$ is given by

$$\overline{W}(\varepsilon, \widetilde{\varepsilon}) = \Sigma_{t=0}^{\infty} \left[w_t(\widetilde{\varepsilon}) - \psi(\varepsilon, e(\widetilde{\varepsilon})) \right] \delta^t,$$

where

$$\psi(\varepsilon, e(\widetilde{\varepsilon})) \equiv \psi(e(\widetilde{\varepsilon}) - \frac{\varepsilon - \widetilde{\varepsilon}}{\gamma}).$$

Incentive compatibility requires that $\varepsilon = \arg\max_{\widetilde{\varepsilon}} \overline{W}(\varepsilon, \widetilde{\varepsilon})$. Let $W(\varepsilon) \equiv \overline{W}(\varepsilon, \varepsilon)$.

The optimal dynamic contract solves

$$\max_{\{w_t(\varepsilon), e_t(\varepsilon)\}_{t=0}^{\infty}} E^{\varepsilon} \Sigma_{t=0}^{\infty} \pi_t(\varepsilon, e_t)\delta^t$$

subject to

- Incentive compatibility: $\varepsilon = \arg\max_{\widetilde{\varepsilon}} \overline{W}(\varepsilon, \widetilde{\varepsilon})$.
- Individual rationality: $W(\varepsilon) \geq U$ for all ε.. This constraint only binds for $\underline{\varepsilon}$.

Note that the participation constraint regards the expected discounted utility of all future periods. It does not require that the utility flow of employed workers is higher than the utility flow of unemployed workers in all periods. Thus, deferred compensation with an increasing wage-tenure profile is allowed for.

Let $C^d = \{w_t^d(\varepsilon), e_t^d(\varepsilon)\}_{t=0}^{\infty}$ denote an optimal contract within the larger set of time-dependent contracts, and let $C^* = \{w^*(\varepsilon), e^*(\varepsilon)\}_{t=0}^{\infty}$ denote the time-independent contract. We want to show that C^d is equivalent to C^*, in the sense that it implements the same effort level in each period, the same discounted expected profit to the firm and the same expected discounted rents to the workers.

Suppose that $C^d \neq C^*$. Then, C^d cannot implement a time independent effort level, as this contract is, by definition, dominated by the optimal static contract C^*. Therefore, suppose that C^d does not implement a time independent effort level. We will show that this leads to a contradiction.

To this end, consider the random time-independent stochastic mechanism C^{dS}, defined as follows: in each period, the contract $(w_t^d(\varepsilon), e_t^d(\varepsilon))$ is implemented with probability $\frac{\delta^t}{1-\delta}$. By definition, this contract is both incentive compatible and satisfies the individual rationality constraint. Furthermore, it yields a higher expected profit to the firm than the static contract $(w^*(\varepsilon), e^*(\varepsilon))$, since C^d dominates C^* and thus, contradicts the optimality of the latter mechanism in the class of time-independent contracts. Thus, it follows that $C^d = C^*$.

Finally, the same argument holds for any given cut-off value ε_c and hence, the optimal cut-off level with time-dependent contracts must be equal to the optimal cut-off level with time-independent contracts.

Appendix 5: Equation (25)

Taking the first-order condition for the problem of maximizing V defined by equation (19) gives

$$q'(p)p'(R)(S^{\max}(R;U) - R) - q(1 - S_R^{\max}) = 0,$$

or, by simple manipulation,

$$el_p q(p) el_R p(R) = (1 - S_R^{\max}) \frac{R}{S^{\max} - R},$$

analogous to (31). By taking exactly the same steps as in Appendix 1, equation (25) follows.

Appendix 6

Let $\widetilde{\varepsilon}$ be a stochastic variable with finite support and define the stochastic matching term as $\varepsilon = k\widetilde{\varepsilon}$. We will show that there exists an interval $(\underline{k}, \overline{k})$ such that for any k in this interval the following holds: 1) $\overline{R} > R^F$ and 2) the cut-off level is equal to $\underline{\varepsilon}$.

