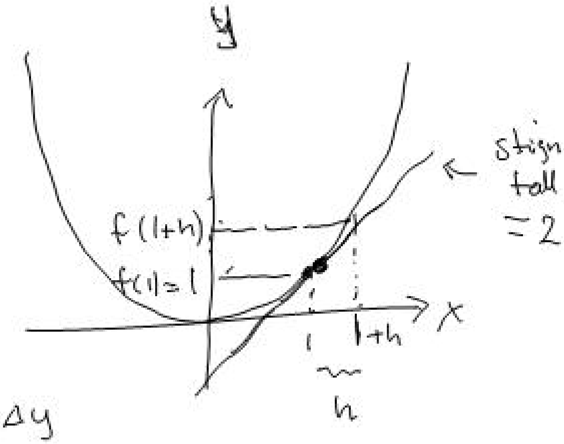
Derivasjon



Eks: $f(x) = x^2$
$a = 1$

$$f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h} \quad \begin{matrix} \overset{!}{=} \Delta y \\ = \Delta x \end{matrix}$$

$$= \ldots = \underline{2}$$

Gen. defn: $$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \lim_{h \to 0} \frac{(a+h)^2 - a^2}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{a^2} + 2ah + h^2 - \cancel{a^2}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h} \cdot (2a+h)}{\cancel{h}} = \lim_{h \to 0} (2a+h) = \underline{\underline{2a}}$$

$$\underline{\underline{f'(a) = 2a}}$$

Den deriverte.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \cdots = \underline{\underline{2x}}$$

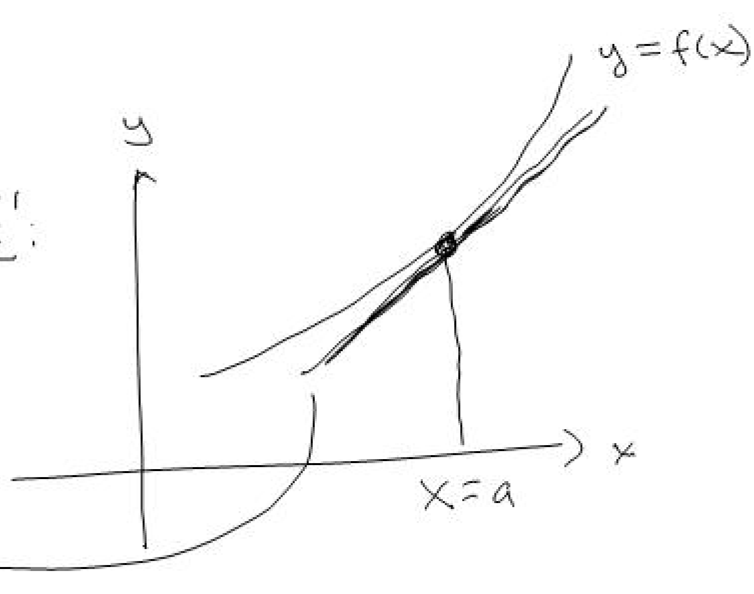
<u>Defn:</u> Den deriverte til funksjonen $f(x)$

er

$$\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

<u>Eks:</u> $f(x) = x^2$

$f'(x) = \underline{2x}$



Geometrisk tolkning av f':

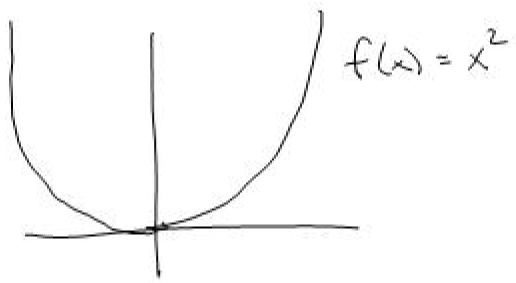
Stigningstallet til tangenten i $x=a$ er $f'(a)$.

