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Efficient Ways to Finance Human Capital Investments

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Standard theory predicts that if wages are determined by bargaining workers underinvest in human capital, as they bear all the investment costs yet receive only a share less than one of the return. I show that this result depends on the way the investments are financed. I introduce *contingent loans*, which do not accumulate interest if the borrower is unemployed. When the investments are financed by such loans, the interest payments are regarded as a (negative) part of the surplus the agents bargain over. As a result, a worker pays the same share of the interest as he receives of the return.

Introduction

It is a well-known result in the economics literature that the market may generate an insufficient level of irreversible investments, for instance in education. Grout (1984) points out that if an agent invests in irreversible relationship-specific capital before complete contracts are written, and then bargains with his trading partner over the return, underinvestments may occur. This is because the investor bears the full share of the costs yet receives only a share less than one of the return. This is referred to as the hold-up problem. Acemoglu (1996b) applies this argument to show that frictions in the labour market may result in underinvestment in education. Although investments in education typically are general investments, in the sense that they increase a worker's productivity in many firms, search costs still give room for bargaining between a worker and his current employer. Thus, a worker receives a share less than one of his investments and underinvests only if he has to carry the full share of the costs.

A considerable body of research has developed on how to obtain efficient investment with the help of simple, incomplete contracts (e.g. MacLeod and Malcomson 1993; Aghion *et al.* 1994). The results obtained do not, however, apply to the problems of underinvestment in education. When a worker invests in education, he does not know which firms he will work for after finishing his education (Acemoglu 1996a, b). It is therefore impossible to write any contract at all when the investments are made. Thus, remedies other than contracts have to be found in order to restore efficiency.

In this paper, I bring a new aspect into the analysis of investment in education and explicitly model how workers *finance* their investments. I show that the investment financing influences the bargaining game between a worker and his employer later on, and thus also affects the incentives to invest in education. In particular, it is possible to design simple schemes for investment financing that eliminate search-induced underinvestments.

I describe the labour market by an equilibrium search model of the Diamond–Mortensen–Pissarides type, where both workers and firms have to undertake time-consuming search to find a trading partner, and where wages are determined by bargaining. Before workers enter the labour market, they make irreversible investments in human capital. If the investments are financed up-front, underinvestment prevails owing to hold-ups. I show, however, that if the investments are financed by infinitely running *contingent loans*, which do not accumulate interest as long as the borrower in question in unemployed, this alters the outcome of the bargaining game. The interest payments are regarded as a part of the surplus the agents bargain over, and they are divided according to the same sharing rule as applies for the investment returns. As a result, the hold-up problem is eliminated and efficiency restored. The optimality result turns out to be robust, and I show that it holds for a wide range of extensions to the model, including the situation where both workers and firms invest and when the agents are heterogeneous.

To gain intuition for the efficiency result, note that, although the investments are intrinsically irreversible, the investment costs are not irreversible from the worker's perspective, as he pays back only the loans that finance the investments when his human capital is utilized. Just as with reversible investments, the market therefore generates an optimal amount of investments.

In most countries in Europe, the government plays an active role in financing education. In addition to direct subsidies, it is common for the government to provide students with contingent loans that do not accumulate interest if the borrower is unemployed. The intention behind this arrangement is probably social, that is, to provide students with an insurance in case they do not find a job. However, my results suggest that it may be productivity-enhancing as well and could reduce problems related to hold-ups.

The main idea in this paper is that, if the returns on investments are determined by bargaining, investment financing may infuence the incentives to invest. I think this insight is relevant for a wide class of economic problems concerning irreversible investments and hold-ups. However, the contingent loans described in this paper do not yield efficiency in Grout's or in MacLeod and Malcomson's models. The reason has to do with the way the wage bargaining is modelled. In this paper, as in most of the matching literature, the agents' outside options serve as their disagreement points in the bargaining game. The investment return can therefore be altered by contingent debt contracts which manipulate the outside options. However, in Grout's and MacLeod and Malcomson's papers the disagreement points in the bargaining game are the agents' incomes during a conflict. To restore efficiency in these models, the contingent loans must be designed so as to manipulate the income to the investor during a conflict. In Moen (1996) it is shown that *optimal investments in this case can be obtained using debt contracts which specify that no interest accumulates during a conflict in the bargaining game.*

On the other hand, it can be shown that decentralized markets do not necessarily lead to wage bargaining (Moen 1997). If not, the hold-up problem may not arise in the first place.

The paper is organized as follows: the model and the main result are presented in Section I. In Section II, I show how this result can be extended in various directions. In Section III, I analyse how contingent contracts may influence (negatively) other aspects of worker behaviour, and study how these distortions may be eliminated by policy measures. Section IV concludes.

