

Harmoniske svingninger

$$f(x) = A \cdot \sin(\omega(x - \phi)) + C$$

Standard
form

$A > 0$: amplitude

$\omega = k > 0$: vinkelhastighet

ϕ : faseforskyvning

C : $y=c$ likevektslinjen

Eks:

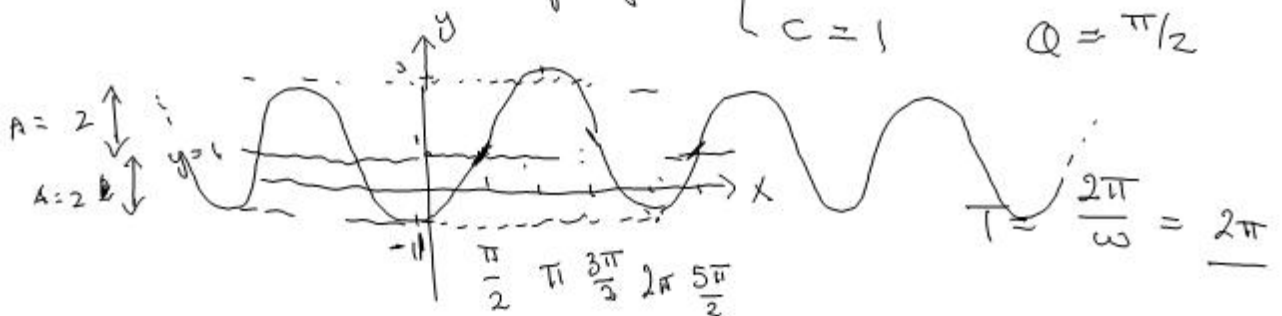
$$f(x) = 2 \sin(x - \pi/2) + 1$$

harmonisk svingning

$$\begin{cases} A = 2 \\ C = 1 \end{cases}$$

$$\omega = 1$$

$$\phi = \pi/2$$

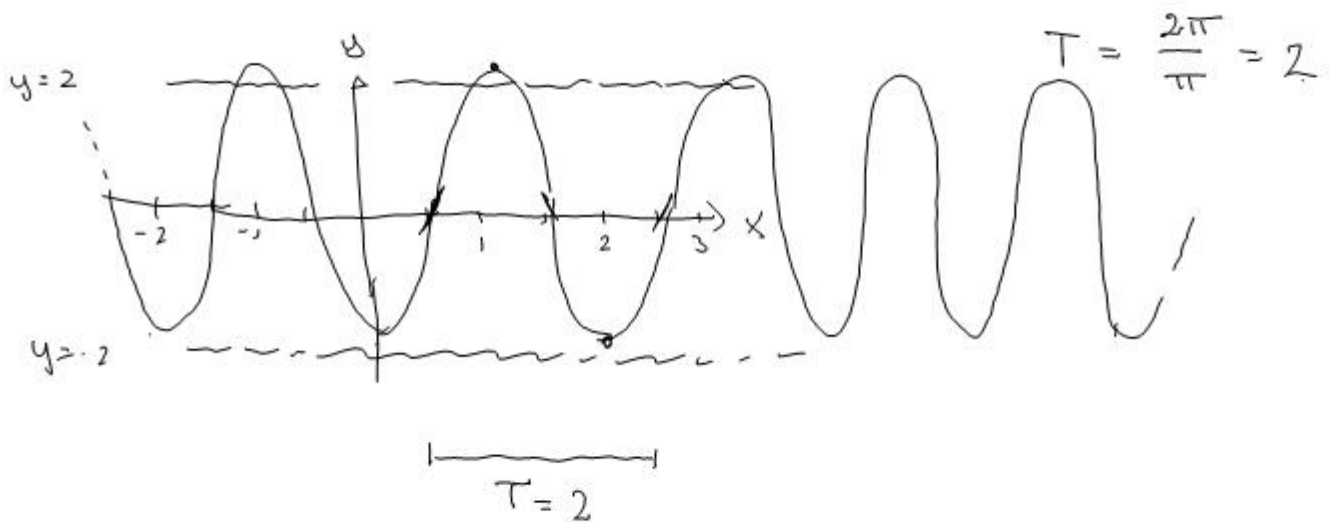


$$T = 2\pi$$

Ex:

$$f(x) = 2 \sin(\pi x - \pi/2) \\ = 2 \sin(\pi(x - 1/2))$$

$A = 2$	$c = 0$
$\omega = \pi$	$\varphi = 1/2$



a) Nullpunkt für f:

$$x = 1/2 + n \cdot 1 = \underline{\underline{1/2 + n}} \quad (n \text{ beliebig})$$

$$f(x) = 0$$

$$2 \sin(\pi x - \pi/2) = 0$$

$$u = \pi x - \pi/2$$

$$2 \sin(u) = 0$$

$$\sin(u) = 0$$

$$u = \sin^{-1}(0) + n \cdot 2\pi \quad \text{oder} \quad u = \pi - \sin^{-1}(0) + n \cdot 2\pi$$

$$= 0 + n \cdot 2\pi$$

$$= \pi + n \cdot 2\pi$$

$$\dots, -4\pi, -2\pi, 0, 2\pi, 4\pi, 6\pi, \dots$$

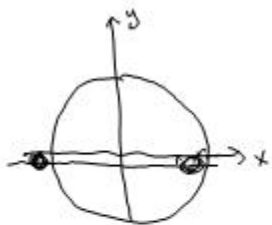
$$\dots, -\pi, \pi, 3\pi, 5\pi, \dots$$

$$u = 0 + n \cdot \pi$$

$$\pi x - \pi/2 = n \cdot \pi$$

$$\cancel{\pi} x = \frac{n\pi + \pi/2}{\cancel{\pi}}$$

$$x = n + 1/2 \quad \underline{\underline{x = 1/2 + n}}$$



b) Lokale topp-plt for f:

$$y = 2 \text{ og } y = f(x) \quad \Rightarrow \quad \underline{\underline{x = 1 + n \cdot 2}}$$

