Software:

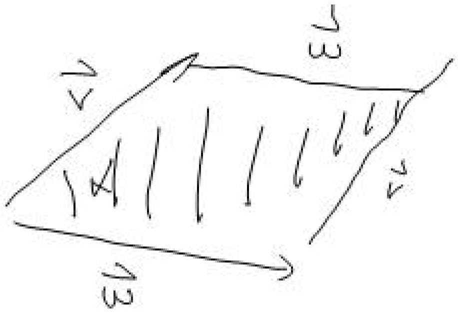
Geogebra: (i planet)

I rommet:

Dyre programmer	Gratis prog:
Mathematica	gnuplot
Maple	winplot
Matlab	

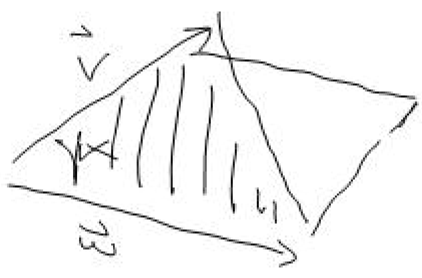
Areal- og volumberegning vha vektorregning

* Arealberegning:



parallellogram utspent av $\vec{v}$ og $\vec{w}$

$$\boxed{A = |\vec{v}| \cdot |\vec{w}| \cdot \sin(\alpha)}$$



trekant utspent av $\vec{v}$ og $\vec{w}$.

$$\boxed{A = \frac{1}{2} \cdot |\vec{v}| \cdot |\vec{w}| \cdot \sin(\alpha)}$$

Arealberegning i planet:

$\vec{v} = (v_1, v_2)$

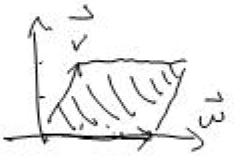
$\vec{w} = (w_1, w_2)$

Arealet av parallellogrammet:

$$A = \left| \begin{vmatrix} v_1 & v_2 \\ w_1 & w_2 \end{vmatrix} \right| = |v_1 w_2 - v_2 w_1|$$

Eks: $\vec{v} = (1,2)$

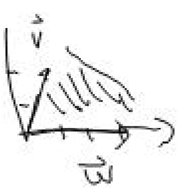
$\vec{w} = (3,0)$



$$A = \left| \begin{vmatrix} 1 & 2 \\ 3 & 0 \end{vmatrix} \right| = |1 \cdot 0 - 2 \cdot 3| = |-6| = \underline{\underline{6}}$$

$$A = \left| \begin{vmatrix} 3 & 0 \\ 1 & 2 \end{vmatrix} \right| = |3 \cdot 2 - 0 \cdot 1| = |6| = \underline{\underline{6}}$$

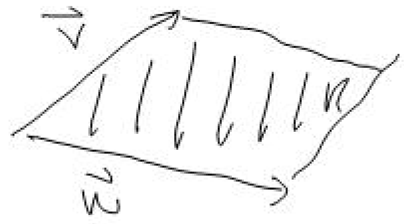
Arealet av trekanten:



$$A = \frac{1}{2} \cdot \left| \begin{vmatrix} v_1 & v_2 \\ w_1 & w_2 \end{vmatrix} \right|$$

$$= \frac{1}{2} \cdot 6 = \underline{\underline{3}}$$

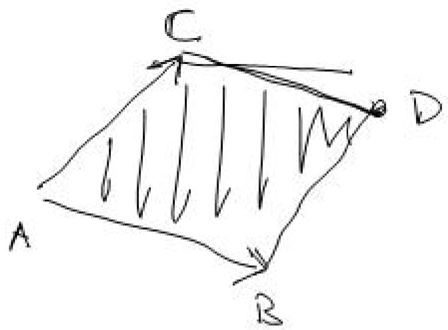
Arealberegning i rommet:



$$\vec{v} = (v_1, v_2, v_3)$$
$$\vec{w} = (w_1, w_2, w_3)$$

Areal av parallellogram:

$$A = |\vec{v} \times \vec{w}|$$



Eks:

$A = (-2, 1, 1)$

$B = (1, -1, 2)$

$C = (-1, 3, 3)$

$\vec{AB} = (3, -2, 1)$

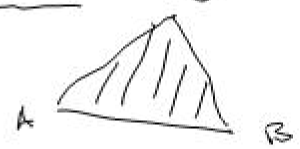
$\vec{AC} = (1, 2, 2)$

$$A = |\vec{v} \times \vec{w}| = |(-6, -5, 8)| = \sqrt{(-6)^2 + (-5)^2 + 8^2}$$
$$= \sqrt{36 + 25 + 64} = \underline{\underline{\sqrt{125}}}$$

$$\vec{v} \times \vec{w} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 3 & -2 & 1 \\ 1 & 2 & 2 \end{vmatrix} = \left(-2 \cdot 2 - 1 \cdot 2, -(3 \cdot 2 - 1 \cdot 1), 3 \cdot 2 - (-2) \cdot 1\right)$$

$$= (-4 - 2, -5, 8) = \underline{(-6, -5, 8)}$$

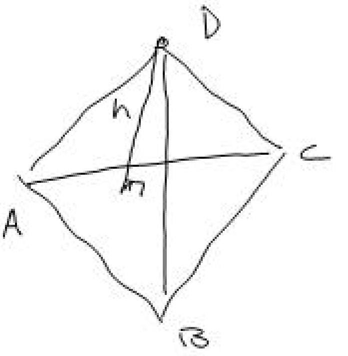
Areal av trekant:



$$A = \frac{1}{2} \cdot |\vec{v} \times \vec{w}|$$
$$= \frac{\sqrt{125}}{2} = \underline{\underline{\frac{5\sqrt{5}}{2}}} \simeq \underline{\underline{5.6}}$$

Volumberegning:

Eks:



Trekantet pyramide

Finn V = volum.

$A = (-2,1,1)$

$B = (1,-1,2)$

$C = (-1,3,3)$

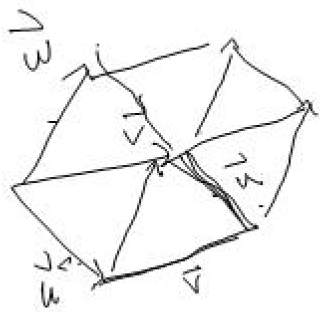
$D = (3,7,1)$

$$V = \frac{1}{3} \cdot \text{Grunnflate} \cdot \text{høgde}$$

$$V = \frac{1}{3} \cdot \frac{5\sqrt{5}}{2} \cdot h$$

$$V = \frac{1}{6} \text{ Volum(parallellepiped)}$$

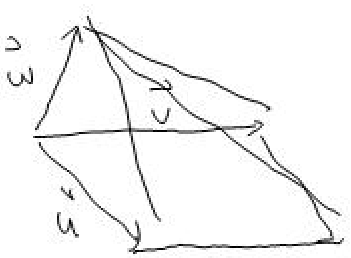
Ved hjelp av vektorregning:



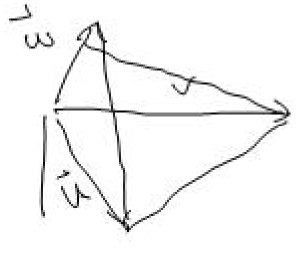
prisme der alle sidene er parallellogram

= parallellepiped

utspent av $\vec{u}, \vec{v}, \vec{w}$.

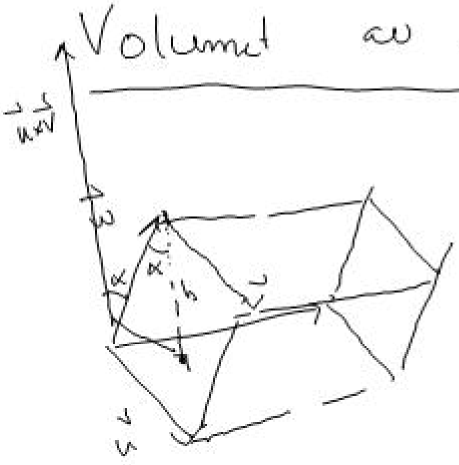


$$V = \frac{1}{3} \text{ Volum(parallellepiped)}$$



$$V = \frac{1}{6} \text{ Volum(parallellepiped)}$$

Volumet av parallellepiped



Parallellepiped utspent av $\vec{u}, \vec{v}, \vec{w}$.

