26/02/09

Trigonometriske likninger

$\begin{cases} \text{kap. 10.1-10.2} \\ \text{kap. 6} \end{cases}$

Repetisjon:

(1) Grunnleggende likninger

$\sin x = c \qquad \cos x = c \qquad \tan x = c$

(2) Likninger med én trigonometrisk funksjon

Eks: $2\sin x - 3 = 1$

$\sin^2 x - 1 = 2\sin x$

$\cos(2x - 3) = -1$

(3) Likninger med flere trigonometriske funksjoner

Løsningsmetoder:

(a) Faktorisering: Eks: $2\sin x \cdot \cos x = 0$

$\sin x = 0$ eller $\cos x = 0$

(b) Bruk $\tan x = \frac{\sin x}{\cos x}$:

Eks: $\sin x - 2\cos x = 0 \quad | : \cos x$

$\tan x - 2 = 0$

(c) Bruk $\sin^2 x + \cos^2 x = 1$: Eks: $\cos^2 x = 1 - \sin x$

$1 - \sin^2 x = 1 - \sin x$

$\sin^2 x = 1 - \cos^2 x$

$\cos^2 x = 1 - \sin^2 x$

Likningen $a \cdot \sin x + b \cdot \cos x = c$ (a, b, c er gitte tall)

$c = 0$: Løser likningen ved å dividere med $\cos x$ i likningen.

$c \neq 0$: Eks: $\sin x + \sqrt{3}\cos x = 1$

Metode 1:

$\cos^2 x = 1 - \sin^2 x$

$\cos x = \pm\sqrt{1-\sin^2 x}$

$$\sin x + \sqrt{3}\cos x = 1$$
$$\sqrt{3}\cos x = 1 - \sin x$$
$$(\sqrt{3}\cos x)^2 = (1-\sin x)^2$$
$$3\cos^2 x = 1 - 2\sin x + \sin^2 x$$
$$3\cdot(1-\sin^2 x) = 1 - 2\sin x + \sin^2 x$$
$$3 - 3\sin^2 x = 1 - 2\sin x + \sin^2 x$$
$$-4\sin^2 x + 2\sin x + 2 = 0$$
$$\sin x = \frac{-2 \pm \sqrt{4 - 4(-4)\cdot 2}}{2(-4)}$$
$$= \frac{-2 \pm 6}{-8}$$

$\sin x = -1/2$ eller $\sin x = 1$

~~$x = -\frac{\pi}{6} + n\cdot 2\pi$~~

eller

~~$x = \frac{7\pi}{6} + n\cdot 2\pi$~~

eller

$x = -\frac{\pi}{6} + n\cdot 2\pi$

$x = \frac{\pi}{2} + n\cdot 2\pi$

Sett prøve:

$x = -\frac{\pi}{6}$: VS $= -\frac{1}{2} + \frac{3}{2} = 1$ ok
HS $= 1$

$x = \frac{7\pi}{6}$: VS $= -\frac{1}{2} - \frac{3}{2} = -2$ falsk
HS $= 1$

$x = \frac{\pi}{2}$: VS $= 1 + \sqrt{3}\cdot 0 = 1$ ok
HS $= 1$

<u>Metode 2:</u> $\sin x + \sqrt{3}\cos x = 1$

Addisjon av bølgefunksjoner (harmoniske svingninger)

$$\sin x = 1 \cdot \sin(1\cdot(x-0)) + 0$$
$$\sqrt{3}\cos x = \sqrt{3}\cdot \sin(1\cdot(x+\tfrac{\pi}{2})) + 0$$
$$\sin x + \sqrt{3}\cos x = A \cdot \sin(1\cdot(x-\varphi)) + 0$$

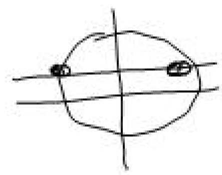
$$\boxed{\cos x = \sin(x+\tfrac{\pi}{2})}$$

<u>Formler</u>: $a\cdot \sin x + b \cdot \cos x = A \cdot \sin(x-\varphi)$

der $\boxed{\begin{aligned} A &= \sqrt{a^2+b^2} \\ \varphi &= \arctan(-b/a) \end{aligned}}$

<u>Eks:</u> $\underline{\sin x + \sqrt{3}\cos x = 2\sin(x+\pi/3)}$

$$A = \sqrt{1^2+(\sqrt{3})^2} = \underline{2}$$
$$\varphi = \arctan(-\sqrt{3}/1)$$
$$= \arctan(-\sqrt{3})$$
$$= \underline{-\pi/3}$$



<u>Løse likningen:</u> $\sin x + \sqrt{3}\cos x = 1$

$$\frac{2\cdot\sin(x+\pi/3)}{2} = \frac{1}{2}$$

$$\sin(x+\pi/3) = \frac{1}{2}$$

$$x+\frac{\pi}{3} = \sin^{-1}(1/2) + n\cdot 2\pi$$
$$= \frac{\pi}{6} + n\cdot 2\pi$$

eller

$$x+\frac{\pi}{3} = \frac{5\pi}{6} + n\cdot 2\pi$$

$$x + \frac{\pi}{3} = \frac{\pi}{6} + n \cdot 2\pi$$

$$x = \frac{\pi}{6} + n \cdot 2\pi - \frac{\pi}{3} = \underline{\underline{-\frac{\pi}{6} + n \cdot 2\pi}}$$

eller

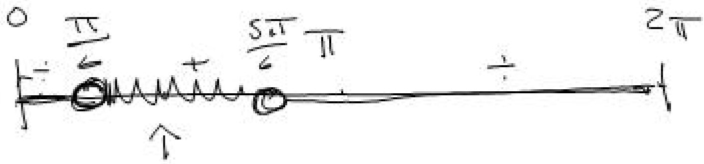
$$x + \frac{\pi}{3} = \frac{5\pi}{6} + n \cdot 2\pi$$

$$x = \frac{5\pi}{6} - \frac{\pi}{3} + n \cdot 2\pi = \underline{\underline{\frac{\pi}{2} + n \cdot 2\pi}}$$

Ulikheter: (kap. 10.7)

Eks. $\underline{2\sin x - 1 > 0}$, $x \in [0, 2\pi)$

$2\sin x - 1$



Løsning: $x \in \underline{(\pi/6, 5\pi/6)}$

$2\sin x - 1 = 0$, $0 \le x < 2\pi$

$$\frac{2\sin x}{2} = \frac{1}{2}$$

$$\sin x = 1/2$$

$$x = \frac{\pi}{6} + n \cdot 2\pi = \underline{\frac{\pi}{6}}$$

eller

$$x = \frac{5\pi}{6} + n \cdot 2\pi = \underline{\frac{5\pi}{6}}$$

Finner fortegn på hvert delintervall:

(a) $[0, \pi/6)$: velger $x = 0$

$2 \cdot \sin 0 - 1 = -1 < 0$

(b) $(\pi/6, \frac{5\pi}{6})$: velger $x = \pi/2$

$2 \cdot \sin \frac{\pi}{2} - 1 = 1 > 0$

(c) $(\frac{5\pi}{6}, 2\pi)$: velger $x = \pi$

$2 \cdot \sin \pi - 1 = -1 < 0$