For sufficiently small values of k, we have that $\overline{R} < R^F$ and first best effort and hiring are feasible with $\varepsilon_c = \underline{\varepsilon}$. Define $\underline{k}$ as the value of k such that $\overline{R} = R^F$. As workers have full incentives, $w'(y) = 1$. Since firms have a positive profit, it thus follows that $y(\underline{\varepsilon}) > w(\underline{\varepsilon})$; otherwise firms would obtain zero profits. Thus, increasing the cut-off level has a first-order effect on the expected surplus. Slightly reducing the incentive power of the contract only gives a second-order effect on the expected surplus. Thus, for values of k on an interval above $\underline{k}$, firms reduce the incentive power of the contract below the first best and still hire all types.

Appendix 7: Proofs related to information-neutral shifts when $\varepsilon_c = \underline{\varepsilon}$

Consider a positive shift in $\overline{y}$. From Lemma 3, we know that in equilibrium, U^* is maximized; hence, it is trivial to show that U^* is increasing in $\overline{y}$.

Suppose that S^* shifts downwards following an increase in $\overline{y}$. From equation (27) and (28) and the following discussion it follows that R^* decreases, the worker's share of the surplus increases, and thus that $S^* - R$ shifts downwards. The free-entry condition then implies that p falls. But then from equation (5) U^* must fall, which is a contradiction.

The proofs regarding shifts in z, c and A are analogous.

Appendix 8: Proof of Proposition 5

Consider first a shift in $\overline{y}$. We want to show that ε_c decreases in $\overline{y}$. Suppose first that α is constant, independent of $\overline{y}$. Then, effort is also independent of $\overline{y}$. Taking derivatives of (32) gives

$$\frac{\partial \Psi}{\partial \overline{y}} = 1 - (r+s)\frac{dU^*}{d\overline{y}},$$

(where the partial derivatives reflect that ε_c is kept constant). Since U^* is maximized it follows from the envelope theorem that $(r+s)\frac{dU^*}{d\overline{y}} < 1$, and thus that $\frac{\partial \Psi}{\partial \overline{y}} > 0$. Differentiating the cut-off equation $\Psi = 0$ and using that $\frac{\partial \Psi}{\partial \varepsilon_c} > 0$ gives

$$\frac{d\varepsilon_c}{d\overline{y}} = -\frac{\frac{\partial \Psi}{\partial \overline{y}}}{\frac{\partial \Psi}{\partial \varepsilon_c}} < 0.$$

Suppose then that $\frac{d\alpha}{d\overline{y}} < 0$. From Property 1 in Appendix 3, we know that ε_c is increasing in α (even when the effect of an increase in α on e is taken into account). Thus, a fall in α exacerbates the negative effect of $\overline{y}$ on ε_c.

Finally, suppose that $\frac{d\alpha}{d\overline{y}} > 0$ (this cannot be ruled out). We want to show that ε_c is still decreasing in $\overline{y}$. Suppose not. From equation (20), it follows that $e(\varepsilon)$ is decreasing in α for all worker types. From (C6) it then follows that R^* decreases and from (6) that S^* falls. Furthermore, if α increases and ε_c increases, using exactly the same argument as in Appendix 7, this leads to a contradiction.

To show the results for z, c and A, we proceed in exactly the same way, and it is therefore sufficient to study the effects of changes keeping α constant. Taking the derivative of Ψ defined by (32) with respect to z , c and A then gives

$$\begin{aligned}
\frac{\partial \Psi}{\partial z} &= -(r+s)\frac{dU^*}{dz} < 0. \\
\frac{\partial \Psi}{\partial c} &= -(r+s)\frac{dU^*}{dc} > 0 \\
\frac{\partial \Psi}{\partial A} &= -(r+s)\frac{dU^*}{dA} < 0.
\end{aligned}$$

Differentiating the cut-off equation $\Psi = 0$ and using that $\frac{\partial \Psi}{\partial \varepsilon_c} > 0$ gives that ε_c is increasing in z and A and decreasing in c. The argument if $\frac{d\alpha}{dz} \neq 0$, $\frac{d\alpha}{dc} \neq 0$, $\frac{d\alpha}{dA} \neq 0$ proceeds in exactly the same way as for changes in $\overline{y}$.

