Derivasjon - flere regneregler (kap. 8.8 - 8.9)

Regneregler:

(1) $(x^n)' = n \cdot x^{n-1}$ for alle tall n

(2) $(u \pm v)' = u' \pm v'$ for alle uttrykk u og v

(3) $(c \cdot u)' = c \cdot u'$ for alle tall c og alle uttrykk u

(4) Produktregelen:

$$(u \cdot v)' = u' \cdot v + u \cdot v'$$ for alle uttrykk u og v

(5) Brøkregelen / kvosientregelen:

$$\left(\frac{u}{v}\right)' = \frac{u' \cdot v - u \cdot v'}{v^2}$$ for alle uttrykk u og v

(6) Kjerneregelen:

$$\left(f(u(x))\right)' = f'(u) \cdot u'(x)$$

Eks:

$$f(x) = (x+1)\cdot\sqrt{x} = u\cdot v, \quad x \geq 0$$

$$\boxed{\begin{array}{ll} u = x+1 & v = \sqrt{x} \\ u' = 1 & v' = 1/(2\sqrt{x}) \end{array}}$$

$$f'(x) = u'\cdot v + u\cdot v'$$

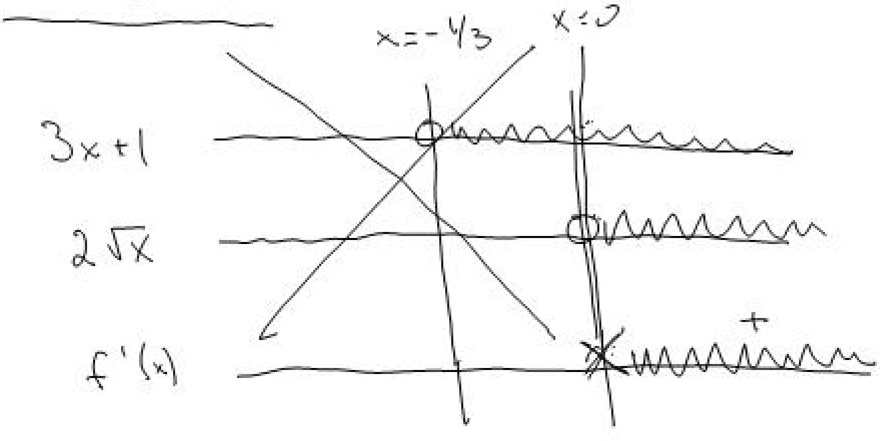
$$= 1\cdot\sqrt{x} + (x+1)\cdot\frac{1}{2\sqrt{x}}$$

$$= \sqrt{x} + \frac{x+1}{2\sqrt{x}}$$

$$= \frac{\sqrt{x}\cdot 2\sqrt{x}}{2\sqrt{x}} + \frac{x+1}{2\sqrt{x}}$$

$$= \frac{2x+(x+1)}{2\sqrt{x}} = \underline{\underline{\frac{3x+1}{2\sqrt{x}}}}$$

Fortegnsskjema:



Es:

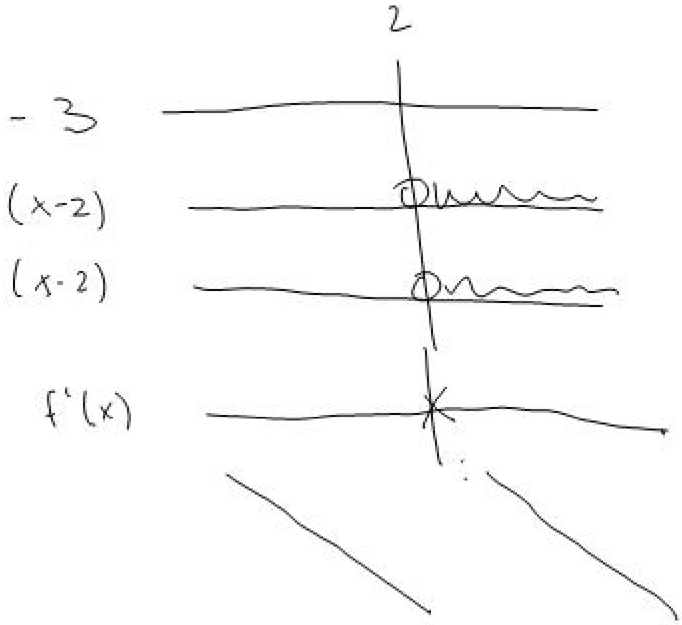
$$f(x) = \frac{x+1}{x-2} = \frac{u}{v}, \quad x \neq 2$$

