Ulikheter (kap. 10.7)

Eks: $\dfrac{\sin x}{\cos x - 1} < 1$, $\boxed{x \in (0, 2\pi)}$

$$\frac{\sin x}{\cos x - 1} - 1 < 0$$

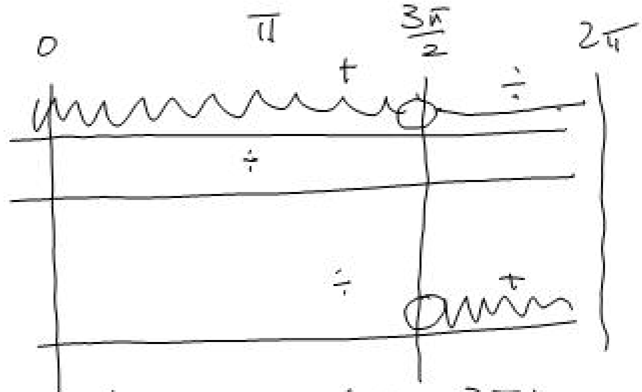
$$\frac{\sin x - 1\cdot(\cos x - 1)}{\cos x - 1} < 0$$

$$\boxed{\frac{\sin x - \cos x + 1}{\cos x - 1} < 0}$$

① $\sin x - \cos x + 1$

② $\cos x - 1$

$$\frac{\sin x - \cos x + 1}{\cos x - 1}$$



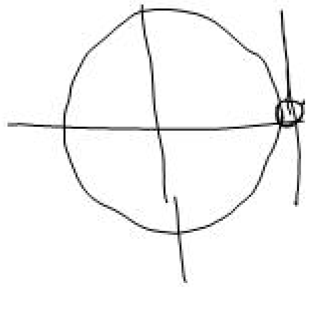
Løsning: $x \in \left(0, \frac{3\pi}{2}\right)$

②: $\cos x - 1 = 0$

$\cos x = 1$

$x = \cos^{-1}(1) + n \cdot 2\pi$

$= 0 + n \cdot 2\pi$ ingen løsn



setter inn $x = \frac{\pi}{2}$: $\cos\left(\frac{\pi}{2}\right) - 1 = -1 < 0$

(1)

$$\sin x - \cos x + 1 = 0$$
$$\sin x - \cos x = -1$$
$$A \cdot \sin(x - \varphi) = -1$$
$$\frac{\cancel{\sqrt{2}} \cdot \sin(x - \pi/4)}{\cancel{\sqrt{2}}} = \frac{-1}{\sqrt{2}}$$
$$\sin(x - \pi/4) = -1/\sqrt{2}$$

$$\boxed{\begin{aligned} A &= \sqrt{1^2 + (-1)^2} = \sqrt{2} \\ \varphi &= \arctan\left(-\frac{(-1)}{1}\right) \\ &= \arctan(1) = \pi/4 \end{aligned}}$$

$$x - \pi/4 = \sin^{-1}(-1/\sqrt{2}) + n \cdot 2\pi$$
$$x - \pi/4 = -\pi/4 + n \cdot 2\pi \implies x = \underline{0 + n \cdot 2\pi}$$

ingen løsn.

eller

$$x - \pi/4 = \pi - \sin^{-1}(-1/\sqrt{2}) + n \cdot 2\pi$$
$$x - \pi/4 = \frac{5\pi}{4} + n \cdot 2\pi \implies x = \underline{\frac{3\pi}{2} + n \cdot 2\pi}$$
$$= \frac{3\pi}{2}$$

Setter inn $\underline{x = 2\pi - \pi/6} = 360^\circ - 30^\circ = \underline{330^\circ}$

$$\sin x - \cos x + 1 = \sin(-30^\circ) - \cos(-30^\circ) + 1$$
$$= -1/2 - \sqrt{3}/2 + 1 = \frac{1}{2} - \frac{\sqrt{3}}{2} < 0$$

Derivasjon av trigonometriske funksjoner (kap. 10.8, 10.9)

$$\lim_{x\to 0} \frac{\sin x}{x} = 1$$

$$\lim_{x\to 0} \frac{\cos x - 1}{x} = 0$$

viktige grenseverdier

$(\sin x)' = ?$

$f(x) = \sin x$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\sin(x+h) - \sin(x)}{h}$$

$$= \lim_{h\to 0} \frac{\sin x \cdot \cos h + \cos x \cdot \sin h - \sin x}{h}$$

$$= \lim_{h\to 0} \left(\sin x \cdot \underbrace{\left(\frac{\cos h - 1}{h} \right)}_{= 0} + \cos x \cdot \underbrace{\left(\frac{\sin h}{h} \right)}_{= 1} \right)$$

$$= \sin x \cdot 0 + \cos x \cdot 1 = \underline{\cos x}$$

Regneregler:

① $(\sin x)' = \cos x$

② $(\cos x)' = -\sin x$

③ $(\tan x)' = 1 + \tan^2 x = \dfrac{1}{\cos^2 x} \quad \left(= \sec^2 x\right)$

Eks:

$$(2 \sin x - 1)' = 2 \cdot (\sin x)' - 0 = \underline{2\cos x}$$

$$\left(\sin\left(\tfrac{\pi}{2} - x\right)\right)' = (\sin(u))' = \cos(u) \cdot u'$$

$$\boxed{\begin{aligned} u &= \tfrac{\pi}{2} - x \\ u' &= -1 \end{aligned}}$$

$$= \cos(u) \cdot (-1) = \underline{-\cos\left(\tfrac{\pi}{2} - x\right)}$$

$$(\cos x)' = -\sin x$$

$$\sin\left(\tfrac{\pi}{2} - x\right) = \cos x$$
$$\cos\left(\tfrac{\pi}{2} - x\right) = \sin x$$

$$\left(\frac{\sin x}{\cos x}\right)' = \frac{(\sin x)' \cdot \cos x - \sin x \cdot (\cos x)'}{(\cos x)^2} = \frac{\cos x \cdot \cos x - \sin x\,(-\sin x)}{(\cos x)^2}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = 1 + \tan^2 x = \frac{1}{\cos^2 x}$$

Eks:

$$(x\cdot\tan x)' = (x)'\cdot\tan x + x\cdot(\tan x)'$$

$$= 1\cdot\tan x + x\cdot(1+\tan^2 x)$$

$$= \underline{\tan x + x + x\tan^2 x}$$

$$\left(\frac{\sin x}{\cos x - 1}\right)' = \frac{(\sin x)'\cdot(\cos x-1) - \sin x\,(\cos x-1)'}{(\cos x-1)^2}$$

$$= \frac{\cos x\cdot(\cos x-1) - \sin x\cdot(-\sin x)}{(\cos x-1)^2}$$

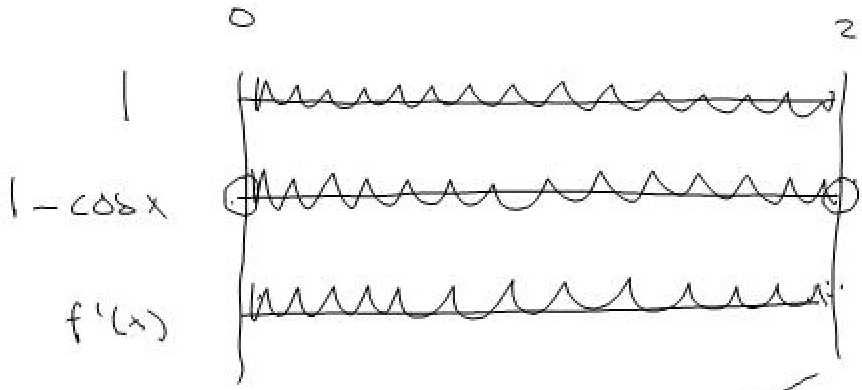
$$= \frac{\cos^2 x - \cos x + \overset{=1}{\sin^2 x}}{(\cos x-1)^2} = \frac{\cancel{1-\cos x}^{\,-1}}{(\cos x-1)^{2}}$$

$$= \cancel{\cos x+1}\quad \frac{1-\cos x}{(\cos x-1)^2} = \frac{-1\cdot\cancel{(\cos x-1)}}{(\cos x-1)^{\cancel{2}}}$$

