Education, Ranking, and Competition for Jobs

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I study workers' incentives to invest in general human capital (education) in the presence of search-induced unemployment. Workers queue for jobs, and firms prefer to hire the most productive applicants because of rent sharing. As a result, an unemployed worker's ranking relative to other job seekers will influence his job-finding rate. This creates a "rat race," where workers invest in education partly in order to achieve a better ranking. In equilibrium, identical workers may have incentives to diversify in terms of education, and the investments in education may exceed the socially optimal level.

I. Introduction

In a situation with unemployment, what are the incentives for workers to invest in general human capital, that is, in education? In this article, I argue that one part of the private gains from education may be that it reduces the probability of being unemployed. I show that this is not matched by a social gain. As a result, the underinvestment in human capital usually associated with unemployment is reduced or eliminated, and we may actually have *over*investment in human capital compared to the socially optimal level. Becker (1964) was one of the first to consider

This article is a revised version of a chapter in my Ph.D. thesis at the London School of Economics. I would like to thank my supervisor Chris Pissarides for encouragement and support and Christian Brink, Rolf Golombek, Steinar Holden, Kalle Moene, Dale Mortensen, Christian Riis, and, in particular, Åsa Rosén for encouragement and valuable comments. Finally, I would like to thank participants at the Society of Labor Economists meeting in Washington, D.C., May 1997, for helpful comments. Financial support from the Research Council of Norway is gratefully acknowledged.

[*Journal of Labor Economics*, 1999, vol. 17, no. 4, pt. 1]

0734-306X/1999/1704-0001$02.50

education as an investment in a person's human capital. Becker argued that if the labor market is competitive and education is not subsidized, a worker will invest a socially optimal amount in education. Becker's reasoning is simple: in a competitive labor market, any gain from education in terms of higher productivity will accrue to the worker in the form of higher wages. Thus, in a competitive market with no education subsidies, a worker both carries the full costs of and receives the full share of the return on his investments. It follows that there are no external effects associated with education, and the investments in human capital are socially optimal.

Becker's results contrast with those of Acemoglu (1996) and others, who argue that the existence of unemployment leads to underinvestment in education. Their reasoning is based on a hold-up argument: the existence of unemployment and search costs creates room for wage bargaining and rent sharing between worker and employer. The more productive the worker, the more rent there will be to share. This then means that a worker who invests in education receives less than a full share of the return. There is a positive externality from education on firms, and as a result there is underinvestment in education.

This argument, however, rests on the assumption that the arrival rate of job offers is independent of a worker's education. Empirical studies indicate that the unemployment rate, which is closely linked to the average arrival rate of job offers, is lower for workers with high education than for those with low education (see, e.g., Björklund and Eriksson [1996] in regard to the Nordic countries). Furthermore, the implications of this assumption for the relationship between overall unemployment in the economy and the incentives to invest in education are not convincing. If the arrival rate of job offers is in fact independent of the level of education, we would expect a negative relationship between the steady-state unemployment rate in the economy and the incentives to invest in education. The reason is twofold: first, a high level of unemployment implies that the human capital remains idle a larger proportion of the time. Second, a high unemployment rate weakens the workers' bargaining position and thus reduces the share of the return from education allocated to the worker.[1] However, a negative relationship between unemployment

[1] A high unemployment rate may reduce the costs of investing in education as it reduces the shadow price of a worker's time. However, within a standard matching model where exit rates from unemployment are independent of education level, there still is an inverse relationship between the steady-state unemployment rate and incentives to invest in education as long as there are some direct costs of education. On the other hand, the fact that a high unemployment rate reduces the costs of education may explain why a *temporary* increase in unemployment may temporarily increase investments in education.

and education is not in line with actual experience in Europe in recent decades. In the 1970s, Western Europe experienced a marked increase in unemployment, and unemployment has stayed high there since then. During the same period, educational levels have soared.[2] The Scandinavian countries had the same experience in the late 1980s and early 1990s. The model that I propose is consistent with these facts.

I assume that a firm will often have more than one applicant for each vacant position. As wages are determined by rent sharing, the firm is most likely to hire the worker with the highest productivity. As a result, a worker's productivity relative to other unemployed workers—his ranking—matter for his economic well-being. By investing in education, a worker improves his ranking, and this in turn increases his incentives to invest in human capital. Since ranking is a relative measure, however, a worker's investment in education now has a negative external effect on other workers, reducing their rankings. This is what I call the "rat-race" element of education (in the terminology of Akerlof 1976), as it reflects that a part of the gain from education is to jump forward in the job queue. I show that, when this rat-race effect is taken into account, identical workers may have an incentive to diversify by choosing different levels of investment in education. Furthermore, not only does the ranking effect reduce the underinvestment due to rent sharing between workers and firms, it may actually dominate this positive external effect (which gives rise to the ranking effect in the first place) and lead to overinvestment in education for some parameter values.

I begin by deriving the matching technology. In the literature on search and matching, the matching process is almost always assumed to be sequential, so that workers arrive at vacancies one at a time. As a result, a worker never experiences "face-to-face" competition with other workers. Recently, some authors (Blanchard and Diamond 1994; Rosén 1997) have argued that this description of the matching process is inappropriate. When a firm announces a vacancy, it may obtain zero, one, or any number of applicants, and the difficulty for an unemployed worker is generally not to find an advertisement for a job he or she is qualified for but to emerge as the preferred candidate among the applicants for that job slot. I apply a matching function similar to the one employed in Blanchard and Diamond (1994). However, I distinguish between two different sources of friction in the market: time spent locating a vacancy and the average number of applicants for each advertised job. I can then vary the

[2] The average unemployment rate in the European Union was 2.6% in 1970, 6.4% in 1980, and 8.4% in 1990. The number of university students as percentage of the population who were 5–24 years old was 4.7% in 1975 (comparable numbers for 1970 were not available), 5.3% in 1980 and 8.3% in 1990 (Basic Statistics of the European Union 1993).

average number of matches for a given aggregate matching rate. This I call the *degree of competition for jobs.*

When a firm with a vacancy obtains several applicants, I assume that the firm first selects one of them and then bargains with that applicant individually over wages. The firm hires the worker for whom the difference between productivity and wage is the greatest. I show that for relatively low degrees of competition for jobs, firms prefer high-productivity workers to low-productivity workers. In this case, I find that otherwise identical workers diversify in terms of education. However, given a sufficiently high degree of competition for jobs, this is not necessarily the case, as productivity differences between workers are exactly offset by wage differences. In this case, the distribution of investments proves to have a mass point.

As mentioned above, investments in general human capital yield both positive externalities on future employees and negative externalities on other job seekers. When there is little job competition, ranking is not important, and the positive externality dominates. We thus have underinvestment in education. On the other hand, if the job competition is sufficiently strong, wages increase as fast as productivity on the margin, so the positive externality on firms is eliminated. What remains is the negative ranking externality on other firms, and this leads to overinvestment in education.

The outline of the article is as follows: after the introduction, I first derive the matching technology in Section II and then present the rest of the model in Section III. In Section IV, I characterize equilibrium, while the welfare analysis follows in Section V. In Section VI, I discuss my assumptions about wage determination, the matching process, and the education technology in some detail before I conclude in Section VII.

II. Matching Technology

The matching technology is important for the analysis. In this section, I therefore focus on the process of matching a given set of unemployed workers with a given set of firms with vacancies. The economic environment in which this happens is presented in the next section. A natural starting point when deriving a specific matching function is the urn-ball process, first demonstrated by Butters (1977) and Hall (1979). In the urn-ball process, balls are allocated to urns at random, the same probability there being always for each urn to receive each ball. An urn may thus end up containing zero, one, or any number of balls. The number of balls in any urn is binominally distributed, with the total number of balls and the inverse of the total number of urns in the market as parameters. From mathematical statistics, we know that this binominal distribution converges to the Poisson distribution as the numbers of urns and balls go to infinity, with the fraction of balls to urns being the Poisson parameter.

