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Performance Pay and Adverse Selection*

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Abstract

It is well known in personnel economics that firms may improve the quality of their workforce by offering performance pay. We analyze an equilibrium model where worker productivity is private information and show that the firms' gain from worker self-selection may not be matched by a corresponding social gain. In particular, the equilibrium incentive contracts are excessively high-powered, thereby inducing the more productive workers to exert too much effort and increasing agency costs stemming from the misallocation of effort.

Keywords: Performance pay; selection; efficiency

JEL classification: *D*82; *J*30

I. Introduction

In his book on personnel economics, Lazear (1998) stresses the importance of performance pay for firm profitability. He argues that the gains from performance pay are twofold. First, and most obviously, performance pay mitigates moral hazard problems. Second, performance contracts affect the quality of workers applying to a firm. When workers have private information about their productivity at the hiring stage, firms can attract higher quality workers by offering more high-powered contracts. That is, good workers self-select into jobs offering more performance-sensitive compensations, e.g. large bonus packages; see Lazear (1986).

Incentive contracts may also give rise to agency costs. In a broad sense, agency costs can be divided into three categories: suboptimal risk sharing, as in Hart and Holmstrom (1987); rent extraction, as in Laffont and Tirole (1993) and Moen and Rosén (2004); and misallocation of effort across multiple tasks, as in Holmstrom and Milgrom (1991). In this paper, we focus on misallocation of effort across tasks, which is relevant if not all

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aspects of a worker's output can be adequately measured and compensated for. Workers then concentrate too much on tasks that give rise to performance pay while neglecting the tasks that do not. Standard examples include too much focus on quantity relative to quality, indifference to cooperation, inadequate maintenance of productive assets, and possibly too much focus on short-term rather than long-term performance.[1]

The contribution of this paper is to analyze the welfare properties of markets with heterogeneous workers in which firms offer incentive contracts, as often recommended by personnel economists. We argue that the private gains associated with the selection effect may not reflect social gains, because the number of talented people in the economy is limited. We examine to what extent a rat-race between firms for talented workers may lead to excessive use of performance pay, overly high agency costs, and thus an inefficient allocation of resources.

Our central proposition is that the incentive power of the equilibrium wage contract exceeds its socially efficient level. The excessive incentive power of the equilibrium wage contract can be dampened by a tax on high incomes. We derive these results in a simple adverse selection model *à la* Rothschild and Stiglitz (1976).

The paper is organized as follows. In Section II we describe the model and derive the equilibrium with observable worker types. Section III focuses on an analysis of the equilibrium outcome when the workers' type is private information and firms offer linear wage contracts. Section IV addresses the robustness of the results when allowing for non-linear wage contracts, and Section V concludes. Mathematical derivations and proofs are provided in the Appendix.

II. Model and Benchmark

We now introduce the basic features of the model with two observable types of workers. The model is deliberately constructed in such a way that the optimal contract is linear over the relevant intervals, as in Holmstrom and Milgrom (1987).

Framework

There are many firms and many employees in each firm. All agents are risk neutral. Each worker undertakes two tasks. The total value of production of any given worker is given by

$$y = e_1 + e_2 + \varepsilon, \tag{1}$$

[1] The latter point requires that the workers have a shorter time horizon than the firm, which is typically the case due to worker turnover.

where e_1 and e_2 denote the effort spent on tasks one and two, respectively. For instance, task 1 may be related to product quality and task 2 to quantity. The term ε reflects a random influence on output such as the difficulty of the particular task in question, or a worker–firm-specific productivity component. The distribution of ε is continuous and symmetric on the interval $[-\bar{\varepsilon}, \bar{\varepsilon}]$. Firms do not observe the realization of ε but they know its distribution. A worker observes ε after the contract is signed, but before he chooses his effort levels.

The existence of ε implies that a simple, non-linear contract with a bonus for output values above a certain threshold is not efficient. As will be made clear below, optimal effort provision requires a worker to have the same incentives on the margin for all realizations of ε. This can only be achieved by a linear contract.

The firm cannot observe e_1, e_2, or y, but only a distorted measure $\tilde{y}$ of their output, given by

$$\begin{aligned}\tilde{y} &= (1-\gamma)e_1 + (1+\gamma)e_2 + \varepsilon \\ &= \tilde{e} + \varepsilon,\end{aligned}$$

where $\tilde{e} = (1-\gamma)e_1 + (1+\gamma)e_2$, and where $\gamma > 0$ reflects the measurement error. The firm thus observes a distorted measure of e_1 and e_2. Due to the measurement problems, e_1 carries less weight than e_2 in the evaluation of performance.

Effort above a certain level is costly for workers, and this cost depends on the type of the worker, which is either high (h) or low (l). The expected utility of type $k = \{l, h\}$ is

$$u^k = Ew - C^k(e_1, e_2),$$

where w is the wage and C^k the effort cost, the latter given by

$$C^k(e_1, e_2) = \frac{(e_1 - e^{0k})^2}{2} + \frac{(e_2 - e^{0k})^2}{2}, \tag{2}$$

for e_1, $e_2 \geq e^{0k}$. We assume that $e^{0l} < e^{0h}$, reflecting that the latter type is more productive.

The sequence of moves is as follows:

(i) The firm signs contracts with all its employees (individually).
(ii) Each worker learns the realization of ε which is independent across workers. Due to unmodeled costs of changing jobs, a worker does not want to quit even when a low value of ε is realized.[2]
(iii) Each worker chooses effort levels e_1 and e_2.

[2] Switching costs could be endogenized in a search-and-matching context; see, for instance, Pissarides (2000) or Moen (1997).

(iv) Firms observe the distorted output measure $\widetilde{y} = \widetilde{e} + \varepsilon$ for each of their employees and remunerate them accordingly.

Optimal Contracts with Observable Types

With observable worker types, the optimal contract leads to a segmentation into submarkets, one for each type. In order to simplify notation we suppress the superscript k.

