Trigonometriske likninger

(kap. 10.1 – 10.2 + kap. 6)

① Grunnleggende likninger:

* $\sin x = c$
* $\cos x = c$
* $\tan x = c$

② Likninger med én trigonometrisk funksjon

Eks:

$$2\tan x + 1 = 3$$

$$\frac{\cancel{2}\tan x}{\cancel{2}} = \frac{2}{2}$$

$$\tan x = 1$$

$$x = \arctan(1) + n\cdot\pi$$

$$= \pi/4 + n\cdot\pi$$

Eks:

$u = \sin x$

$$2 + \sin^2 x = 4\sin x$$

$$2 + u^2 = 4u$$

$$u^2 - 4u + 2 = 0$$

$$u = \frac{4 \pm \sqrt{16 - 4\cdot 1\cdot 2}}{2} = \frac{4 \pm \sqrt{8}}{2} = 2 \pm \frac{2\sqrt{2}}{2} = 2 \pm \sqrt{2}$$

$\sin x = 2 + \sqrt{2}$ eller $\sin x = 2 - \sqrt{2}$

ingen løsning.

$$x = \arcsin(2 - \sqrt{2}) + n\cdot 2\pi$$

$$= 0.626 + n\cdot 2\pi$$

eller

$$x = \pi - \arcsin(2-\sqrt{2}) + n\cdot 2\pi$$
$$\simeq \underline{2.516 + n\cdot 2\pi}$$

Svar: $x \cong \underline{0.626 + n\cdot 2\pi}$

eller

$x \cong \underline{2.516 + n\cdot 2\pi}$

Ex: $2\boxed{\cos(x/2+\pi)} - 1 = 0$

$T = \frac{2\pi}{1/2} = 4\pi \longrightarrow$ $\cos(x/2+\pi) = 1/2$

$\cos(u) = 1/2$

$\boxed{u = x/2+\pi}$

$u = \cos^{-1}(1/2) + n\cdot 2\pi$

eller

$u = -\cos^{-1}(1/2) + n\cdot 2\pi$

① $x/2+\pi = \cos^{-1}(1/2) + n\cdot 2\pi$

eller

② $x/2+\pi = -\cos^{-1}(1/2) + n\cdot 2\pi$

① $x/2+\pi = \pi/3 + n\cdot 2\pi$

$x/2 = \pi/3 - \pi + n\cdot 2\pi = -\frac{2\pi}{3} + n\cdot 2\pi$

$x = 2\cdot\left(-\frac{2\pi}{3} + n\cdot 2\pi\right) = \underline{-\frac{4\pi}{3} + n\cdot 4\pi}$

② $x/2+\pi = -\pi/3 + n\cdot 2\pi$

$x/2 = -4\pi/3 + n\cdot 2\pi$

$x = \underline{-8\pi/3 + n\cdot 4\pi}$

~

(3) Likninger med flere trigonometriske funksjoner.

Eks:

$$\sin x + 3\cos x = 0$$
$$\sin x \cdot \cos x = 0$$
$$\sin x + 3\cos x = 1$$

Metode 1: Triks: $\tan x = \dfrac{\sin x}{\cos x}$

Eks: $\dfrac{\sin x}{\cos x} + \dfrac{3\cancel{\cos x}}{\cancel{\cos x}} = \dfrac{0}{\cos x}$ $\quad | : \cos x$

$$\tan x + 3 = 0$$
$$\tan x = -3$$
$$x = \tan^{-1}(-3) + n \cdot \pi$$
$$\approx -1.249 + n \cdot \pi$$

Eks: $\dfrac{\sin x}{\cos x} + \dfrac{3\cancel{\cos x}}{\cos x} = \dfrac{1}{\cos x}$ $\quad | : \cos x$

$$\tan x + 3 = 1/\cos x$$

hjelper ikke

$$\sin x + 3\cos^2 x = 0$$
$$\tan x + 3\cos x = 0$$

hjelper ikke

Typiske tilfeller hvor vi bruker denne metoden:

$a \cdot \sin x + b \cdot \cos x = 0$ a, b tall

$a \cdot \sin^2 x + b \cdot \sin x \cdot \cos x + c \cdot \cos^2 x = 0$ a, b, c tall