I. The Model

In this section, I introduce human capital investments into a Diamond–Mortensen–Pissarides type of matching model (Diamond 1982; Mortensen 1986; Pissarides 1990). I first study a stripped-down version of the model, which is almost identical to the model in Pisarides (1990, ch. 2). Generalizations are deferred to later sections.

The model is set in continuous time. The economy consists of a continuum of identical and risk-neutral workers and firms, and the numbers (measure) of both workers and firms are exogenously given.[1] I assume that workers exit the market at a constant, exogenous rate s, and are replaced by a flow of workers entering the market, so that the number (measure) of workers in the market is constant. Each firm hires at most one worker, and a worker is equally productive in all firms and stays with the same firm until he leaves the market for exogenous reasons.

The labour market is characterized by *frictions*, meaning that all agents have to go through a time-consuming search process in order to find a trading partner. The frictions in the market are captured by a constant-returns-to-scale matching technology. For our purpose, it is sufficient to focus on the arrival rate p of job offers to workers and the arrival rate q of applicants to a firm with a vacancy. Since the numbers of workers and firms are exogenously given, and I consider only steady-state analysis, the rates p and q are constant and can be treated as exogenous.

Just before new workers enter the labour market, they invest in human capital (education) in order to increase their productivity. I assume that the cost of receiving a certain level of education can be conceptualized by an exogenous, pecuniary variable i, which I refer to as the amount invested in education. The productivity of a worker is written as $H(i)$, with $H' > 0$ and $H'' < 0$. When the investments are undertaken, the worker joins the unemployment pool and starts searching for a job.

In contrast to the literature on hold-ups, I explicitly focus on how the investments are financed. I assume that all investments are financed by loans. The interest rate on the loan may be contingent on the borrower's status in the labour market. A *debt contract* is characterized by a pair of interest rates (r_u, r_e), specifying the interest rate on the loan when the borrower is unemployed and employed, respectively. A *standard debt contract* specifies a constant interest rate which is independent of the borrower's status in the labour market. As we will see, applying a standard debt contract is equivalent to paying the investment costs up-front. A (*pure*) *contingent debt contract* $(0, r_e)$, in contrast, specifies zero interest during unemployment, so that interest accumulates only if the borrower is matched. I assume that all loans are infinitely running (or run until the borrower leaves the market). I also require that all debt contracts *break even*, so that the expected net present value of the flow of repayments, discounted by a (non-contingent) exogenous discount factor r is equal to the amount borrowed.

Returns from education

Workers invest in education so as to maximize their expected lifetime income. In this subsection, I derive the expected discounted value of an unemployed worker's future income, or his 'asset value', which I denote by $U(i)$. I assume that a worker receives no unemployment benefits (this is innocuous, since the investment decision is independent of exogenous income flows) and normalize his income after leaving the market to zero. The Bellman equation determining $U(i)$ is then

$$(1) \qquad rU(i) = -r_u i + p[W(i) - U(i)] - sU(i),$$

where $W(i)$ is the expected discounted value of an employed worker's future income flow, while $r_u i$ is the interest payments the worker pays during search. The equation states that the interest rU on the 'asset' must be equal to the return for an unemployed worker conducting a search. The latter is equal to the current income during search, $-r_u i$, plus the return from search activity, $p[W(i) - U(i)]$, minus the expected loss of income associated with leaving the market, $-sU$. The income after leaving the market is normalized to zero.

Similarly, the asset vaue $W(i)$ is determined by the equation

$$(2) \qquad rW(i) = w(i) - r_e i - sW(i),$$

which can be interpreted analogously to (1). I also need to characterize the expected discounted value of the income flow for a firm with a vacancy, V, and with an employee, J. The Bellman equation for V can be written as

$$(3) \qquad rV = -c + q[E^i J(i) - V],$$

where the expectations are taken with respect to the education of the future employee.[2] The left-hand side gives the normal return on an asset with value V, and the right-hand side gives actual return for a searching firm. The latter is equal to the current income $-c$ plus the gains from search.

Finally, the Bellman equation determining J is given by

$$(4) \qquad rJ(i) = H(i) - w(i) - s(J(i) - V),$$

where the last term, $s(J - V)$, gives the expected loss associated with the worker exiting the market, in which case the job becomes vacant.