$$= \frac{-1}{\cos x-1} = \underline{\underline{\frac{1}{1-\cos x}}}$$

Eks I: $f(x) = \frac{\sin x}{\cos x - 1}$, $x \in (0, 2\pi)$

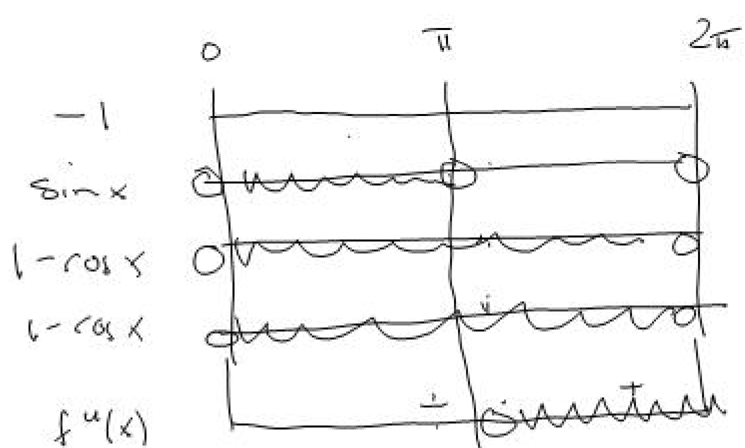
$$f'(x) = \frac{1}{1 - \cos x}$$



$$\left(\frac{u}{v}\right)' = \frac{u' \cdot v - u \cdot v'}{v^2}$$

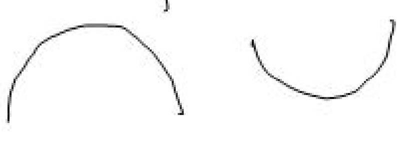
Vendepkt: $f''(x) = \left(\frac{1}{1-\cos x}\right)' = \frac{0 \cdot (1-\cos x) - 1 \cdot (1-\cos x)'}{(1-\cos x)^2}$

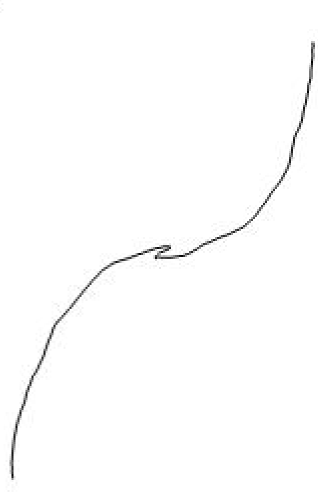
$$= \frac{-1 \cdot \sin x}{(1-\cos x)^2} = \frac{-\sin x}{(1-\cos x)^2} = \frac{(-1) \cdot (\sin x)}{(1-\cos x)(1-\cos x)}$$



$x = \pi$ er et vendepkt.

$y = 0$





Eks 2:

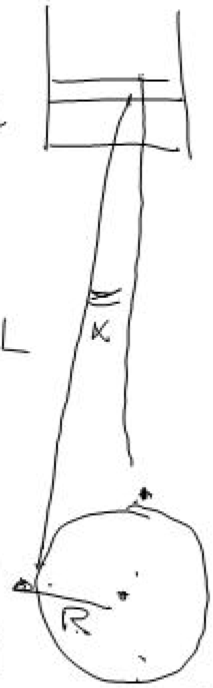
n ~~omdr.~~ /min
runder

$$\omega = \frac{2\pi n}{60} \ \mathrm{rad/s} \quad \text{(vinkelhastighet)}$$

$$v(\alpha) = \omega R \cdot \left(\sin(\alpha) + \frac{R}{2L}\sin(2\alpha)\right)$$

α: vinkelen i sirkelbevegelsen (rad)
v: hastigheten til stempelet (m/s)

(m/s) $v \downarrow$



$$\omega R = \frac{2\pi R \cdot n}{60} \ \mathrm{m/s} = \underline{7.0} \ \mathrm{m/s}$$

$$\frac{R}{2L} = \underline{0.12}$$

$$v(\alpha) = 7.0 \cdot (\sin\alpha + 0.12 \cdot \sin(2\alpha))$$
$$= \underline{7.0 \cdot \sin\alpha + 0.84 \cdot \sin(2\alpha)}$$

$$v'(\alpha) = 7.0 \cdot \cos\alpha + 0.84 \cdot \cos(2\alpha) \cdot 2$$
$$= \underline{7.0 \cdot \cos\alpha + 1.68 \cdot \cos(2\alpha)}$$
$$= 7.0 \cdot \cos\alpha + 1.68 \cdot (\cos^2\alpha - \sin^2\alpha)$$
$$= 7.0 \cos\alpha + 1.68 \cdot \cos^2\alpha - 1.68 \cdot \sin^2\alpha$$

$R = 0.10\,\mathrm{m}$
$\omega = 70 \ \mathrm{rad/s}$
$L = \dfrac{0.10\,\mathrm{m}}{24} \sim 0.4\,\mathrm{m}$

$\underline{v'(\alpha) = 0}$:

$$7.0 \cos\alpha + 1.68\cos^2\alpha - 1.68\sin^2\alpha = 0$$
$$7.0 \underline{\cos\alpha} + \underline{1.68\cos^2\alpha} - 1.68\,\underline{(1-\cos^2\alpha)} = 0$$
$$3.36\cos^2\alpha + 7.0\cos\alpha - 1.68 = 0$$
$$\cos\alpha \cong 0.217 \quad \text{eller} \quad \cancel{\cos\alpha \cong -2.30}$$
$$\alpha = \cos^{-1}(0.217) \cong \underline{1.352\,\mathrm{rad}} \cong 77.5^\circ$$