07/05/09:

Oblig 8: 5dj) : hopp over.

De neste ukene:

11 - 15. mai
18 - 22. mai
25 - 29. mai

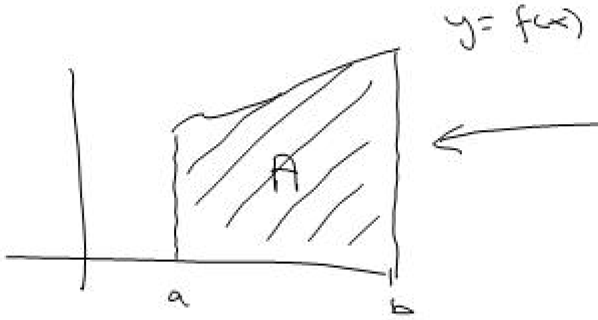
Repetisjon:
- integral med $\cos^2$, $\sin^2$, ...
- logaritmer / eksponensialf.
- vektorregning / trigonometri

fredag 15. mai: begynner 09.30.

prøve-eksamen: mandag 25. mai.

2. juni → eksamen

Integrasjon og arealberegning

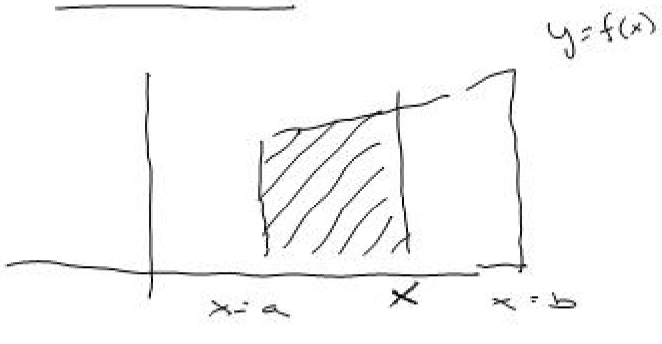


Areal:

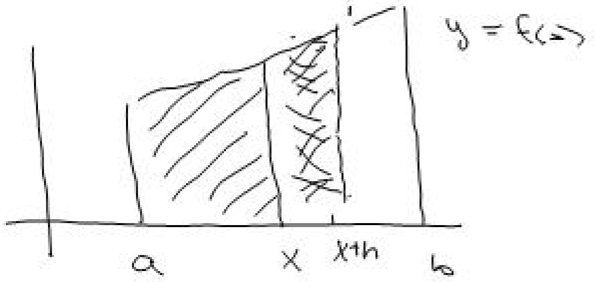
$$\boxed{A = \int_a^b f(x)\,dx}$$

$$\begin{cases} f \text{ er kontinuerlig på } [a,b] \\ f(x) \geq 0 \quad \text{for } x \in [a,b] \end{cases}$$

Sammenheng: arealfunksjon



$A(x)$ = arealet under grafen fra a til x.



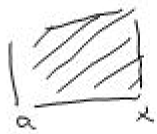
$A(x)$ = arealet under grafen fra a til x = skravert areal

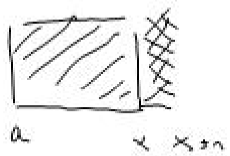
$$A(a) = 0$$

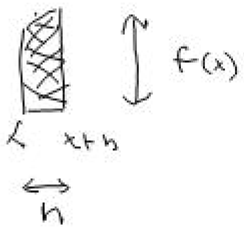
$$A(b) = A$$

$$\boxed{A'(x) = f(x)}$$

$$A'(x) = \lim_{h \to 0} \frac{A(x+h) - A(x)}{h}$$

$A(x) =$ 

$A(x+h) =$ 

$A(x+h) - A(x) =$ 

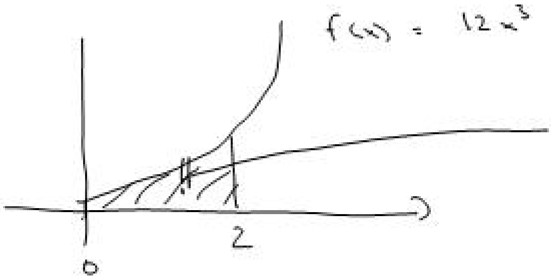
$$\simeq h \cdot f(x)$$

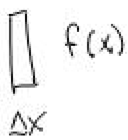
$$\lim_{h \to 0} \frac{A(x+h) - A(x)}{h} = \lim_{h \to 0} \frac{\cancel{h} \cdot f(x)}{\cancel{h}} = f(x)$$

