Noen beskjeder.

Øvingstimer $\left\{ \begin{array}{lll} \text{Mandag} & 12.30-14.15 & \text{Janicke} \\ \text{—"—} & 14.30-16.15 & \text{Anastasia } \underline{\text{PI246}} \\ \text{torsdag} & & \text{Janicke.} \end{array} \right.$

Mandag 10. november: Forelesning <u>12.30-14.15</u>.

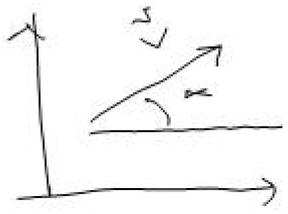
<u>Vektorregning:</u> Vektorer i planet (kap. 13)

1. <u>Vektoraddisjon/subtraksjon:</u> $\vec{v}+\vec{w}$, $\vec{v}-\vec{w}$

2. <u>Skalar multiplikasjon:</u> $c \cdot \vec{v}$

3. <u>Lengden til en vektor:</u> $|\vec{v}|$

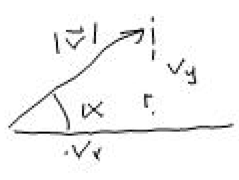
4. <u>Vinkelen mellom en vektor og positiv x-akse</u>



5. <u>Finne koordinatene til en vektor:</u>

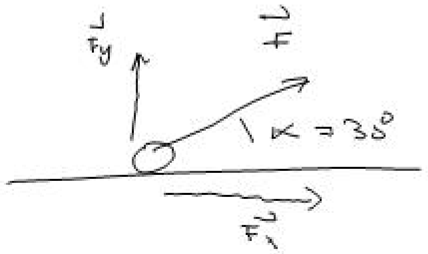
$\vec{v}$: Vet $\left\{ \begin{array}{l} |\vec{v}| \text{ - lengden} \\ \alpha \text{ - vinkel i.f.t. pos. x-akse} \end{array} \right.$

$$\vec{v} = (v_1, v_2) = (v_x, v_y)$$



$$\boxed{\begin{array}{l} v_x = v_1 = |\vec{v}| \cdot \cos\alpha \\ v_y = v_2 = |\vec{v}| \cdot \sin\alpha \end{array}}$$

Eks:



$|\vec{F}| = 50N$

$\vec{F} = (F_x, F_y)$

$F_x = |\vec{F}| \cdot \cos 30° = 50N \cdot \frac{\sqrt{3}}{2} = \dots$

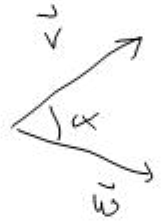
$F_y = |\vec{F}| \cdot \sin 30° = 50N \cdot \frac{1}{2} = 25N$

⑥ Skalar produkt (prikkprodukt)

$\vec{v} = (v_1, v_2)$

$\vec{w} = (w_1, w_2)$

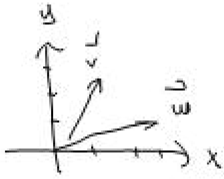
$$\boxed{\begin{aligned} \vec{v} \cdot \vec{w} &= v_1 \cdot w_1 + v_2 \cdot w_2 \\ \vec{v} \cdot \vec{w} &= |\vec{v}| \cdot |\vec{w}| \cdot \cos(\alpha) \end{aligned}}$$



(α er vinkelen mellom $\vec{v}$ og $\vec{w}$)

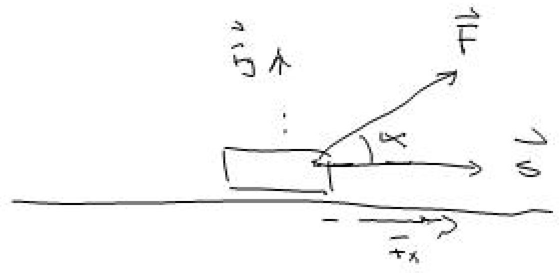
Eks: $\vec{v} = (1,2)$

$\vec{w} = (3,1)$



$\vec{v} \cdot \vec{w} = 1 \cdot 3 + 2 \cdot 1 = \underline{5}$

Eks:



$F_x = |\vec{F}| \cdot \cos\alpha$

$F_y = |\vec{F}| \cdot \sin\alpha$

$$W_{\vec{F}} = F_x \cdot s = |\vec{F}| \cdot \cos\alpha \cdot |\vec{s}|$$

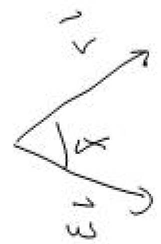
$$= |\vec{F}| \cdot |\vec{s}| \cdot \cos\alpha = \underline{\underline{\vec{F} \cdot \vec{s}}}$$

Hva brukes prikkproduktet til?