Appendix 9:

Here we show that an increase in k increases $S_R^{\max}(R, U)$ (i.e., increases α) if $\overline{b} \geq 1/2$. In the proof of Proposition 2 part b (see Appendix 3) we showed that α is decreasing in R. It is thus sufficient to show that for a given α, an increase in k implies that the rent-constraint defined in equation (C6) is no longer satisfied as the right-hand side increases.

From equation (24), it follows that

$$(r+s)R = \int_{-k}^{k} b(\varepsilon)(1-H)d\varepsilon.$$

As $\varepsilon = k\mu$, $H(\varepsilon) = \widetilde{H}(\varepsilon/k)$, it follows that

$$(r+s)R = \int_{-1}^{1} kb(k\mu)(1-\widetilde{H}(\mu))d\mu. \tag{33}$$

From equation (23) we have that (since $h(\varepsilon) = \widetilde{h}(\varepsilon/k)/k$)

$$b(k\mu) = 1 - \alpha k \frac{1-\widetilde{H}(\mu)}{\widetilde{h}(\mu)} \frac{\psi''(e(k\mu))}{\gamma^2}. \tag{34}$$

We will show that $b(k\mu)$ is decreasing in k. Suppose not. Then $e(k\mu)$ increases and hence, also $\psi''(e(k\mu))$, in which case the right-hand side of equation (34) decreases, and we have derived a contradiction.

Inserting (34) into equation (33) gives

$$(r+s)R = \int_{-1}^{1} k[1 - \alpha k \frac{1-\widetilde{H}(\mu)}{\widetilde{h}(\mu)} \frac{\psi''(e(k\mu))}{\gamma^2}](1-\widetilde{H}(\mu))d\mu.$$

Taking the derivative with respect to k gives

$$\begin{aligned}
&\frac{d(r+s)R}{dk} \\
&= \int_{-1}^{1} [1 - 2\alpha k \frac{1-\widetilde{H}(\mu)}{\widetilde{h}(\mu)} \frac{\psi''(e(k\mu))}{\gamma^2} - k^2 \alpha \frac{1-\widetilde{H}(\mu)}{\widetilde{h}(\mu)} \frac{\psi'''(e(k\mu))}{\gamma^2} \frac{de(k\mu)}{dk}](1-\widetilde{H}(\mu))d\mu \\
&> \int_{-1}^{1} [1 - 2\alpha k \frac{1-\widetilde{H}(\mu)}{\widetilde{h}(\mu)} \frac{\psi''(e(k\mu))}{\gamma^2}](1-\widetilde{H}(\mu))d\mu \\
&= \int_{-1}^{1} [2b(k\mu) - 1](1-\widetilde{H}(\mu))d\mu.
\end{aligned}$$

To obtain this result, we have used that $\frac{de(k\mu)}{dk} < 0$ (since $b(k\mu)$ is decreasing in k) and that ψ''' is positive. Finally we have inserted b from equation (34). Hence, a sufficient condition ensuring that an increase in k increases R for a given α is $\overline{b} \geq 1/2$.

Appendix 10: Proof of Proposition 6.

First, we show that $\frac{d\widetilde{y}(\overline{k})}{dk} = 0$. Write $\widetilde{y}(k)$ as

$$\widetilde{y}(k) = \int_{-1}^{1} [\overline{y} + k\mu + \gamma e^*(k\mu, k) - \psi(e^*(k\mu, k))] d\widetilde{H}(\mu),$$

where $e^*(k\mu, k)$ denotes the effort level prescribed by the optimal contract as a function of μ and k. At $k = \overline{k}$, e^* maximizes $\widetilde{y}(k)$, and due to the envelope theorem, it follows that we can ignore the effects of a change in k on e^*. Thus,

$$\frac{d\widetilde{y}(\overline{k})}{dk} = \int_{-1}^{1} \mu d\widetilde{H}(\mu) = 0.$$

