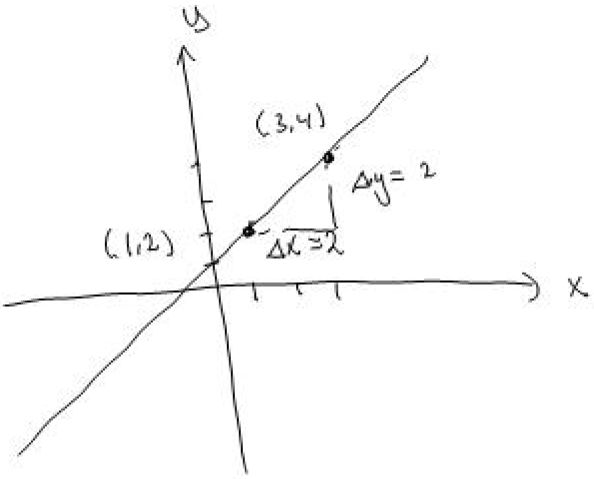
Rette linjer



Likningen til en rett linje:

$$\boxed{y = ax + b}$$

a = stigningstall
b = skjæringspkt med y-aksen

Eks: Linjen $\underline{l}$ går gjennom (1,2) og (3,4).

Finn likningen til $\underline{l}$.

$(x_1, y_1) = (1,2)$
$(x_2, y_2) = (3,4)$

Finner stigningstall:

$$a = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2}{2} = \underline{1}$$

Finner skjæringspkt. med y-aksen:

$(1,2) \in \underline{l}$:

$$y = ax + b$$
$$2 = 1 \cdot 1 + b$$
$$\underline{b = 1}$$

a = 1, b = 1:

$$y = 1 \cdot x + 1$$
$$\underline{\underline{y = x + 1}}$$

Ettpunktsformelen:

Hvis linjen $\underline{\ell}$ går gjennom (x_0, y_0) og har stigningstall a, så har $\underline{\ell}$ likning

$$\boxed{y - y_0 = a \cdot (x - x_0)}$$

Eks:

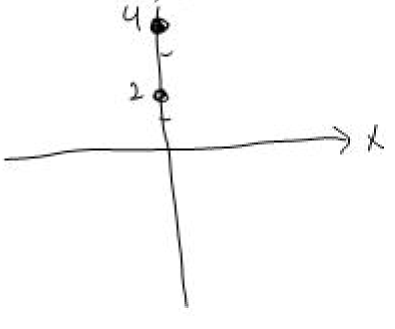
$$\left.\begin{array}{l} a = 1 \\ (x_0, y_0) = (1, 2) \end{array}\right\} \quad \begin{aligned} y - 2 &= 1 \cdot (x - 1) \\ y - 2 &= x - 1 \\ \underline{\underline{y}} &\underline{\underline{= x + 1}} \end{aligned}$$

Oppsummering:
- formel for stigningstall
- ettpunktsformel
- likningen for en rett linje.

Eks: Linja $\underline{\ell}$ går gjennom $(0,2)$ og $(0,4)$.

$$a = \frac{\Delta y}{\Delta x} = \frac{2}{0} \;(= \pm\infty)$$

$\underline{\ell = y\text{-aksen}}$

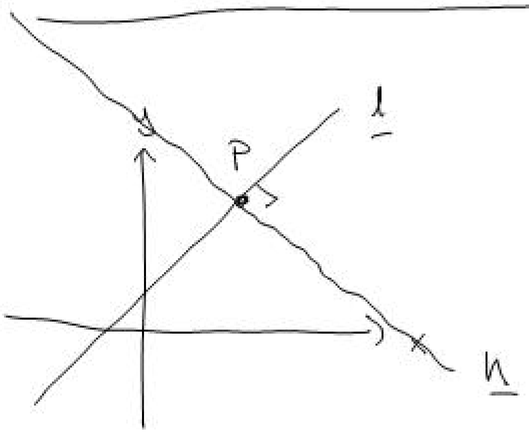


Likning: $\underline{x = 0}$

Likning for vertikal linje: $x = c$

Likning for skrå / horisontale: $y = ax + b$

Normalen til en rett linje:

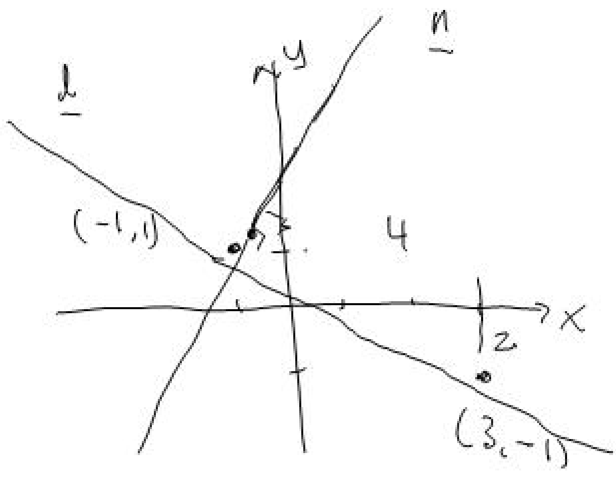


l : en rett linje

$P = (x_0, y_0)$: punkt på l.

n : normalen til l i P.

Eks: Linje l går gjennom $(-1, 1)$ og $(3, -1)$. Finn likningen til l og til normalen til l gjennom $(-1, 1)$.



$$a_l = \frac{\Delta y}{\Delta x} = \frac{-2}{4} = -\frac{1}{2}$$

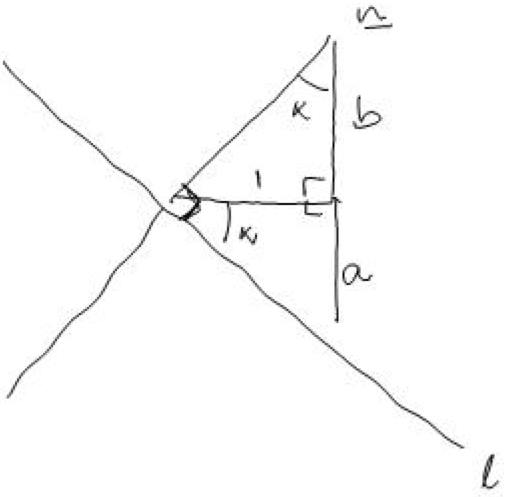
$$y - 1 = -\frac{1}{2}(x - (-1))$$

$$y - 1 = -\frac{1}{2}x - \frac{1}{2}$$

$$l: \; y = -\frac{1}{2}x + \frac{1}{2}$$

n: $(-1,1) \in n$

$$a_n = -\frac{1}{a_l}$$



$$a_l = -a$$
$$a_n = b$$

$$\tan(\alpha_1) = \frac{1}{b} = \frac{a}{1}$$

$$\frac{1}{b} = \frac{a}{1} \quad | \cdot b \cdot 1$$

$$1 = ab \quad | : a$$

$$\underline{b = \frac{1}{a}}$$

$\underline{n}$: $(x_0, y_0) = (-1, 1)$

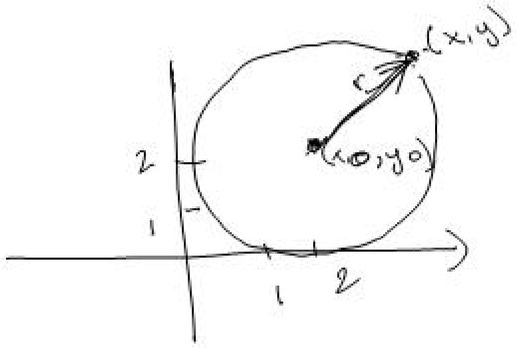
$$a_{\underline{n}} = -\frac{1}{a_{\underline{l}}} = -\frac{1}{-1/2} = \underline{2}$$

$$y - y_0 = a_n(x - x_0)$$
$$y - 1 = 2 \cdot (x - (-1))$$
$$y - 1 = 2x + 2$$
$$\underline{y = 2x + 3}$$

Formel for stigningstall for normalen $\underline{n}$ til $\underline{l}$ gjennom (x_0, y_0):

$$\boxed{a_{\underline{n}} = -\frac{1}{a_l}}$$

Sirkelen.