$\frac{\sin^2 x}{\cos^2 x} = \tan^2 x \qquad \frac{\sin x \cdot \cos x}{\cos^2 x} = \tan x \qquad \frac{\cos^2 x}{\cos^2 x} = 1$

Metode 2: Faktorisering:

$\boxed{u \cdot v = 0 \text{ gir } u = 0 \text{ eller } v = 0}$

Eks: $\sin x \cdot \cos x = 0$

$\sin x = 0$ eller $\cos x = 0$

Men: $\frac{\sin x \cdot \cancel{\cos x}}{\cancel{\cos x}} = \frac{0}{\cos x}$ $| : \cos x$

$\sin x = 0$

Husk:

$\cos x = 0$ er løsning

Metode 3: Bruk $\boxed{\sin^2 x + \cos^2 x = 1}$.

$$\begin{cases} \sin^2 x = 1 - \cos^2 x \\ \sin x = \pm\sqrt{1-\cos^2 x} \\ \\ \cos^2 x = 1 - \sin^2 x \\ \cos x = \pm\sqrt{1-\sin^2 x} \end{cases}$$

Eks: $\boxed{1 + 4\cos x = \sin^2 x}$

$$1 + 4\cos x = 1 - \cos^2 x$$

$$\cos^2 x + 4\cos x + 0 = 0$$

$$\cos x \cdot (\cos x + 4) = 0$$

$\cos x = 0$ $\qquad$ $\cos x = -4$

$x = \frac{\pi}{2} + n 2\pi$ $\qquad$ ingen løsning

eller

$x = -\frac{\pi}{2} + n \cdot 2\pi$

$\sin x = \pm\sqrt{1-\cos^2 x}$

Eks: $\sin x + 3\cos x = 1$

$$\sin x = 1 - 3\cos x \qquad | \ ( )^2$$

$$\sin^2 x = (1 - 3\cos x)^2 = 1 - 6\cos x + 9\cos^2 x$$

$$1 - \cos^2 x = 1 - 6\cos x + 9\cos^2 x$$

$$-10\cos^2 x + 6\cos x = 0$$

$$\cos x\,(-10\cos x + 6) = 0$$

$\cos x = 0$ $\qquad$ eller $\qquad$ $\cos x = 0.6$

$\cos x = 0$ eller $-10\cos x + 6 = 0$

$\cos x = 0.6$

$x = \cos^{-1}(0) + n\cdot 2\pi$

$= \underline{\pi/2 + n\cdot 2\pi}$

eller

$x = \underline{-\pi/2 + n\cdot 2\pi}$

$x_0 \approx \cos^{-1}(0.6) + n\cdot 2\pi$

$= \underline{0.927 + n\cdot 2\pi}$

eller

$x \simeq \underline{-0.927 + n\cdot 2\pi}$

Sett prøve: $\sin x + 3\cos x = 1$

$x = \pi/2$: VS: $\sin \pi/2 + 3\cdot\cos \pi/2 = 1 + 3\cdot 0 = 1$

HS: 1

$x = \underline{\pi/2 + n\cdot 2\pi}$

$x = -\pi/2$: VS: $-1 + 3\cdot 0 = -1$ HS $= 1$ falsk

$x \simeq 0.927$: VS: $0.8 + 3\cdot 0.6 = 2.6$ HS $= 1$ falsk.

$x \simeq -0.927$: VS: $-0.8 + 3\cdot 0.6 = 1$ HS $= 1$

$x \approx \underline{-0.927 + n\cdot 2\pi}$

Svar: $\underline{x = \pi/2 + n\cdot 2\pi}$ eller $x \approx \underline{-0.927 + n\cdot 2\pi}$