The next step is to show that the equilibrium responses of U and S to a change in k at $k = \overline{k}$ are zero. Taking the derivative of (19) for a given U^* and using the envelope theorem gives

$$r\frac{dV^{\max}}{dk} = q(R^*)\frac{\partial S^{\max}(R, \mu, \overline{k})}{\partial k} = \frac{d\widetilde{y}(\overline{k})/(r+s)}{dk} = 0.$$

Hence, $V^{\max}(U)$ does not shift, and from the equilibrium equation (15), we have that $U^{*\prime}(\overline{k}) = 0$. Finally, we can write $S^*(k) = S^{\max}(R^*, U^*, k)$, and since $S_R^{\max} = 0$ at $R = \overline{R}$ it follows that $S^{*\prime}(\overline{k}) = 0$.

Thus, it is sufficient to show that R^* increases (and thus p decreases and hence u increases, see equation 7). We write $R^* = R^*(k)$. At $k = \overline{k}$, the derivative of $R^*(k)$ may not exist. Define $R^{*\prime}(\overline{k}) = \lim_{k \to \overline{k}^+} R^{*\prime}(k)$. We want to show that $R^{*\prime}(\overline{k}) > 0$. Write the shadow value of rents, α as a function of R and k, $\alpha = \alpha(R, k)$. Since S^* is constant, we know from equation (26) that $R^{*\prime}(\overline{k}) > 0$ if and only if

$$\frac{\partial \alpha(\overline{R}, \overline{k})}{\partial k} \equiv \lim_{k \to \overline{k}^+} \frac{\partial \alpha(\overline{R}, k)}{\partial k} > 0.$$

From equation (24), it follows that

$$R = \int_{-k}^{k} b(\varepsilon)(1 - H(\varepsilon)) d\varepsilon.$$

Suppose now, contrafactually, that $b(\varepsilon) = \hat{b}$ for all ε, where $\hat{b}$ is a constant in $(0, 1]$. Then

$$R = \hat{b}k.$$

At $k = \overline{k}$, we know that $b = 1$ for all ε and thus, that $R^* = \overline{R} = \overline{k}$. Taking the derivative of $\widehat{b}$ with respect to k, still assuming $\hat{b}$ to be constant over types gives

$$\hat{b}'(k)|_{R=\bar{k}} = -\frac{1}{\bar{k}} < 0.$$

For $k < \overline{k}$, we know that b is not constant in ε. However, it follows that $\frac{db(\varepsilon)}{dk}|_{R=\bar{k}} \geq \hat{b}'(k)|_{R=\bar{k}}$ for some ε. Furthermore, since $b(\varepsilon)$ is increasing in ε for all $k < \overline{k}$, it follows that the absolute value of $\frac{db(\varepsilon)}{dk}|_{R=\bar{k}}$ is largest at $\varepsilon = \underline{\varepsilon}$; hence $\frac{db(\varepsilon)}{dk}|_{R=\bar{k}} < -\frac{1}{\bar{k}} < 0$. From equation (23) we know that this can only be true if $\frac{\partial\alpha(\overline{R},\bar{k})}{\partial k} > 0$. The result thus follows.

Appendix 11: Proof of Lemma 4

Here, we show that an increase in γ increases $S_R^{\max}(R, U)$ (i.e., increases α). In the proof of Proposition 2 part b (see Appendix 3), we showed that α is decreasing in R. It is thus sufficient to show that for a given α, an increase in γ implies that the rent-constraint defined in equation (C6) is no longer satisfied as the right-hand side increases. Thus, it is sufficient to show that a positive shift in γ, for a given α, increases $b(\varepsilon)$ for all ε.