An optimal wage contract $f(\widetilde{y})$ maximizes the firm's expected profit subject to the following constraints:

(i) Incentive-compatibility constraint: workers choose effort levels so as to maximize their utility.
(ii) Individual rationality: the contract provides each worker with at least his reservation expected utility, denoted by u^0.

While the firm can affect a worker's measured effort level ($\widetilde{e}$), it cannot control how the worker allocates effort across the two tasks. Among all effort combinations that yield the worker the same wage w, the worker always chooses the pair (e_1, e_2) that minimizes his effort costs. The Lagrangian associated with this minimization problem is

$$L = (e_1 - e^0)^2/2 + (e_2 - e^0)^2/2 - \lambda[e_1(1-\gamma) + e_2(1+\gamma) - \widetilde{e}].$$

Minimizing L with respect to e_1 and e_2 gives

$$\frac{e_2 - e^0}{e_1 - e^0} = \frac{1+\gamma}{1-\gamma}, \tag{3}$$

which is independent of measured total effort $\widetilde{e}$ and the chosen wage contract.

The gain to the firm from the worker's effort is $e_1 + e_2$. In the Appendix we show that

$$e_1 + e_2 = 2e^0 \frac{\gamma^2}{1+\gamma^2} + \frac{1}{1+\gamma^2}\widetilde{e}. \tag{4}$$

The firm's (gross) benefit from a higher measured effort is largest in the absence of distortion ($\gamma = 0$), and its marginal benefit decreases proportionally with $1 + \gamma^2$.

Equations (3) and (4) have several interesting implications. The production function (1) and the effort cost (2) imply that optimal effort allocation across tasks is $e_1 = e_2$. If $\widetilde{e} = 2e^0$, the worker sets $e_1 = e_2 = e^0$, and there is no misallocation on tasks. For larger values of $\widetilde{e}$, (3) shows that effort in excess of e^0 is distorted in the direction of e_2. Equation (4) shows that the productivity $e_1 + e_2$ is less than proportional to $\tilde{e}$. In relative terms, the misallocation of effort increases with $\tilde{e}$. Wage contracts with high incentive power give rise to a high value of $\tilde{e}$ and thus large distortions in effort allocation across tasks.

The gain to the firm from measured effort is given by (4). On the margin, the gain to the firm from measured effort is constant and equal to $1/(1 + \gamma^2)$. Consider a linear wage contract $w = \alpha + \beta\tilde{y}$ and choose $\beta = 1/(1 + \gamma^2)$. For any realization of ε, this contract gives the worker the right incentives: on the margin, the worker is paid the entire marginal gain from his effort, given by $1/(1 + \gamma^2)$. As the firm can extract all the rent by adjusting α in such a way that the participation constraint binds, this linear contract is optimal.

Due to the stochastic term ε, the firm cannot implement the constrained optimal measured effort level (which we denote by $\tilde{e}^*$) with a trigger contract in which the wage is discontinuous at some point. In order to implement the optimal effort level for all values of ε, the wage contract must be such that the marginal effect of increased $\tilde{y}$ on wages has to be equal to $1/(1 + \gamma^2)$ over the entire interval $[\tilde{e}^* - \bar{\varepsilon}, \tilde{e}^* + \bar{\varepsilon}]$. Thus, any optimal contract is linear with slope $1/(1 + \gamma^2)$ on this interval.[3] Finally, the zero-profit condition determines the equilibrium value of α.

Proposition 1. *The unique optimal contract is linear (on the relevant intervals) and can be written as*

$$w = \alpha + \beta\tilde{y},$$

where

$$\alpha = 2e^0 \frac{\gamma^2}{1 + \gamma^2} \tag{5}$$

$$\beta = \frac{1}{1 + \gamma^2}. \tag{6}$$

The value of α is derived in the Appendix, where we also show that the equilibrium expected utility u^0 is given by

$$u^0 = 2e^0 + \frac{1}{1 + \gamma^2}. \tag{7}$$

[3] Note the resemblance to Holmstrom and Milgrom (1987). They study the trade-off between incentive provision and risk sharing in a continuous time model in which workers have more information about the state of the world. In this setting the optimal contract is linear if the workers have constant absolute risk aversion.

III. Unobservable Worker Types

When firms cannot observe worker types, they may induce workers to self-select just as in insurance markets with adverse selection; see Rothschild and Stiglitz (1976). For expositional simplicity, we continue to consider linear contracts. (In the next section we argue that our main result also holds with non-linear contracts.)

An equilibrium in this market must satisfy the following conditions:[4]

(i) Workers apply to firms that offer them the best contract.
(ii) Firms choose contracts to maximize their profits, given the workers' behavior.
(iii) Free entry of firms.

We derive the competitive equilibrium outcome with market clearing, and do not allow for rationing as a sorting mechanism; see Gale (1992). With observable types, firms offer equally high-powered incentive components to both types ($\beta^h = \beta^l$), but pay the high-type a higher fixed salary ($\alpha^h > \alpha^l$). This is no longer an equilibrium when types are not observable, as low-type workers would have an incentive to take jobs intended for high-type workers.

A pooling equilibrium does not exist in the present setting. To see this, suppose firms offer a pooling contract. Consider a firm that deviates slightly and offers a contract with stronger incentives and lower fixed pay. This firm can attract only high-type workers. The reason is that high-type workers are more willing to accept a lower fixed-salary component in return for stronger incentives (higher production-related bonuses). Thus, by increasing β slightly above $1/(1 + \gamma^2)$, and lowering α so that low-type workers are marginally better off with the initial contract, the firm attracts high-type workers but not low-type workers.

Lemma 1. *Suppose the firms in the market offer a pooling contract* (α^p, β^p). *There then exists another contract, arbitrarily close to* (α^p, β^p) *that attracts high-type workers only.*

The next step is to show that for a given contract, firms prefer to attract high-type workers. From (A12) in the Appendix it follows that

$$\frac{\partial \pi}{\partial e^0} = 2(1 - \beta), \tag{8}$$

where π denotes expected profits. Thus, as long as $\beta < 1$, firms strictly prefer to hire high-type workers rather than low-type workers on a given contract. But

[4] We assume that each firm offers at most one contract. This assumption is not important, as firms have constant returns to scale production technology.

this rules out any pooling contracts in which $\beta < 1$. Furthermore, a pooling equilibrium with $\beta = 1$ does not exist either, as low-type workers would prefer a contract with $\beta = 1/(1 + \gamma^2)$ and α set according to Proposition 1.

