16/04/09:

Repetisjon: Ubestemt integral (15.1–15.5)

Regneregler:

(1) $\int (u \pm v)\,dx = \int u\,dx \pm \int v\,dx$

(2) $\int c \cdot u\,dx = c \cdot \int u\,dx$ (c konstant)

(3) $\int x^n\,dx = \frac{1}{n+1} \cdot x^{n+1} + C$ $(n \neq -1)$

(4) $\int e^x\,dx = e^x + C$

(5) $\int \sin x\,dx = -\cos x + C$

(6) $\int \cos x\,dx = \sin x + C$

(7) $\int \frac{1}{x}\,dx = \ln|x| + C$

Metoder:

(a) Substitusjon: Bruk $du = u' \cdot dx$

Eks: $\int x \cdot \sin(x^2)\,dx = \int x \cdot \sin(u) \cdot \frac{du}{2x} = \int \frac{1}{2} \sin(u)\,du$

$u = x^2$
$du = u' \cdot dx$
$du = 2x \cdot dx$

$= \frac{1}{2} \cdot (-\cos u) + C$

$= -\frac{1}{2} \cdot \cos(x^2) + C$

Formel: $\int f(u) \cdot u' \cdot dx = \int f(u)\,du$

Eks: $\int \tan x \, dx = \int \frac{\sin x}{\cos x} dx = \int \frac{-1}{u} \cdot du$

$u = \cos x$
$du = -\sin x \cdot dx$

$= -\ln|u| + C = -\ln|\cos x| + C$

$dx = \frac{du}{-\sin x} \Rightarrow \int \frac{\sin x}{u} \cdot \frac{du}{-\sin x} = \int -\frac{1}{u} du$

$\sin x \cdot dx = -du \Rightarrow \int \frac{\sin x}{\cos x} \cdot dx = \int \frac{\sin x \cdot dx}{\cos x} = \int \frac{-du}{u}$

$= \int -\frac{1}{u} du$

(b) Delvis integrasjon:

Produkt regel for derivasjon: $(u \cdot v)' = u' \cdot v + u \cdot v'$

$\Rightarrow$ Integrasjons regel: $\int (u' \cdot v + u \cdot v') dx = u \cdot v + C$

$\int u' \cdot v \, dx + \int u \cdot v' dx = uv + C$

$$\boxed{\int u' \cdot v \, dx = u \cdot v - \int u \cdot v' dx}$$

Eks: $\int x \cdot \sin(x) \, dx = \frac{1}{2}x^2 \cdot \sin x - \int \frac{1}{2}x^2 \cdot \cos x \, dx$

$u = \frac{1}{2}x^2 \quad v = \sin x$
$u' = x \quad v' = \cos x$

Eks: $\int x \cdot \sin x \, dx = \int \underset{u'}{\sin x} \cdot \underset{v}{x} \, dx = \int u' \cdot v \, dx$

$u = -\cos x$	$v = x$
$u' = \sin x$	$v' = 1$

$$= u \cdot v - \int u \cdot v' dx$$
$$= -\cos x \cdot x - \int (-\cos x) \cdot 1 \cdot dx$$
$$= -x \cdot \cos x + \int \cos x \, dx$$
$$= -x \cdot \cos x + \sin x + C$$
$$= \underline{\underline{\sin x - x \cdot \cos x + C}}$$

$$(\sin x - x \cdot \cos x + C)' = \cos x - (1 \cdot \cos x + x \cdot (-\sin x)) + 0$$
$$= \cancel{\cos x} - \cancel{\cos x} + x \cdot \sin x = x \cdot \sin x$$

$\int x \cdot e^{x^2} dx = \longrightarrow$ substitusjon

$$\int \cancel{x \cdot e^{x}} \, dx = \int u' \cdot v \, dx = u \cdot v - \int u \cdot v' dx$$

$u = e^x$	$v = x$
$u' = e^x$	$v' = 1$

$$= x \cdot e^x - \int e^x \cdot 1 \, dx$$
$$= \underline{\underline{x \cdot e^x - e^x + C}}$$

Eks: $\int (x \cdot \ln x) \, dx = \int u' \cdot v \, dx = u \cdot v - \int u \cdot v' dx$

~~$u = 2$, $v = x$, $u' = \ln x$, $v' = 1$~~

$u = \frac{1}{2}x^2$	$v = \ln x$
$u' = x$	$v' = \frac{1}{x}$

$$= \frac{1}{2}x^2 \cdot \ln x - \int \frac{1}{2}x^2 \cdot \frac{1}{x} \, dx$$
$$= \frac{1}{2}x^2 \ln x - \frac{1}{2} \int x \, dx$$
$$= \frac{1}{2}x^2 \ln x - \frac{1}{2} \cdot \frac{1}{2}x^2 + C$$
$$= \underline{\underline{\frac{1}{2}x^2 \cdot \ln x - \frac{1}{4}x^2 + C}}$$

Eks:

$$\int \ln x \, dx = \int 1 \cdot \ln x \, dx = x \cdot \ln x - \int x \cdot 1/x \, dx$$

$$= x \cdot \ln x - \int 1 \, dx$$

$$= x \cdot \ln x - x + C$$

$u = x$	$v = \ln x$
$u' = 1$	$v' = 1/x$

Eks:

$$\int x^2 \cdot \cos x \, dx = \sin x \cdot x^2 - \int 2x \cdot \sin x \, dx$$

$$= x^2 \cdot \sin x - 2 \int x \cdot \sin x \, dx$$

$$= x^2 \cdot \sin x - 2 \cdot (-x \cos x + \sin x) + C$$

$$= x^2 \cdot \sin x + 2x \cos x - 2 \sin x + C$$

$u = \sin x$	$v = x^2$
$u' = \cos x$	$v' = 2x$

$$\int x \cdot \sin x \, dx = -\cos x \cdot x - \int (-\cos x) \cdot 1 \, dx$$

$$= -x \cdot \cos x + \sin x + C$$

$u = -\cos x$	$v = x$
$u' = \sin x$	$v' = 1$