The first-order condition for effort (20) reads

$$\gamma[1 - \psi'(e(\varepsilon))/\gamma - \alpha\frac{1 - H(\varepsilon)}{h(\varepsilon)}\psi''(e(\varepsilon))/\gamma^2] = 0.$$

It is sufficient to show that an increase in γ, keeping $b(\varepsilon) = \psi'(e(\varepsilon))/\gamma$ constant, increases the LHS of the above equation. The second-order conditions then ensure that $\psi'(e(\varepsilon))/\gamma$ increases. Substituting in $b(\varepsilon) = \psi'(e(\varepsilon))/\gamma$ gives

$$\gamma[1 - b(\varepsilon) - \alpha\frac{1 - H(\varepsilon)}{h(\varepsilon)}\frac{\psi''(e(\varepsilon))b(\varepsilon)^2}{\psi'(e(\varepsilon))^2}] = 0.$$

For a given b, e is increasing in γ, and it follows that the left-hand side is increasing in e provided that $\frac{\psi''(e(\varepsilon))}{\psi'(e(\varepsilon))^2}$ is decreasing in e.

Appendix 12: Proof of Lemma 5

a) Note that $H(\varepsilon) = \widetilde{H}(\varepsilon/k) = \widetilde{H}(\mu)$ and that $h(\varepsilon) = \widetilde{h}(\mu)/k$. Inserting this into (32) gives

$$\Psi = \overline{y} + k\mu_c + \gamma e(k\mu_c) - \psi(e(k\mu_c)) - (r+s)U^* - \alpha k \frac{(1-\widetilde{H}(\mu_c))}{\widetilde{h}(\mu_c)} \frac{\psi'(e(k\mu_c))}{\gamma}. \qquad (35)$$

We proceed in the same way as in Appendix 8. We first want to show that Ψ decreases in k for a given α. From Appendix 8 (and Appendix 3), we know that if Ψ falls, so do ε_c (for a given α). Denote the first terms in (35) by FT,

$$FT = \overline{y} + k\mu_c + \gamma e(k\mu_c) - \psi(e(k\mu_c)) - (r+s)U^*$$

It follows that

$$\frac{\partial FT}{\partial k} = \mu_c + (\gamma - \psi'(e)) \frac{\partial e(k\mu_c)}{\partial k}.$$

From Appendix 9, we know that $b(k\mu_c)$ decreases in k and hence that $e(k\mu_c)$ decreases in k. Since $\gamma - \psi'(e) > 0$, FT is decreasing in k provided that $\mu_c \leq 0$.

Denote the last term in (35) by ST.

$$\begin{aligned}
ST &= -\alpha k \frac{(1-\widetilde{H}(\mu_c))}{\widetilde{h}(\mu_c)} \frac{\psi'(e(k\mu_c))}{\gamma} \\
&= -\alpha k \frac{(1-\widetilde{H}(\mu_c))}{\widetilde{h}(\mu_c)} b(k\mu_c) \\
&= -\alpha k \frac{(1-\widetilde{H}(\mu_c))}{\widetilde{h}(\mu_c)} [1 - \alpha k \frac{(1-\widetilde{H}(\mu_c))}{\widetilde{h}(\mu_c)} \frac{\psi''(e(k\mu_c))}{\gamma^2}] \\
&= -a_1 k[1 - a_2 k].
\end{aligned}$$

where $a_1 = \alpha \frac{(1-\widetilde{H}(\mu_c))}{\widetilde{h}(\mu_c)}$ and $a_2 = \frac{(1-\widetilde{H}(\mu_c))}{\widetilde{h}(\mu_c)} \frac{\psi''(e(k\mu_c))}{\gamma^2}$. Now

$$\frac{\partial ST}{\partial k} = -a_1[1 - 2ka_2] + a_1 k^2 \frac{\partial a_2}{dk}.$$

Since $e(k\mu)$ is decreasing in k we know that $\psi''(e(k\mu_c))$ is decreasing in k and **hence,** that $\frac{\partial a_2}{dk} < 0$. A sufficient condition for ST being decreasing in k is thus that $1 - 2a_2 k > 0$, or $a_2 k < 1/2$. Since $b(k\mu_c) = 1 - a_2 k$**,** this holds if and only if $b(k\mu_c) > 1/2$, which is true by assumption.