Lemma 2. *There exists no pooling equilibrium.*

Thus, any equilibrium of the model has to be a separating equilibrium. Denote the equilibrium wage contracts by (α^k, β^k), with $k = l, h$. In a separating equilibrium, the contract offered to low-type workers coincides with the optimal contract with observable types. If not, a firm that offers a contract with $\beta = 1/(1 + \gamma^2)$ and α slightly below $2e^{0l}\gamma^2/(1 + \gamma^2)$ would attract workers and earn positive profits. It follows that $\beta^l = 1/(1 + \gamma^2)$ and $\alpha^l = 2e^{0l}\gamma^2/(1 + \gamma^2)$.

The contracts for high-type workers must be such that low-type workers do not apply for the jobs intended for high-type workers. To ensure separation, the fixed salary must be smaller and the performance component larger than in the optimal contract with observable types. The indirect utility function of a low-type worker is given by (see (A8) in the Appendix):

$$u^l(\alpha, \beta) = \alpha + \beta 2e^{0l} + \beta^2(1 + \gamma^2). \tag{9}$$

In a separating equilibrium, only high-type workers apply for high-type jobs. In (A13) in the Appendix we show that the zero-profit condition for firms implies the following relationship between α^h and β^h:

$$\alpha^h = 2(1 - \beta^h)e^{0h} + 2\beta^h(1 - \beta^h(1 + \gamma^2)). \tag{10}$$

By combining (9) and (10), we obtain the expected utility of a low-type worker who applies for a high-type job:

$$u^l(\alpha^h, \beta^h) = 2e^{0l} + 2(1 - \beta^h)(e^{0h} - e^{0l}) + [2\beta^h - (\beta^h)^2(1 + \gamma^2)]. \tag{11}$$

The first term of this expression is the same as in (7). The second term in (11) reflects the additional income due to higher average productivity of high-type workers. When $\beta^h = 1$, this additional income vanishes. However, due to the distorted measure of effort, there are costs associated with providing such strong incentives. The last term in (11) is maximized for $\beta^h = 1/(1 + \gamma^2)$, in which case it takes the value of $1/(1 + \gamma^2)$, and is equal to the last term of (7). For $\beta^h > 1/(1 + \gamma^2)$, the term decreases in β^h reflecting that it is costly to increase the incentives above the optimal level. Due to the free entry of firms, this cost is ultimately borne by the workers.

In a separating equilibrium $u^l(\alpha^l, \beta^l) \geq u^l(\alpha^h, \beta^h)$ must hold. In (A18) in the Appendix, we show that this condition is equivalent to:

$$\left(\beta^h - \frac{1}{1 + \gamma^2}\right)^2 (1 + \gamma^2) \geq 2(1 - \beta^h)(e^{0h} - e^{0l}). \tag{12}$$

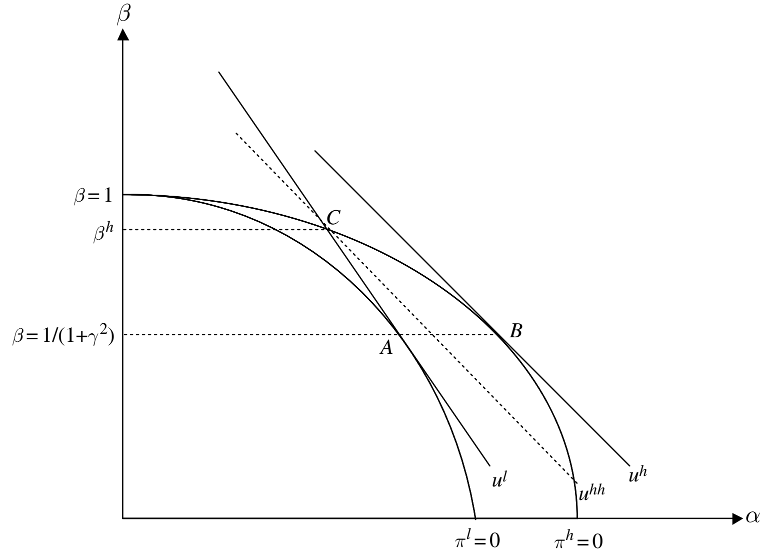

Fig. 1.

Since the incentive compatibility constraint binds in a separating equilibrium, β^h lies in the interval $(1/(1+\gamma^2), 1)$.[5]

Figure 1 shows the indifference curves of low-type and high-type workers in the α, β space, given by (A8). The iso-profit curves $\pi^h = 0$ and $\pi^l = 0$ are the combinations of α and β that yield zero profits to a firm attracting high-type and low-type workers; cf. equation (A13). If the worker type were observable, the equilibrium contract would be given by point A for low-type workers and point B for high-type workers. With unobservable types, the equilibrium contract for high-types is given by point C.

We know from Rothschild and Stiglitz (1976) that a separating equilibrium may not exist for all parameter values. The reason is that an efficient contract that attracts both types may be more profitable than the optimal contracts in a separating equilibrium. This happens if the share of low-productivity workers in the economy is sufficiently low. In this case no equilibrium exists, since pooling can never be an equilibrium.

