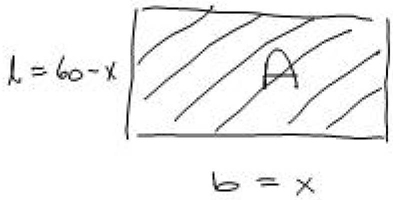
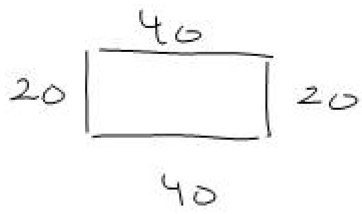
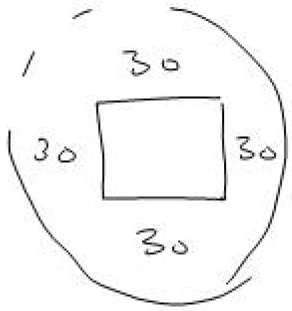
Optimering (kap. 9.3)

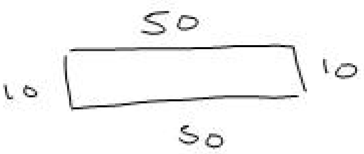
Eks:



Rektangulært gjerde, maks. 120 m omkrets.
Ønsker størst mulig areal.







$A = 20 \cdot 40 = 800$ $\quad$ $A = 30 \cdot 30 = 900$ $\quad$ $A = 50 \cdot 10 = 500$

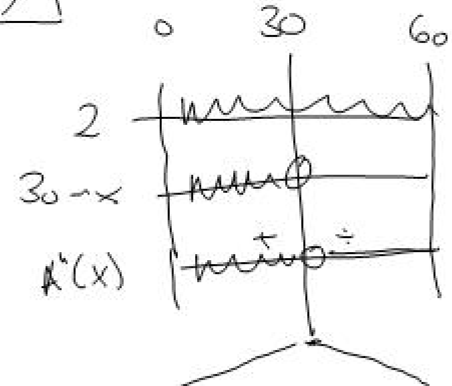
$$O = 2b + 2l = 2 \cdot (b + l) = 120 \implies \underline{b + l = 60}$$

$$A = b \cdot l \qquad l = 60 - b = \underline{60 - x}$$

$$A(x) = b \cdot l = x \cdot (60 - x) = 60x - x^2$$

$$\boxed{A(x) = 60x - x^2, \quad x \in (0, 60)}$$

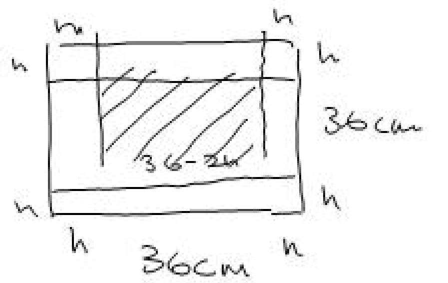
$$A'(x) = 60 - 2x = 2(30 - x)$$



Størst areal:

$x = 30\text{ m}$ $\quad$ $A = 900\text{ m}^2$

Ex:



Hva er det største volumet esken kan få.

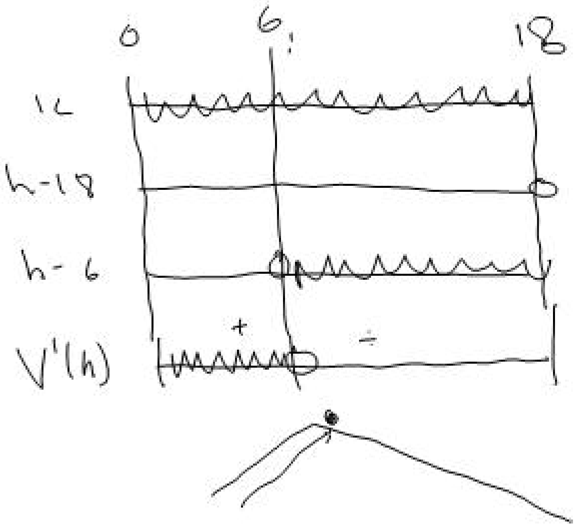
$$V = G \cdot h = (36-2h)^2 \cdot h$$
$$= 2^2 \cdot (18-h)^2 \cdot h$$
$$= 4h \cdot (18^2 - 36h + h^2)$$
$$= 4h^3 - 144h^2 + 4 \cdot 18^2 \cdot h$$

$$\boxed{V(h) = 4h^3 - 144h^2 + \underset{1296}{\underbrace{4 \cdot 18^2}} \cdot h \;, \quad h \in (0, 18)}$$

$$V'(h) = 12h^2 - 288h + 1296$$
$$= 12 \cdot (h-18)(h-6)$$

$$12h^2 - 288h + 1296 = 0$$
$$h = \frac{288 \pm \sqrt{288^2 - 4 \cdot 12 \cdot 1296}}{2 \cdot 12}$$
$$= \frac{288 \pm 144}{24}$$

