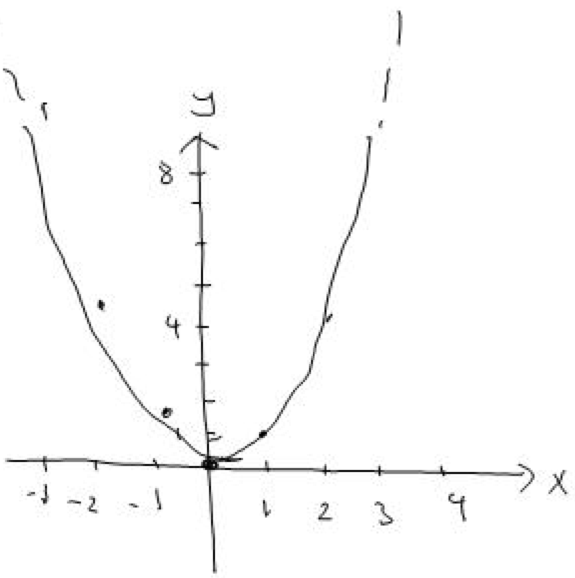
Kvadratiske funksjoner

Defn: $f(x) = ax^2 + bx + c$

$(a, b, c$ er gitte tall $, a \neq 0)$

(polynomfunksjon av grad 2).

Grafen er en parabel.



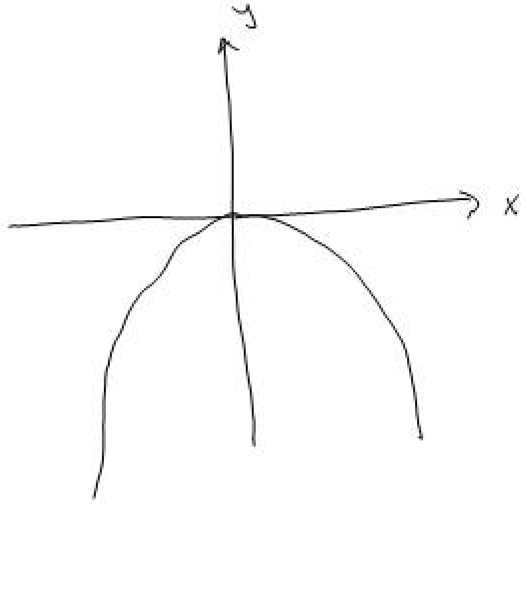
Eks: $f(x) = x^2$

x	0	1	2	3	4
y	0	1	4	9	16

x	-1	-2	-3	-4
y	1	4	9	16

$x = 0$ symmetrilinje

Eks: $f(x) = -x^2$



x	0	1	2
y	0	-1	-4

Eks: $f(x) = x^2 + 4x + 5$

$= \underbrace{x^2 + 4x + 4} + \underbrace{5-4}$

$= (x+2)^2 + 1$

$f(x) = (x+2)^2 + 1$ Standard form

Standard form:

Enhver kvadratisk funksjon kan alltid skrives som

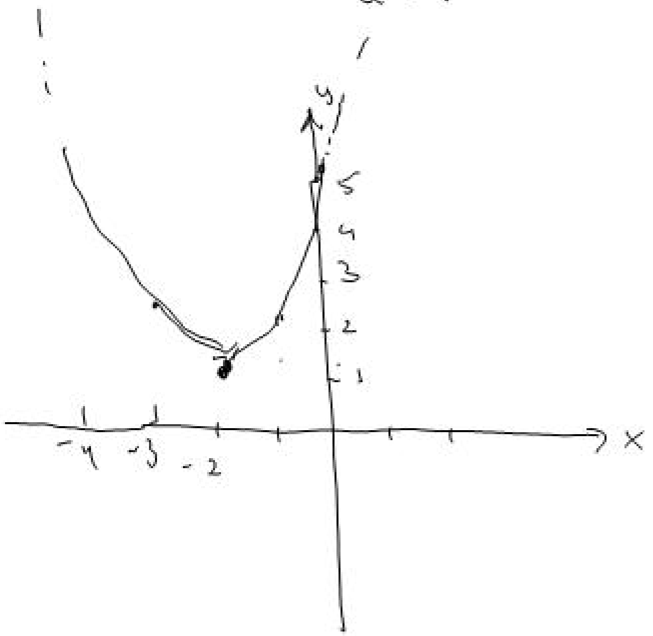
$$\boxed{f(x) = a \cdot (x - x_0)^2 + y_0}$$

Eks: $f(x) = (x+2)^2 + 1$ $\begin{cases} a = 1 \\ x_0 = -2 \\ y_0 = 1 \end{cases}$

* (x_0, y_0) er et topp-pkt / bunn pkt for parabelen
* $\begin{cases} a > 0 : (x_0, y_0) \text{ bunn pkt} \\ a < 0 : (x_0, y_0) \text{ topp-pkt} \end{cases}$
* $|a|$ forteller hvor bratt parabelen er.