Applied to the labor market, the balls can be interpreted as unemployed workers sending off an application at random, while the urns are interpreted as the firms (see Butters [1977] for details). The urn-ball process is a particularly convenient tool for describing the labor market when workers are heterogeneous, as it makes it possible to specify a worker's exit rate from unemployment as a function of his characteristics, as noted by Blanchard and Diamond (1994). I model the matching process somewhat differently than Blanchard and Diamond do, for two reasons: to obtain time-invariant arrival rates of trading partners both for firms and workers and, more important, to be able to parametrize the degree of job competition.

Consider a situation where there is a continuum of workers searching for a continuum of vacant jobs. The measures of workers and jobs are denoted by u and v, respectively. I assume that the market is in steady state, so that the number of workers finding a job is equal both to the number of new workers and new vacancies entering the market at any point in time. Hence u and v are constant over time. I start out by dividing each unit of time into n periods and obtain a continuous-time matching function as the limit when n goes to infinity. In each period, a fraction γ/n of the vacancies are advertised for one period only, and a fraction a/n of the workers respond and apply for one of the vacancies at random. Let $\lambda = au/\gamma v$ The parameter λ then gives the ratio of applications sent to jobs advertised in each period and is independent of n.

Let us start with a situation where all employers have the same strict ranking over the set of workers.[3] In the appendix, I show that the arrival rate of job offers to a worker with ranking Π, p_r, then is given by

$$p_r = ae^{-(1-\Pi)\lambda}. \tag{1}$$

Note that the arrival rate of job offers to a worker is an increasing function of his ranking, Π. The arrival rate is a when $\Pi = 1$; this is independent of λ, the average number of applicants per vacancy. The most preferred worker gets any job he or she applies for and is not affected by congestion effects from other workers. Note that for a given value of Π, the arrival rate of job offers depends on a and λ only, while the parameters u, v, and γ only influence p_r through λ.

Define the parameter c as the average arrival rate of jobs to workers, that is, the aggregate number of matches per unit of time divided by the number of unemployed workers. In the appendix, I show that p_r from (1) can be written as

[3] By strict ranking I mean that a firm is never indifferent between two applicants.

$$p_r(\Pi; \lambda, c) = \frac{c\lambda}{1 - e^{-\lambda}} e^{-\lambda(1-\Pi)}. \tag{2}$$

In this equation, λ expresses the number of workers per advertised vacancy for a given level of friction, that is, for a given average job-finding rate. I will refer to λ as the *degree of competition for jobs.* Below, I analyze how the equilibrium of the model depends on λ. Equation (2) shows how an increase in the competition for jobs changes the job-finding rate for workers, depending on their rankings when the average job-finding rate is held constant. Intuitively, an increase in the average number of job applicants can be expected to reduce the probability of getting a job (this is formally shown in the appendix), and in order to keep c constant, such an increase must be followed by an increase in a, the frequency at which workers send off applications.[4] It is now easy to show (see the appendix) that the higher the degree of competition for jobs, the more important is ranking. As the competition for jobs approaches zero, ranking does not influence the job-finding rate. As the degree of competition for jobs goes to infinity, a worker at the lower end of the distribution never finds a job, while a worker at the top of the distribution will find a job immediately.

Finally, suppose that the distribution of workers has a mass point at the top of the distribution and that all firms, if they receive more than one applicant from this mass point, choose one of them randomly. Further, suppose that a fraction ξ of the unemployed workers belong to this mass point. In the appendix, I show that the arrival rate of jobs for these workers is given by

$$p_a(\xi; \lambda, c) = \frac{c}{\xi} \frac{1 - e^{-\xi\lambda}}{1 - e^{-\lambda}}, \tag{3}$$

which, of course, is equal to c when $\xi = 1$.

III. The Model and Asset Value Equations

A. The Model

Except for the matching technology, our model is standard, following the lines set out by Mortensen (1986), Pissarides (1990), and others. The labor market consists of risk-neutral workers and firms. Workers exit the market at a constant rate s, and new workers enter the market at the same speed, so that the number (measure) of workers remains constant. There is no entry or exit of firms, so the number of firms is constant and

[4] In order to obtain an increase in both $\lambda = au/\gamma v$ and a, it follows that γ must increase (for given values of u and v).

exogenous. Each firm has a limited number of job openings, normalized to one. A firm is thus either searching for a worker or having one employee and producing. I assume that all firms are identical. When a worker is hired by a firm, he stays with that firm until he exits the market.[5]

At the outset, all workers are identical. Entering workers first invest in education. For simplicity, I assume that the costs of a certain level of education can be conceptualized by an exogenous, pecuniary variable k—the amount invested in education. After finishing their education, the workers have productivity $H = H(k)$.[6] In order to ensure internal solutions, I assume that $H' > 0$, $H'' < 0$, $H(0) = 0$, $H'(0) = \infty$, and $H'(\infty) = 0$. After the investment in education, the worker in question enters the labor market as unemployed. I assume that the search intensities of both workers and firms are exogenous. This assumption is discussed in Section VI.

B. Asset Value Equations

Workers invest in education so as to maximize their expected lifetime income. In this subsection, I derive the expected discounted value $U(k)$ of an unemployed worker's future income, or his "asset value," as a function of his investment k in education. I also derive the asset values for firms with an employee and for firms with a vacancy. For simplicity, I assume that a worker receives no unemployment benefits, and I normalize the worker's income after leaving the market to zero. The Bellman equation determining $U(k)$ is then

$$rU(k) = p[W(k) - U(k)] - sU(k)$$

or

$$U(k) = p\,\frac{[W(k) - U(k)]}{r + s}, \tag{4}$$

where $W(k)$ is the expected discounted value of an employed worker's future income flow, $\Pi(k)$ is his ranking, and $p = p(\Pi(k))$ is the arrival

[5] I believe this is an innocuous simplification. If workers undertake an on-the-job search in order to get a better job, a worker's ranking (which is influenced by his education) matters *both* for unemployed and employed job seekers.

[6] I assume that investment in education is a continuous variable. Since a worker can choose different levels of education, attend schools or take courses with different qualities, and choose different levels of commitment and effort in his or her studies, I think this is a reasonable approximation. The importance of having a continuous set of education levels is studied in Sec. VI.

rate of job offers, which may depend on the worker's ranking.[7] Note that the ranking $\Pi(k)$ also can be interpreted as the cumulative distribution function over k. The equation states that the interest rU on the "asset" must be equal to the return from the job search. The latter is equal to the return from the search activity, $p[W(k) - U(k)]$, minus the expected loss of income associated with leaving the market, $-sU$. Similarly, the asset value, $W(k)$, for an employed worker is given by the equation

$$W(k) = \frac{w(k)}{r + s}, \tag{5}$$

where w denotes the wage he receives as employed.