$h = 18$ eller $h = 6$

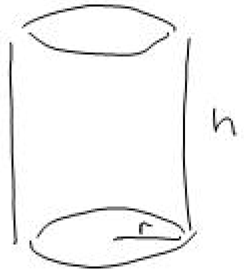


Størst volum:

$h = 6\text{cm}$

$$V = 24^2 \cdot 6 \text{ cm}^3$$
$$= 3456 \text{ cm}^3$$
$$= 3.456\ l$$

Eks:



$(700\ cm^3)$

$V = 350\ cm^3 = 0.35\ l$

Minst mulig overflate.

$$V = \pi r^2 \cdot h = 350 \implies \underline{h = \frac{350}{\pi r^2}}$$

$$O = 2 \cdot \pi r^2 + h \cdot 2\pi r$$

$$= 2\pi r^2 + \frac{350}{\pi r^2} \cdot 2\pi r$$

$(1400/r)$

$$= 2\pi r^2 + \frac{350 \cdot 2\cancel{\pi}\cancel{r}}{\cancel{\pi}r^{\cancel{2}}} = 2\pi r^2 + \frac{700}{r}$$

$$\boxed{O(r) = 2\pi r^2 + \frac{700}{r}, \quad r \in (0, \infty)}$$

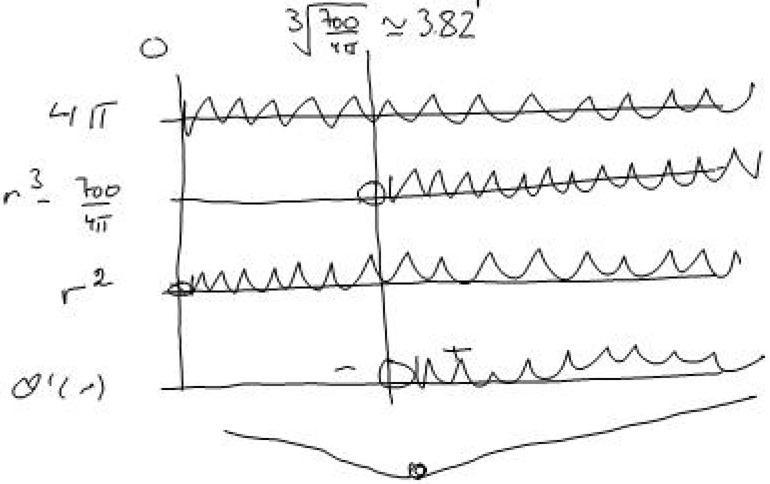
$$O'(r) = 2\pi \cdot 2r + 700 \cdot \left(-\frac{1}{r^2}\right)$$

$$= 4\pi r - \frac{700}{r^2}$$

$$= \frac{4\pi r \cdot r^2}{r^2} - \frac{700}{r^2} = \frac{4\pi r^3 - 700}{r^2}$$

$$O'(r) = \frac{4\pi\left(r^3 - \frac{700}{4\pi}\right)}{r^2} \longrightarrow r = \sqrt[3]{\frac{1400}{4\pi}} = \sqrt[3]{2} \cdot 3.82 = 4.81 cm$$

1400



$$r^3 - \frac{700}{4\pi} = 0$$

$$r^3 = \frac{700}{4\pi}$$

$$\underline{r = \sqrt[3]{\frac{700}{4\pi}} \cong 3.82}$$

<u>Minst overflate:</u>

$r = \sqrt[3]{\frac{700}{4\pi}} \simeq \underline{3.82\ cm}$ $\quad O \simeq \underline{275\ cm^2}$

Eks: Opps. 4, Eksamen Julen 2006/07

$$f(x) = \frac{6x^2+3x}{3x+2} = \frac{u}{v}, \quad x \neq -2/3$$

$$\boxed{\begin{array}{ll} u = 6x^2+3x & v = 3x+2 \\ u' = 12x+3 & v' = 3 \end{array}}$$

b) $f'(x) = \dfrac{u' \cdot v - u \cdot v'}{v^2}$

$$= \frac{(12x+3)\cdot(3x+2) - (6x^2+3x)\cdot 3}{(3x+2)^2}$$

$$= \frac{(36x^2 + \cancel{9x} + 24x + 6) - (18x^2 + \cancel{9x})}{(3x+2)^2} = \frac{18x^2+24x+6}{(3x+2)^2}$$

