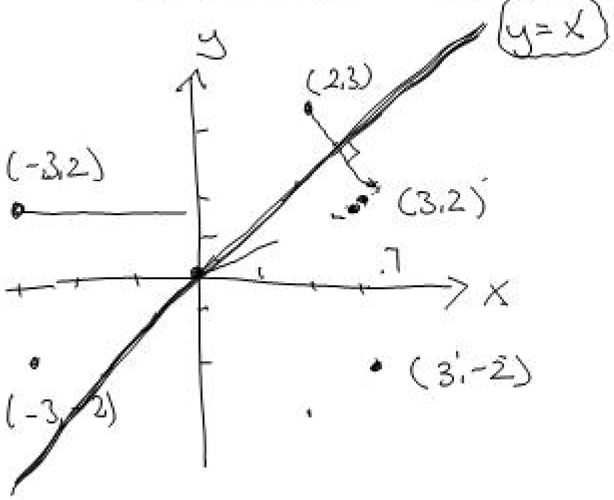
20/03/09:

Speiling og symmetri (kap. 11.8)



① **Speiling om x-aksen**

$(x, y) \longmapsto (x, -y)$

② **Speiling om y-aksen**

$(x, y) \longmapsto (-x, y)$

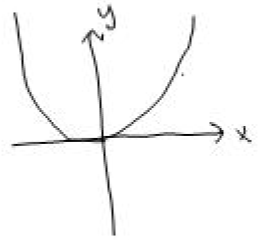
③ **Speiling om origo**

$(x, y) \longmapsto (-x, -y)$

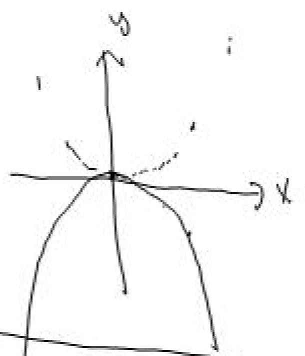
④ **Speiling om linjen $y = x$**

$(x, y) \longmapsto (y, x)$

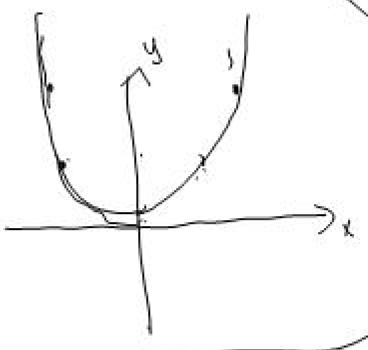
Eks: Grafen til $y = f(x) = x^2$



Speiling om x-aksen:

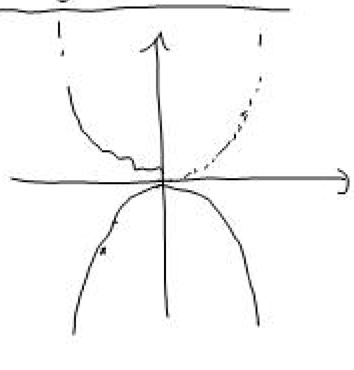


Speiling om y-aksen

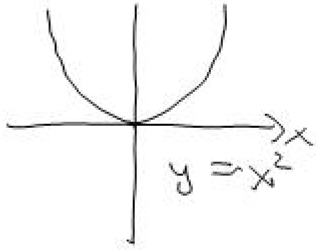


$y = x^2$ er symmetrisk om y-aksen

Speiling om origo:



Speiling av $y = x^2$ ved regning:



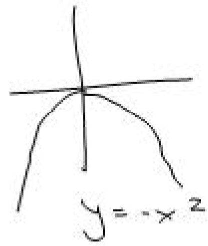
Om x-aksen:

$(x, y) \mapsto (x, -y)$

Før: $y = x^2$

Etter: $-y = x^2$

$y = -x^2$



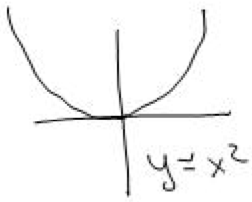
Om y-aksen:

