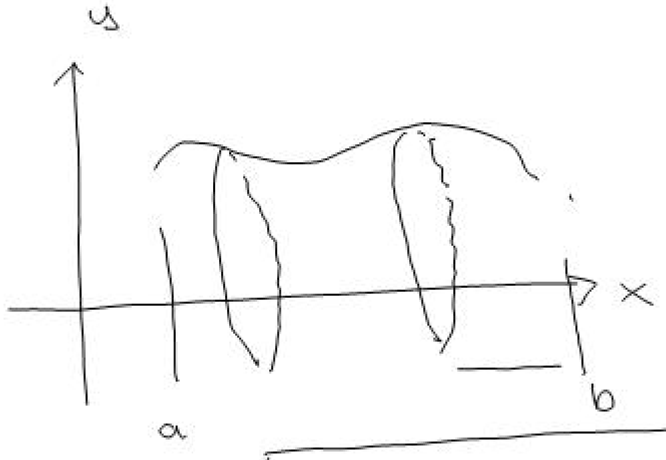
08/05/09:

Volumberegning vha integraler (16.6–16.7)

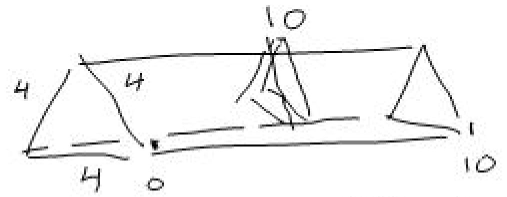


tverrsnitt som står normalt på x-aksen har areal $A(x)$

Volum: $\boxed{V = \int_a^b A(x)\cdot dx}$

$G = \frac{4\cdot 4\cdot \sin 60^\circ}{2} = 4\sqrt{3}.$

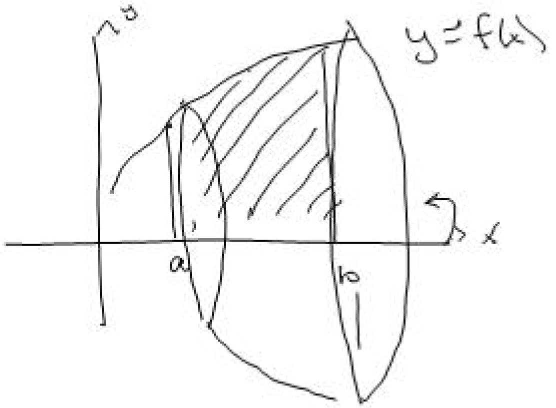
Eks:



$$V = G\cdot h = \frac{4\cdot 4\cdot \sin 60^\circ}{2}\cdot 10$$
$$= 4\cdot\sqrt{3}\cdot 10 = \underline{\underline{40\sqrt{3}}}$$

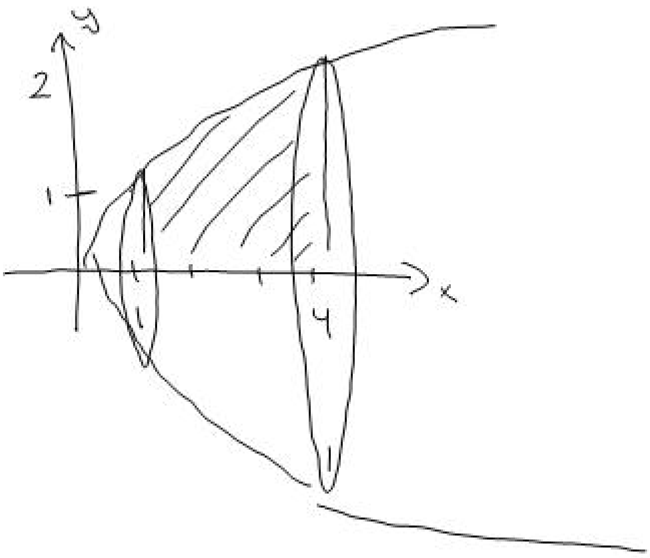
$$V = \int_0^{10} 4\sqrt{3}\cdot dx = \left[4\sqrt{3}x\right]_0^{10}$$
$$= (4\sqrt{3}\cdot 10 - 4\sqrt{3}\cdot 0) = \underline{\underline{40\sqrt{3}}}$$

Sirkelskivemetoden: Alle tverrsnitt er sirkelskiver.



