27/03/09:

Integrasjon:

(1) $\int (u \pm v)\,dx = \int u\,dx \pm \int v\,dx$

(2) $\int (cu)\,dx = c\int u\,dx$

(3) $\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C \quad , n \neq -1$

(4) $\int \sin x\,dx = -\cos x + C$

(5) $\int \cos x\,dx = \sin x + C$

(6) $\int e^x\,dx = e^x + C$

(7) $\int \frac{1}{x}\,dx = \ln|x| + C$

Eks: $\int \left(7e^x - 2\sin x + 35x^2\right) dx$

$= 7\cdot e^x - 2(-\cos x) + 35\cdot\frac{1}{3}x^3 + C$

$= \underline{7e^x + 2\cos x + \frac{35}{3}x^3 + C}$

Eks: $\int \frac{x-1}{x}\,dx = \int \left(\frac{x}{x} - \frac{1}{x}\right) dx$

$= \int \left(1 - \frac{1}{x}\right) dx = \underline{x - \ln|x| + C}$

$$\int \frac{x^2+4}{x-1}\,dx$$

Alt 1:

$$\left.\begin{array}{l} (x^2+4):(x-1) = x+1 \\ -(x^2-x) \\ \quad x+4 \\ -(x-1) \\ \quad 5 \end{array}\right\} \quad \frac{x^2+4}{x-1} = \underline{x+1+\frac{5}{x-1}}$$

Alt 2:

$$\frac{x^2+4}{x-1} = \frac{(x^2-1)+5}{x-1} = \frac{x^2-1}{x-1} + \frac{5}{x-1}$$

$$= \underline{x+1+\frac{5}{x-1}}$$

$$\int \frac{x^2+4}{x-1}\,dx = \int \left(x+1+\frac{5}{x-1}\right)dx$$

$$= \frac{1}{2}x^2 + x + 5\int \frac{1}{x-1}\,dx$$

$$= \underline{\frac{1}{2}x^2 + x + 5\cdot \ln|x-1| + C}$$

Sjekk: $\left(\ln|x-1|\right)' = \frac{1}{u}\cdot u' = \frac{1}{x-1}\cdot 1 = \frac{1}{x-1}$

$u = x-1$
$u' = 1$

$$\left(\ln|2x+1|\right)' = \frac{1}{2x+1}\cdot 2 = (2)\frac{1}{2x+1}$$ Kjerneregeden

$$\int \frac{1}{2x+1}\,dx = \underline{\frac{1}{2}\cdot\ln|2x+1| + C}$$

$$\int \frac{1}{1-x}\,dx = \frac{1}{-1}\cdot\ln|1-x| + C = \underline{-\ln|1-x| + C}$$

Omvendt kjerneregel

Hvis $u' =$ en konstant, så har vi

$$\boxed{\int f(u)\,dx = \frac{1}{u'}\cdot F(u) + C}$$

Eks:
$$\int e^{2x}\,dx = \int e^{u}\,dx = \frac{1}{2}e^{u} + C$$
$$= \underline{\frac{1}{2}e^{2x} + C}$$

$u = 2x$
$u' = 2$

$$\int \sin(2x+1)\,dx = \frac{1}{2}\cdot\left(-\cos(2x+1)\right) + C$$
$$= \underline{-\frac{1}{2}\cos(2x+1) + C}$$

$$\int (1-x)^5\,dx = -\frac{1}{6}(1-x)^6 + C$$

Substitusjon: (kap. 15.5)

Eks: $\int \frac{x}{x^2+1}\,dx =$

Substitusjon:

$u = x^2+1$

$u' = 2x$

$\boxed{du = u' \cdot dx}$

$\frac{du}{2x} = \frac{\cancel{2x} \cdot dx}{\cancel{2x}}$

Generell regel for substitusjon.

$\frac{du}{dx} = u'$ (Leibniz notasjon)

$du = u' \cdot dx$

$$\int \frac{x}{x^2+1}\,dx = \int \frac{\cancel{x}}{u} \cdot \frac{du}{2\cancel{x}} = \int \frac{1}{u} \cdot \frac{du}{2}$$

$$= \frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2} \cdot \ln|u| + C$$

$$= \frac{1}{2} \ln|x^2+1| + C = \underline{\frac{1}{2}\ln(x^2+1) + C}$$

Eks: $\int \frac{x}{x^3+1}\,dx = \int \frac{\cancel{x}}{u} \cdot \frac{du}{3x^{\cancel{2}}}$

$\boxed{\begin{array}{l} u = x^3+1 \\ du = 3x^2 \cdot dx \end{array}}$

$= \frac{1}{3}\int \frac{1}{u \cdot x}\,du$ går ikke, må prøve en annen metode.

Eks: $\int \frac{x}{\sqrt{x-1}}\,dx$

Alt I: $\boxed{u = \sqrt{x-1}}$

$u' = \frac{1}{2\sqrt{x-1}}$

$du = \frac{1}{2\sqrt{x-1}}\,dx$

$dx = 2\sqrt{x-1}\,du$

$u^2 = x-1$

$x = u^2+1$

$$\int \frac{x}{\sqrt{x-1}}\,dx$$

$$= \int \frac{x}{u}\cdot 2\sqrt{x-1}\,du$$

$$= \int \frac{x}{\cancel{u}}\cdot 2\cancel{u}\,du$$

$$= \int 2x\,du = 2\int x\,du$$

$$= 2\int (u^2+1)\,du$$

$$= 2\left(\tfrac{1}{3}u^3 + u\right) + C$$

$$= \frac{2}{3}u^3 + 2u + C$$

$$= \frac{2}{3}\left(\sqrt{x-1}\right)^3 + 2\sqrt{x-1} + C$$

Alt. 2: $u = x-1$

$du = dx$

$x = u+1$

$$\int \frac{x}{\sqrt{x-1}}\,dx = \int \frac{x}{\sqrt{u}}\,du$$

$$= \int \frac{u+1}{\sqrt{u}}\,du = \int\left(\frac{u}{\sqrt{u}} + \frac{1}{\sqrt{u}}\right)du$$

$$= \int \left(u^{1/2} + u^{-1/2}\right)du$$

$n+1 = 1/2$

$n+1 = 3/2$

$$= \frac{2}{3}\cdot u^{3/2} + 2\cdot u^{1/2} + C$$

$$= \frac{2}{3}(x-1)^{3/2} + 2(x-1)^{1/2} + C$$