$(x, y) \mapsto (-x, y)$

Før: $y = x^2$

Etter: $y = (-x)^2$

$y = x^2$



Om origo:

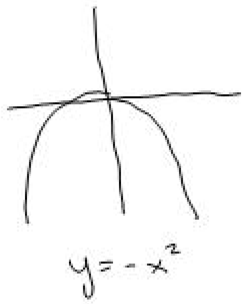
$(x, y) \mapsto (-x, -y)$

Før: $y = x^2$

Etter: $-y = (-x)^2$

$-y = x^2$

$y = -x^2$



Om linjen $y = x$:

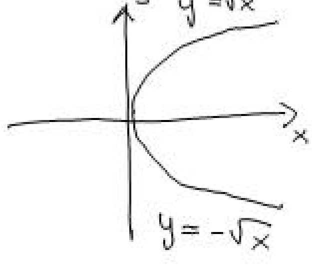
$(x, y) \mapsto (y, x)$

Før: $y = x^2$

Etter: $x = y^2$

$y^2 = x$

$y = \pm\sqrt{x}$



Definisjon:

Figuren $y = f(x)$ er symmetrisk om $\begin{cases} \text{origo} \\ x\text{-aksen} \\ y\text{-aksen} \end{cases}$

hvis figuren ikke forandrer seg under speiling om

$\begin{cases} y\text{-aksen} \\ x\text{-aksen} \\ \text{origo} \end{cases}$

Resultat:

$y = f(x)$ er symmetrisk om x-aksen: kun hvis $f(x) = 0$

—— || ———— om y-aksen: $f(-x) = f(x)$

—— || ———— om origo: $f(-x) = -f(x)$

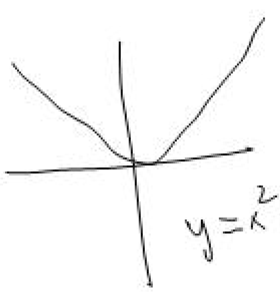
Eks:

$f(x) = x^2$

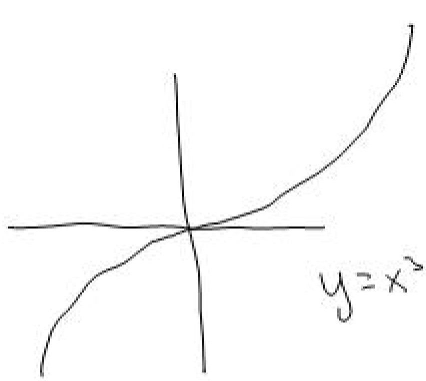
$f(-x) = (-x)^2 = x^2$

$f(x) = f(-x) \Rightarrow$ symm. om y-aksen

$\left.\begin{array}{l} f(x) = x^3 \\ f(-x) = (-x)^3 = -x^3 \end{array}\right\}$ $f(-x) = -f(x) \Rightarrow$ symm. om origo



$y = x^2$

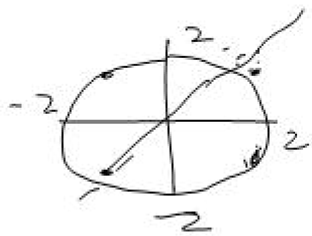


$y = x^3$

Eks: $x^2+y^2=4$

$y^2 = 4-x^2$

$y = \pm\sqrt{4-x^2}$



X-aksen:

$(x,y) \mapsto (x,-y)$

$x^2+y^2=4$

$x^2+(-y)^2=4$

$x^2+y^2=4$

y-aksen:

$(x,y) \mapsto (-x,y)$

$x^2+y^2=4$

$(-x)^2+y^2=4$

$x^2+y^2=4$

Origo:

$(x,y) \mapsto (-x,-y)$

$x^2+y^2=4$

$(-x)^2+(-y)^2=4$

$x^2+y^2=4$

om $x=y$:

$(x,y) \mapsto (y,x)$

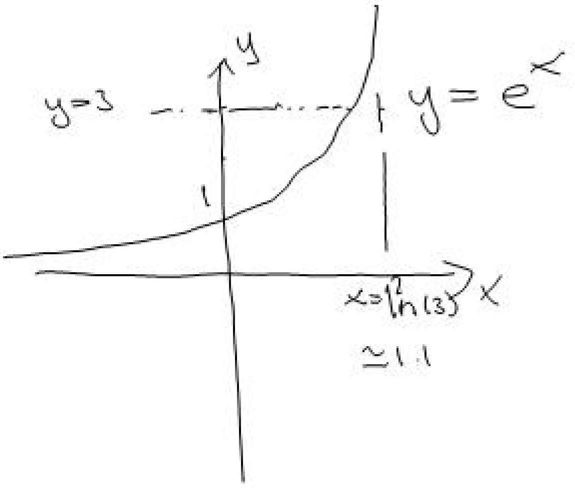
$x^2+y^2=4$

$y^2+x^2=4$

$x^2+y^2=4$

Omvendte / inverse funksjoner (kap. 11.9)

Eks! $\ln x$ er den omvendte funksjonen til e^x



$$f(x) = e^x$$

Likningen. $e^x = 3$.
$x = \ln(3)$

Omvendt funksjon: $e^x = y$
$x = \ln(y)$

$$x = f^{-1}(y) = \ln(y)$$

er den omvendte funksjonen til $y = f(x) = e^x$.

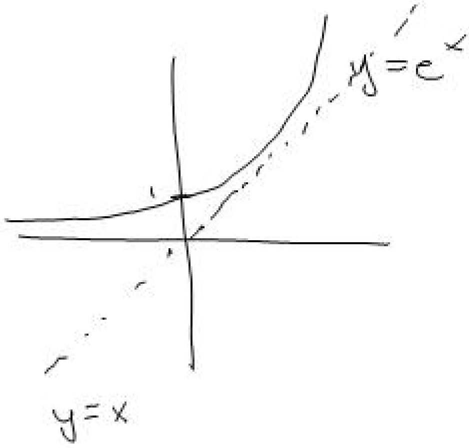
Egenskaper til omvendte funksjoner:

$$\boxed{\begin{aligned} f^{-1}(f(x)) &= x \\ f(f^{-1}(y)) &= y \end{aligned}}$$

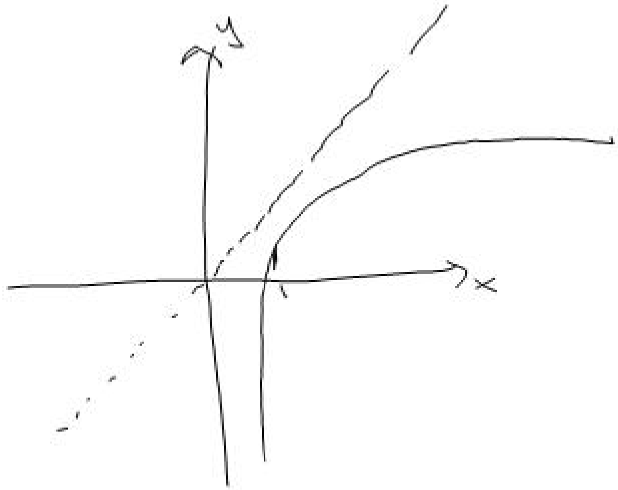
Grafen til $f^{-1}(x)$ er speilig av grafen til $f(x)$ om linjen $y = x$.