As in the rest of the matching literature, I assume that the incomes are split according to the Nash sharing rule. I thus have that

$$(5) \qquad W - U = \beta S,$$

where $S = W + J - U - V$ can be referred to as the *match surplus*. To get an expression for S, I first subtract rV from both sides of (4) and rearrange to find that

$$(6) \qquad J - V = \frac{H - w - rV}{r + s}.$$

From (2), it follows that $W = (w - r_e i)/(r + s)$. Substituting this and (6) into the definition of S gives

$$(7) \qquad S = \frac{H - ir_e - rV}{r + s} - U.$$

From (1), it follows that $(r+s)U = -r_u i + p\beta S$, or that

$$U(i) = \frac{-ir_u(r+s) + p\beta(H(i) - ir_e - rV)}{(r+s)(r+s+p\beta)}. \tag{8}$$

From the perspective of an individual worker, his choice of investments does not influence his employer's outside option V. It thus follows from (8) that the derivative $U'(i)$ is independent of V, and hence that the investment decision is independent of V as well. Still, I derive an explicit expression for V for later reference and show how V depends on the *equilibrium* value of i. Note that S can be written as (from (7))

$$S = \frac{H - r_e i - (r+s)U}{r+s} - \frac{r}{r+s}V.$$

From (3), it follows that $rV = -c + q(1-\beta)S$. Substituting for S and rearranging gives

$$V = \frac{-(r+s)c + q(1-\beta)[H - r_e i - (r+s)U]}{(r+s)(r + q(1-\beta)r/(r+s))}. \tag{9}$$

Since the choice of education is related to lifetime income U and not wages, it is not necessary to derive an explicit expression for the latter.

Determination of interest rates

I now derive the relationship between the contingent interest rates r_u and r_e and the risk-free interest rate r. Let A^u and A^e denote the expected discounted values of the interest payments from a one-unit loan when the borrower is unemployed and employed, respectively. As mentioned above, I require that all loans break even at the interest rate r. Furthermore, because the borrowers are just about to join the labour market as unemployed when the debt contracts are undertaken, it follows that $A^u = 1$. Now A^u and A^e are given by the Bellman equations

$$rA^u = r_u + p(A^e - A^u) - sA^u$$

$$rA^e = r_e - sA^e.$$

The left-hand side of the equation for A^u gives the normal return from an asset of value A^u. The right-hand side of the equation gives the return to the lender from the loan. This contains the interest payments r_u and the capital gains associated with the event that the borrower finds a job (which happens at a rate p), less the capital loss associated with the event that the worker exits the market. The second equation can be given the same interpretation. Substituting the second equation into the first and utilizing that $A^u = 1$ gives (after some simple manipulations)

$$\frac{r_u(r+s) + r_e p}{r+s+p} = r+s. \tag{10}$$

In a *standard contract*, where $r_u = r_e$, it follows that

$$r_u = r_e = r+s. \tag{11}$$

Thus, the interest rate is equal to the sum of the market rate r and the worker's exit rate s. In a *pure contingent contract*, with $r_u = 0$, it follows that

$$(12) \qquad r_e = (r+s)\frac{r+s+p}{p}.$$

In this case, r_e is higher than $r+s$ to compensate for the period with no interest payments before the worker is employed.

Investments in education

I first derive the socially optimal level of education. The simplest way of doing this is by realizing that the social planner will choose i so as to maximize $U(i)$ when $\beta = 1$, since $U(i)$ in this case captures all the gains from education (on the margin). Substituting $\beta = 1$ into (8) gives

$$U(i) = \frac{p(H(i) - rV)}{(r+s)(r+s+p)} - i\frac{r_u(r+s) + r_e p}{(r+s)(r+s+p)}.$$

From (10), it follows that the last term is equal to i independent of the specification of the debt contract (as long as it breaks even). It thus follows that the planner chooses a value of i that solves the problem

$$(13) \qquad \max_i \frac{p(H(i) - rV)}{(r+s)(r+s+p)} - i$$

with first-order condition

$$(14) \qquad \frac{p}{r+s+p} H'(i) = r+s.$$

Since H is concave, the first-order condition uniquely determines i. Note that since $p < \infty$, the first factor on the left-hand side of (14) is less than one, reflecting that the human capital will stay idle until the worker in question finds a job.