Hvordan regne ut f':

Metode I: direkte fra definisjonen

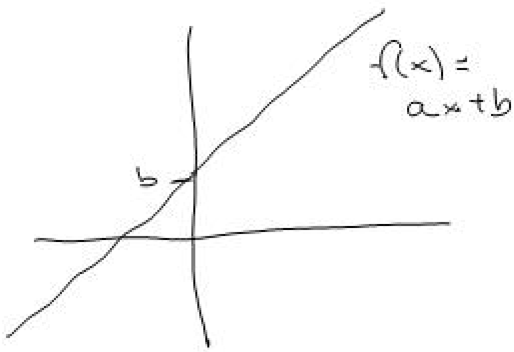
$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Eks:



$$f'(x) = 2x$$

Eks:



$$f'(x) = a$$

Metode 2: regneregler

Regneregel I:

$$\boxed{f(x) = x^n \quad \text{gir} \quad f'(x) = n \cdot x^{n-1}}$$ for alle n.

Skrivemåte: $(x^n)' = n \cdot x^{n-1}$ $\qquad (x^2)' = 2x$

Eks:

$$(x^n)' = n \cdot x^{n-1}$$

$n=2$: $(x^2)' = 2x$

$n=3$: $(x^3)' = 3x^2$

$n=4$: $(x^4)' = 4x^3$

$n=1$: $(x)' = 1$

$n=0$: $(1)' = 0$

$f(x) = x^n$

Hvorfor er $(x^n)' = n \cdot x^{n-1}$?

$$(x^n)' = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^n - x^n}{h}$$

$$= \lim_{h \to 0} \frac{(\cancel{x^n} + n \cdot x^{n-1} \cdot \cancel{h} + \dots) - \cancel{x^n}}{\cancel{h}}$$

$$= \lim_{h \to 0} (n \cdot x^{n-1} + h \cdot \dots)$$

$$= n \cdot x^{n-1}.$$

Binomial formel:

$$(a+b)^n =$$

$$a^n + n \cdot a^{n-1} \cdot b$$

$$+ \frac{n \cdot (n-1)}{2} \cdot a^{n-2} \cdot b^2$$

$$+ \dots$$

$$+ b^n$$

Es:

$$\boxed{(x^n)' = n \cdot x^{n-1}}$$

$$(x^{17})' = \underline{17 \cdot x^{16}}$$

$$(x^{-1})' = -1 \cdot x^{-2} = -x^{-2}$$

$$(x^{-2})' = -2 \cdot x^{-3} = -2x^{-3}$$

$$\left(\frac{1}{x}\right)' = -\frac{1}{x^2}$$

$$\left(\frac{1}{x^2}\right)' = -\frac{2}{x^3}$$

$$\left(\frac{1}{x^3}\right)' = (x^{-3})' = -3 \cdot x^{-4}$$

$$= \underline{-\frac{3}{x^4}}$$

$$(x^{1/2})' = \frac{1}{2} \cdot x^{-\frac{1}{2}}$$

$$(\sqrt{x})' = (x^{1/2})' = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2} \cdot \frac{1}{x^{1/2}} = \frac{1}{2} \cdot \frac{1}{\sqrt{x}} = \underline{\underline{\frac{1}{2\sqrt{x}}}}$$

$$(x\sqrt{x})' = (x^{1+\frac{1}{2}})' = (x^{3/2})' = \frac{3}{2} x^{1/2} = \underline{\underline{\frac{3}{2}\sqrt{x}}}$$

Regneregler for derivasjon

1. $(x^n)' = n \cdot x^{n-1}$ for alle tall n
2. $(u \pm v)' = u' \pm v'$ for alle uttrykk u og v
3. $(c \cdot u)' = c \cdot u'$ for alle tall c og alle uttrykk u

(Kap. 8.1 – 8.3).

Eks:

$(x^2 + x^3)' = (x^2)' + (x^3)' = \underline{2x + 3x^2}$

$(x - x^2 + x^4)' = \underline{1 - 2x + 4x^3}$

$(1 + 2x + 4x^2)' = 0 + 2 \cdot 1 + 4 \cdot 2x = \underline{2 + 8x}$

$(4x^3)' = 4 \cdot (x^3)' = 4 \cdot 3x^2 = \underline{12x^2}$

$(x + 17)' = 1 + 17 \cdot 0 = \underline{1}$