Let V and J denote the expected discounted incomes for a firm with a vacancy and for a firm with a worker, respectively. First, look at a firm that has an employee. If this employee exits the market, the job becomes vacant, and the firm suffers a "capital loss" $J - V$. The asset value equation determining $J(k)$ is thus given by $rJ(k) = H(k) - w(k) - s(J - V)$, or

$$J(k) = \frac{H(k) - w(k) + sV}{r + s}. \tag{6}$$

Finally, define the *match surplus* $S(k)$ as $S(k) = J(k) + W(k) - U(k) - V$. By using (5) and (6), it follows that $S(k)$ is given by

$$\begin{aligned} S(k) &= \frac{H(k) + sV}{r + s} - U(k) - V \\ &= \frac{H(k) - rV}{r + s} - U(k). \end{aligned} \tag{7}$$

Now we can analyze the wage determination process. If a firm receives exactly one application, it will always hire that applicant.[8] If there are several applicants, I assume that the firm first takes its first-choice applicant and then starts to bargain with that applicant over pay. As is standard in the matching literature, I assume that the wage is given by the Nash sharing rule, with the agents' outside options being their disagreement (or

[7] In the section on matching technology, the arrival rate of job offers was written as $p_r(\Pi)$. However, we do not know yet whether or not all the firms rank their applicants, so I will write the arrival rate of job offers by $p(\Pi)$ (omitting the subscript r).

[8] It is never optimal for a worker to choose an education level so low that no firm will hire him.

threat) points. Clearly, for the worker, the outside option is to be unemployed. More problematic is the relevant outside option for the firm when there are several applicants. One possibility is that the outside option for the firm is to call in its second-choice applicant and start to bargain with him. However, the resulting bargaining outcome will not be renegotiation proof. Sooner or later, and relatively quickly compared to the duration of the employment relationship, the other applicants will leave the scene. At this point, the outside option for the firm becomes to have a vacancy, and the worker will therefore renegotiate the wage. It may therefore be more reasonable to assume that the firm's outside option is to have a vacancy. For an extended treatment of wage determination, I refer to Section VI.

I thus assume that the outside option for the firm is to have a vacancy and to be searching for an employee. It then follows from the Nash sharing rule that the expected discounted future income for an employed worker with education k is $W(k) = U(k) + \beta S$, where S is the match surplus defined above, and β is the (exogenous) share of the match surplus allocated to the worker.[9] From this and equation (7), it follows that $W(k) - U(k) = \beta S(k) = \beta[H(k) - rV]/(r + s) - \beta U(k)$. Substituting this into the asset value equation (4) yields

$$U(k) = p\,\frac{\beta[H(k) - rV]/(r + s) - \beta U(k)}{r + s}.$$

Solving for U gives

$$\begin{aligned} U(k) &= \frac{p\beta[H(k) - rV]}{(r + s)(r + s + p\beta)} \\ &= A(p)\,\frac{H(k) - rV}{r + s}, \end{aligned} \tag{8}$$

where $A(p) = p\beta/(r + s + p\beta)$ and where $p = p(\Pi(k))$. Here $H(k) - rV$ is the production per unit of time less the flow equivalent of the firm's outside option, while $r + s$ is the relevant discount factor. The proportionality factor $A(k)$ is less than one, for two reasons: first, because it takes valuable time for an unemployed worker to find a job, and second, because the employer receives an income greater than V.

Finally, the equilibrium of the model also depends on the value of a

[9] As in the rest of the literature on bargaining and matching, I assume that β is constant and exogenous. Note, however, that an increase in p, the arrival rate of job offers, still increases the share of the return of human capital investments allocated to the worker since it increases the value of the worker's outside option. In the limit, as $p \to \infty$, the worker gets a full share of the return on his education.

vacancy, V. To derive the expression for the value of a vacancy given by (11), first note that it follows from the Nash solution that $J(k) = V + (1 - \beta)S(k)$. Replacing S by using (7) then gives

$$J(k) - V = (1 - \beta)\left[\frac{H(k) - (r + s)U}{r + s} - \frac{r}{r + s}V\right]. \tag{9}$$

In the appendix, I show that, in situations where a firm receives several applicants, the education of the most productive applicant is distributed according to the distribution function

$$F(k) = \frac{e^{-\lambda[1-\Pi(k)]} - e^{-\lambda}}{1 - e^{-\lambda}}. \tag{10}$$

Let $\hat{F}$ denote the probability that a firm that announces a vacancy obtains at least one applicant. Define $q = \gamma\hat{F}$ as the arrival rate of workers to a firm with a vacancy (equal to the frequency at which the vacancy is announced times the probability of getting at least one applicant when announcing it). Assume also that the firm's search costs are zero. In the appendix, I show that the value of a vacancy then can be written as

$$V = q\,\frac{(1 - \beta)\displaystyle\int_k (H(k) - (r + s)U(k))dF}{(r + s)(r + q(1 - \beta)r/(r + s))}. \tag{11}$$

IV. Equilibrium

In this section, I first characterize the behavior of workers and firms. Then I derive the equilibrium of the model.

All workers choose education so as to maximize lifetime income $U(k) - k$. Thus, all workers choose k so that

$$U'(k) = 1. \tag{12}$$

A firm's outside option does not depend on the productivity of the worker it is currently bargaining with. Taking derivatives of (8) therefore gives (since $dp/dk = p'(\Pi)\pi(k)$, where $\pi(k) = \Pi'(k)$ is the density of k)

$$U'(k) = A\frac{H'(k)}{r + s} + A'(p)p'(\Pi)\pi(k)\left[\frac{H(k) - rV}{r + s}\right]. \tag{13}$$

The return on education consists of two parts, represented by the two terms in equation (13). The first term shows the direct effect of the worker becoming more effective (for a given p). Of this, a share less than one is allocated to the worker through higher wages, while the rest is allocated to the firm. The second term reflects the fact that higher investments in education improve a worker's ranking and thereby his job-finding rate. This increases the worker's lifetime income both because it reduces his unemployment spell and because it increases his outside options and thus his bargaining position in the bargaining game with the firm. This is the rat-race effect of education, as it does not result in a social gain; instead, it both reduces the positive externality on the firm and, since ranking is a relative measure, produces a negative externality on other workers.

Suppose that firms, when choosing among agents with investments in education that lie within a certain interval, strictly prefer the applicant with the highest education. In this case, I say that firms *rank* their applicants on this interval. It follows that firms rank their applicants if and only if $J(k)$ is monotonically increasing. From (9) we have

$$J'(k) = (1 - \beta)\left[\frac{H'(k)}{r + s} - U'(k)\right]. \tag{14}$$

We can show that $J'(k) \geq 0$ for all k. Suppose that this is not the case, so that $J'(k) < 0$. Then all firms would prefer a worker with low productivity over a worker with high productivity. But then the arrival rate of job offers would be decreasing on this interval, and from (13), we would have that $U'(k) < H'(k)/(r + s)$ (since $A < 1$). From (14) it then follows that $J'(k) > 0$, which is a contradiction. The fact that $J'(k) \geq 0$ implies that (from [14])

$$U'(k) \leq H'(k)/(r + s). \tag{15}$$

On intervals where $J'(k) = 0$, firms are indifferent as to whom to hire, and I say that they *randomize* when choosing a worker. Suppose that the distribution Π has a mass point, say at k'. Then firms randomize on an interval around k'. To show this, suppose firms ranked their applicants on this interval. Then an arbitrarily small increase in investments k from just below to just above the mass point would lead to a discrete jump in the arrival rate of job offers to the worker in question and thus also in that worker's net present income U. This would induce a discrete jump in the wage. But then a firm would be better off selecting the worker just below the mass point than the one just above the mass point, which is a contradiction.[10]

Suppose that some workers invest an amount k that lies in an interval at which firms do randomize. Then k must satisfy both $U'(k) = H'(k)/(r + s)$ (from [14]) and the first-order condition $U'(k) = 1$. Denote the corresponding level of education by k^*. The value of k^* is thus given by the equation

$$H'(k^*) = r + s. \tag{16}$$

Since $H'(k)$ is strictly concave, it follows from (15) and (16) that $U'(k) < 1$ for all $k > k^*$, and no workers invest more than k^* in equilibrium. In the appendix, I show that, if $k < k^*$ is in the support of Π, firms strictly prefer the worker with education k^* to the worker with education k. Lemma 1 summarizes our findings so far.