Eks.:

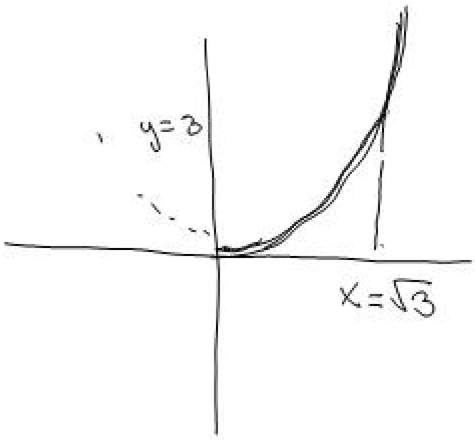
$f(x) = e^x$



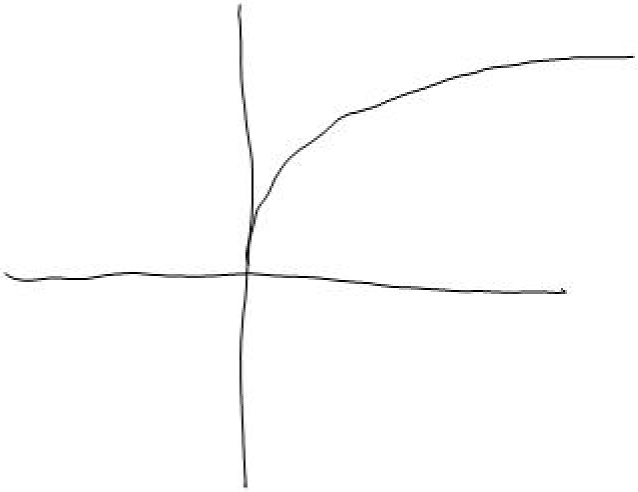
$f^{-1}(x) = \ln x$



$f(x) = x^2, \; x \geq 0$



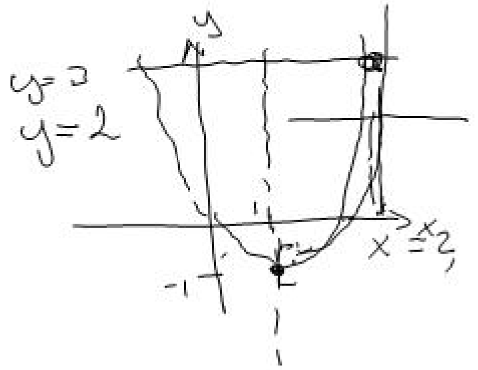
$f^{-1}(x) = \sqrt{x}$



$$\left.\begin{aligned} D_f &= V_{f^{-1}} \\ V_f &= D_{f^{-1}} \end{aligned}\right\}$$ x og y bytter roller

Eks: $f(x) = x^2 - 2x$, $x \geq 1$

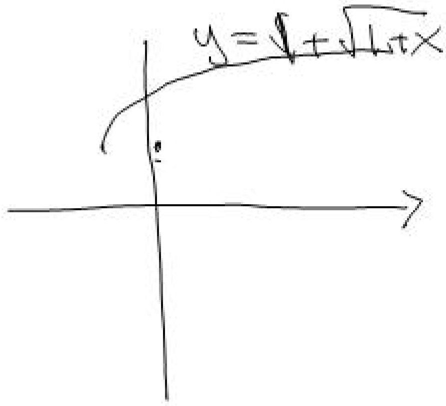
$= (x-1)^2 - 1$



$f(x) = x^2 - 2x$

$D_f = [1, \infty)$

$V_f = [-1, \infty)$



$$y = x^2 - 2x$$

$$0 = x^2 - 2x - y$$

$$x = \frac{2 \pm \sqrt{4 - 4\cdot 1\cdot(-y)}}{2\cdot 1}$$

$$= \frac{2 \pm \sqrt{4+4y}}{2}$$

$$= \frac{2 \pm \sqrt{4}\cdot\sqrt{1+y}}{2}$$

$$= 1 \pm \sqrt{1+y}$$

$$x = 1 + \sqrt{1+y}$$

$$f^{-1}(y) = 1 + \sqrt{1+y}$$

Eks: $y = 3$

$x = 3$