- I fysikk, f.eks. arbeid
- I matematikk, f.eks. for å finne vinkler

$$\vec{v} \cdot \vec{w} = v_1 \cdot w_1 + v_2 \cdot w_2$$

$$\vec{v} \cdot \vec{w} = |\vec{v}| \cdot |\vec{w}| \cdot \cos\alpha$$



$$v_1 \cdot w_1 + v_2 \cdot w_2 = |\vec{v}| \cdot |\vec{w}| \cdot \cos\alpha$$

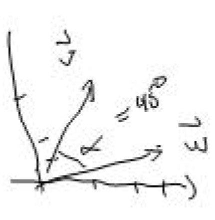
$$\cos\alpha = \frac{v_1 \cdot w_1 + v_2 \cdot w_2}{|\vec{v}| \cdot |\vec{w}|} = \frac{\vec{v} \cdot \vec{w}}{|\vec{v}| \cdot |\vec{w}|}$$

Eks: $\left.\begin{matrix} \vec{v} = (1,2) \\ \vec{w} = (3,1) \end{matrix}\right\}$

$\vec{v} \cdot \vec{w} = (1,2) \cdot (3,1) = 1 \cdot 3 + 2 \cdot 1 = 5$

$|\vec{v}| = \sqrt{1^2 + 2^2} = \sqrt{5}$

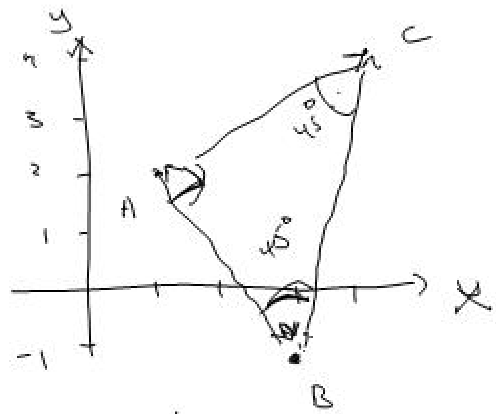
$|\vec{w}| = \sqrt{3^2 + 1^2} = \sqrt{10}$



$$\cos\alpha = \frac{5}{\sqrt{5} \cdot \sqrt{10}} = \frac{5}{\sqrt{5} \cdot \sqrt{5}\sqrt{2}} = \frac{\cancel{5}}{\cancel{5} \cdot \sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\alpha = \underline{\underline{45°}}$$

Eks: $A = (1,2)$
$B = (3,-1)$
$C = (4,4)$



$$\vec{AB} = (2,-3) \qquad \vec{BA} = -\vec{AB} = (-2,3)$$
$$\vec{AC} = (3,2) \qquad \vec{CA} = (-3,-2)$$
$$\vec{BC} = (1,5) \qquad \vec{CB} = (-1,-5)$$

$$AB = |\vec{AB}| = \sqrt{2^2 + (-3)^2} = \sqrt{13}$$
$$AC = |\vec{AC}| = \sqrt{3^2 + 2^2} = \sqrt{13}$$
$$BC = |\vec{BC}| = \sqrt{1^2 + 5^2} = \sqrt{26}$$

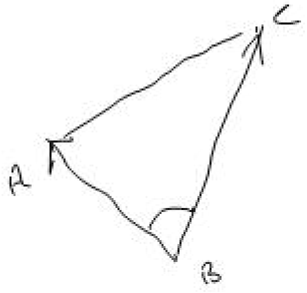
$\underline{\angle A}$: $$\cos A = \frac{\vec{AB} \cdot \vec{AC}}{|\vec{AB}| \cdot |\vec{AC}|} = \frac{(2,-3)\cdot(3,2)}{\sqrt{13}\cdot\sqrt{13}} = \frac{2\cdot 3 + (-3)\cdot 2}{13} = 0$$

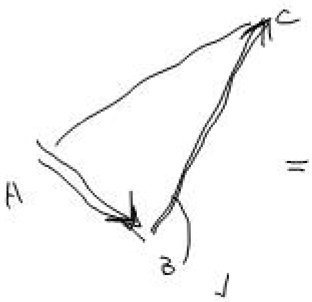
$\angle A = \underline{90^\circ}$

$\angle B = \angle C = 45^\circ$ siden $AC = AB$ og $\angle A = 90^\circ$.