$h = \Delta x$

$$\int_a^b f(x)\,dx = \left[A(x)\right]_a^b = A(b) - A(a) = A$$

<u>Eks:</u>

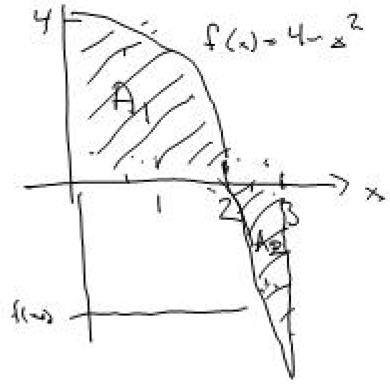




$$A = \int_0^2 f(x) \cdot dx = \int_0^2 12x^3\,dx = \left[12 \cdot \tfrac{1}{4}x^4\right]_0^2$$

$$= \left[3x^4\right]_0^2 = 3 \cdot 2^4 - 0 = \underline{\underline{48}}$$

Eks: A_1 = Arealet begrenset av grafen til $f(x) = 4 - x^2$, x-aksen, $x = 0$, og $x = 2$.



$$A_1 = \int_0^2 f(x)\,dx = \int_0^2 (4 - x^2)\,dx$$

$$= \left[4x - \frac{1}{3}x^3 \right]_0^2 = \left(8 - \frac{8}{3}\right) - 0$$

$$= \underline{\frac{16}{3}} \simeq \underline{5.33}$$

A_2 = Arealet begrenset av grafen til $f(x) = 4 - x^2$, x-aksen $x = 2$ og $x = 3$

$$A_2 = \int_2^3 - f(x)\,dx \quad \left(= \int_3^2 f(x)\,dx \right)$$

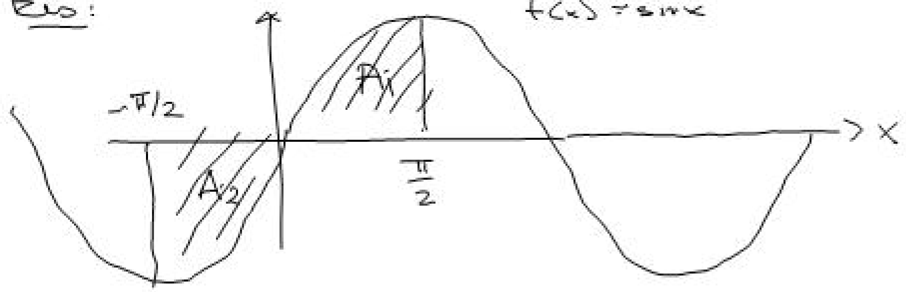
$$= - \int_2^3 (4 - x^2)\,dx = - \left[4x - \frac{1}{3}x^3 \right]_2^3$$

$$= - \left(\left(4 \cdot 3 - \frac{1}{3} \cdot 3^3\right) - \left(4 \cdot 2 - \frac{1}{3} \cdot 2^3\right) \right)$$

$$= -3 + \frac{16}{3} = \underline{\underline{\frac{7}{3}}} \simeq \underline{\underline{2.33}}$$

$$\int_2^3 f(x)\,dx = -\frac{7}{3} \Rightarrow A_2 = - \int_2^3 f(x)\,dx = \underline{\underline{\frac{7}{3}}}$$

Eks:



$A = A_1 + A_2$

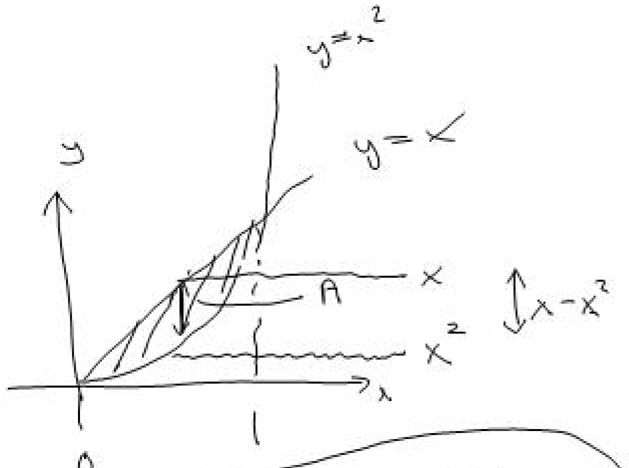
$$\int_{-\pi/2}^{\pi/2} f(x)\,dx = \int_{-\pi/2}^{\pi/2} \sin x\,dx = \left[-\cos x\right]_{-\pi/2}^{\pi/2} = 0 - 0 = \underline{0}$$

$$\int_{-\pi/2}^{\pi/2} f(x)\,dx = -A_2 + A_1 = A_1 - A_2 \Longrightarrow \underline{A_1 = A_2}$$

$$A = A_1 + A_2 = 2A_1 = 2 \cdot \int_0^{\pi/2} \sin x\,dx$$

$$= 2\left[-\cos x\right]_0^{\pi/2} = 2(-0+1) = \underline{\underline{2}}$$

Eks:

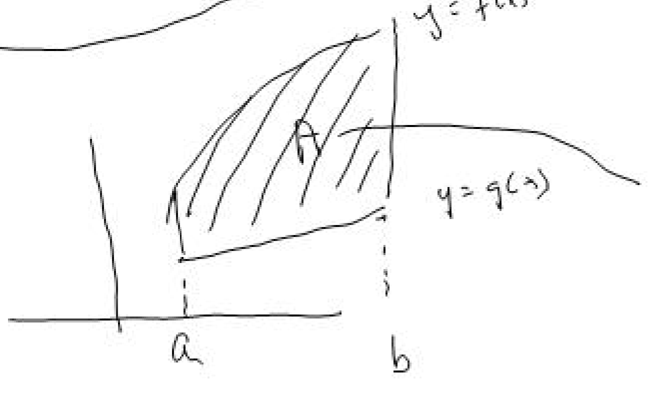


$$A = \int_0^1 (x - x^2)\,dx$$

$$= \left[\tfrac{1}{2}x^2 - \tfrac{1}{3}x^3\right]_0^1$$

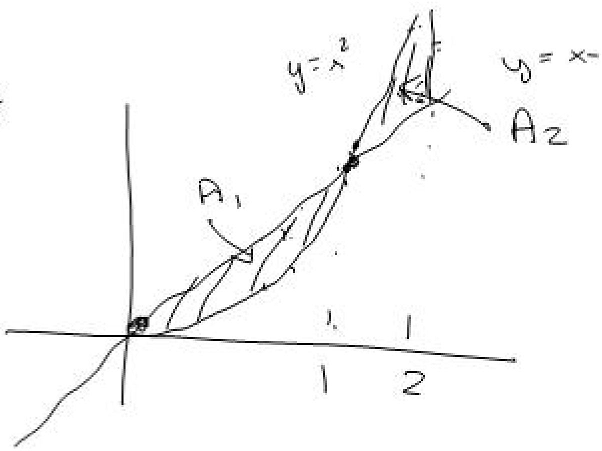
$$= \left(\tfrac{1}{2}\cdot 1 - \tfrac{1}{3}\cdot 1\right) - 0$$

$$= \underline{\underline{\tfrac{1}{6}}}$$



$$A = \int_a^b \left(f(x) - g(x)\right)dx$$

Eks:



$$x^2 = x$$
$$x=0 \quad \text{eller} \quad x=1$$

A = arealet begrenset av $y=x$, $y=x^2$, $x=0$ og $x=2$

$$\underline{A = A_1 + A_2}$$

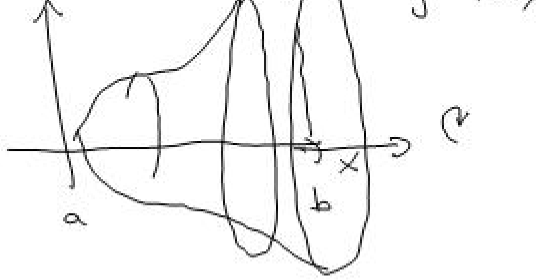
$$A_1 = \int_0^1 (x-x^2)\,dx = \left[\tfrac{1}{2}x^2 - \tfrac{1}{3}x^3\right]_0^1 = \underline{\tfrac{1}{6}}$$

$$A_2 = \int_1^2 (x^2-x)\,dx = \left[\tfrac{1}{3}x^3 - \tfrac{1}{2}x^2\right]_1^2 = \left(\tfrac{1}{3}\cdot 2^3 - \tfrac{1}{2}\cdot 2^2\right) + \tfrac{1}{6}$$

$$= \left(\tfrac{8}{3} - 2\right) + \tfrac{1}{6} = \tfrac{2}{3} + \tfrac{1}{6} = \tfrac{5}{6}$$

$$A = \tfrac{1}{6} + \tfrac{5}{6} = \underline{\underline{1}}$$

Volumberegning ved skivemetode:



$$V = \int_a^b \pi \cdot f(x)^2\,dx$$