Let us consider the existence of equilibrium in more detail. Consider a separating equilibrium candidate, where the low-type workers choose a

[5] If we solve this equation, we find that β^h is given by

$$\beta^h = \sqrt{\Gamma^2[(1-\Delta)^2 - 1] + 2\Delta\Gamma} + \Gamma(1-\Delta),$$

with $\Gamma = 1/(1+\gamma^2)$ and $\Delta = e^{0h} - e^{0l}$.

contract (α^l, β^l) with β^l equal to $1/(1 + \gamma^2)$, while the high-type workers choose a contract (α^h, β^h). Consider a firm that deviates and offers a pooling contract (α^p, β^p). The optimal pooling contract is characterized by an incentive power $\beta^p = 1/(1 + \gamma^2)$. The constant $\alpha^p > \alpha^l$ is set so as to satisfy the participation constraint of the high-type workers ($u^h(\alpha^p, \beta^p) = u^h(\alpha^h, \beta^h)$). The firm offering this pooling contract earns a positive profit if it hires a high-type worker, but a negative profit if it attracts a low-type worker (since $\alpha^p > \alpha^l$). Following Rothschild and Stiglitz (1976), we assume that the probability of attracting a given type is equal to the proportion of that type in the market. Let a denote the share of low-type workers in this economy. In the Appendix we show that the expected gain $\Delta\pi^d$ from deviating and offering the pooling contract is equal to

$$\Delta\pi^d = (1-a)(1+\gamma^2)\left(\beta^h - \frac{1}{1+\gamma^2}\right)^2 - 2a\left(\beta^h - \frac{1}{1+\gamma^2}\right)(e^{0h} - e^{0l}). \quad (13)$$

The first term reflects the expected gain from hiring a high-type worker, and the second term the expected loss from hiring a low-type worker. A separating equilibrium exists whenever $\Delta\pi^d \le 0$. Since β^h is independent of the proportion of low-type workers a, the next lemma follows directly:

Lemma 3. *For any parameter constellation* e^{0h}, e^{0l} *and* γ *there exists an* a', $0 < a' < 1$ *such that a separating equilibrium exists whenever* $a \ge a'$ *and does not exist whenever* $a < a'$.

In what follows we assume that the parameter values are such that the separating equilibrium exists.[6]

Welfare Analysis

We define welfare as the sum of the workers' and firms' payoffs,

$$W = au^l + (1-a)u^h + \pi, \quad (14)$$

[6] Alternatively, the existence of an equilibrium can be ensured by refining the equilibrium concept. Riley (1979), Cho and Kreps (1987), Hellwig (1987), Mailath, Okuno-Fujiwarwa, and Postlewaite (1993) and Asheim and Nissen (1996) are contributions to the literature on equilibrium refinements in signaling games. For instance, Riley (1979) develops an equilibrium concept (reactive equilibrium) that constrains the set of admissible deviating strategies that can break an equilibrium. A set of contracts is a reactive (Riley) equilibrium if no other contract exists that remains profitable even after yet another new (deviating) contract is offered. It can be shown that the separating equilibrium derived here is the unique reactive equilibrium for all parameter values.

where π is the firms' expected profit per worker (the measure of workers is normalized to 1). We now consider a social planner who chooses contracts so as to maximize welfare, subject to the workers' incentive-compatibility and individual-rationality constraints.

Suppose first that there is only one worker type in the economy. As wages cancel out in (14), it follows that $W = e_1 + e_2 - C(e_1, e_2)$ or, from (4):

$$W = 2e^0 \frac{\gamma^2}{1+\gamma^2} + \frac{1}{1+\gamma^2}\widetilde{e} - C(\widetilde{e}),$$

where $C(\widetilde{e})$ is given by (A4) in the Appendix. The first-order condition is thus that $C'(\widetilde{e}) = 1/(1+\gamma^2)$. In equilibrium workers choose $\widetilde{e}$ such that $C'(\widetilde{e}) = \beta = 1/(1+\gamma^2)$. Hence, the equilibrium with one worker type is constrained efficient.

With two worker types, the constrained optimal values of $\widetilde{e}^h$ and $\widetilde{e}^l$ maximize W given by

$$\begin{aligned} W = a&\left[2e^{0l}\frac{\gamma^2}{1+\gamma^2} + \frac{1}{1+\gamma^2}\widetilde{e}^l - C^l(\widetilde{e}^l)\right] \\ &+ (1-a)\left[2e^{0h}\frac{\gamma^2}{1+\gamma^2} + \frac{1}{1+\gamma^2}\widetilde{e}^h - C^h(\widetilde{e}^h)\right]. \end{aligned}$$

The first-order conditions are given by $C^{h\prime}(\widetilde{e}^h) = C^{l\prime}(\widetilde{e}^l) = 1/(1+\gamma^2)$. Hence, when types are unobservable, the planner can obtain the optimal allocation of resources by offering a pooling contract with $\beta = 1/(1+\gamma^2)$ and set α in accordance with the zero-profit condition in a pooling equilibrium.

Proposition 2. *The separating equilibrium is not efficient, as the high-type workers are offered contracts that are too high powered (β^h is too high).*

The overall output in the separating equilibrium exceeds the output in the efficient allocation. From a welfare point of view, this additional output comes at too high a cost (in terms of effort). Compared to the equilibrium with observable types, the high-type workers suffer: in order to obtain separation, high-type workers are offered contracts that provide them with overly strong incentives. They receive high salaries, but have to exert excessive effort, which reduces their utility.[7]

Note that Proposition 2 does not depend on our assumption of free entry. Suppose first that firms incur a cost F of opening a job. It is straightforward

[7] When using the refinement concept of "undefeated equilibrium", as in Mailath *et al*. (1993), a pooling equilibrium exists if the fraction a of low-type workers is sufficiently small. The equilibrium pooling contract is such that the utility of high-type workers is maximized. Applied to our setting, this refinement concept implies that the value of β in the pooling equilibrium exceeds the constrained efficient level.

to show that this only reduces the fixed payment α^h and α^l by F units, otherwise leaving the equilibrium unchanged. Suppose then that the number of firms is exogenously given. If there are more firms than workers, the zero-profit condition still applies. If there are more workers than firms, but fewer high-type workers than firms, low-type workers obtain zero utility and α^l adjusts accordingly, while β^l and β^h remain unchanged. Thus, Proposition 2 still applies. Proposition 2 does not break down unless there are fewer firms than high-type workers, in which case only high-type workers are employed and receive zero utility.

