19/03/09: **Derivasjon av** $\begin{cases} \text{eksponensialfunksjoner} \\ \text{logaritmer} \end{cases}$

(a) $(e^x)' = e^x$

(b) $(\ln x)' = \frac{1}{x}$

Eks: $(x \cdot e^x)' = 1 \cdot e^x + x \cdot e^x$

$= e^x + x e^x = \underline{\underline{(1+x)e^x}}$

$$\left(\frac{e^x}{e^x+1}\right)' = \frac{e^x \cdot (e^x+1) - e^x \cdot (e^x)}{(e^x+1)^2}$$

$$= \frac{\underbrace{\cancel{(e^x)^2}}_{=e^{2x}} + e^x - \underbrace{\cancel{(e^x)^2}}_{=e^{2x}}}{(e^x+1)^2} = \underline{\underline{\frac{e^x}{(e^x+1)^2}}}$$

$$(x \cdot \ln x)' = 1 \cdot \ln x + \cancel{x} \cdot \frac{1}{\cancel{x}} = \underline{\underline{\ln x + 1}}$$

$$((\ln x)^2)' = 2 \ln x \cdot \frac{1}{x} = \underline{\underline{\frac{2 \ln x}{x}}}$$

$$(\ln(x^2+1))' = \frac{1}{x^2+1} \cdot 2x = \underline{\underline{\frac{2x}{x^2+1}}}$$

Eks: $(2^x)' = 2^x \cdot \ln 2 \simeq 0.69 \cdot 2^x$

$(\log x)' = ?$

Omforming av eksponensial funksjoner:

Eks: $2^x = (e^{\ln 2})^x$

$= e^{\ln 2 \cdot x}$ eksakt

$\simeq \underline{\underline{e^{0.69x}}}$

$2 = e^{\ln 2}$

$f(x) = 1000 \cdot e^{0.4x}$

$= C \cdot e^{kx} \quad \left(\begin{array}{l} C = 1000 \\ k = 0.4 \end{array}\right)$

$\cong 1000 \cdot 1.49^x$

$e^{0.4} \cong 1.49$

$\underline{r = 49\%}$

$(a^x)' = ((e^{\ln a})^x)' = (e^{\ln a \cdot x})'$

$u = \ln a \cdot x$
$u' = \ln a$

$= (e^u)' = e^u \cdot u'$

$= e^{\ln a \cdot x} \cdot \ln a$

$= a^x \cdot \ln a$

$\boxed{(a^x)' = a^x \cdot \ln a \quad \text{når } a \text{ er et gitt tall } (a>0)}$

$$(a^x)' = a^x \cdot \left(\lim_{h \to 0} \frac{a^h - 1}{h} \right)$$

$$= a^x \cdot \ln a$$

$$(\log_a x)' = ?$$

Omforming av logaritmer

$$\boxed{\log_a (x) = \frac{\ln(x)}{\ln(a)}} = \frac{1}{\ln a} \cdot \ln x$$

Eks: $\log_3 (10) = \frac{\ln 10}{\ln 3}$

$$\log_3 (x) = \frac{\ln x}{\ln 3} = \frac{1}{\ln 3} \ln x$$

$$3^{\frac{\ln 10}{\ln 3}} = \left(e^{\ln 3}\right)^{\frac{\ln 10}{\ln 3}} = e^{\cancel{\ln 3} \cdot \frac{\ln 10}{\cancel{\ln 3}}}$$

$$= e^{\ln 10} = \underline{\underline{10}}$$

$$(\log_a (x))' = \left(\frac{1}{\ln a} \cdot \ln x \right)' = \frac{1}{\ln a} \cdot \frac{1}{x}$$

Formel: $\boxed{(\log_a x)' = \frac{1}{\ln a} \cdot \frac{1}{x}}$ når $a > 0$ er et gitt tall

Derivasjonsregler

$$(e^x)' = e^x \qquad (a^x)' = a^x \cdot \ln a$$

$$(\ln x)' = \frac{1}{x} \qquad (\log_a x)' = \frac{1}{x} \cdot \frac{1}{\ln a}$$

Eks: Funksjonsdrøfting av

$f(x) = \ln(x^2+1)$, $D_f = (-\infty, \infty)$

(a) Skjæringspunkter med aksene

x-aksen: $y = 0$ (nullpkt)

$\ln(x^2+1) = 0$ | bruker e^x

$e^{\ln(x^2+1)} = e^0$

$x^2+1 = 1$

$x^2 = 0$

$\underline{\underline{x = 0}} \longrightarrow \underline{\underline{(0,0)}}$

y-aksen: $x = 0$

$f(0) = \ln(0^2+1) = \ln(1) = 0 \rightarrow (0,0)$

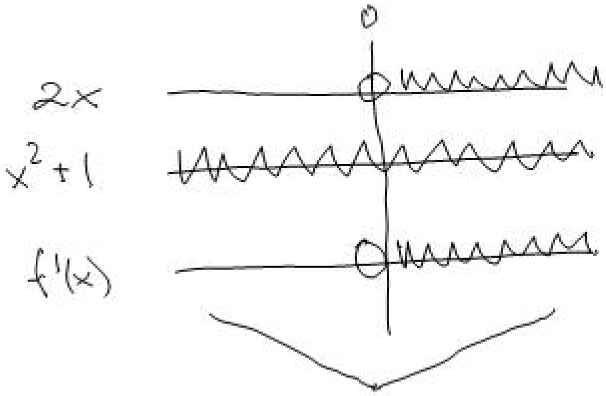
(b) Finn $f'(x)$.

$$f'(x) = \left(\ln(x^2+1)\right)' = \frac{1}{x^2+1} \cdot 2x = \underline{\underline{\frac{2x}{x^2+1}}}$$

(c) Finn lokale topp/bunnpkt

$$f'(x) = \frac{2x}{x^2+1}$$

$x^2+1=0$
ingen løsning.