Likningen $\boxed{a \cdot \sin x + b \cdot \cos x = c \quad , c \neq 0}$

Eks: $\sin x + 3\cos x = 1$

$$\boxed{\begin{aligned} f(x) &= \sin x + 3\cos x \\ &\cong 3.18 \cdot \sin(x + 1.24) \end{aligned}}$$

$$\frac{3.18 \cdot \sin(x+1.24)}{3.18} = \frac{1}{3.18}$$

$$\sin(x+1.24) = \frac{1}{3.18}$$

$$x + 1.24 \simeq \sin^{-1}(1/3.18) + n \cdot 2\pi$$

$$x \simeq \underbrace{\sin^{-1}(1/3.18) - 1.24}_{-0.927} + n \cdot 2\pi$$

eller

$$x + 1.24 \simeq \pi - \sin^{-1}(1/3.18) + n \cdot 2\pi$$

$$x \simeq \underbrace{\pi - \sin^{-1}(1/3.18) - 1.24}_{\pi/2} + n \cdot 2\pi$$

Addisjon av bølger med samme periode

$$f(x) = A_1 \cdot \sin(\omega(x-\varphi_1)) + c_1$$
$$g(x) = A_2 \cdot \sin(\omega(x-\varphi_2)) + c_2$$

to bølger med samme periode $T = 2\pi/\omega$

$$h(x) = f(x) + g(x) = A \cdot \sin(\omega(x-\varphi)) + c$$

summen er en bølge med samme periode

Formler:

$$c = c_1 + c_2$$

$$A = \sqrt{A_1^2 + A_2^2 - 2A_1A_2 \cdot \cos(\omega(\varphi_1-\varphi_2))}$$

$$\varphi = \frac{1}{\omega}\arctan\left(\frac{A_1 \cdot \sin(\omega\varphi_1) + A_2 \cdot \sin(\omega\varphi_2)}{A_1 \cdot \cos(\omega\varphi_1) + A_2 \cdot \cos(\omega\varphi_2)}\right)$$

Eks: $f(x) = \sin x = 1 \cdot \sin(1 \cdot (x-0)) + 0$

$A_1 = 1 \quad \omega = 1 \quad \varphi_1 = 0 \quad c_1 = 0$

$g(x) = 3\cos x = 3 \cdot \sin(x + \pi/2) = 3 \cdot \sin(1 \cdot (x-(-\pi/2))) + 0$

$A_2 = 3 \quad \omega = 1 \quad \varphi_2 = -\pi/2 \quad c_2 = 0$

$c = c_1 + c_2 = 0 \qquad A = \sqrt{1^2 + 3^2 - 2 \cdot 1 \cdot 3 \cdot \cos(\pi/2)} = \sqrt{10} \simeq 3.16$

$$\Phi = \frac{1}{1} \cdot \arctan\left(\frac{1 \cdot \underbrace{\sin(1 \cdot 0)}_{=0} + 3 \cdot \underbrace{\sin(1 \cdot (-\pi/2))}_{=-1}}{1 \cdot \underbrace{\cos(1 \cdot 0)}_{=1} + 3 \cdot \underbrace{\cos(1 \cdot (-\pi/2))}_{=0}} \right)$$

$$= \arctan(-3/1) = \arctan(-3) \simeq \underline{-1.249}$$

$$\sin x + 3 \cos x = A \cdot \sin(x - \varphi)$$
$$= A \cdot (\underline{\sin x} \cdot \cos \varphi - \underline{\cos x} \cdot \sin \varphi)$$

$$\sin x + 3 \cos x = (A \cdot \cos \varphi) \cdot \sin x - (A \sin \varphi) \cdot \cos x$$

$$\boxed{\begin{aligned} 1 &= A \cdot \cos \varphi \\ 3 &= -A \cdot \sin \varphi \end{aligned}}$$

$$\frac{-\cancel{A} \sin \varphi}{\cancel{A} \cos \varphi} = \frac{3}{1}$$

$$-\tan \varphi = 3$$
$$\tan \varphi = -3$$
$$\varphi = \arctan(-3)$$
$$\varphi \simeq \underline{-1.249}$$

$$A = 1/\cos \varphi \approx 1/\cos(-1.249)$$
$$A \simeq \underline{\underline{3.16}}$$

$$\underline{a \cdot \sin x + b \cdot \cos x} = A \sin(x - \varphi)$$

$$\boxed{\begin{aligned} A &= \sqrt{a^2 + b^2} \\ \varphi &= \arctan(-b/a) \end{aligned}}$$

$A_1 = a \quad \varphi_1 = 0$

$A_2 = b \quad \varphi_2 = -\pi/2$

$$1 \cdot \sin x + 3 \cdot \cos x = \underline{\sqrt{10} \cdot \sin(x - \arctan(-3))}$$