As shown, the constrained welfare-maximizing contract is the pooling contract with $\beta = 1/(1 + \gamma^2)$. One way to implement this contract for all workers is to have unions negotiate wages at the industry level.[8] If the union maximizes the expected (or average) utility of its members, i.e., $au^l + (1 - a)u^h$, efficient bargaining results in a single wage contract with $\beta = 1/(1 + \gamma^2)$.[9]

Absent such wage negotiations at the industry level, progressive taxes may improve welfare.

Figure 2 illustrates the effect of a marginal tax on incomes exceeding w^l, the low-type equilibrium income. Note first that an increase in β along the indifference curve (i.e., matched by a decrease in α) leads to higher worker

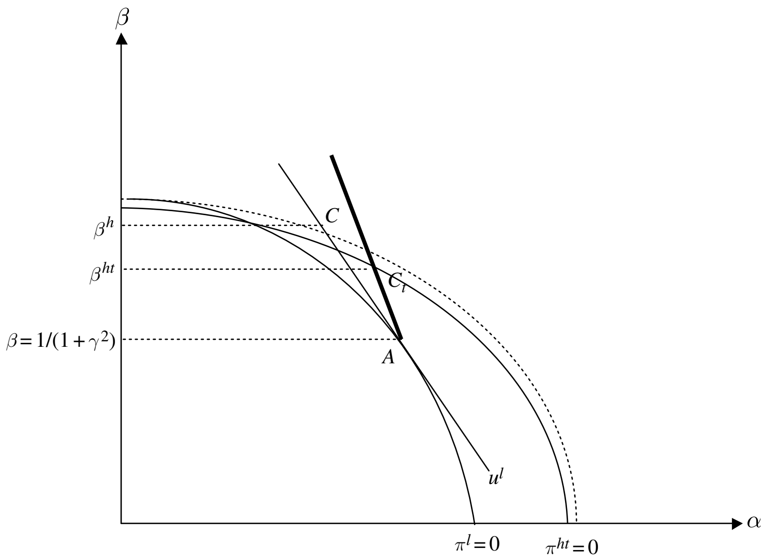

Fig. 2.

[8] This point was suggested to us by an anonymous referee.
[9] Although this contract would be socially efficient, high-type workers lose compared to the separating equilibrium.

income as well as higher effort. The tax affects the low-type workers' indifference curve only for β values above $\beta^l = 1/(1 + \gamma^2)$; the curve therefore shifts outwards (though not necessarily in a linear way). In addition, the iso-profit curve of firms attracting high-type workers shifts inwards, as the tax lowers effort (for a given β). But this implies that the equilibrium value of β falls from β^h to β^{ht}. Thus the excessive incentives for high-type workers are reduced. For small values of the tax rate it thus follows that welfare increases.[10] In the next section we argue that this result does not crucially depend on our restriction to linear contracts.

Of course, progressive taxes have other effects that are not captured in our model. Still, self-selection of high-type workers on high-powered incentive contracts provides a new argument in favor of progressive taxes.

IV. Non-linear Contracts

We know from Section II that (constrained) efficiency requires the contract to be linear on the relevant interval. However, a firm that attracts high-type workers may try to prevent low-type workers from applying to their firm by specifying a very low wage if output falls below a certain threshold $\widetilde{y}^t$. To avoid distortions for the high-type workers, such a threshold must satisfy the condition $\widetilde{y}^t \leq \widetilde{e}^h/(1+\gamma^2) - \overline{\varepsilon}$, where $-\overline{\varepsilon}$ is the lower bound of the support of ε.

Consider a firm that offers a contract $(\alpha^h, 1/(1 + \gamma^2))$, where α^h satisfies the zero-profit constraint with high-type workers. In addition, the firm sets a wage w' if output is less than $\widetilde{y}^t$. Even in the absence of a lower bound on w' (which seems unreasonable as long as slavery is forbidden), this may not be sufficient to satisfy the low-type workers' individual rationality constraint, because a low-type worker can always obtain $\widetilde{y}^t$ by working sufficiently hard.

Define $\Delta e^0 \equiv e^{0h} - e^{0l}$. Our last proposition shows that if Δe^0 is not too large, there exists no contract for high-type workers that implements first best:

Proposition 3. *There exists a value $\overline{\Delta e} > 0$ such that if $\Delta e^0 < \overline{\Delta e}$, there is no contract for high-type workers that implements a constrained efficient effort level for high-type workers and at the same time satisfies the low-type workers' incentive-compatibility constraint.*

The value of $\overline{\Delta e}$ depends on the distribution of ε. In order to do comparative statics, we define $\varepsilon = \sigma\widehat{\varepsilon}$, where $\widehat{\varepsilon}$ is a single-peaked stochastic variable and σ is a shift parameter (noise term). In the Appendix we show that $\overline{\Delta e}$ goes to infinity as σ goes to infinity. Thus, if the support of the error term is

[10] A similar result can be found in Moen (2003), where search frictions lead to excessive wages for high-type workers that can be dampened by marginal taxes.

unbounded, the inefficiency result holds for all parameter values. Note, however, that the value of $\overline{\Delta e}$ remains strictly positive also when σ goes to zero.

Thus, our result that the separating equilibrium is inefficient carries over to the general case with non-linear contracts, although the welfare loss may be smaller. For large differences between worker types, separation may be possible without distorting the incentives for high-type workers. However, this may require an unreasonably low (negative) wage if output falls below $\tilde{y}^t$.

We conjecture that the welfare-improving effect of progressive taxes also carries over to the case with non-linear contracts. Given that first-best contracts do not implement separation, the logic underlying Lemma 2 implies that the equilibrium (if it exists) is a separating equilibrium. In order to obtain separation, firms attracting high-type workers have to lower base pay and increase performance pay (at least on some intervals). Again, progressive taxes relax the incentive-compatibility constraint of low-type workers, reduce the excessive effort among high-type workers, and may thus improve welfare.

V. Concluding Remarks

At the firm level, performance pay may give rise to a positive selection effect, as highly productive workers are more attracted by performance pay than less productive workers. This paper demonstrates that the private gains from selection are not matched by social gains. The equilibrium-incentive contracts provide excessively strong incentives, which induce high-type workers to exert too much effort and increase agency costs resulting from the misallocation of effort. A tax on high incomes may reduce the incentive power of the contract targeted at high-type workers and thereby increase welfare.