LEMMA 1. In equilibrium, the following holds:

1. No worker invests more than k^*.
2. If the distribution $\Pi(k)$ has a mass point, it is at k^*.
3. On all parts of the support of Π, firms rank their applicants.

Recall that, although firms rank their applicants on the support of Π, we know that they may randomize between applicants with education on an interval around k^*. The point is that in equilibrium, all workers who invest in this interval invest k^*.

The continuous part of the distribution, where there are no mass points, I will refer to as the tail, and in the appendix, I show that the tail is connected. Lemma 1 implies that $p(\Pi) = p_r(\Pi)$ given by equation (2) on the tail, while it is given by $p = p_a(\xi)$ from equation (3) at k^*, where ξ is the probability mass concentrated at k^* (i.e., the proportion of unemployed workers who have invested an amount k^* in education).

Suppose now that the distribution has a tail, and define k^0 as the minimum of that tail. It follows that k^0 is given by

$$k^0 = \operatorname*{argmax}_k U(H(k), p_r(0)) - k. \tag{17}$$

From (8) it follows that

[10] However, even if firms randomize, the arrival rate of job offers is increasing in k. If not, it follows from (14) that $J'(k) > 0$, since we then have $(r + s)U'(k) = AH'(k) < H'(k)$ (from [13]). Thus, equilibrium hiring strategies must be such that the probability that a high-productivity worker will be chosen is greater than the probability that a low-productivity worker will be chosen also on intervals where firms randomize.

$$A(p_r(0))H'(k^0) = r + s. \tag{18}$$

Obviously, the minimum of the support cannot be below k^0 since workers at the minimum then could improve their situation by increasing k up to k^0. But the minimum cannot be above k^0 either. Suppose it were, and denote the minimum of the support by k'. Since the arrival rate of jobs at k' is still $p_r(0)$, it follows from the very definition of k^0 that the worker in question would be better off by reducing his investments to k^0—which is a contradiction. Since $A < 1$, it follows from (16) and (18) that $k^0 < k^*$ for all λ. Let $U^0 \equiv U(H(k^0), p_r(0))$. It follows that the worker at the infimum of the distribution invests k^0 in education and obtains a payoff $U^0 - k^0$.

Suppose first (in contrast with what is assumed in the rest of this article) that there is no competition for jobs, that is, that $\lambda = 0$. Then the probability rate of finding a job is independent of k, and $p(\Pi) = c$ for all Π. In this case, all workers will choose k according to (17) with $p_r(0)$ replaced by c. The workers will have no incentive to increase their rankings, as this will not increase the arrival rate of job offers. We are thus back to the standard matching model.

Suppose then that λ is strictly positive. Then we know from lemma 1 that there is no mass point at k^0. Furthermore, if λ is sufficiently small, the difference between $p_r(0)$ and $p_a(0) = p_r(1)$ will also be small, and a worker will be strictly better off investing k^0 rather than k^*. Thus the entire probability mass is continuously distributed below k^*. Since $U'(k) = 1$ on the entire support of Π, we have that $U(k) = U^0 + k - k^0$. We can thus derive the distribution of k explicitly (given V) by inserting the algebraic expressions for $U(k)$ and $p_r(\Pi)$ (from [8] and [2]). In the appendix, I show that $\Pi(k)$ is given by

$$\Pi(k) = 1 - \ln\left[\frac{c\lambda\beta}{1 - e^{-\lambda}} \frac{H(k) - rV - (r+s)(U^0 + k - k^0)}{(r+s)^2(U^0 + k - k^0)}\right] \Big/ \lambda, \tag{19}$$

where k^0 still is defined by (17) and V is given by (11). The top of the support, k^1, is defined by the equation $\Pi(k^1) = 1$.

To see why the investment distribution obtained is not degenerated, suppose all workers did invest k^0. Suppose also, hypothetically, that firms ranked their applicants around k^0. Then an arbitrarily small increase in k from k^0 would imply a discrete increase in the job-finding rate for the worker in question since he would be preferred above all other applicants for all jobs. This is not consistent with equilibrium. Now we know that, if there were a mass point at k^0, firms *would* randomize between workers with different education levels, and we would thus have $w'(k) = H'(k)$ around k^0. However, from the definition of k^* and the fact that $H(k)$ is strictly concave, we know that $U'(k) > 1$ on intervals below k^* at which firms randomize. From this it follows that the workers would be better off increasing their investments in education, and having a mass point at k^0 is therefore not consistent with equilibrium.

For higher levels of job competition, the arrival rate of job offers to the least efficient workers is lower. It follows from (17) that k^0 is lower, and the worker at the infimum of the support is worse off than when the job competition level was lower. For the worker on the supremum of the distribution, on the other hand, a higher value of λ means that the arrival rate of job offers will be higher, and this will tend to increase his expected income. As a result, the costs in terms of education of getting on the top of the distribution is higher for high than for low values of λ, and at some point the supremum k^1 reaches k^*.[11] This happens at the point λ_1, where $U(k_0;\ \lambda_1) - k^0$ is exactly equal to $U(k^*;\ \lambda_1) - k^*$ provided that the $\Pi(k^*) = 1$. The point λ_1 is thus given by the equation

$$A(p_r(1;\lambda_1))\,\frac{H(k^*) - rV}{r+s} - k^* = A(p_r(0;\lambda_1))\,\frac{H(k^*) - rV}{r+s} - k^0. \quad (20)$$

For values of λ above λ_1, k^1 does *not* increase above k^* since, as we have seen, $U'(k) < 1$ for all $k > k^*$. Instead, a mass point is established at k^* around which the firms randomize. The probability mass at ξ of workers at k^* adjusts so that the expected income $U(k^*) - k^*$ is exactly equal to the expected income at the infimum of the support. Since $U(k^*)$ is strictly decreasing in ξ (as shown in the appendix), it follows that ξ is unique.

However, as λ increases further, $p_r(0)$ still falls, and at some point a worker will be better off investing k^* rather than k^0, even when $\xi = 1$. Since the arrival rate of job offers at the mass point then is equal to c, λ_2 must be given by

$$A(p_a(1;\lambda_2))\,\frac{H(k^*) - rV}{r+s} - k^* = A(p_r(0;\lambda_2))\,\frac{H(k^*) - rV}{r+s} - k^0. \quad (21)$$

Since the arrival rate of job offers is independent of λ for $\lambda > \lambda_2$, it follows that a further increase in the degree of competition for jobs will have no

[11] Note that we cannot rule out that the steady-state value k_1 may increase nonmonotonically in λ. The reason is that an increase in λ increases the average number of applicants for each job (while the average speed at which a vacant job is filled remains constant as γ adjusts), which ceteris paribus increases V. This hurts high-ranked workers more than low-ranked workers, as the former are employed a larger proportion of the time than are the latter. This tends to decrease the value of having a high ranking.

effect on the equilibrium. The results obtained so far are stated in proposition 1. The remaining part of the proof is given in the appendix.

PROPOSITION 1. Suppose the expected discounted income, $U(k^*) - k^*$, is positive when the probability mass at k^*, ξ, equals one (a restriction on primitives is given in the appendix). Then a unique equilibrium of the model exists, and the associated distribution function $\Pi(k)$ will take one of the following forms:

- Π is continuously distributed on an interval $[k^0, k^1]$, where k^0 is determined by equation (17), $\Pi(k)$ by equation (19), and k^1 by the requirement that $\Pi(k^1) = 1$.
- $\Pi(k)$ has a mass point at k^* with mass $\xi < 1$ and a continuous tail with the same distribution function as above on an interval $[k^0, k^1]$, where k^1 is determined such that $\Pi(k^1) = 1 - \xi$.
- Π consists of a singleton at k^* with probability mass 1.