Lokalt bunnpkt

$$\left.\begin{aligned} x &= 0 \\ y &= f(0) = 0 \end{aligned}\right\} \quad \underline{\underline{(0,0)}}$$

(d) Finn f''(x).

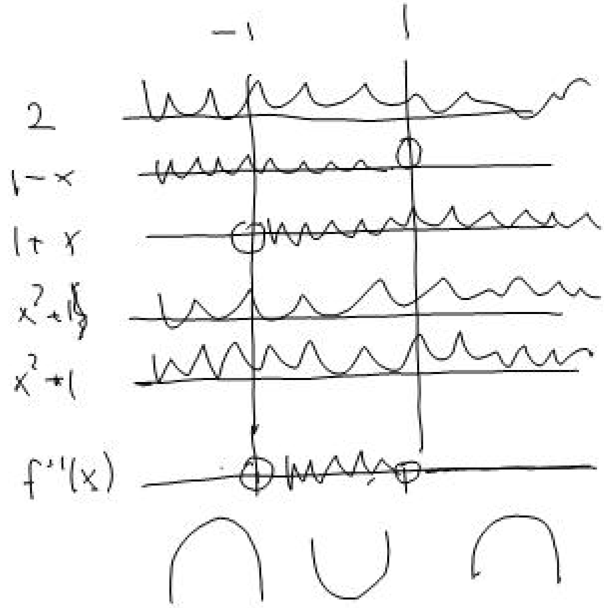
$$f''(x) = \left(\frac{2x}{x^2+1}\right)' = \frac{2\cdot(x^2+1) - 2x\cdot 2x}{(x^2+1)^2}$$

$$= \frac{2x^2+2-4x^2}{(x^2+1)^2} = \frac{2-2x^2}{(x^2+1)^2}$$

$$= \frac{2\cdot(1-x^2)}{(x^2+1)^2} = \underline{\underline{\frac{2\cdot(1-x)(1+x)}{(x^2+1)^2}}}$$

(e) Finn vendepkt / når f er konkav / konveks

$$f''(x) = \frac{2(1-x)(1+x)}{(x^2+1)^2}$$



<u>Vendepkt</u>: $(-1, \ln 2)$, $(1, \ln 2)$

$x = -1$: $y = f(-1) = \ln((-1)^2+1) = \ln(2)$

$x = 1$: $y = f(1) = \ln(1^2+1) = \ln(2)$

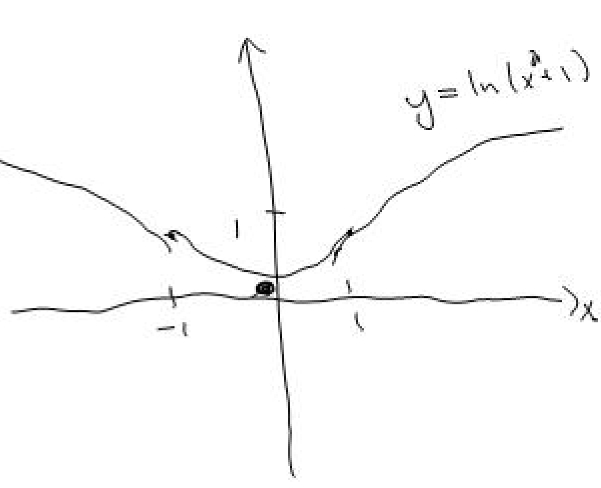
$(-\infty, -1]$ $\qquad$ $[1, \infty)$

<u>Konkav</u>: $x \leq -1$ eller $x \geq 1$

<u>Konveks</u>: $-1 \leq x \leq 1$

$[-1, 1]$

(f) Tegn grafen.

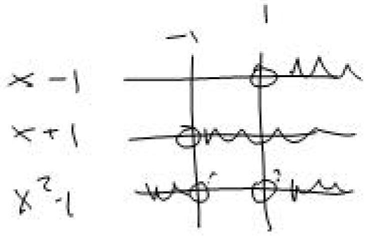


Asymptoter og logaritmer

Eks: $f(x) = \ln(x^2-1)$

$D_f = ?$ $\quad x^2 - 1 > 0 \Rightarrow x > 1$ eller $x < -1$

$x^2 - 1 = (x-1)(x+1)$



$D_f = (-\infty, -1) \cup (1, \infty)$

$x = -1$ og $x = 1$ er vertikale asymptoter

ingen skrå eller horisontale asymptoter

$\lim\limits_{\substack{x \to \infty \\ -\infty}} \ln(x^2+1) = \infty \rightarrow$ ingen horisontal

$\lim\limits_{\substack{x \to \infty \\ -\infty}} \dfrac{\ln(x^2+1)}{x} = 0 \rightarrow$ ingen skrå