Next, I turn to the market solution and study the situation where $0 < \beta < 1$. First, I assume that the investments are financed by standard debt contracts, with $r_u = r_e = r+s$. Inserted into (8), this gives

$$\begin{aligned} U(i) &= \frac{-i(r+s)^2 + p\beta(H - i(r+s) - rV)}{(r+s)(r+s+p\beta)} \\ &= \frac{p\beta(H - rV)}{(r+s)(r+s+p\beta)} - i\frac{(r+s)^2 - p\beta(r+s)}{(r+s)(r+s+p\beta)} \\ &= \frac{p\beta(H - rV)}{(r+s)(r+s+p\beta)} - i. \end{aligned}$$

The worker's choice of i is thus given by the first-order condition

$$(15) \qquad \frac{p\beta}{r+s+p\beta} H'(i) = r+s.$$

If we compare this with the first-order condition for the planner's problem (14), we find that the only difference between the two is that p in the planner's problem is replaced by $p\beta$ in the worker's problem. Since the first coefficient on the left-hand side of (15) is decreasing in β, it immediately follows that the *investment level in the market solution is strictly less than the socially optimal investment level.* This is what is referred to as the 'hold-up problem' in the literature. The intuition for the result is clear: when there are frictions in the labour market, there is a surplus associated with a worker–firm match, and this surplus is shared between the worker and the firm. The more productive the worker is, the higher is the surplus and the income to the firm. The worker thus receives a share less than one of the investment return, and, since he bears all the costs, underinvestment occurs.

Now assume that the education investment is financed by contingent loans, where r_u is zero and r_e is given by (12). The expected income U from (8) is then

$$U(i) = \frac{\beta p[H(i) - i(r+s)(r+s+p)/p - rV]}{(r+s)(r+s+p\beta)} \tag{16}$$

$$= \frac{\beta}{(r+s)(r+s+p\beta)}[p(H(i) - rV) - (r+s)(r+s+p)i].$$

The first factor in the last part of this equation is independent of i, and the first-order condition for the worker's problem of maximizing $U(i)$ is thus

$$\frac{p}{r+s+p}H'(i) = (r+s), \tag{17}$$

which is identical to (14), the first-order condition in the planner's problem. We have thus shown our main result:

Proposition 1. Suppose the investments in human capital are financed with a contingent loan that accumulates interest only when the borrower is employed. Then the investment level undertaken is socially optimal.

When contingent debt contracts are employed, the worker's problem is actually equivalent to the planner's problem. To see this, note that (16) can be rewritten as

$$U(i) = C\left[\frac{pH(i) - rV}{(r+s)(r+s+p)} - i\right], \tag{18}$$

where the constant C is given by

$$C = \frac{\beta(r+s)(r+s+\beta)}{(r+s)(r+s+p\beta)}.$$

By comparing (18) with the planner's objective function (13), we find that they are equal save for the constant C.

To gain intuition for the result, we must first carefully examine what a worker and a firm bargain over in the wage-setting game. We know that the agents bargain over the match surplus, defined as their joint income when matched less the sum of their outside options. Interest that the worker pays if and only if he is employed are included as a (negative) part of the match surplus. Thus, the firm in effect pays a share of the investment costs exactly equal to the share the firm receives from the return on the investments. As a result, the positive externalities for future employers from the investments are eliminated, and optimal investments are obtained.

When the investments are financed by contingent loans, the investments are in effect paid for only when the human capital is utilized. In this sense, financing irreversible investments by contingent loans is very similar to *renting* the capital. Our result is thus in line with the result in Pissarides (1990), which shows that when the firms rent capital they choose the socially optimal capital intensity.

I have assumed that the number of firms is exogenously given. In some circumstances it may be more plausible to assume that new firms may enter the market and will do so until the profitability of entering the market is zero.

The workers' investment decisions influence firm profits, and with free entry they will also influence the number of firms entering the market. As shown in Hosios (1990), the entry decisions made by firms are generally not optimal. This may again give second-best effects regarding human capital investments. Suppose, for instance, that the market solution implies that too few firms enter the market. An increase in the education level among unemployed workers increases the profitability of entry (which is good), and the socially optimal level of education is higher than in my model. On the other hand, if the entry decision is optimal, the effect of education on the entry decisions of firms has no welfare implications at the margin. In this case, all my results still hold.

Finally, if workers finance their investments by standard loans, free entry may lead to multiple equilibria (Moen 1995, ch. 4). If many firms enter the market, p and thus i are high, and as the investments in this case yield positive externalities on firms, profits may increase. With contingent loans the equilibrium is unique, as there are no positive externalities from human capital investments on firms at the margin.

II. Extensions

As we have seen, a worker who invests in human capital and finances the investments by using contingent debt contracts in effect acts so as to maximize the social value of his investments. This fact implies that our optimality result is quite robust and holds in a wide range of situations. In this section I show that the optimality result still applies (1) when both workers and firms invest, (2) when the agents are heterogeneous, (3) when I allow for direct externalities (as explained below), and finally (4) when workers may have more than one job before they retire from the market.