Furthermore, for sufficiently small values of λ, the entire distribution $\Pi(k)$ will be below k^*, and as $\lambda \rightarrow 0$, the distribution converges to k^0. On the other hand, for sufficiently large values of λ, the entire distribution will be concentrated at the mass point at k^*.

V. Welfare Analysis

In this section, I first derive the socially optimal distribution of k as a function of λ. Then I compare it with the market solution.

To calculate the optimal allocation of resources, I follow the standard route in economics and assume that all the decisions previously made by the agents in the economy now are made by a benevolent social planner, otherwise facing the same technology and information constraints as the agents. I thus assume that the planner faces the same information constraints as the workers regarding which workers apply to which firms and thus cannot coordinate the applications sent. Hence, the planner determines investment levels and hiring strategies, taking as given the matching technology and the exogenous variables in the model.[12]

The first thing to note is that the socially optimal hiring rule is to always employ the worker with the highest education, simply because this person will be the most productive one. Less obvious is that the socially optimal distribution Π^* has no mass point. To see this, suppose Π^* had a mass point at k'. Welfare will then not be influenced by who is

[12] Note that there exist both private and public employment agencies that facilitate search. An interesting issue is to what extent the private and social incentives to establish such agencies coincide. This question, however, lies outside the scope of this article.

employed when choosing between workers with education k'. Let us therefore divide the set of agents investing k' into two subsets, B and B^c, both with strictly positive probability mass, and let our hiring rule be to always choose applicants from B rather than from B^c. Then workers in subset B will move more quickly into jobs than those in subset B^c, and one can increase welfare by letting workers from B invest more in education than workers from B^c, which is a contradiction. The planner will thus take advantage of the fact that the matching technology allows for discrimination between workers so that the most productive workers will be employed more quickly than the others.

Define $k^s(\Pi^*)$ as the socially optimal value of k for a worker with ranking Π^*. Then $k^s(\Pi^*)$ solves the problem

$$\max_{k^s} U(k^s, p_r(\Pi^*)) - k^s, \tag{22}$$

where Π^* is held constant and where β is set equal to one so that U captures the entire return on investments in education. The value $k^s(\Pi^*)$ thus maximizes the expected income of a worker, given that he receives the entire return and given his ranking. Thus, from (8) it follows that

$$\begin{aligned}\frac{p_r(\Pi^*)}{r + s + p_r(\Pi^*)} H'(k^s) &= A^*(\Pi^*) H'(k^s) \\ &= r + s.\end{aligned} \tag{23}$$

If we compare this with equation (18), which determines k^0, it follows that $k^0 < k^s(0)$ (since $A(p_r(0))$ in [18] is less than $A^*(0)$). Thus, for all values of λ (including $\lambda = 0$) the socially optimal distribution will be situated strictly above k^0. Furthermore, since k^* is defined by the equation $H'(k^*) = r + s$ and $A^* < 1$ for all Π^*, it follows that the entire socially optimal distribution will be situated strictly below k^*. When $\lambda \to 0$, we know that the distribution $\Pi(k)$ generated by the market collapses to k^0. On the other hand, for sufficiently high values of λ, the market distribution is a singleton at k^*. We have thus shown the following proposition.

PROPOSITION 2. When there is sufficiently weak competition for jobs, all workers will underinvest in education. With sufficiently intense competition for jobs, all workers will overinvest in education.

The intuition is as follows: as noted, there are in this model two types of externalities associated with investments in education. First, education has a positive impact on future employers, as they may receive a part of the return from education in the form of higher profits. Second, education increases a worker's ranking, thereby reducing the ranking for some other workers and their probability of finding a job. Increasing one's own ranking is beneficial from an individual point of view, but it has no positive impact on overall welfare, as one person's gain is another person's loss. This is the rat race effect of education. From the definition of k^0 given by (17), it follows that workers at the bottom of the distribution will not take into account the effect of higher education on their ranking. Hence, the negative externality on other agents is absent at this point. We are left with only a positive externality from education; as a result, the level of investment is less than optimal at the lower end of the distribution. More generally, if λ is low, the importance of ranking will be low, and the investments in education will be less than optimal. At k^*, the positive externality for future employers is eliminated, as any increase in productivity is fully reflected in the wage offered. Since the arrival rate of job offers increases with a worker's ranking also at intervals where the firms randomize (see n. 10 above), the negative ranking externality prevails. Thus at the mass point, the level of education undertaken is above the socially optimal level. If job competition becomes sufficiently intense, all workers will invest k^*, thereby eliminating the positive externality from education.

A. A Special Case

In the limit case, where $\beta \to 1$, the equilibrium has a particularly simple form. Since firms rank their applicants on the support of k for all $\beta < 1$, this also holds in the limit as $\beta \to 1$. Since $J(k)$ and V converge to zero as $\beta \to 1$, this model becomes simple. First, $A(p)$ in (8) converges to

$$A(p) = \frac{p}{(r+s)(r+s+p)}. \tag{24}$$

Thus $k(0)$ is given by the equation

$$\frac{p_r(0)}{r+s+p} H'(k^0) = r + s. \tag{25}$$

For $\lambda < \lambda_1$, the first-order condition (13) can be written as

$$\frac{p_r}{r+s+p_r} H'(k) + \frac{r+s}{(r+s+p_r)^2} p_r(\Pi)\pi(k) = r + s. \tag{26}$$

The distribution function $\Pi(k)$ is given by (19) with $V = 0$. We thus have an explicit solution for $\Pi(k)$. Furthermore, it follows that k^1 is monotonically increasing in λ. For a given function $H(k)$, it is also straightforward to derive explicit expressions for λ_1 and λ_2. In the limit equilibrium, the positive externality from education on firms is eliminated for all values of λ, and we are left with the negative ranking externality only. By comparing (23) and (25), we can see that the optimal and the actual level of education coincide at the infimum of the support, at which point there are no ranking externalities. However, for all other levels of education, the ranking externality will lead to overinvestment in education.

B. Changing the Disagreement Point

In much of the literature on labor economics (and other fields of economics as well), it is common to assume that the relevant disagreement points are the agents' incomes during wage negotiations rather than their outside options, while the latter only define lower bounds of the agents' payoffs. I will therefore briefly study the model under this assumption. If both workers and firms have zero incomes during the negotiations, it follows that the wage is given by[13]

$$w = \min[\beta H(w), H(w) - rV]. \tag{27}$$

The first thing to note is that firms will always rank their applicants as long as their outside options do not bind. By applying exactly the same argument as above, we can see that the equilibrium distribution of k has no mass point when $H'(k) > 0$ (provided that the firms' outside options do not bind on this interval). This indicates that problems with overinvestment can be much more severe than before. To illustrate this, suppose $H(k)$ has a maximum at $\hat{k}$, above which $H(k)$ is constant or declining. Then the following will hold.

Proposition 3. Suppose the disagreement points in the bargaining game are the agents' incomes during wage negotiations. Then, for sufficiently high values of λ, all workers will invest in education up to the point $\hat{k}$ at which the return from education is zero (provided that $U(\hat{k}) - \hat{k} > 0$).

The proof is given in the appendix. When the agents' outside options serve as disagreement points, this strengthens the relationship between a worker's education and his wage. For sufficiently high levels of education, it will make firms indifferent between workers with different levels of education. This restrains the overinvestments. When the agents' incomes during the bargaining game serve as disagreement points, firms always rank their applicants according to productivity, and no such restraint exists.