The economic significance of this market failure depends on the prevalence of incentive contracts and on the importance of self-selection. Based on the National Longitudinal Survey of Youth, Lazear (2000) reports that the fraction of employees working on piece-rate contracts is quite small, only 3.3 percent among young workers in the US in 1990. Using the same data source, MacLeod and Malcomson (1998) conclude that this number rises to 24 percent when bonuses and commissions are included. According to Millward, Stevens, Smart, and Hawes (1992), the fraction of workers in the UK who received some kind of merit pay was 34 percent in 1990.[11] Still, the relevant fraction may be even higher, as our notion of performance pay also includes both promotions based on performance and fixed salaries based on past performance. Thus, performance pay, broadly interpreted, seems to be common.

[11] Merit pay is defined as payment that depends on subjective judgment of a supervisor or a manager regarding the individual's performance.

How important is the effect of self-selection on contracts? Lazear (1986) was the first to propose that firms offering performance pay can attract high-ability workers. This reason for performance pay has received considerable attention in economic theory, as well as in the management and organization literature; cf. e.g. Lazear (1998), and in the financial press, cf. *The Economist* (1998). As regards the empirical evidence, a number of papers examine the effects of introducing performance pay on worker productivity; see Prendergast (1999) for a survey. In most of these studies, however, the data do not allow the incentive effects and the selection effects to be disentangled.

A notable exception is Lazear (2000), who analyzes the effects of introducing piece-rate payments in one particular firm. Lazear finds that the selection effect accounts for almost 50 percent of the improvement in worker productivity. However, when the firm introduced performance pay, it simultaneously increased average wages (for given performance). Hence, the selection effect may reflect the effect of higher average salaries rather than of performance pay.

Another exception is a study by Sørensen and Grytten (2003). They study differences in performance among Norwegian physicians, who chose to work on either a fixed-wage contract or a performance-based contract. Sørensen and Grytten find that physicians with performance pay have around 35 percent more patients than those with fixed pay and attribute about one-third of the difference to the selection effect. A later study by Grytten, Skau, Sørensen, and Aasland (2004) examines a reform in the Norwegian health system that imposed performance-based remuneration on all physicians. After the reform, the average number of patients per physician was lower than the average number per physician who, prior to the reform, had voluntarily chosen to be on performance pay. The authors estimate that 30 percent of the productivity increase associated with performance pay is due to the selection effect, thus confirming the previous findings in Sørensen and Grytten (2003).

Eriksson and Villeval (2004) report results from a laboratory experiment in which fixed salaries and performance pay were offered by firms and chosen by low-skilled and high-skilled workers. The participants in the experiment were randomly assigned the roles of firm, high-skilled worker, and low-skilled worker. The experiment shows a concentration (self-selection) of high-skilled workers in firms offering performance pay.

Our results crucially depend on some kind of agency costs associated with performance pay. In this paper, the agency cost of performance pay is a misallocation of effort across tasks, and it increases with the incentive power of the wage contract. As actual output grows less than proportionally to measured output (see Section II), firms reduce the variable part and increase the fixed part of the salary compared to the situation with no agency costs. As a result, firms prefer to hire high-type workers to low-type workers on a given contract. If there were no misallocation of effort ($\gamma = 0$), then $\beta = 1$ and $\alpha = 0$ would be optimal, independent of worker type. In this case, the firm is indifferent as to which type of worker it hires, and there is no welfare loss due to excessive effort by high-type workers.

We conjecture that similar inefficiency results obtain with other kinds of agency costs that also reduce the optimal incentive power below 1. For instance, if workers are risk averse and firms risk neutral, the optimal contract trades off incentives and worker insurance, and the resulting contract provides workers with less than full incentives. As a result, the firm prefers to hire a high-type worker to a low-type worker on a given contract. We conjecture that the resulting equilibrium is separating, with excessive effort provision and too little insurance for high-type workers.

To sum up, our analysis suggests that inefficiencies created by competition among firms to attract high-type workers are most prevalent when performance pay is present but when the incentive power of the contracts is reduced due to agency costs.

Appendix

Derivation of Equation (4)

From the definition of $\tilde{e}$, it follows that

$$(e_1 - e^0)(1-\gamma) + (e_2 - e^0)(1+\gamma) = \tilde{e} - 2e^0.$$

Substituting $e_2 - e^0$ by using equation (3) gives

$$(e_1 - e^0)\left[1 - \gamma + \frac{(1+\gamma)^2}{1-\gamma}\right] = \tilde{e} - 2e^0$$

or

$$e_1 - e^0 = \frac{(\tilde{e} - 2e^0)}{2}\frac{1-\gamma}{1+\gamma^2}. \tag{A1}$$

It thus follows that

$$e_2 - e^0 = \frac{(\tilde{e} - 2e^0)}{2}\frac{1+\gamma}{1+\gamma^2} \tag{A2}$$

$$\begin{aligned} e_1 + e_2 &= 2e^0 + \frac{(\tilde{e} - 2e^0)}{2}\left[\frac{1-\gamma}{1+\gamma^2} + \frac{1+\gamma}{1+\gamma^2}\right] \\ &= 2e^0 + (\tilde{e} - 2e^0)\left[\frac{1}{1+\gamma^2}\right] \\ &= 2e^0\frac{\gamma^2}{1+\gamma^2} + \tilde{e}\frac{1}{1+\gamma^2}. \end{aligned} \tag{A3}$$

Deriving Equations (5) and (7)