Eks: $f(x) = (x+2)^2 + 1$

$a = 1 \qquad (x_0, y_0) = (-2, 1)$



x	-2	-1	0
y	1	2	5

$-2 \cdot \frac{9}{4} = -\frac{9}{2}$

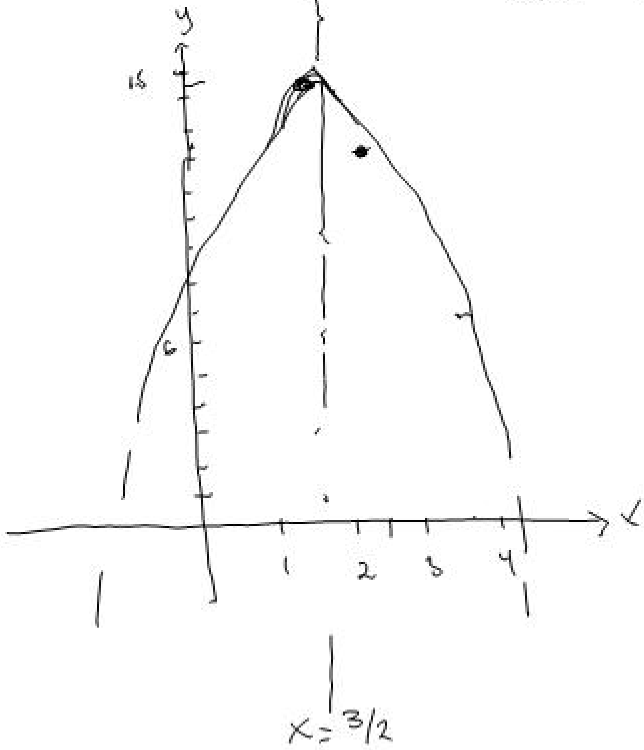
Eks: $f(x) = -2x^2 + 6x + 10$

$= -2\left(x^2 - 3x + 9/4\right) + \left(10 + 9/2\right)$

$= -2 \cdot \left(x - 3/2\right)^2 + \frac{29}{2}$

$a = -2 \qquad (x_0, y_0) = \left(3/2, 29/2\right)$

$= (1.5, 14.5)$



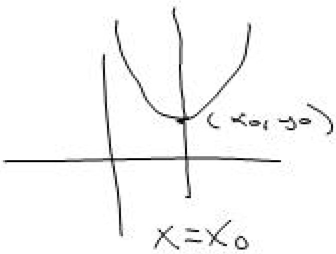
x	1.5	2.5	3.5
y	14.5	12.5	6.5

Oppsummering:

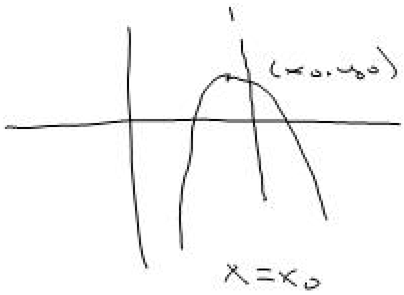
En kvadratisk funksjon $f(x) = ax^2 + bx + c$ kan alltid skrives på standard form

$$\boxed{f(x) = ax^2 + bx + c = a \cdot (x - x_0)^2 + y_0}$$

$a > 0$:



$a < 0$:



Nullpkt:

Et nullpkt for funksjonen $f = f(x)$ er et pkt slik at $f(x) = 0$.

$f(x) = 0 \longleftrightarrow$ skjæringspkt med x-aksen

Eks: $f(x) = -2x^2 + 6x + 10$

$f(x) = 0$

$-2x^2 + 6x + 10 = 0$

$$x = \frac{-6 \pm \sqrt{36 - 4 \cdot (-2) \cdot 10}}{2 \cdot (-2)}$$

$$= \frac{-6 \pm \sqrt{116}}{-4}$$

$x \simeq \underline{-1.19}$ eller $x \simeq \underline{4.19}$

Hvis $f(x) = ax^2 + bx + a$ er kvadratisk, så blir nullpkt.

$f(x) = 0$

$ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$x = \frac{-b}{2a}$ ligger midt mellom nullpkt.

$\Downarrow$

$$\boxed{x_0 = \frac{-b}{2a}}$$

Konklusjon: $f(x) = ax^2 + bx + c$

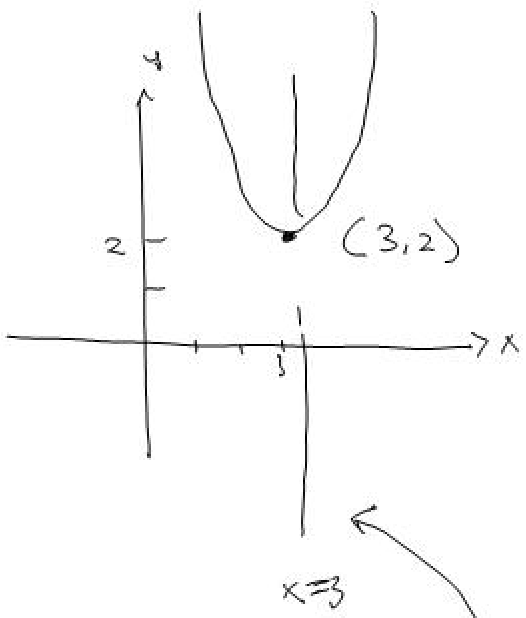
har symmetrilinje $x = x_0$

$$\boxed{\begin{aligned} x_0 &= \frac{-b}{2a} \\ y_0 &= f\left(\frac{-b}{2a}\right) \end{aligned}}$$