Defn: Sirkel med radius r og sentrum (x_0,y_0) er alle punkt som har avstand r til (x_0,y_0).

$$d\big((x,y),(x_0,y_0)\big)=r$$

$$\sqrt{(x-x_0)^2+(y-y_0)^2}=r$$

Kap. 4.7 Eks. 2

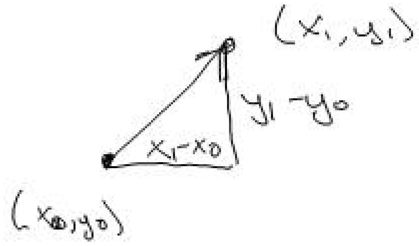
$$\boxed{(x-x_0)^2+(y-y_0)^2=r^2}$$

Likningen for en sirkel med radius r og sentrum (x_0,y_0)

$$d\big((x_0,y_0),(x_1,y_1)\big)$$

$$=\|((x_1-x_0),(y_1-y_0))\|$$

$$=\sqrt{(x_1-x_0)^2+(y_1-y_0)^2}$$



Eks:

$r=2$

$(x_0,y_0)=(2,2)$ sentrum

$$(x-2)^2+(y-2)^2=4$$

$$x^2-4x+4+y^2-4y+\cancel{4}=\cancel{4}$$

$$x^2-4x+y^2-4y+4=0$$

Eks:

$r=1$

$(x_0,y_0)=(0,0)$ sentrum

$$x^2+y^2=1$$

Eks:

$$x^2+4x+y^2-6y = 6$$

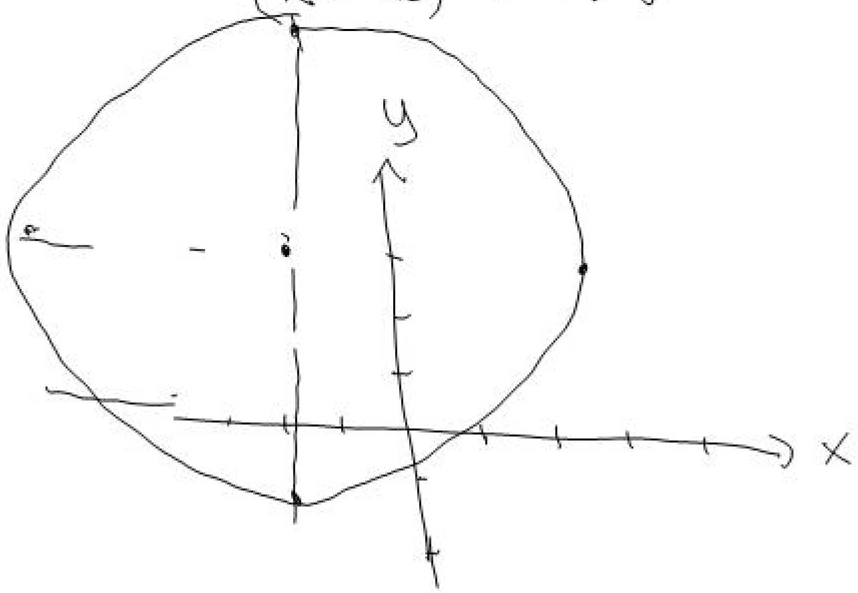
$$\underbrace{x^2+4x+4}_{(x+2)^2}+\underbrace{y^2-6y+9}_{(y-3)^2} = 6+4+9$$

$$\underline{(x+2)^2+(y-3)^2 = 19}$$

$$(x_0,y_0) = \underline{(-2,3)}$$

$$r = \underline{\sqrt{19}}$$

$$(x-x_0)^2+(y-y_0)^2 = r^2$$



Eks:

$$x^2-2x+y^2+y+5=0$$

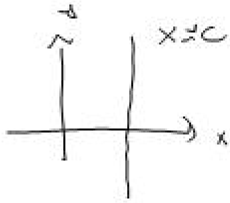
$$x^2-2x+1+y^2+y+\frac{1}{4} = -5+1+\frac{1}{4}$$

$$\boxed{(x-1)^2+(y+\tfrac{1}{2})^2 = -3.75 = -\frac{15}{4}}$$

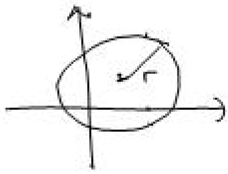
$$\left.\begin{aligned}(x_0,y_0) &= (1,-1/2)\\ r^2 &= -\frac{15}{4}\end{aligned}\right\}\quad \text{ingen løsning}$$