[13] Since firms are identical, the worker's outside option never binds.

VI. Discussion

In this section, I discuss some of the assumptions in the model regarding wage determination, the matching framework, and the education technology.

A. Wage Determination

In the wage bargaining game, I assume that the relevant outside option for the firm is to continue to have a vacancy, not to hire one of the other applicants. This I rationalize by renegotiations: the chosen worker can renegotiate his or her wage later on when the other applicants have left the scene. The same assumption is made in Blanchard and Diamond (1994), who analyze a situation where employers rank workers according to their unemployment spells. A similar assumption is also made in Mortensen and Pissarides (1994), who model wage negotiations when a worker may have more than one job offer at the same time. Furthermore, in models where the focus is on the dynamic properties of matching models, it is standard to assume that wages are continuously renegotiated; see Pissarides (1985, 1987) and Bertola and Caballero (1994).

Even if the agents may sign a wage contract covering a substantial period of time, it is not obvious that this would prevent the worker from renegotiating wages later on. If a worker refuses to work for the wage contracted on and instead demands renegotiations, there is little the firm can do except fire him since slavery is forbidden. Nor will firing the worker be a credible threat, as long as the firm can expect to obtain more by renegotiating the wage than by having a vacancy.

The question is then how the initial contract will influence renegotiations. In MacLeod and Malcomson (1993), a renegotiation game is constructed in which the initial contract does influence the bargaining outcome. However, as pointed out in Holden (1994), their result hinges on the game being set in finite time. In order to analyze whether an initial contract influences the bargaining game in our case, we will have to study the strategic bargaining game that underpins the Nash solution. It can be shown that the Nash solution with the outside options as the disagreement points is the (limit) equilibrium of a strategic bargaining game of the Stål-Rubinstein type, with an exogenous risk of breakdown of the negotiations as the driving force toward agreement. If no production takes place (and no salaries are paid) during the negotiations, it is easy to show that the initial wage has no influence on the equilibrium outcome.

Even if long-term wage contracts can be enforced, the mechanisms described in this article may still hold, at least in a modified form. Suppose that, in case of a negotiation breakdown with the most productive worker, the firm calls in the second most productive worker. If the firm incurs a small cost if it changes its bargaining partner, it will never *choose* to start bargaining with the second worker, and any threat to do so will not be credible. However, if the negotiations do break down, the firm will call in the second worker, so it follows that the outside option for the firm is to call for the second applicant. In this case, it can be shown (Moen 1996) that there is still a surplus associated with getting a job, and the negative ranking externality prevails. Finally, as an alternative to wage bargaining, firms may simply post wages, as in Moen (1997). This may reduce the problems associated with overinvestment.

B. Search Externalities

I have assumed that at the stage when a worker is looking for a job, the arrival rate of job offers is exogenous (although this may, of course, depend on the worker's human capital investments). I thus ignore other means by which the worker may speed up the job-finding process, most importantly by choosing a high search intensity. Here, the most natural interpretation of a high search intensity is that the worker in question applies for jobs at a high frequency (a is high). Workers can then choose an optimal mix of investment in education and search intensity in order to improve their job-finding rate. Furthermore, as shown by Hosios (1990), a worker's choice of search intensity yields both a negative external effect on other workers and a positive external effect on firms with job vacancies, and the market may induce too low or too high search activity compared to the socially optimal amount. In order to make the analysis tractable, I have focused on education, and therefore I have taken the search intensity as given. Note, however, that the overinvestment result is likely to hold even when search intensity is included as a choice variable. The reason is that the arrival rate of job offers to the least efficient worker still goes to zero as the job competition goes to infinity (provided that the search intensity does not go to infinity). In this case, the expected income at the infimum of the distribution goes to zero as well, and this forces all workers to invest an amount at which firms will randomize among the applicants.

C. Entry of Firms

In my model, the value of a firm with a vacancy is endogenous while the number (measure) of firms is exogenous. Alternatively, one may assume that the number of firms is endogenous, as new firms may enter the market. If there is a cost C associated with entering the market (buying capital, etc.), the number of vacancies in the market adjusts so that $V = C$.

Entry of firms will complicate the welfare analysis of the model. The reason is that the entry decisions made by firms are generally not optimal, and we may have too much or too little entry (Hosios 1990). Since human capital investments influence the entry decision, this will give rise to second-best effects. If there is too little entry, a social planner will take into account the effect of education on entry, while the opposite holds if there is too much entry. On the other hand, if the entry decision is optimal, the effect of education on the entry will have no welfare implications at the margin. With optimal entry, the efficiency results obtained in this article are likely to carry over to a model with entry. In particular, the model with entry will yield overinvestment in education for some parameter values and underinvestment for others. To see this, note that any equilibrium allocation obtained with a fixed number of firms can be obtained as an equilibrium allocation in a model with entry, simply by setting the entry cost C equal to the equilibrium value of V.

D. Education Technology

I have assumed that education takes no time and that the costs of education are simply a pecuniary variable k. This is a simplification. Obviously, education does take time, and a significant part of the costs of taking education will be linked to the shadow price of time. Thus, if the unemployment rate is high, the costs of investing in education will be low, and this reduces both the private and the social costs of education. If the shadow price of time is included in the analysis, this may possibly increase the degree of overinvestment. The reason is that, if other workers overinvest in education, the private cost of overinvestment for an individual worker will be low, as the alternative to investing in education will be to stay unemployed (with a high probability).

I have also assumed that investment in education is a continuous variable. From a technical point of view this is important: the fact that the arrival rate of jobs is independent of the behavior of other workers at the infimum of the support pins down an equilibrium uniquely. With a discrete set of possible education levels, we cannot rule out multiple equilibria. To see this, suppose the only options are to undertake zero or one unit of education. Suppose also that firms always prefer workers with education to workers with no education. Finally, suppose that both c and λ are large. If no workers invest in education, the return to education may be relatively low, since the job-finding rate c is high anyway. On the other hand, if almost all workers invest in education, a worker who does not will have to wait a very long time to find a job, and the value of having education and obtaining the high job-finding rate c is high. This may lead to multiple equilibria.

Finally, since I have assumed that all jobs are identical, people with very different education levels will be competing for the same jobs. Obviously, this, too, is a simplification. If firms were heterogeneous and the comparative advantage of having higher education differed between job types, we would obtain an endogenous segmentation of the labor market into submarkets. However, within each submarket, workers with education within certain ranges would be competing for the same jobs, and presumably the main arguments in this article would still apply. This is among the issues that I hope to address in future work.

VII. Conclusion

In this article, I have analyzed the incentives to invest in human capital in situations where the labor market contains frictions and where an unemployed worker's chances of getting a job depend on his productivity relative to other workers, that is, on his ranking. I show that the equilibrium of the model can take different forms, depending on the degree of competition for jobs. If the competition is not too intense, workers will choose to diversify, and the distribution of investments in education will be continuous and without mass points. If competition for jobs is sufficiently intense, however, all workers will choose the same education level, which is above the socially optimal level. A hybrid equilibrium exists for an intermediate degree of competition.

Although this analysis has been concerned with the labor market and matching, I believe that the underlying mechanism is more general and will apply in a wide range of situations where hold-up problems create underinvestment. What most previous analyses of hold-up problems typically have failed to take into account is that ex ante investments may influence the likelihood of finding a trading partner. Because a trading partner gains from the investments, these investments make the investor more attractive and thus increase his probability of actually getting involved in trade. I show that this may create a rat race between the investors, which may lead to diversification and to overinvestment. If overinvestment occurs, this means that the negative externalities associated with the rat race are actually stronger than the positive externalities that initiated the hold-up problem and the rat race in the first place.