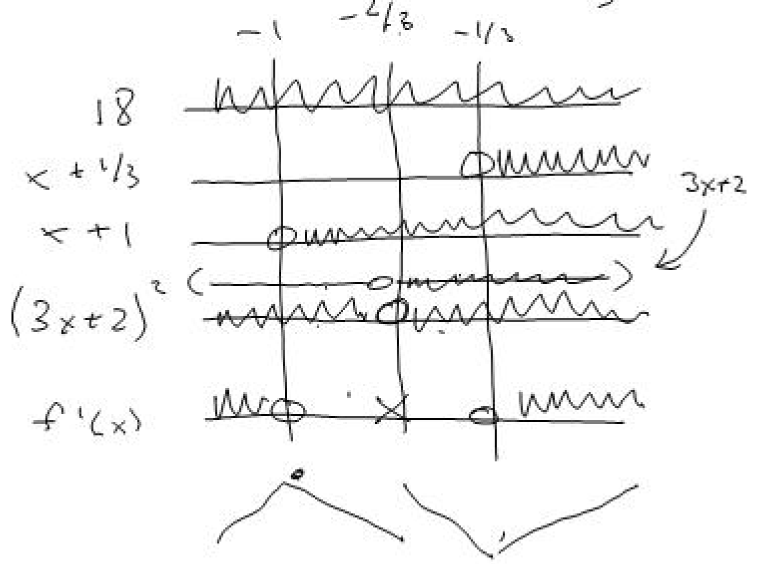
$$= \frac{6(3x^2+4x+1)}{(3x+2)^2} = \frac{6(3)\cdot(x+1/3)(x+1)}{(3x+2)^2} = \frac{18(x+1/3)(x+1)}{(3x+2)^2}$$

c) $3x^2+4x+1 = 0$

$x = \dfrac{-4 \pm \sqrt{16-12}}{6}$

$x = -1/3 \qquad x = -1$

$3x^2+4x+1 = 3(x+1/3)(x+1)$



Lokale topp-pkt:

$x = -1 \qquad y = f(-1) = -3$

Lokale bunnpkt:

$x = -1/3 \qquad y = f(-1/3) = -1/3$

f)

$x_0 = -\frac{1}{2}$

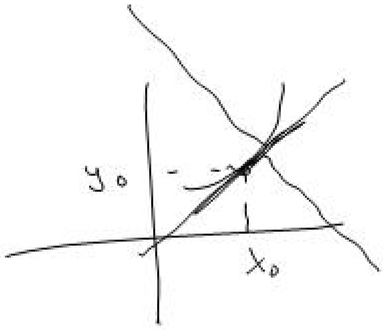
$y_0 = f(-\frac{1}{2}) = 0$

$a = f'(-\frac{1}{2}) = \frac{-\frac{3}{2}}{\frac{1}{2}} = -3$

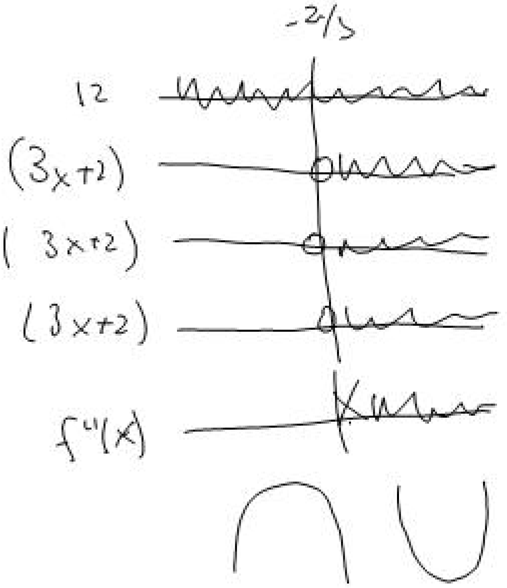
$y - y_0 = a \cdot (x - x_0)$

$y - 0 = -3(x + \frac{1}{2})$

$\underline{\underline{y = -3x - \frac{3}{2}}}$



g) $f''(x) = \frac{12}{(3x+2)^3}$



<u>Vendepkt:</u>

Ingen vendepkt, fordi eneste mulige vendepunkt er $x = -\frac{2}{3}$, men $x = -\frac{2}{3}$ er ikke med i D_f.