The costs (for workers) of providing measured effort. Let us denote the cost function by $C(\tilde{e})$. From (A1) and (A2), it follows that

$$\begin{aligned} C(\tilde{e}) &= \left[\frac{\tilde{e}-2e^0}{2}\frac{1-\gamma}{1+\gamma^2}\right]^2/2 + \left[\frac{\tilde{e}-2e^0}{2}\frac{1+\gamma}{1+\gamma^2}\right]^2/2 \\ &= \frac{(\tilde{e}-2e^0)^2}{8}\left[\frac{(1-\gamma)^2}{(1+\gamma^2)^2}+\frac{(1+\gamma)^2}{(1+\gamma^2)^2}\right] \\ &= \frac{(\tilde{e}-2e^0)^2}{4}\frac{1}{1+\gamma^2}. \end{aligned} \tag{A4}$$

Choice of measured effort. We first derive a worker's choice of measured effort $\tilde{e}$ for an arbitrary contract. The worker chooses $\tilde{e}$ to maximize

$$\begin{aligned} u(\varepsilon) &= \alpha + \beta\tilde{e} - C(\tilde{e}) + \varepsilon \\ &= \alpha + \beta\tilde{e} - \frac{(\tilde{e}-2e^0)^2}{4}\frac{1}{1+\gamma^2} + \varepsilon, \end{aligned} \tag{A5}$$

and the solution is

$$\tilde{e} = 2e^0 + 2\beta(1+\gamma^2). \tag{A6}$$

With an optimal contract ($\beta = 1/(1+\gamma^2)$) this expression simplifies to $\tilde{e} = 2e^0 + 2$. Using (A6) and (A3) yields

$$e_1 + e_2 = 2e^0 + 2\beta. \tag{A7}$$

The (expected) indirect utility function. Inserting the optimal value of $\tilde{e}$ (A6) into the maximand (A5), gives

$$\begin{aligned} u &= \alpha + \beta(2e^0 + 2\beta(1+\gamma^2)) - \frac{[2\beta(1+\gamma^2)]^2}{4}\frac{1}{1+\gamma^2} \\ &= \alpha + \beta(2e^0 + 2\beta(1+\gamma^2)) - \beta^2(1+\gamma^2) \\ &= \alpha + \beta 2e^0 + \beta^2(1+\gamma^2). \end{aligned} \tag{A8}$$

For an optimal contract, it follows that u is given by

$$u = \alpha + 2e^0\frac{1}{1+\gamma^2} + \frac{1}{1+\gamma^2}. \tag{A9}$$

Reservation utility u^0 and the value of α. The firm sets α such that $u = u^0$, the equilibrium value of u. From (A8), it therefore follows that the firm sets α such that

$$\alpha = u^0 - \beta 2e^0 - \beta^2(1+\gamma^2). \tag{A10}$$

Given the optimal value of β, $\beta = 1/(1+\gamma^2)$, we thus have that

$$\alpha = u^0 - \frac{2e^0+1}{1+\gamma^2}. \tag{A11}$$

Free entry and the value of α. For any given contract, (A6) and (A3) imply that expected profits are given by

$$\begin{aligned}\pi &= e_1 + e_2 - \beta\widetilde{e} - \alpha \\ &= 2e^0 + 2\beta - \beta(2e^0 + 2\beta(1+\gamma^2)) - \alpha \\ &= 2(1-\beta)e^0 + 2\beta(1-\beta(1+\gamma^2)) - \alpha. \end{aligned} \tag{A12}$$

Thus, the zero-profit condition amounts, for a given contract, to

$$\alpha = 2(1-\beta)e^0 + 2\beta(1-\beta(1+\gamma^2)), \tag{A13}$$

and for the optimal contract to

$$\alpha = 2e^0 \frac{\gamma^2}{1+\gamma^2}. \tag{A14}$$

We have thus derived equation (5). Combining (A14) and (A9) gives equation (7).

Proof of Lemma 1

Let u^{0h} and u^{0l} be the expected income of high-type workers and low-type workers in the pooling equilibrium. Let $\delta > 0$ be arbitrarily small. We want to show that for any δ there exists a k such that the contract $(\alpha^* - k\delta, \beta^* + \delta)$ attracts high-type workers only, i.e., such that $u^h(\alpha^* - k\delta, \beta^* + \delta) \geq u^{0h}$ and $u^l(\alpha^* - k\delta, \beta^* + \delta) < u^{0l}$. From the envelope theorem, it follows that $\partial u^k/\partial\beta = \widetilde{e}^k$, $k = l, h$. From (A6), we know that $\widetilde{e}^h = \widetilde{e}^l + 2(e^{0h} - e^{0l})$ for any given contract, and thus that

$$\frac{\partial u^h}{\partial\beta} = \frac{\partial u^l}{\partial\beta} + 2(e^{0h} - e^{0l}).$$

Thus, increasing β by δ and reducing α by $\widetilde{e}^h\delta$ yields the same expected utility to the high-type workers, while low-type workers are strictly worse off under this modified contract. ■

Derivation of Equation (12)

In the high-type market, we know that the zero-profit condition holds. Let $u^k(\alpha, \beta)$ denote the expected utility of a k-type worker applying for a job with a contract (α, β). Combining (A8) and (A13) implies that a high-type worker applying for a high-type contract may expect a utility

$$\begin{aligned} u^h(\alpha^h, \beta^h) &= 2(1-\beta^h)e^{0h} + 2\beta^h(1-\beta^h(1+\gamma^2)) + \beta^h 2e^{0h} + \beta^{h2}(1+\gamma^2) \\ &= 2e^{0h} + 2\beta^h - \beta^{h2}(1+\gamma^2), \end{aligned} \tag{A15}$$

which (of course) is maximized for $\beta^h = 1/(1+\gamma^2)$. The expected utility of a low-type worker applying for a high-type job is given by (from (A8), then replacing α with (A13))

$$\begin{aligned}u^l(\alpha^h,\beta^h) &= \alpha^h + \beta^h 2e^{0l} + \beta^{h2}(1+\gamma^2)\\ &= 2(1-\beta^h)e^{0h} + 2\beta^h(1-\beta^h(1+\gamma^2)) + \beta^h 2e^{0l} + \beta^{h2}(1+\gamma^2)\\ &= 2(1-\beta^h)e^{0h} + \beta^h 2e^{0l} + 2\beta^h - \beta^{h2}(1+\gamma^2). \qquad \text{(A16)}\end{aligned}$$