Appendix

Omitted Proofs

The Matching Function and the Distribution Function $F(k)$

Derivation of equation (1).—By construction, it follows that the matching technology is of the urn-ball type. Since there is a continuum of agents of each type, it follows that infinitely many agents on either side of the market are active for any number n of periods. Thus the number of applicants a firm with a vacancy obtains will be Poisson distributed with parameter λ (see Butters [1977] for details). Consider a worker who is ranked above a fraction Π and below a fraction $1 - \Pi$ of the unemployed workers in the market. When applying for a job, this worker will be successful if and only if no workers with a higher ranking apply for the same job. The number of applicants with ranking above Π is Poisson distributed with parameter $(1 - \Pi)\lambda$ (since there are $(1 - \Pi)u$ workers with ranking above Π). The probability that a worker with ranking Π is the preferred candidate for a job is thus equal to the probability that a Poisson distributed variable with parameter Π is zero. The probability is thus $e^{-(1-\Pi)\lambda}$. The arrival rate of job offers to a worker, denoted by p_r, is equal to the rate at which he applies for a job times the probability that he obtains a job when applying. For high values of n, the probability that a worker sends off an application within a short time interval a is approximately $a\Delta t$, and in the limit as n goes to infinity, the rate at which he applies is a. The arrival rate of job offers for this worker will thus be

$$p_r = ae^{-(1-\Pi)\lambda}, \tag{A1}$$

which is identical to (1).

Derivation of equation (2).—A vacancy will be filled if and only if the firm gets at least one applicant, and since the number of applicants is Poisson distributed with parameter λ, the probability of this event is $1 - e^{-(1-\Pi)\lambda}$. The total number of matches in each period is thus $v\gamma[1 - e^{-(1-\Pi)\lambda}]$. The fraction of the workers who send off applications within this period and get a job is equal to the total number of matches divided by the number of workers applying, or $[1 - e^{-(1-\Pi)}\lambda]/\lambda$. The average arrival rate of jobs to workers, c, is then obtained by multiplying this expression by a:

$$c = a[1 - e^{-(1-\Pi)\lambda}]/\lambda. \tag{A2}$$

Solving for a and then replacing a in (1) gives

$$p = \frac{c\lambda}{1 - e^{-\lambda}} e^{-\lambda(1-\Pi)},$$

which is equal to (2).

Derivation of equation (3).—All workers have the same job-finding rate at the mass point. The expression for this job-finding rate is the same as the expression for c, with u replaced by ξu in the expression for the Poisson parameter. From (29) we thus get

$$p = \frac{c}{\xi}\frac{1 - e^{-\xi\lambda}}{1 - e^{-\lambda}}, \tag{A3}$$

which corresponds to (3).

Properties of the matching function.—Let us now see how the matching function depends on λ for different values of Π and, too, how the matching technology behaves in the limits when λ goes to zero and to infinity. First, note that the derivative of $p_r(0; \lambda)$ with respect to λ is given by

$$p_{r\lambda}(0; \lambda, c) = c\,\frac{e^{-\lambda}(1 - \lambda - e^{-\lambda})}{(1 - e^{-\lambda})^2},$$

which is strictly negative for $\lambda > 0$. Thus, $p_r(0)$ is decreasing in λ. Since $p_r(1; \lambda) = c\lambda/(1 - e^{-\lambda})$, it follows that the derivative $p_{r\lambda}(1, \lambda)$ is given by

$$p_{r\lambda}(1, \lambda) = \frac{1 - \lambda e^{-\lambda}}{(1 - e^{-\lambda})^2} \tag{A4}$$

Since $\lambda e^{-\lambda} < 1$ for all λ, it follows that $p_r(1; \lambda)$ is increasing in λ. When $\lambda \rightarrow 0$, both the numerator and the denominator in (2) go to zero for all values of Π, and we can apply l'Hôpital's rule. Hence

$$\begin{aligned}\lim_{\lambda\to 0} p_r(\Pi, \lambda, c) &= \lim_{\lambda\to 0} c\,\frac{-\lambda(1 - \Pi)e^{-\lambda(1-\Pi)} + e^{-\lambda(1-Pi)}}{e^{-\lambda}}\\ &= c.\end{aligned}$$

For $\Pi < 1$ and $\lambda \rightarrow \infty$, we find that

$$\begin{aligned}\lim_{\lambda\to\infty} p_r(\Pi, \lambda, c) &= \lim_{\lambda\to\infty} \frac{c\lambda/(1 - e^{-\lambda})}{e^{\lambda(1-\Pi)}}\\ &= c \lim_{\lambda\to\infty} c\,\frac{\lambda}{e^{\lambda(1-\Pi)}}\\ &= \lim_{\lambda\to\infty} c\,\frac{1}{(1 - \Pi)e^{\lambda(1-\Pi)}}\\ &= 0,\end{aligned} \tag{A5}$$

where we used L'Hôpital's rule between the second and the third equation. For $\Pi = 1$, we find that

$$\begin{aligned}\lim_{\lambda\to 0} p_r(1; \lambda) &= \lim_{\lambda\to\infty} c\,\frac{\lambda}{1 - e^{-\lambda}}\\ &= \infty.\end{aligned} \tag{A6}$$

That p_a defined by (3) is increasing in λ follows directly from the facts that the average job-finding rate is independent of λ and that an increase in λ increases the job-finding rate at the top of the distribution (i.e., at the mass point) while it reduces the job-finding rate for workers at the bottom of the distribution.

Derivation of the distribution F(k) and the value of a vacancy V.—For each advertised vacancy, the number of applicants with education above k is Poisson distributed with parameter $\lambda(1 - \Pi(k))$. The probability that the worker with the highest amount of human capital will have a ranking above Π is thus $1 - e^{-\lambda(1-\Pi)}$. Let $\hat{F} = 1 - e^{-\lambda}$ denote the probability that the firm receives any applications at all. The (unconditional) probability of getting an applicant with education above k must be equal to the probability of getting at least one worker times the probability of getting a worker with education above k contingent on getting at least one application. Since $F(k)$ is the distribution contingent on the event that at least one job seeker applies,

$$1 - e^{-\lambda(1-\Pi(k))} = (1 - F(k))\hat{F},$$

which gives

$$\begin{aligned} F(k) &= 1 - (1 - e^{-\lambda(1-\Pi(k))}/\hat{F} \\ &= \frac{e^{-\lambda(1-\Pi)} - e^{-\lambda}}{1 - e^{-\lambda}}, \end{aligned} \tag{A7}$$

which is the expression used in the text.

To derive the expression for the value of a vacancy given by (11), first note that the asset value equation determining V can be written as

$$rV = q \int_k (J(k) - V)dF. \tag{A8}$$

If I substitute in for $J(k) - V$ from (9) and rearrange, then

$$V = q \frac{(1 - \beta) \int_k (H(k) - (r + s)U(k))dF}{(r + s)(r + q(1 - \beta)r/(r + s))}, \tag{A9}$$

which is what I wanted to prove.

Proofs Related to the Equilibrium of the Model (Section IV)

Proof of claim about firms' ranking associated with lemma 1.—I want to prove that, if $k' < k^*$ is in the support of Π, then a firm would strictly prefer a worker with education k^* to a worker with education k'. Suppose that this were not true. Then we must have $H(k^*) - (r + s)U(k^*) = H(k') - (r + s)U(k')$. We know that $H'(k) \geq (r + s)U'(k)$ for all k. These two combined imply that $H'(k) = (r + s)U'(k)$ for all k. On the interval $[k', k^*)$, it then follows from the definition of k^* that $U'(k) = H'(k)/(r + s) > 1$. But then the worker would be strictly better off investing k^* rather than k', so that k' must be outside the support of Π, and we have derived a contradiction.