Both workers and firms invest

Suppose both workers and firms make irreversible investments prior to search. Let the productivity of a worker–firm pair be given by a concave function $H(i,j)$, where i and j denote the worker's and the firm's investments, respectively. It is straightforward to show that U is still given by (8), except that we have to include the interest payments made by the firm. Thus,

$$U(i)=\frac{-(r+s)r_u^w i+p\beta(H(i,j)-r_e^w i-r_e^f j-rV)}{(r+s)(r+s+p\beta)}, \tag{19}$$

where superscript w indicates that the interest is on a worker's loan and superscript f indicates that the interest is on a firm's loan.

Now I turn to the firm. I assume that the 'search cost' c is equal to $r_u^w j$, the firm's interest payments during search. From (9) it follows that

$$V = \frac{-(r+s)r_u^f j + q(1-\beta)[H - r_e^f j - r_e^w i - (r+s)U]}{(r+s)[r+(1-\beta)qr/(r+s)]}. \tag{20}$$

I require that the firm's debt contract breaks even. By reasoning as I did when deriving (10), I find that the relationship between r_u^f and r_e^f is given by

$$\frac{r_u(r+s)+r_e q}{r+qr/(r+s)} = r+s. \tag{21}$$

The planner chooses i so as to maximize U with β substituted out by 1, and j so as to maximize V with $1-\beta$ substituted out by 1. We proceed as in the last section, and substitute $\beta=1$ and $\beta=0$ into (19) and (20), respectively. When we apply the break-even requirements (10) and (21), it follows that the planner solves the problems

$$\max_i \frac{p(H(i,j)-rV-r_e^f j)}{(r+s)(r+s+p)} - i$$

and

$$\max_j \frac{q[H(i,j)-(r+s)U-r_e^w i]}{(r+s)[r+qr/(r+s)]} - j.$$

The first-order conditions for a maximum are thus given by the equations

$$\frac{p}{r+s+p} H_i(i,j) = r+s \tag{22}$$

$$\frac{q}{r+qr/(r+s)} H_j(i,j) = r+s.$$

Now I turn to the market solution. I assume that both workers and firms use contingent debt contracts, so that $r_u^w = r_u^f = 0$. It follows that $r_e^w = [(r+s+p)/p](r+s)$ as before and that $r_e^f = \{[(r+qr)/(r+s)]/q\}(r+s)$ (from (21)). Substituting the expression for r_u into (19) gives (analogous to (16))

$$U = \frac{\beta}{(r+s+p\beta)(r+s)}[pH(i,j^e)-(r+s+p)(r+s)i - r_e^w j^e - rV],$$

where j^e is the worker's beliefs about the investment level undertaken by firms. Analogously, substituting the expression for r_e^f into (20) gives

$$V = \frac{1-\beta}{(r+(1-\beta)qr/(r+s))(r+s)} \times \left[qH(i,j) - \frac{r+qr}{(r+s)}(r+s)j - r_e^w i^e - (r+s)U\right],$$

where i^e is the firm's beliefs about the investment levels undertaken by workers. In equilibrium I require beliefs to be rational, so that $i^e = i$ and $j^e = j$. Taking derivatives of U and V with respect to i and j, respectively, leads us to (22). It thus follows that the investment levels generated by the market are optimal.

Acemoglu (1996b) finds that, if both workers and firms invest and the investments are complementary in production, there may exist multiple equilibria. The intuition is that if, say, firms increase their investments, this increases the workers' investments, which in turn may make the firms' increased investments profitable. With contingent loans, this is no longer true as there are no positive externalities associated with physical or human capital investments at the margin, and the equilibrium is unique. Formally, this follows from the first-order conditions (22) and the fact that $H(i,j)$ is concave. A proof of this claim is given in the Appendix.

Heterogeneous agents

Let y^w denote worker-specific and y^f firm-specific characteristics, and write the productivity of a match as $H(i,j,y^w,y^f)$. Let $i(y^w)$ denote the equilibrium investments undertaken by workers as a function of the worker's type, and let $j(y^f)$ denote the same for the firms. To make notation easier, I define $H^w(i,y^w)$ as the expected value of H from a worker's perspective, given that he is of type y^w and has invested an amount i. More precisely, I define H^w as $H^w(i,y^w)\equiv E^{y^f}H(i,j(y^f),y^w,y^f)$. Similarly, let $H^f(y^f,j)$ denote the expected value of H from a firm's perspective, given that the firm is of type y^f and has invested an amount j. Formally, $H^f(j,y^f)\equiv E^{y^w}H(i(y^w),j,y^w,y^f)$. Finally, let $i^e=E^{y^w}i(y^w)$ and $j^e=E^{y^f}j(y^f)$, and analogously $U^e=E^{y^w}U(i(y^w),y^w)$ and $V^e=E^{y^f}V(j(y^f),y^f)$. I assume that all matches are acceptable to both types of agent. Since the value of finding a trading partner is linear in H for both workers and firms, it follows that U and V from (19) and (20) can be written as