The expected utility of a low-type worker applying for a low-type job is (the same formula as (A15) with $\beta = 1/(1+\gamma^2)$ and e^{0l} instead of e^{0h}):

$$u^l(\alpha^l,\beta^l) = 2e^{0l} + \frac{1}{1+\gamma^2}. \qquad \text{(A17)}$$

In order to obtain separation, we must have that

$$u^l(\alpha^l,\beta^l) \geq u^l(\alpha^h,\beta^h)$$

or

$$2e^{0l} + \frac{1}{1+\gamma^2} \geq 2(1-\beta^h)e^{0h} + \beta^h 2e^{0l} + 2\beta^h - \beta^{h2}(1+\gamma^2)$$

or

$$\frac{1}{1+\gamma^2} - [2\beta^h - \beta^{h2}(1+\gamma^2)] \geq 2(1-\beta^h)(e^{0h} - e^{0l}).$$

As the LHS can be written as $(\beta^h - 1/(1+\gamma^2))^2(1+\gamma^2)$, the inequality reduces to

$$\left(\beta^h - \frac{1}{1+\gamma^2}\right)^2(1+\gamma^2) \geq 2(1-\beta^h)(e^{0h} - e^{0l}). \qquad \text{(A18)}$$

In equilibrium, (A18) holds with equality and hence β^h lies in the interval $(1/(1+\gamma^2), 1)$.

Derivation of Equation (13)

When offering a pooling contract, the firm gains if it hires a high-type worker and loses if it hires a low-type worker. Expected profits from hiring a high-type worker are

$$\pi^{ph} = e_1 + e_2 - Ew = e_1 + e_2 - C^h(e_1,e_2) - u^h(\alpha^h,\beta^h).$$

For any given β, (A4) and (A6) imply that the cost of effort is $\beta^2(1+\gamma^2)$. Hence, expected profit is $\pi^{ph} = e_1 + e_2 - \beta^2(1+\gamma^2) - u^h(\alpha^h,\beta^h)$, or using (A7),

$$\pi^{ph} = 2e^{0h} + 2\beta^p - \beta^{p2}(1+\gamma^2) - u^h(\alpha^h,\beta^h),$$

where β^p denotes the incentive power of the pooling contract. Inserting for $u^h(\alpha^h,\beta^h)$ from (A15) gives

$$\pi^{ph} = 2\beta^p - \beta^{p2}(1+\gamma^2) - (2\beta^h - \beta^{h2}(1+\gamma^2)).$$

We now derive the loss if the firm attracts a low-type worker. From (A8) it follows that the difference in the utilities of a high- and low-type worker under the same arbitrarily chosen linear contract, is given by

$$u^h - u^l = 2\beta(e^{0h} - e^{0l}).$$

By definition, the low-type worker is indifferent between the two separating contracts, $u^l(\alpha^l, \beta^l) = u^l(\alpha^h, \beta^h)$. Since the high-type is indifferent between the high-type separating contract and the pooling contract, $u^h(\alpha^h, \beta^h) = u^h(\alpha^p, \beta^p)$, it follows that the low-type obtains a utility of $u^l(\alpha^l, \beta^l) + 2(\beta^h - \beta^p)(e^{0h} - e^{0l})$ in a deviating pooling contract. We thus have that

$$\alpha^p = \alpha^l + 2(\beta^h - \beta^p)(e^{0h} - e^{0l}).$$

Since firms make zero profit in the separating equilibrium, it follows that a firm earns a negative profit of $2(\beta^h - \beta^p)(e^{0h} - e^{0l})$ if it offers a pooling contract and hires a low-type worker. Using that $\beta^p = 1/(1 + \gamma^2)$, equation (13) follows.

Proof of Proposition 3

Suppose the high-type workers are given a contract that induces efficiency, i.e., $\beta^h = 1/(1 + \gamma^2)$ and $\alpha^h = 2e^{0h}\gamma^2\beta^h$. Let $u^{lh}(e^{0h}, e^{0l})$ denote the expected utility of a low-type worker who works under this contract and chooses, for any ε, an effort level that generates exactly the same measurable output as that of high-type workers. As this choice is not optimal for the low-type worker, his pay-off from choosing the high-type's contract is at least $u^{lh}(e^{0h}, e^{0l})$. Due to the envelope theorem, it follows that the derivative of u^{lh} with respect to e^{0h} evaluated at $e^{0h} = e^{0l}$ is equal to $2\gamma^2\beta^h > 0$. Hence, there exists an interval for $e^{0h} - e^{0l}$ such that the individual-rationality constraint of the low-type worker cannot be satisfied.

Proof of Claims Following Proposition 3

As noted in the main text, the firm punishes the worker arbitrarily hard if the measured output falls short of $\tilde{y}^t$. Thus, a low-type worker chooses his first-best effort level if the resulting observed productivity exceeds $\tilde{y}^t$, and the effort level necessary to reach $\tilde{y}^t$ otherwise. The maximum cost of excessive effort (obtained when $\hat{\varepsilon} = -\bar{\varepsilon}$) is equal to $(e^{0h} - e^{0l})^2/(1 + \gamma^2)$ (using (A4) and the high-type's optimal choice of e_1 and e_2).

The probability that the observable output with a first-best effort level of the low-type worker falls short of $\tilde{y}^t$ is given by

$$\rho(\sigma) = \Pr[\hat{\varepsilon} \leq -\bar{\varepsilon} + (e^{0h} - e^{0l})/\sigma].$$

The expected cost of excessive effort is bounded from above by $(e^{0h} - e^{0l})^2/(1 + \gamma^2)\rho(\sigma)$, which goes to zero as σ goes to infinity. Hence, $\overline{\Delta e}$ goes to infinity as σ goes to infinity.

The claim that $\overline{\Delta e} > 0$ as $\sigma \rightarrow 0$ follows directly from the proof of Proposition 3. The derivative of u^{lh} with respect to e^{0h} evaluated at $e^{0h} = e^{0l}$ is equal to $2\gamma^2\beta^h > 0$.

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