23/04/09: Delbrøkoppspaltning

Oppgave:

$$\begin{cases} \int \frac{12x}{x^2-4}\,dx & (a) \\ \int \frac{12x}{(x-2)^2} & (b) \\ \int \frac{12}{(x-2)^2}\,dx & (c) \end{cases}$$

(a) $\frac{12x}{x^2-4} = \frac{12x}{(x-2)(x+2)} = \frac{A}{x-2} + \frac{B}{x+2} \quad \Big| \cdot (x-2)(x+2)$

$12x = A\cdot(x+2) + B(x-2)$

$\underline{x=-2}$: $\quad -24 = A\cdot 0 + B\cdot(-4)$

$-24 = -4B \Rightarrow \underline{B=6}$

$\underline{x=2}$: $\quad 24 = A\cdot 4 + B\cdot 0$

$24 = 4A \Rightarrow \underline{A=6}$

$$\int \frac{12x}{x^2-4}\,dx = \int \frac{6}{x-2} + \frac{6}{x+2}\,dx = \underline{6\ln|x-2| + 6\ln|x+2| + C}$$

Alt: $\boxed{\begin{array}{l} u = x^2-4 \\ du = 2x\cdot dx \end{array}}$

$$\int \frac{12x}{x^2-4}\,dx = \int \frac{\cancel{12x}^{6}}{u}\cdot\frac{du}{\cancel{2x}} = \int \frac{6}{u}\,du$$

$$= 6\ln|u| + C = \underline{6\ln|x^2-4| + C}$$

(b) og (c):

$$\int \frac{12x}{(x-2)^2}\,dx$$

$$\int \frac{12}{(x-2)^2}\,dx$$

$$\frac{12x}{(x-2)^2} = \frac{A}{(x-2)} + \frac{B}{(x-2)^2} \quad \Big| \cdot (x-2)^2$$

$$12x = A\cdot(x-2) + B$$

Metode 2:

$x=2$: $24 = A\cdot 0 + B$

$24 = B \Rightarrow \underline{B = 24}$

$x=0$: $0 = A\cdot(-2) + B$

$0 = -2A + B$

$0 = -2A + 24 \Rightarrow \underline{A=12}$

$$\int \frac{12x}{(x-2)^2}\,dx = \int \frac{12}{x-2} + \frac{24}{(x-2)^2}\,dx = \underline{12\ln|x-2| + \frac{-24}{x-2} + C}$$

$$\int \frac{24}{(x-2)^2}\,dx = \int \frac{24}{u^2}\,du = 24\int u^{-2}\,du = 24\cdot\left(\frac{1}{-1}\cdot u^{-1}\right) + C$$

$$\boxed{\begin{array}{l} u = x-2 \\ du = dx \end{array}} \qquad = -24\,\frac{1}{x-2}$$

Husk:

$$\int \frac{c}{ax+b}\,dx = \frac{c}{a}\cdot \ln|ax+b| + C$$

(fordi $\int \frac{1}{u}\,du = \ln|u| + C$)

$$\int \frac{c}{(ax+b)^2} = \frac{c}{a}\cdot\frac{-1}{ax+b}$$

(fordi $\int \frac{1}{u^2}\,du = -\frac{1}{u} + C$)

(c) $$\int \frac{12}{(x-2)^2}\,dx = \frac{12}{1}\cdot\frac{-1}{x-2} + C = \frac{-12}{x-2} + C$$

Eks:

$\frac{12}{(x-2)^2}$ → integreres direkte $\frac{A}{x-2} + \frac{B}{(x-2)^2}$

$$\frac{2x+3}{(x-2)^2} = \frac{A}{(x-2)} + \frac{B}{(x-2)^2}$$

$$\frac{12}{x^2-4} = \frac{A}{x-2} + \frac{B}{x+2}$$

$$\frac{x^2+4}{x\cdot(x-1)^2} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$$

<u>Eks:</u> $\int \frac{12x}{(x-2)^2}\,dx = \int \frac{12x-24+24}{(x-2)^2}\,dx$

$$= \int \frac{12x-24}{(x-2)^2}\,dx + \int \frac{24}{(x-2)^2}\,dx$$

$$= 12\cdot \int \frac{\cancel{x-2}}{(x-2)^{\cancel{2}}}\,dx + \int \frac{24}{(x-2)^2}\,dx$$

$$= \int \frac{12}{x-2}\,dx + \int \frac{24}{(x-2)^2}\,dx$$

<u>Generell regel:</u> $\frac{ax+b}{(x-\alpha)^2} = \frac{A}{x-\alpha} + \frac{B}{(x-\alpha)^2}$

Tilfellet der nevner ikke kan faktoriseres

$$\int \frac{12x}{x^2+4}\,dx = \int \frac{\cancel{12x}^{6}}{u} \cdot \frac{du}{\cancel{2x}} = \int \frac{6}{u}\,du$$

$$\boxed{\begin{aligned} u &= x^2+4 \\ du &= 2x \cdot dx \end{aligned}}$$

$$= \underline{\underline{6 \ln(x^2+4) + C}}$$

$x^2+4=0$
$x^2=-4$
ingen løsning

$$\left(6\ln(x^2+4)+C\right)' = 6 \cdot \frac{1}{x^2+4} \cdot 2x$$
$$= \frac{12x}{x^2+4}$$

$$\int \frac{12}{x^2+4}\,dx = \int \frac{\cancel{12}^{6}}{u} \cdot \frac{du}{\cancel{2x}} = \int \frac{6}{u \cdot x}\,du \rightarrow \text{går ikke}$$

$$12\int \frac{1}{x^2+4}\,dx = \text{noe med arctan} \ldots$$

$$\left((\arctan x)' = \frac{1}{x^2+1} \right)$$