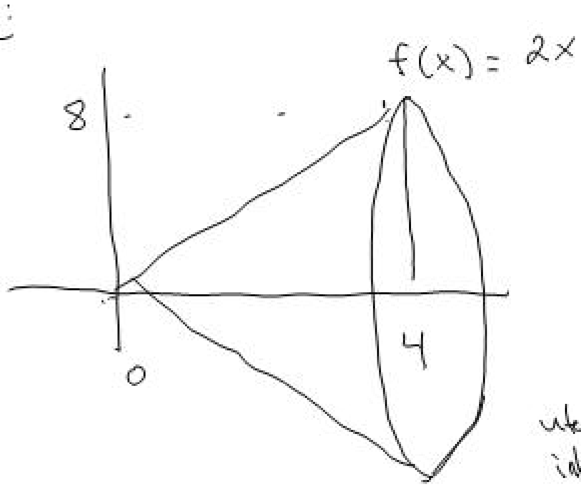
$$V = \int_a^b \underbrace{\pi \cdot f(x)^2}_{A(x)}\,dx$$

Eks. $f(x) = \sqrt{x}$, $a = 1$, $b = 4$



$$V = \int_1^4 \pi \cdot (\sqrt{x})^2 \, dx$$
$$= \pi \int_1^4 x \, dx$$
$$= \pi \left[\tfrac{1}{2} x^2 \right]_1^4$$
$$= \pi \cdot \left(\tfrac{1}{2} \cdot 4^2 - \tfrac{1}{2} \cdot 1^2 \right)$$
$$= \pi \, (8 - 1/2) = \underline{\underline{\tfrac{15}{2}\pi}}$$

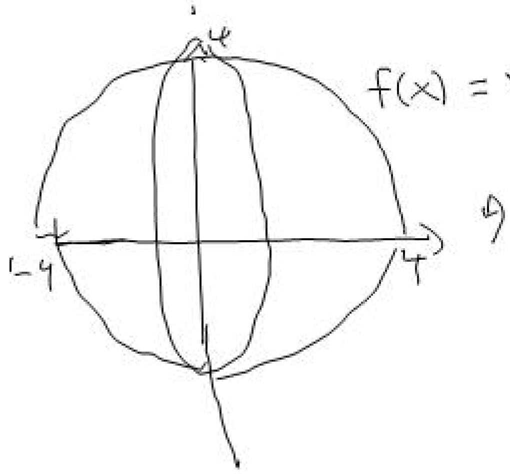
Eks:



$$V = \int_0^4 \pi \cdot (2x)^2 \, dx$$
$$= 4\pi \left[\tfrac{1}{3} x^3 \right]_0^4$$
$$= 4\pi \left(\tfrac{1}{3} \cdot 4^3 - 0 \right) = \underline{\underline{\tfrac{256}{3}\pi}}$$

uten integral: $V = \frac{1}{3} \cdot G \cdot h = \frac{1}{3} \cdot \pi \cdot 8^2 \cdot 4 = \underline{\underline{\frac{256}{3}\pi}}$

Volumet av en kule: med radius 4



$f(x) = \sqrt{16 - x^2}$

$$x^2 + y^2 = 4^2$$
$$y^2 = 16 - x^2$$
$$y = \sqrt{16 - x^2}$$

Volum: $$V = \int_{-4}^{4} \pi \cdot \left(\sqrt{16 - x^2}\right)^2 \, dx$$

$$= \pi \int_{-4}^{4} (16 - x^2) \, dx$$

$$= \pi \left[16x - \frac{1}{3}x^3 \right]_{-4}^{4}$$

$$= \pi \left(\left(16 \cdot 4 - \frac{1}{3} \cdot 4^3\right) - \left(-16 \cdot 4 + \frac{1}{3} \cdot 4^3\right) \right)$$

$$= \pi \left(64 - \frac{64}{3}\right) - \left(-64 + 64/3\right))$$

$$= \pi \cdot \left(\frac{128}{3} + \frac{128}{3} \right) = \underline{\underline{\frac{256}{3} \pi}}$$

Formel: $$V = \frac{4}{3} \pi R^3 = \frac{4}{3} \cdot \pi \cdot 4^3 = \underline{\underline{\frac{256}{3} \pi}}$$