$\underline{\angle B}$: $\angle B$ = vinkelen mellom $\vec{BA}$ og $\vec{BC}$

$180 - \angle B$ = vinkelen mellom $\vec{AB}$ og $\vec{BC}$





$$\cos B = \frac{\vec{BA}\cdot\vec{BC}}{BA \cdot BC} = \frac{(-2,3)\cdot(1,5)}{\sqrt{13}\cdot\sqrt{26}}$$
$$= \frac{-2\cdot 1 + 3\cdot 5}{\sqrt{13}\cdot\sqrt{13}\cdot\sqrt{2}} = \frac{\cancel{13}}{\cancel{13}\cdot\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$\angle B = \underline{45^\circ}$

To spesialtilfeller:

$\vec{v} = (v_1, v_2)$
$\vec{w} = (w_1, w_2)$

(a) $\vec{v} \perp \vec{w}$? ($\vec{v}$ og $\vec{w}$ er normale)

$$\left\{ \begin{array}{l} \text{vinkelen mellom} \\ \vec{v} \text{ og } \vec{w} \text{ er } 90^\circ \end{array} \right\} \longleftrightarrow \left\{ \vec{v} \cdot \vec{w} = 0 \right.$$

(b) $\vec{v} \parallel \vec{w}$ ($\vec{v}$ og $\vec{w}$ er parallelle)

$$\left\{ \begin{array}{l} \text{vinkelen mellom} \\ \vec{v} \text{ og } \vec{w} \text{ er } 0^\circ \\ \text{eller } 180^\circ \end{array} \right\} \longleftrightarrow \left\{ \begin{array}{l} \vec{w} = c \cdot \vec{v} \\ \text{for et tall } c \end{array} \right\}$$

Eks: $\vec{v} = (1,2)$
$\vec{w} = (3,-1)$

$\vec{v} \perp \vec{w}$?

$\vec{v} \cdot \vec{w} = (1,2) \cdot (3,-1)$
$= 1 \cdot 3 + 2 \cdot (-1) = 1 \neq 0$

$\vec{v}$ og $\vec{w}$ ikke normale

$\vec{v} \parallel \vec{w}$?

$\vec{w} = c \cdot \vec{v}$

$(3,-1) = c \cdot (1,2)$

$(3,-1) = (c, 2c)$

$$\boxed{\begin{array}{l} 3 = c \\ -1 = 2c \end{array}} \begin{array}{l} \Rightarrow c = 3 \\ \Rightarrow -1 = 6 \text{ umulig.} \end{array}$$

$\Rightarrow$ ingen løsning.
$\Rightarrow$ $\vec{v}$ og $\vec{w}$ ikke parallelle.

$$\left\{\begin{array}{l}\text{Vinkelen mellom}\\ \vec{v} \text{ og } \vec{w} \text{ er } 0^\circ\\ \text{eller } 180^\circ\end{array}\right\} \longleftrightarrow \left\{\begin{array}{l}\vec{w} = t\cdot\vec{v}\\ \text{for en } t\end{array}\right\} \longleftrightarrow \left\{\begin{array}{l}\begin{vmatrix} v_1 & v_2 \\ w_1 & w_2 \end{vmatrix} = 0\\ (\text{determinant})\end{array}\right\}$$

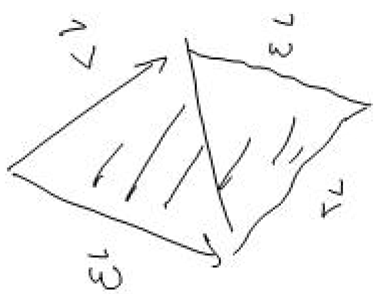
$$\left.\begin{array}{l}\vec{v} = (v_1, v_2)\\ \vec{w} = (w_1, w_2)\end{array}\right\} \quad \boxed{\begin{array}{c}\begin{vmatrix} v_1 & v_2 \\ w_1 & w_2 \end{vmatrix} = v_1\cdot w_2 - v_2\cdot w_1 \\ 2\times 2 - \underline{\text{determinant}}\end{array}}$$

<u>Eks:</u>

$$\left.\begin{array}{l}\vec{v} = (1,2)\\ \vec{w} = (3,-1)\end{array}\right\} \quad \begin{vmatrix} 1 & 2 \\ 3 & -1 \end{vmatrix} = 1\cdot(-1) - 2\cdot 3 = -7 \neq 0$$

$\vec{v}$ og $\vec{w}$ er <u>ikke</u> parallelle.

<u>Areal:</u>



Areal av parallellogrammet utspent av $\vec{v}$ og $\vec{w}$:

$$A = \left|\begin{vmatrix} v_1 & v_2 \\ w_1 & w_2 \end{vmatrix}\right|$$

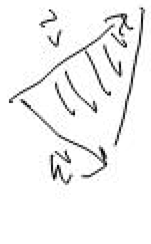
$$= |v_1 w_2 - v_2 w_1|$$

<u>Absolutt verdi:</u>

$|7| = 7$

$|-7| = 7$

<u>Areal av trekant:</u>



$$\text{Areal} = \frac{1}{2}\cdot\left|\begin{vmatrix} v_1 & v_2 \\ w_1 & w_2 \end{vmatrix}\right|$$