$$\boxed{\begin{array}{ll} u = x+1 & v = x-2 \\ u' = 1 & v' = 1 \end{array}}$$

$$f'(x) = \frac{u' \cdot v - u \cdot v'}{v^2}$$

$$= \frac{1 \cdot (x-2) - (x+1) \cdot 1}{(x-2)^2}$$

$$= \frac{(\cancel{x}-2) - (\cancel{x}+1)}{(x-2)^2} = \frac{-3}{(x-2)^2}$$



Es:

$$f(x) = \frac{x^2-3x+2}{x-3} = \frac{u}{v}, \quad x \neq 3 \qquad \boxed{\begin{array}{ll} u = x^2-3x+2 & v = x-3 \\ u' = 2x-3 & v' = 1 \end{array}}$$

$$f'(x) = \frac{u' \cdot v - u \cdot v'}{v^2}$$

$$= \frac{(2x-3)\cdot(x-3) - (x^2-3x+2)\cdot 1}{(x-3)^2}$$

$$= \frac{(2x^2-3x-6x+9) - (x^2-3x+2)}{(x-3)^2}$$

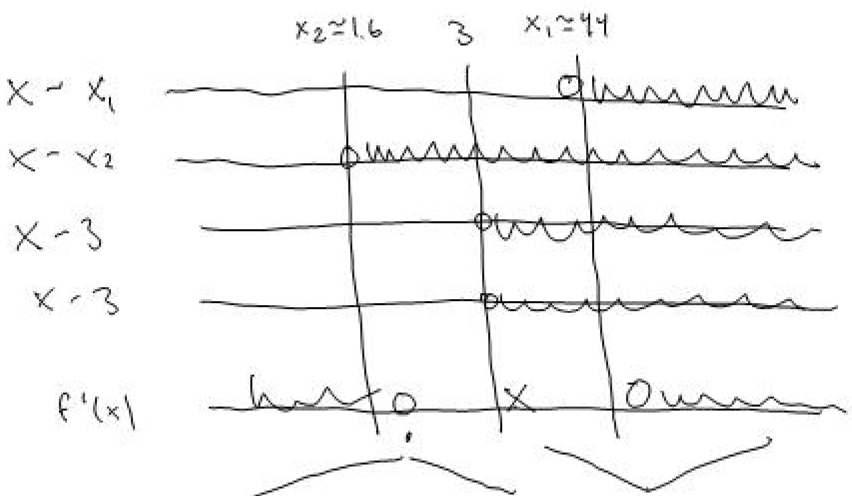
$$= \frac{x^2-6x+7}{(x-3)^2}$$

$$= \boxed{\frac{x^2-6x+7}{(x-3)^2}} = \frac{(x-x_1)(x-x_2)}{(x-3)^2}$$

$$x^2-6x+7=0$$

$$x = \frac{6 \pm \sqrt{36-28}}{2} = \frac{6 \pm \sqrt{8}}{2} = 3 \pm \sqrt{2}$$

$$x_1 = 3+\sqrt{2} \simeq 4.4 \qquad x_2 = 3-\sqrt{2} \simeq 1.6$$



Bsp: $f(x) = \dfrac{x^2-3x+2}{(x-3)}, \quad x \neq 3$

$$f'(x) = \frac{x^2-6x+7}{(x-3)^2} = \frac{u}{v}$$

$$\boxed{\begin{array}{ll} u = x^2-6x+7 & v = (x-3)^2 \\ & = x^2-6x+9 \\ u' = 2x-6 & v' = 2x-6 \\ & = 2\cdot(x-3) \end{array}}$$