Prisme betyr at

$V =$ Grunnflate $\cdot$ høgde

$$(\vec{u}\times\vec{v})\cdot\vec{w} = |\vec{u}\times\vec{v}|\cdot|\vec{w}|\cdot\cos(\alpha)$$

$$= \text{Grunnflate}\cdot\underbrace{|\vec{w}|\cdot\cos\alpha}$$

$$= \text{Grunnflate}\cdot\text{høgde}$$

$$\frac{h}{|\vec{w}|} = \cos\alpha$$

$$h = \underline{|\vec{w}|\cdot\cos\alpha}$$

<u>Konklusjon:</u> $$\boxed{V = |(\vec{u}\times\vec{v})\cdot\vec{w}|}$$

volum av parallellepiped

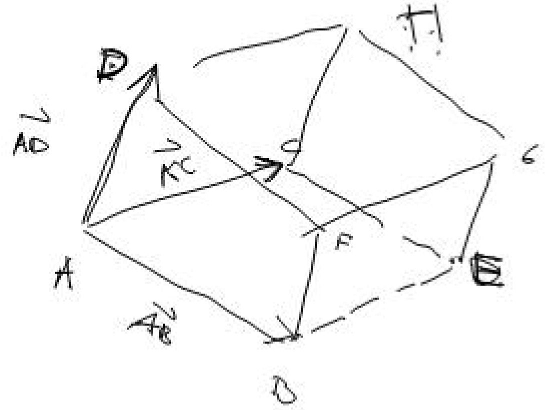
Eks:

$A = (-2,1,1)$

$B = (1,-1,2)$

$C = (-1,3,3)$

$D = (3,7,1)$



$$\left.\begin{aligned} \vec{AB} &= (3,-2,1) \\ \vec{AC} &= (1,2,2) \\ \vec{AD} &= (5,6,0) \end{aligned}\right\} \quad V = \left| \left(\vec{AB} \times \vec{AC} \right) \cdot \vec{AD} \right|$$

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 3 & -2 & 1 \\ 1 & 2 & 2 \end{vmatrix}$$

$$= \left(-2\cdot 2 - 1\cdot 2 \,,\, -(3\cdot 2 - 1\cdot 1) \,,\, 3\cdot 2 - (-2)\cdot 1 \right)$$

$$= (-6, -5, 8)$$

$$V = \left| \left(\vec{AB} \times \vec{AC} \right) \cdot \vec{AD} \right| = \left| (-6,-5,8) \cdot (5,6,0) \right|$$

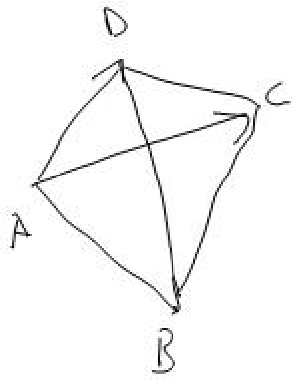
$$= \left| -6\cdot 5 - 5\cdot 6 + 8\cdot 0 \right| = |-60| = \underline{\underline{60}}$$

Merk: $\left| \left(\vec{AB} \times \vec{AD} \right) \cdot \vec{AC} \right| = 60$

$\left| \left(\vec{CA} \times \vec{CE} \right) \cdot \vec{CH} \right| = 60$

etc

Volum av trekantet pyramide



$$V = \frac{1}{6} \cdot \left| \left(\vec{AB} \times \vec{AC} \right) \cdot \vec{AD} \right|$$

$$= \frac{1}{6} \cdot 60 = \underline{\underline{10}}$$