\Downarrow

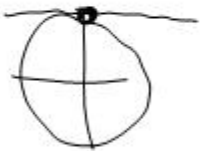
$$f(x) = 2$$

$$2 \sin(\pi(x - 1/2)) = 2$$

$$\frac{2 \sin(u)}{2} = \frac{2}{2}$$

$$\sin(u) = 1$$

$$u = \pi/2 + n \cdot 2\pi$$



$$\frac{\pi \cdot (x - 1/2)}{\pi} = \frac{\pi/2 + n \cdot 2\pi}{\pi}$$

$$x - 1/2 = \frac{1}{2} + 2n$$

$$x = \underline{\underline{1 + n \cdot 2}}$$

Eks:

$$f(x) = 7 - 10 \cos\left(\frac{2\pi}{365}x - \frac{5\pi}{73}\right)$$
$$= A \cdot \sin(\omega(x-\phi)) + C$$

$$f(u) = 7 - 10 \cos(u)$$
$$= 7 - 10 \cdot \sin(u + \pi/2)$$
$$= -10 \sin(u + \pi/2) + 7$$
$$= 10 \sin(u + \pi/2 + \pi) + 7$$
$$= 10 \sin(u + \frac{3\pi}{2}) + 7$$

$$\sin(u + \pi/2) = \cos u$$

$$\sin(u + \pi) = -\sin u$$

$$= 10 \cdot \sin\left(\frac{2\pi}{365}x - \frac{5\pi}{73} + \frac{3\pi}{2}\right) + 7$$

$$\sin(u+c)$$

$$= \sin u \cdot \cos c + \cos u \cdot \sin c$$
$$= \sin u \cdot (-1) + \cos u \cdot 0$$

$$= 10 \cdot \sin\left(\frac{2\pi}{365}(x - \phi)\right) + 7$$

$$\sin(u+\pi)$$

$$= \sin u \cdot (-1) + \cos u \cdot 0$$
$$= -\sin u$$

$$+ \frac{5\pi}{73} \cdot \frac{365}{2} = + \phi \cdot \frac{2\pi}{365} \quad \left| \cdot \frac{365}{2\pi}\right.$$

$$\phi = \frac{5\pi}{73} \cdot \frac{365}{2\pi} - \frac{3\pi}{2} \cdot \frac{365}{2\pi}$$

$$= \frac{365 \cdot 5}{2 \cdot 73} - \frac{365 \cdot 3}{2 \cdot 2} = -261.25$$

$$(\phi = -261.25 + 365 = 103.75)$$

$$f(x) = 10 \cdot \sin\left(\frac{2\pi}{365}(x + 261.25)\right) + 7$$

$$\sin(u + \pi/2) = \cos u$$

$$\sin(u + \pi) = -\sin u$$

Fin n nullpunkt til f:

$$f(x) = 0$$

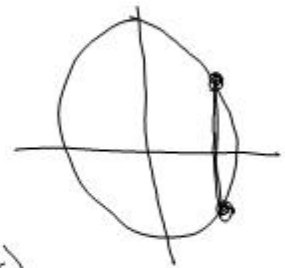
$$u = \frac{2\pi}{365}x - \frac{5\pi}{73}$$

$$7 - 10 \cos\left(\frac{2\pi}{365}x - \frac{5\pi}{73}\right) = 0$$

$$7 - 10 \cos(u) = 0$$

$$\frac{-10 \cos u}{-10} = \frac{-7}{-10}$$

$$\cos u = 0.7$$



$$u = \cos^{-1}(0.7) \quad \text{eller} \quad u = -\cos^{-1}(0.7)$$
$$\approx 0.795 + n \cdot 2\pi \quad \approx -0.795 + n \cdot 2\pi$$

$$\frac{2\pi}{365}x - \frac{5\pi}{73} \approx \pm 0.795$$

$$\frac{2\pi}{365}x \approx \pm 0.795 + \frac{5\pi}{73} \quad \left| \cdot \frac{365}{2\pi} \right.$$

$$x \approx \pm 0.795 \cdot \frac{365}{2\pi} + \frac{5\pi}{73} \cdot \frac{365}{2\pi}$$

$$x \approx 58.7 \quad \text{eller} \quad x \approx -33.7$$

$$w = \frac{2\pi}{365}$$
$$T = \frac{365}{2\pi}$$

$$\underline{x \approx 58.7 + n \cdot 365} \quad \text{eller} \quad \underline{x \approx -33.7 + n \cdot 365}$$