$$(23)\qquad U(i,y^w)=\frac{-(r+s)r_u^w i+p\beta[H^w(i,y^w)-r_e^w i-r_u^f j^e-rV^e]}{(r+s)(r+s+p\beta)}$$

$$V(j,y^f)=\frac{-(r+s)r_u^f j+q(1-\beta)[H^w(j,y^f)-r_e^w i^e-r_e^f j-(r+s)U^e]}{(r+s)(r+(1-\beta)qr/(r+s))}.$$

The asset value equations thus have the same form as in (19) and (20) with H replaced by H^w. Again, the planner chooses $i(y^w)$ so as to maximize U with β substituted out by 1, and chooses $j(y^f)$ so as to maximize V with $1-\beta$ substituted out by 1. By applying exactly the same argument as above, it follows that the market solution is optimal when the agents use contingent debt contracts.

Direct externalities

Suppose the effect of investments in education is more complex than in the analysis above, and that the investments have a direct impact on the opponent's utility (direct externalities). More specifically, suppose the joint income depends on both the worker's and the firm's effort levels, and that the disutility of effort depends on the investment levels. Let e^w and e^f represent the effort level of the worker and the firm, respectively, measured in money terms. The (net) outcome to the worker–firm pair is thus

$$(24)\qquad H(i,j,e^w,e^f)=f(i,j,e^w,e^f)-e^w(i,j)-e^f(i,j),$$

where e^w and e^f denote the worker's and the firm's effort level, respectively. Since the agents have full information, we know that the Nash solution is efficient. Thus, the agents choose e^f and e^w so as to maximize H (see e.g. Osborne and Rubinstein 1990). Define $\hat{H}(i,j)$ by the equation

$$\hat{H}(i,j) = \max_{e^w, e^f} [f(i,j,e^w,e^f) - e^w(i,j) - e^f(i,j)].$$

Now we can perform the same analysis as above with $H(i,j)$ replaced by $\hat{H}(i,j)$, and it follows that contingent debt contracts still lead to efficient investments.

Turnover

Suppose firms are hit by productivity shocks at a constant rate t, after which the firm's productivity is zero. A match may then be destroyed for two reasons. First, the worker may exit the market; if so, the firm has to start searching for a new employee. Second, the firm may be hit by a productivity shock, after which the worker has to start searching for a new job while the firm exits the market. I assume that new firms constantly enter the market so that the number of firms in the market stays constant. For simplicity, I return to the case where only workers make investments. To simplify the notation, I define $x = r+s+t$ and $z = p(r+s)/(r+s+t)$. In the Appendix, I show that $U(i)$ is given by

$$U = \frac{-xr_u i + p\beta(H(i) - r_e i - (r+t)V)}{x(r+s+\beta z)}, \tag{25}$$

and that the break-even requirement can be written as

$$\frac{r_u x + r_e p}{r+s+z} = x. \tag{26}$$

As above, a planner chooses the value of i that maximizes U when β is substituted out by 1. The planner thus solves the problem

$$\max_i \frac{p(H(i) - (r+t)V)}{x(r+s+z)} - i \tag{27}$$

with first-order condition given by (analogous to (14))

$$\frac{p}{r+s+z} H'(i) = x. \tag{28}$$

With contingent debt contracts, it follows from (26) that

$$r_e = x(r+s+z)/p.$$

When we insert this into (25), we find that

$$U(i) = \frac{\beta}{x(r+s+\beta z)} [pH(i) - ix(r+s+z)]. \tag{29}$$

The first-order condition for the optimum is thus given by (28), and this shows that the market solution is optimal. Thus, the worker faces the same maximization problem as the planner, and the investment decision is optimal.

III. MATCH-SPECIFIC PRODUCTIVITIES

In the previous sections I have examined how contingent debt contracts increase the agents' incentives to undertake irreversible investments prior to search. In this section I discuss whether contingent debt contracts may bring in new distortions. Typically, a worker makes decisions between the time of the investment decision and the time at which he is employed. Examples may include the choice of search intensity and the choice to accept job offers. Here I focus on the issue of job acceptance. A worker may have different productivities in different jobs, and the productivity in some jobs may be so low that it is optimal for a worker and a firm who have identified each other in the market to continue searching rather than to form a match. The question is, to what extent do contingent debt contracts distort the decision to accept a certain match compared with the situation where the investments are financed by standard debt contracts? The issue concerning the worker's choice of search intensity is very similar. The discussion here is informal; a more formal treatment can be found in Moen (1996).