Eksempler som ikke er grafen til en funksjon

* *vertikale linjer*: $x = c$



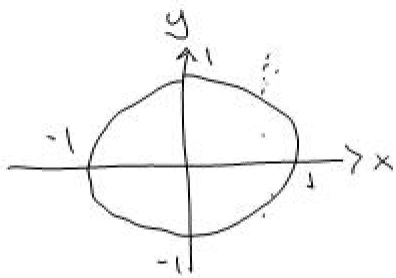
* *sirkelen*: $(x-x_0)^2+(y-y_0)^2=r^2$



Grafen til en funksjon:

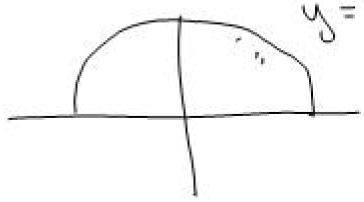
$f(x)$ funksjon $\rightsquigarrow$ $y=f(x)$

EKS: $x^2+y^2=1$



$y^2 = 1-x^2$

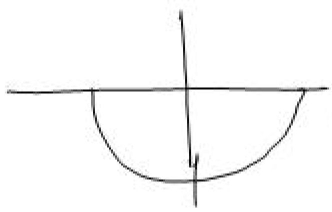
$y = \pm\sqrt{1-x^2}$



$y=\sqrt{1-x^2}$

$f(x) = \sqrt{1-x^2}$

$f(x) = -\sqrt{1-x^2}$

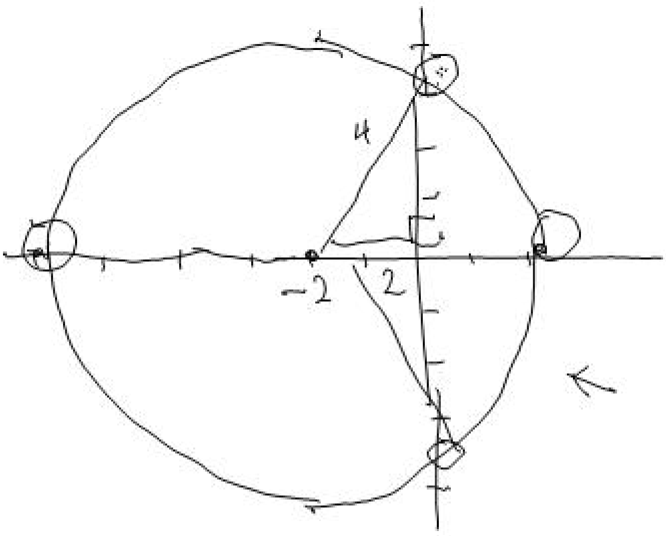


$y = -\sqrt{1-x^2}$

Eks: Sirkelen $x^2+4x+y^2-12=0$
skjærer x-aksen og y-aksen.
Finn skjæringspunktene.

$$x^2+4x+4+y^2 = 12+4$$
$$\boxed{(x+2)^2+y^2 = 16}$$

$r=4$; Sentrum: $(-2,0)$



Ser at vi har to skjæringspunkt med x-aksen og to med y-aksen

$$(x+2)^2+y^2=16$$
$$y^2=16-(x+2)^2$$
$$\boxed{y=\pm\sqrt{16-(x+2)^2}}$$

x-aksen:

$y=0$

$$x^2+4x-12=0$$
$$x=\frac{-4\pm\sqrt{16+48}}{2}=\frac{-4\pm8}{2}=2,-6$$

$\Rightarrow$ $(2,0)$, $(-6,0)$

y-aksen:

$x=0$

$$y^2-12=0$$
$$y=\pm\sqrt{12}=\pm2\sqrt{3}$$
$$\simeq\pm3.4$$

$(0,3.4)$, $(0,-3.4)$

x-aksen:

$(2,0)$, $(-6,0)$

y-aksen:

$$\sqrt{4^2-2^2}=\sqrt{12}=2\sqrt{3}$$
$$\simeq3.4$$

~~$(3.4,0)$, $(-3.4,0)$~~

$(0,3.4)$, $(0,-3.4)$

Oppsummering:

(kap. 3 untatt 3.7)

- Generelt om funksjoner
- Rette linjer (kap. 3)
- Sirkler (4.7)

Neste uke:

- Grafisk løsning (3.7, 4.2)
- Mer om funksjoner.
- Grenseverdier, asymptoter (kap. 7)