Eks: $f(x) = 2x^2 - 12x + 20$

$a = 2$ $\quad x_0 = \frac{-b}{2a} = \frac{-(-12)}{2 \cdot 2} = 3$

$y_0 = f(3) = 2 \cdot 3^2 - 12 \cdot 3 + 20 = 2$



$V_f = [2, \infty)$

Nullpkt: $f(x) = 0$

$2x^2 - 12x + 20 = 0$

$x = \frac{12 \pm \sqrt{144 - 4 \cdot 2 \cdot 20}}{4}$

$= \frac{12 \pm \sqrt{-16}}{4}$

$x = 3 \pm \frac{\sqrt{-16}}{4}$

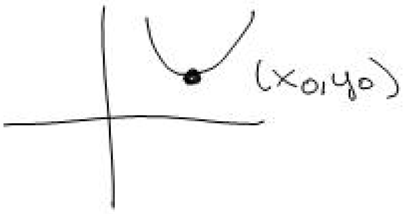
ingen nullpkt.

$x = 3$ symmetrilinje

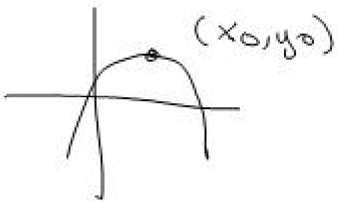
Topp / bunn pkt:

- Vanskelig å regne ut uten derivasjon
- Kan finne topp / bunnpkt for kvadratiske funksjoner:

$$f(x) = ax^2 + bx + c = a \cdot (x - x_0)^2 + y_0$$



$a > 0$



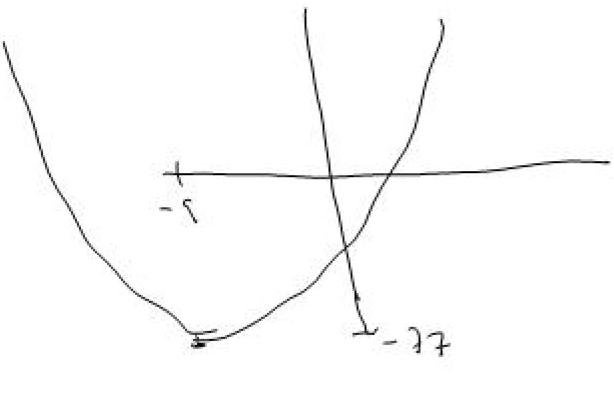
$a < 0$

Eks: $f(x) = x^2 + 18x + 4$

bunnpkt $(x_0, y_0) = (-9, -77)$

$$x_0 = \frac{-b}{2a} = \frac{-18}{2} = -9$$

$$y_0 = f(-9) = (-9)^2 + 18 \cdot (-9) + 4$$
$$= 81 - 162 + 4$$
$$= \underline{-77}$$



$V_f = [-77, \infty)$

(verdimengden)

Eks:

$d = 20 - b$

$= 20 - x$

$b = x$

$$2d + 2b = 40$$

$$d + b = 20$$

$$\boxed{d = 20 - b}$$

Rektangel med omkrets 40.

Finn størst mulig areal.

$$A(x) = b \cdot d = x \cdot (20 - x)$$

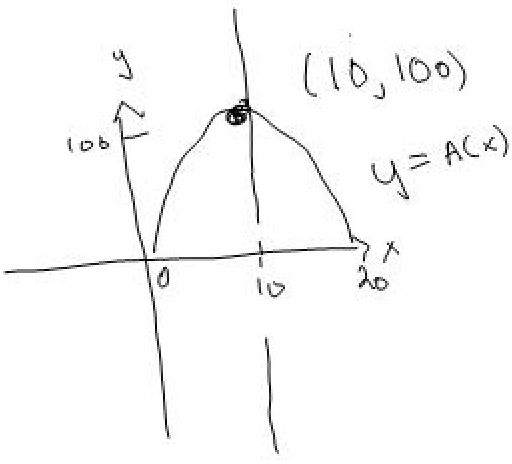
$$= x \cdot (20 - x) = 20x - x^2$$

$$A(x) = 20x - x^2 \quad , \quad D_f = (0, 20)$$

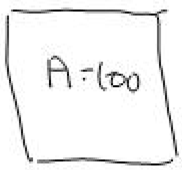
$$= -x^2 + 20x$$

$$a = -1 \, , \quad x_0 = \frac{-b}{2a} = \frac{-20}{-2} = \underline{10}$$

$$y_0 = A(10) = 20 \cdot 10 - 10^2 = \underline{100}$$



$x = 10$.



Svar: Største areal er $A = 100$ når $d = 10$, $b = 10$.