Suppose the worker in question has invested an optimal amount i^* in education. With contingent debt contracts, the cost of continuing the search is lower than with standard debt contracts, since the borrower does not pay interest during search. Turning down a job offer has a negative external effect on the lender, since he does not receive any interest before the borrower is employed. It follows that the worker is more choosy and accepts fewer matches when using contingent debt contracts rather than standard debt contracts (given the investment level i^*).

A countermeasure to this distortion may be a penalty to workers who reject job offers. The penalty should be set independently of the investment made by the worker in question in order not to distort the investment decision. The penalty should be equal to the expected interest savings for a person with education i^* (the equilibrium level) when rejecting a job offer. Denote this penalty by A, and let p^* denote the job finding rate with standard debt contracts (for workers with education i^*). Let A be determined by the asset value equation

$$rA = r_u i^* - sA - p^*A$$

or $A = ru_i^*/(r+s+p^*)$. The penalty then exactly offsets the negative externality on the lender from turning down job offers, and the incentive to accept a job is the same as with standard debt contracts. A similar type of argument can be made by introducing subsidies to workers who find a job or to firms who employ a worker.

Furthermore, Hosios (1990) shows that, in general, the market does not generate optimal acceptance rates of jobs. The fact that contingent debt contracts (and no search tax) makes workers more choosy compared with standard debt contracts does not therefore necessarily imply reduced welfare. More importantly, it is an open issue whether job acceptance is a choice variable for a worker; in many countries unemployment benefits are withdrawn if the person in question does not accept 'proper' job offers. Since 'proper' typically is defined by the authorities, the latter in effect also determines which jobs should be accepted. If so, the introduction of contingent debt contracts will not alter the decision to accept jobs unless they influence the authorities' definition of 'proper' job offers.

IV. Conclusion

I have studied a model where workers and possibly firms undertake irreversible investments before they enter the labour market. Owing to turnover costs, hold-ups and underinvestment prevail if the investments are financed up-front or with standard loans. However, if the investments are financed by contingent loans, which carry interests only if the borrower is employed, optimal investments are obtained in a wide range of situations. Furthermore, contingent loans are not only a theoretical construct; in many European countries the government actually provides contingent loans to students.

APPENDIX

Wage determination

Here I show how the Nash bargaining solution applied in the paper can be derived as the solution to a strategic bargaining game with alternating offers. For a more general treatment of the problem, I refer to Osborne and Rubinstein (1990) or to Moen (1996). The game proceeds in the following way.

1. A random device selects the worker (with probability b) or the firm (with probability $1-b$) as *proposer*. The proposer offers a wage which the opponent accepts or rejects.
2. If the proposal is accepted, the bargaining game ends.
3. If the proposal is rejected, a new offer cannot be made before the next period. In the meantime, there is a positive probability x that the match will be dissolved, in which case the agents have to start over again searching for a new trading partner.
4. If the game continues to the next period, it proceeds as described in step 1.

I assume that the time delay between two successive offers is small, so that discounting and the probability that the worker exits the market can be ignored. I assume that no interest is paid during the negotiations. Since the worker and the firm have preferences over W and J respectively, and W and J are both linear functions of the wage rate w, I can formulate the wage offers in terms of W rather than w. Finally, define $M=J+W$.

I proceed in the standard way and assume that in equilibrium all proposals are accepted immediately. Let W^f denote the equilibrium proposal made by the firm, and let W^e denote the equilibrium proposal made by the worker. The expected wage (before a proposer is chosen) is thus $W^*=bW^e+(1-b)W^f$. If a proposal is rejected, the expected wage next period will be W^*. An agent will always give a wage offer that the opponent is just willing to accept. Thus, W^f and W^e are determined by the equations

$$\begin{aligned} W^f &= (1-x)W^*+xU \\ M-W^e &= (1-x)(M-W^*)+xV \\ W^* &= bW^e+(1-b)W^f. \end{aligned}$$

Solving for W^* gives

$$W^*=b(M-V)+(1-b)U.$$

Setting $b=\beta$ gives the Nash sharing rule applied in the text.

Uniqueness of the equilibrium solution

I want to prove that the equilibrium is unique when both workers and firms invest; that is, I want to show that (22) has a unique solution. The first equation in (22) defines i as a function of j, $i=i(j)$, with derivative $i'(j)=-H_{ij}/H_{ii}$. Now it is sufficient to show that the left-hand side of the second equation in (22), with i replaced by $i(j)$, is strictly decreasing in j. Thus, I require that

$$-H_{ji}H_{ij}/H_{ii}+H_{jj}<0$$

or that $H_{ii}H_{jj}-H_{ij}^2>0$. But this follows from the fact that $H(i,j)$ is strictly concave.