$$f''(x) = \left(\frac{u}{v}\right)' = \frac{u'\cdot v - u\cdot v'}{v^2}$$

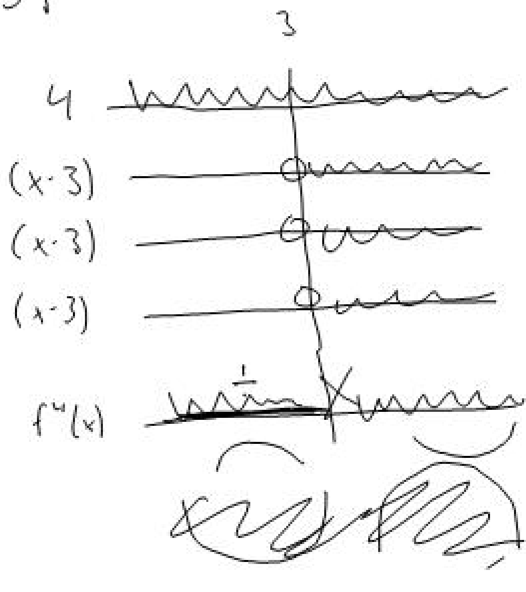
$$= \frac{(2x-6)\,(x-3)^2 - (x^2-6x+7)\cdot 2\cdot(x-3)}{\left((x-3)^2\right)^2}$$

$$= \frac{\cancel{(x-3)}\cdot\left((2x-6)(x-3) - 2(x^2-6x+7)\right)}{(x-3)^{\cancel{4}\,3}}$$

$$= \frac{(\cancel{2x^2} - \cancel{6x} - \cancel{6x} + 18) - (\cancel{2x^2} - \cancel{12x} + 14)}{(x-3)^3}$$

$$= \frac{4}{(x-3)^3}$$

$$= \underline{\underline{\frac{4}{(x-3)^3}}}$$



Sammensatte funksjoner:

Eks: $f(x) = \sqrt{x^2+1}$

$f(1) = \sqrt{1^2+1} = \sqrt{2}$

$$x \longrightarrow \boxed{} \longrightarrow u = x^2+1 \xrightarrow{u} \boxed{} \longrightarrow \sqrt{u}$$

$$1 \longmapsto \qquad \longmapsto 2 \longrightarrow \qquad \longrightarrow \sqrt{2}$$

Sammensatt funksjon:

$u(x) = x^2+1$ indre funksjon, kjerne

$f(u) = \boxed{\sqrt{u}}$ ytre funksjon

$f(x) = \underline{f(u(x))} = f(x^2+1) = \sqrt{x^2+1}$

$$\boxed{\left(f(u(x)) \right)' = \boxed{f'(u)} \cdot u'(x)}$$

kjerne-regelen

$$\left(\sqrt{x^2+1} \right)' = \frac{1}{2\sqrt{u}} \cdot 2x$$

$u = x^2+1$ $\quad f(u) = \sqrt{u}$

$u' = 2x$ $\quad f'(u) = \frac{1}{2\sqrt{u}}$

$$= \frac{\cancel{2}x}{\cancel{2}\sqrt{u}} = \frac{x}{\sqrt{u}} = \underline{\underline{\frac{x}{\sqrt{x^2+1}}}}$$

Eks: $$\left(\sqrt{2x+1}\right)' = \underset{\substack{\uparrow \\ f'(u)}}{\frac{1}{2\sqrt{2x+1}}} \cdot \underset{\substack{\uparrow \\ u'(x)}}{2} \qquad \boxed{\begin{aligned} u &= 2x+1 \\ u' &= 2 \end{aligned}}$$

$$= \frac{\cancel{2}}{\cancel{2}\sqrt{2x+1}} = \underline{\underline{\frac{1}{\sqrt{2x+1}}}}$$

Eks: $$\left.\begin{aligned} f(x) &= (x+1)^8 \\ f'(x) &= \,? \end{aligned}\right\} \text{ Bruk kjerneregelen.}$$