Differentiating the cut-off equation $\Psi = 0$ and using that $\frac{\partial \Psi}{\partial \varepsilon_c} > 0$ then gives that ε_c is increasing in k.

Suppose then that the equilibrium value of α increases. Then, we know from Property 1 in Appendix 3 that this will increase ε_c even further. Suppose then that α decreases. By applying exactly the same argument as in Appendix 8, it follows that ε_c must increase.

Result b). We use the same method as above. Consider $\Psi(\varepsilon_c)$ defined by equation (32). For a given α, we know from Appendix 11 that an increase in γ leads to an increase in $b(\varepsilon)$ for a given α. Hence the last term of (32) decreases in γ. Due to our normalization that U^* is constant, the derivative of the first terms with respect to γ is zero. Thus, ε_c increases in γ. If α increases, we know from Property 1 that this increases ε_c even further. Hence, it only remains to show that ε_c increases even if α falls. If α decreased, an identical argument to that given in Appendix 8 shows that ε_c increases also in this case.

References

Acemoglu, D. and Shimer, R. (1999a), "Efficient Unemployment Insurance", *Journal of Political Economy*, **107**, 893-928.

Acemoglu, D. and Shimer, R. (1999b), "Holdups and Efficiency with Search Frictions", *International Economic Review*, **40**, 827-849.

Baron, D. and Besanko, D. (1984), "Regulation and Information in a Continuing Relationship", *Information Economics and Policy*, **1**, 447-470.

Brugemann, B. (2008), "Volatility Bounds applied to Moen and Rosen (2007)". Manuscript.

Brugemann, B. and Moscarini, G. (2007), "Rent Rigidity, Asymmetric Information and Labor Market Fluctuations", Department of Economics, Yale University, mimeo.

Danthine, J.-P., and Donaldson, J. B. (1990), "Efficiency Wages and the Business Cycle Puzzle", *European Economic Review*, **34**, 1275-1301.

Diamond, P.A. (1982), "Wage Determination and Efficiency in Search Equilibrium", *Review of Economic Studies*, **49**, 217-227.

Faig, M. and Jarez, B. (2005), “A Theory of Commerce”, *Journal of Economic Theory*, **122**, 60-99.

Fudenberg, D. and Tirole, J. (1991), *Game Theory*, MIT Press, Cambridge.

Gertler, M., Sala, L., and Trigari, A. (2007), “An Estimated Monetary DSGE Model with Unemployment and Staggered Nominal Wage Bargaining", Boccony University, mimeo

Guerrieri, V. (2007), "Inefficient Unemployment Dynamics under Asymmetric Information", University of Chicago mimeo.

Hall, R. (2004), “The Labor Market is the Key to Understanding the Business Cycle”, Mimeo, Stanford University.

Hall, R. (2005a), “Employment Fluctuations with Equilibrium Wage Stickiness”, *American Economic Review*, **95**, 50-65.

Hall, R. (2005b), “The Amplification of Unemployment through Self-Selection”, Mimeo, Stanford University.

Hosios, A.J. (1990), “On The Efficiency of Matching and Related Models of Search and Unemployment”, *Review of Economic Studies*, **57**, 279-298.

Jovanovic, B. (1979), “Job Matching and the Theory of Turnover”, *Journal of Political Economy*, **87**, 972-990.

Kennan, J. (2007), “Private Information, Wage Bargaining and Employment Fluctuations”, University of Wisconsin-Madison, Mimeo.

Krause, M. U. and Lubik, T. A. (2007), “On-the-Job Search and the Cyclical Dynamics of the Labor Market”, European Central Bank no 779.

Laffont, J.J. and Tirole, J. (1993), *A Theory of Incentives in Procurement and Regulation*, MIT Press, Cambridge.

Lazear, Edward P. (2000). “Performance Pay and Productivity”, *American Economic Review,* **90**, 1346-1361.