Turnover

First I derive (25). The procedure is the same as when I derived (8). The asset value equation for an employed worker is now given by

$$rW=w-sW-t(W-U)-r_e i.$$

Solving for W gives

$$\text{(A1)} \qquad W=\frac{w-r_e i+tU}{r+s+t}.$$

The asset value equation for a firm with an employee is

$$rJ=H-w-s(J-V)-tJ.$$

Subtracting rV on both sides and rearranging gives

$$\text{(A2)} \qquad J-V=\frac{H-w-(r+t)V}{r+s+t}.$$

From (A1) and (A2), it follows that the match surplus $S=J+W-V-U$ is given by

$$\text{(A3)} \qquad S=\frac{H-r_e i-(r+t)V+tU}{r+s+t}-U$$

$$=\frac{H-r_e i-(r+t)V}{r+s+t}-\frac{r+s}{r+s+t}U.$$

As before, the Nash solution implies that $W-U=\beta S$, which inserted into (1) yields, after some rearranging

$$\text{(A4)} \qquad U=\frac{-r_u i(r+s+t)+p\beta[H-r_e i-(r+t)V]}{(r+s+t)[r+s+p\beta(r+s)/(r+s+t)]}.$$

To obtain (26), I proceed as I did when I derived (10). The relevant asset value equations are now

$$rA^u=r_u+p(A^e-A^u)-sA^u$$
$$rA^e=r_e-t(A^e-A^u)-sA^e.$$

Solving for A^e in the first equation and using that $A^u=1$ gives (26).

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NOTES

1. It is common to assume that the number of jobs is endogenously determined through entry. In this paper the focus is on the incentives to invest in human capital, not on the incentives to enter the market. I therefore assume that the number of firms is exogenously given, and precludes distortions arising from suboptimal entry behaviour. However, my results concerning the efficacy of human capital investments still hold in the presence of entry as long as the entry decision is optimal; see comments below.
2. Note, however, that in equilibrium all workers invest the same amount in education.

REFERENCES

ACEMOGLU, D. (1996a). A microfoundation for social increasing returns in human capital accumulation. *Quarterly Journal of Economics*, **61**, 779–804.

— (1996b). Training and innovation in an imperfect labor market. *Review of Economic Studies*, **64**, 445–64.

AGHION, P., DEWATRIPONT, M. and REY, P. (1994). Renegotiation design with unverifiable information. *Econometrica*, **62**, 257–82.

BECKER, G. T. (1964). *Human Capital: A Theoretical and Empirical Analysis, with Special Reference to Education*. Chicago: University of Chicago Press.

BLANCHFLOWER, D. G. and OSWALD, A. J. (1995). An introduction to the wage curve. *Journal of Economic Perspectives*, **9**, 153–67.

DIAMOND, P. A. (1982). Wage determination and efficiency in search equilibrium. *Review of Economic Studies*, **49**, 217–27.

GROUT, P. A. (1984). Investments and wages in the absence of binding contracts: a Nash approach. *Econometrica*, **52**, 449–70.

HART, O. and MOORE, J. (1988). Incomplete contracts and renegotiation. *Econometrica*, **56**, 755–85.

HOSIOS, A. J. (1990). On the efficiency of matching and related models of search and unemployment. *Review of Economic Studies*, **57**, 279–98.

JOVANOVIC, B. (1979). Job matching and the theory of turnover. *Journal of Political Economy*, **87**, 972–90.

MACLEOD, W. B. and MALCOMSON, J. M. (1993). Investments, holdups, and the form of market contracts. *American Economic Review*, **83**, 811–37.

MOEN, E. R. (1995). Essays on matching models of the labour market. PhD thesis, London School of Economics.

—— (1996). Efficient ways to finance irreversible investments. Working paper, SNF-Oslo.

— (1997). Competitive search equilibrium. *Journal of Political Economy*, **105**, 385–411.

MORTENSEN, D. T. (1986). Job search and labour market analysis. In O. C. Ashenfelter and R. Layard (eds.), *Handbook of Labor Analysis*, vol. 2. Amsterdam: North-Holland.

OSBORNE, M. J. and RUBINSTEIN, A. (1990). *Bargaining and Markets*. San Diego: Academic Press.

PISSARIDES, C. A. (1985). Short run equilibrium dynamics of unemployment, vacancies and real wages. *American Economic Review*, **75**, 677–90.

—— (1990). *Equilibrium Unemployment Theory*. Oxford: Basil Blackwell.