

17/04/09: Integrasjons metoder

- (a) Substitusjon (15.5) \leftrightarrow kjerneregel
(b) Delvis integrasjon (15.6) \leftrightarrow produktregel
(c) Delbrøks oppspaltning (15.7) \leftrightarrow brøkregel

Ex:

$$\frac{1}{x} + \frac{1}{x+1} = \frac{x+1}{x \cdot (x+1)} + \frac{x}{(x+1)x} = \frac{2x+1}{x(x+1)}$$

$$\int \left(\frac{1}{x} + \frac{1}{x+1} \right) dx = \underline{\ln|x| + \ln|x+1| + C}$$

$$\int \frac{c}{ax+b} dx = c \int \frac{1}{ax+b} dx = c \cdot \frac{1}{a} \cdot \ln|ax+b| + C$$

$$\int \frac{2}{3x-1} dx = 2 \cdot \int \frac{1}{3x-1} dx = 2 \cdot \frac{1}{3} \cdot \ln|3x-1| + C$$
$$= \underline{\frac{2}{3} \ln|3x-1| + C}$$

Eks: $\int \frac{3}{x^2-3x+2} dx = ?$

① Ser at $x^2-3x+2 = (x-1) \cdot (x-2)$.

② $\frac{3}{x^2-3x+2} = \frac{3}{(x-1)(x-2)} = \frac{A}{x-1} + \frac{B}{x-2}$

$$\frac{3}{(x-1)(x-2)} = \frac{A}{x-1} + \frac{B}{x-2} \quad | \cdot (x-1)(x-2)$$

$$\boxed{3 = A \cdot (x-2) + B(x-1)}$$

Metode 1:

$$3 = Ax - 2A + Bx - B$$

$$0x + 3 = (A+B)x + (-2A-B)$$

$$A+B = 0 \quad (1)$$

$$-2A-B = 3 \quad (2)$$

$$(1) \quad B = -A$$

$$(2) \quad -2A - (-A) = 3$$

$$-A = 3$$

$$A = \underline{-3}$$

$$B = \underline{3}$$

$$\frac{3}{(x-1)(x-2)} = \frac{-3}{x-1} + \frac{3}{x-2}$$

Metode 2:

Setter inn

$$\underline{x=2}: \quad 3 = A \cdot 0 + B \cdot 1$$

$$\underline{B=3}$$

$$\underline{x=1}: \quad 3 = A \cdot (-1) + B \cdot 0$$

$$3 = -A$$

$$\underline{A=-3}$$

$$\frac{3}{(x-1)(x-2)} = \frac{-3}{x-1} + \frac{3}{x-2}$$

$$\int \frac{3}{(x-1)(x-2)} dx = \int \left(\frac{-3}{x-1} + \frac{3}{x-2} \right) dx = -3 \ln|x-1| + 3 \ln|x-2|$$

Eks:

$$\int \frac{3x}{x^2-3x+2} dx = \int \left(\frac{A}{x-1} + \frac{B}{x-2} \right) dx$$

$$\frac{3x}{(x-1)(x-2)} = \frac{A}{x-1} + \frac{B}{x-2} \quad | \cdot (x-1)(x-2)$$

$$3x = A(x-2) + B(x-1)$$

$$x=2: 6 = B \rightarrow B = 6$$

$$x=1: 3 = -A \rightarrow A = -3$$

$$= \int \left(\frac{-3}{x-1} + \frac{6}{x-2} \right) dx = \underline{-3 \ln|x-1| + 6 \ln|x-2| + C}$$

$$= \ln|x-1|^{-3} + \ln|x-2|^6 + C$$

$$= \underline{\ln \frac{|x-2|^6}{|x-1|^3} + C}$$

Eks:

$$\int \frac{3x^2}{x^2-3x+2} dx : \quad \begin{array}{l} 3x^2 : (x^2-3x+2) = 3 \\ -(3x^2-9x+6) \\ \hline 9x-6 \end{array}$$

$$= \int \left(3 + \frac{9x-6}{x^2-3x+2} \right) dx$$

$$= 3x + \int \frac{9x-6}{x^2-3x+2} dx = ?$$