MacLeod, W. B. and Malcomson, J.M. (1998), “Motivation and Markets”, *American Economic Review*, **88**, 388-411.

MacLeod, W.B., Malcomson, J.M. and Gomme, P. (1994), “Labor Turnover and the Natural rate of Unemployment: Efficiency Wages versus Frictional Unemployment” *Journal of Labor Economics,* **12,** 276-315.

Menzio, G. (2005), “High-Frequency Wage Rigidity”, Northwestern University Mimeo.

Mortensen, D.T (1986), “Job Search and Labour Market Analysis”, In O.C. Ashenfelter and R. Layard (eds.),*Handbook of Labor Economics* Volume 2, Amsterdam, North-Holland, 849-919.

Mortensen, D.T. and Nagypál, É. (2006), “More on Unemployment and Vacancy Fluctuations”, Northwestern University, Mimeo.

Mortensen, D.T, and Pissarides, C.A. (1999), “New Developments in Models of Search in the Labor Market”, in O. Ashenfelter and D. Card (eds.), *Handbook of Labor Economics*, vol 3b, North-Holland, pp 2567-2627.

Mortensen, D. and Wright, R. (2002), “Competitive Pricing and Efficiency in Search Equilibrium”, *International Economic Review,* **43,** 1-20.

Moen, E.R. (1997) “Competitive Search Equilibrium”, *Journal of Political Economy,* **105,** 385-411.

Moen, E.R. and Rosén, Å. (2004), “Does Poaching Distort Training?”, *Review of Economic Studies,* **71,** 1143-1162.

Moen, E.R. and Rosén, Å. (2006a), “Equilibrium Incentive Contracts and Efficiency Wages”, *Journal of European Economic Association*, **4**, 1165-1192.

Moen, E.R. and Rosén, Å. (2006b), "Incentives in Competitive Search Equilibrium and Wage Rigidity", *CEPR Working Paper no 5554.*

Nagypál, É. (2006), "Amplification of Productivity Shocks: Why Vacancies Don't Like to Hire the Unemployed?", in Structural Models of Wage and Employment Dynamics, vol. 275 of '*Contributions to Economic Analysis*', ed. H. Bunzel, B. J. Christensen, G. R. Neumann, and J.-M. Robin, pp. 481-506. Amsterdam: Elsevier

Pissarides, C.A. (1985). "Short-Run Dynamics of Unemployment, Vacancies, and Real Wages", *American Economic Review* , **75**, 675-90.

Pissarides, C.A. (2000). *Equilibrium Unemployment Theory.* MIT press, Cambridge MA.

Ramey, G and Watson, J. (1997), "Contractual Fragility, Job Destruction, and Business Cycles" *Quarterly Journal of Economics,* **112**, 873-911.

Reiter, M. (2007), "Embodied Technical Change and the Fluctuation of Unemployment and Wages". *Scandinavian Journal of Economics* vol 109 No 4, 695-722.

Rocheteau, G. (2001), "Equilibrium Unemployment and Wage Formation with Matching Frictions and Worker Moral Hazard", *Labour Economics,* **8**, 75-102.

Rudanko, L. (2008), "Labor Market Dynamics under Long Term Wage Contracting and Incomplete Markets". Boston University, Mimeo.

Shapiro, C, and Stiglitz, J.E. (1984), "Equilibrium Unemployment as a Worker Discipline Device", *American Economic Review*, **74**, 433-444.

Shimer, R. (2005), "The Cyclical Behavior of Equilibrium Unemployment and Vacancies ", *American Economic Review*, **95**, 25-49.

Shimer, R. and Wright, R. (2004), "Competitive Search Equilibrium with Asymmetric Information", University of Chicago, Mimeo.

Strand, J. (1992), "Business Cycles with Worker Moral Hazard", *European Economic Review*, **36**, 1291-1303.

Weiss, A.W. (1980), “Job Queues and Layoffs in Labor Markets with Flexible Wages”, *Journal of Political Economy*, **88**, 525-538.