Proof of the claim that the tail forms a connected set.—I want to prove that the support situated below k^* (if any) forms a connected set. Suppose that it does not. Then there exists an interval (k', k'') where $\pi = 0$. But then $p(k') = p(k'') = p(k)$ for all $k \in [k', k'']$. Since both k' and k'' are optimal, we must have $U(k_1) - k_1 = U(k_2) - k_2$. Since $H(k)$ is strictly concave in k and U is linear in H for a given p, it follows that $U(k) - k > U(k_1) - k_1$ for all $k \in (k_1, k_2)$, which is a contradiction.

Proof of proposition 1.—The structure of the proof is as follows:[14]

1. Characterize an upper bound $\bar{V}$ for the equilibrium values of V.
2. For a given $V \leq \bar{V}$, show that there exists a unique distribution, $\Pi(k)$.
3. Show that there exists a unique equilibrium value of V.
4. Show that, when $\lambda \to 0$, the distribution collapses to k^0, defined by (17) with $p_r = c$ and that, for sufficiently high values of λ, π is a singleton at k^* (this and continuity ensure the existence of λ_1 and λ_2 referred to in the text).

Proofs of steps 2 and 4 are given in the text, and I therefore concentrate on the first and the third steps.

Step 1. Obviously, if some workers increase k, this increases V. We know that, in any equilibrium, no worker invests more than k^*. We also know that, for workers to be willing to invest k^*, we must have $U(k^*) \geq k^*$. We can therefore define $\bar{V}$ as the value of V that emerges when all workers invest k^* and given that $U(k^*) = k^*$. From (11), then,

$$\bar{V} = B^*[H(k^*) - (r + s)k^*], \tag{A10}$$

where B^* is given by

[14] I do not prove that there exists a set of hiring strategies such *that* $U'(k^*) = 1$ when the distribution has a mass point at k^*. However, this is given in Moen (1996).

$$B^* = \frac{q(1-\beta)}{(r+s)(r+qr(1-\beta)/(r+s)},$$

where, as in the main text, q is the hiring rate for firms. In order to ensure that the parameters are such that $U(k^*) \geq k^*$, we insert $V = \bar{V}$ into (8). This gives the following inequality (with $A^* = A(c)/(r + s)$):

$$U(k) \geq A^*[H(k^*) - rB^*(H(k^*) - (r+s)k^*)].$$

The requirement that $U(k) \geq (r + s)k^*$ is thus satisfied if

$$\begin{aligned} &A^*(H(k^*) - rB^*(H(k^*) - (r+s)k^*) - (r+s)k^* \\ &= A^*(1 - rB^*)H(k^*) - (rA^*B^* + 1)(r+s)k^* \qquad \text{(A11)} \\ &\geq 0. \end{aligned}$$

In the rest of the proof, I assume that the inequality is strict. Note that the inequality is satisfied if the economy is sufficiently productive in the following sense: let $H(k) = Ch(k)$, where C is a parameter. It is easy to show that $rB^* < 1$, and it follows that the inequality is always satisfied for sufficiently high values of C.

Step 3.—It follows from step 2 that I can write $\Pi = \Pi(k; V)$. Now, let the mapping $X(V)$ be defined as the value of a vacancy given that the distribution of investments is $\Pi(k; V)$. By using (11) and the definition of B^* (or, alternatively, eq. [A10]), it follows that

$$X(V) = B^*\left(\int_k H(k)dF - (r+s)U\right), \qquad \text{(A12)}$$

where $F = F(k; \Pi(k; V))$ is given by (10). Equilibrium is then a fixed-point $V = X(V)$. First, I use Brower's fixed-point theorem to show that such a fixed point exists. To this end, I have to show that the range of $X(V)$ is within $[0, \bar{V}]$. Since V is increasing in k and $U(k^*) - k^* > 0$, it follows that

$$\begin{aligned} X(\bar{V}) &\leq B^*(H(k^*) - (r+s)U(k^*)) \\ &< B^*(H(k^*) - (r+s)k^*) \\ &= \bar{V} \end{aligned}$$

since, by assumption, $U(k^*) > k^*$. Furthermore, since the firms have no search costs, we have that $X(0) > 0$. Since $X(V)$ is continuous (which I do not prove), existence follows from Brower's fixed-point theorem.

To show uniqueness, it is sufficient to show that $X'(V) < 1$. Let $k(\Pi, V)$ denote the investment made by a person with ranking Π for a given V. From (19) and the fact that $k \le k^*$, it follows that $\partial k(\Pi, V)/\partial V \le 0$. Thus, the distribution of investments shifts downward, and a person with a given k obtains a higher ranking the higher is V. But then we must have that

$$\begin{aligned} dU/dV &\ge \frac{\partial U}{\partial V} \\ &= -A^* r \\ &> -r/(r+s) \end{aligned}$$

since $A^* < 1/(r+s)$. This implies that

$$\begin{aligned} X'(V) &= \frac{d}{dV} B^* \left(\int_{k \in K} H(k) - (r+s)U \right) dF \\ &\le B^* \left(\int_{k \in K} H(k) - (r+s) \frac{\partial U}{\partial V} \right) dF \\ &< B^* r < 1, \end{aligned} \tag{A13}$$

where the partial derivatives indicate that the distribution of k is kept constant. The last inequality follows from the fact that $B^* < 1/r$. This shows uniqueness.

Explicit derivation of $\Pi(k)$.—To get a closed-form solution for π, I first insert (8) into the equation $U(k) = U^0 + (k - k^0)$. This gives

$$\frac{\beta p}{r + s + \beta p} \frac{H(k) - rV}{r + s} = U^0 + k - k^0,$$

where $\bar{H} = H/r$. Rearranging and inserting for p from (2) yields

$$\begin{aligned} \frac{(r+s)^2(U^0 + k - k^0)}{H(k) - rV - (r+s)(U^0 + k - k^0)} &= \beta p \\ &= \beta \frac{c\lambda}{1 - e^{-\lambda}} e^{-\lambda(1 - \Pi(k))}. \end{aligned}$$

Taking logarithms and rearranging gives

$$\Pi(k) = 1 - \ln\left(\frac{c\lambda}{1 - e^{-\lambda}} \beta \frac{H(k) - rV - (r + s)(U^0 + k - k^0)}{(r + s)^2(U^0 + k - k^0)}\right) \Big/ \lambda. \tag{A14}$$

Proof of proposition 3.—I want to show that a mass point at $\hat{k}$ with probability mass equal to one constitutes an equilibrium if $\lambda \geq \hat{\lambda}$ for some $\hat{\lambda} > 0$. Suppose all workers do invest $\hat{k}$. Their expected discounted income when entering the labor market is then given by the asset value equation

$$(r + s)U(\hat{k}) = c(\beta H(\hat{k})/(r + s) - U),$$

which implies that

$$U(\hat{k}) = \frac{c}{r + s + c} \beta H(\hat{k})/(r + s)$$

By assumption, $U(\hat{k}) - \hat{k} > 0$. Now suppose a worker deviates and invests less than $\hat{k}$. Then he has zero rank, and since the firms always hire the most productive worker, the arrival rate of job offers he faces is given by $p_r(0; \lambda)$, which goes to zero as λ goes to infinity. It thus follows that the payoff to a deviating worker goes to zero when λ goes to infinity. The proposition follows.

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