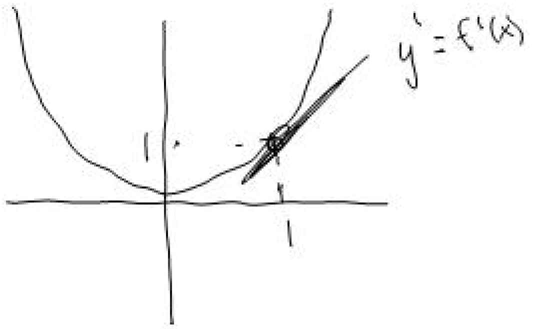
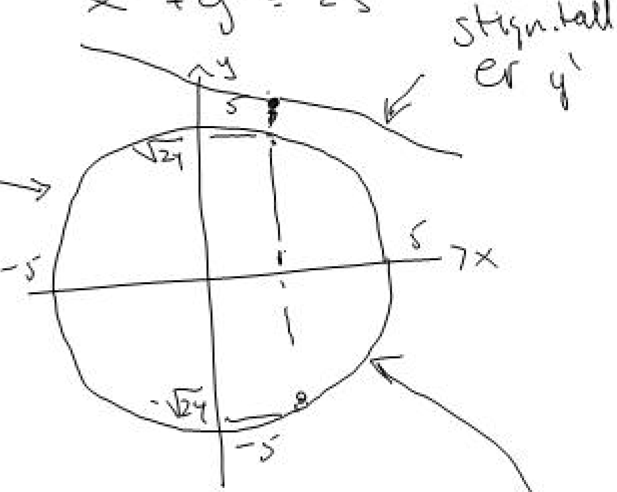
Implisitt derivasjon (Notat, LF)

$y = x^2$



$x^2 + y^2 = 25$



eksplisitt form

$y = f(x)$

$\underline{x=1}$: $y = f(1) = 1^2 = 1$

implisitt form

$F(x,y) = 0$

$\underline{x=1}$: $1^2 + y^2 = 25$

$y^2 = 24$

$y = \pm\sqrt{24}$

$\underline{\text{Eks:}}$ $x^2 + y^2 = 25$

$y^2 = 25 - x^2$

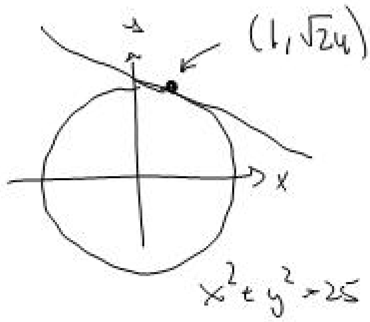
$y = \pm\sqrt{25 - x^2}$

$y = f_1(x) = \sqrt{25 - x^2}$

$y = f_2(x) = -\sqrt{25 - x^2}$

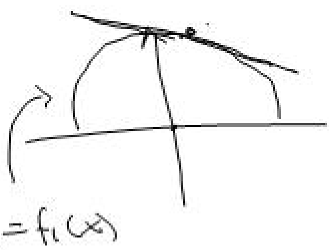
$\underline{\text{Eks:}}$ $y^3 + 2xy^2 = 4x$

<u>Eks:</u> Finn y' i punktet $(1, \sqrt{24})$ til sirkelen $x^2 + y^2 = 25$.



<u>Metode I:</u> $y = \sqrt{25 - x^2} = f_1(x)$

$f_1(x) = \sqrt{25 - x^2}$



$$y' = f_1'(x) = \left(\sqrt{25 - x^2}\right)'$$

$$= \frac{1}{2 \cdot \sqrt{25 - x^2}} \cdot (-2x)$$

$$= \frac{-\cancel{2}x}{\cancel{2}\sqrt{25 - x^2}} = \frac{-x}{\sqrt{25 - x^2}}$$

I punktet $(1, \sqrt{24})$: $y' = f_1'(1) = \dfrac{-1}{\sqrt{25 - 1^2}} = \dfrac{-1}{\sqrt{24}}$

$$\underline{\simeq -0.2}$$

Metode 2: Implisitt derivasjon

$$(x^2+y^2)' = (25)'$$

$$2x + 2y\,y' = 0$$

$$\frac{\cancel{2y}\cdot y'}{\cancel{2y}} = \frac{-\cancel{2}x}{\cancel{2}y}$$

$$y' = -\frac{x}{y}$$

$$\boxed{\,' = \frac{d}{dx}}$$

derivasjon med hensyn til x

$$(y^2)' = \frac{dy^2}{dx} = \frac{dy^2}{dy}\cdot\frac{dy}{dx}$$

$$= 2y\cdot y'$$

I punktet $(1, \sqrt{24})$: $y' = -\frac{1}{\sqrt{24}} \simeq -0.2$

Eks: $y^3 + 2xy^2 = 4x$

$$(y^3 + 2xy^2)' = (4x)'$$

$$3y^2\cdot y' + (2x)'\cdot y^2 + 2x\cdot(y^2)' = 4$$

$$3y^2\cdot y' + \cancel{2y^2} + 2x\cdot 2y\cdot y' = 4 - 2y^2$$

$$(3y^2 + 4xy)\cdot y' = 4 - 2y^2$$

$$y' = \frac{4-2y^2}{3y^2+4xy}$$

Eks: $y^3 - 2y = 4x$

$$(y^3 - 2y)' = (4x)'$$

$$3y^2\cdot y' - 2\cdot y' = 4$$

$$\frac{\cancel{(3y^2-2)}\cdot y'}{\cancel{3y^2-2}} = \frac{4}{3y^2-2}$$

$$y' = \frac{4}{3y^2-2}$$