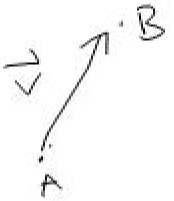
Vektorregning

Generelt (kap. 12)

- I planet (kap. 13)
- I rommet (kap. 14)

<u>Defn:</u> En vektor er <u>rettet linjestykke</u>



$\vec{AB} = \vec{v}$

To vektorer er <u>parallelle</u> hvis linjestykkene er parallelle.

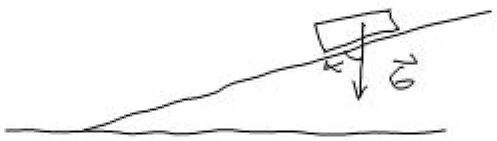
$\vec{u}$ og $\vec{v}$ er parallelle.

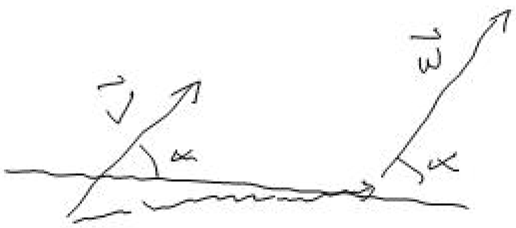
To vektorer er <u>like</u> hvis de har samme <u>størrelse</u> og samme <u>retning</u>.

størrelse: $|\vec{v}|$ = lengden til vektoren

retning: for eksempel angitt med en vinkel

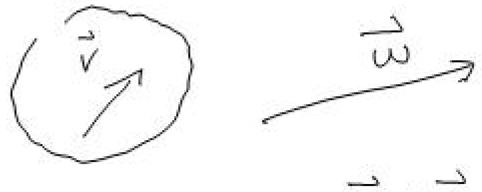
$|\vec{G}| = 50N$





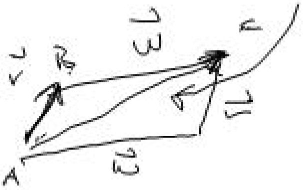
$\vec{v} = \vec{w}$

parallell-forskyvning.



Operasjoner:

* Addisjon:

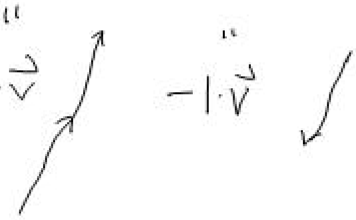


$\vec{v} + \vec{w}$

$\vec{AB} + \vec{BC} = \vec{AC}$

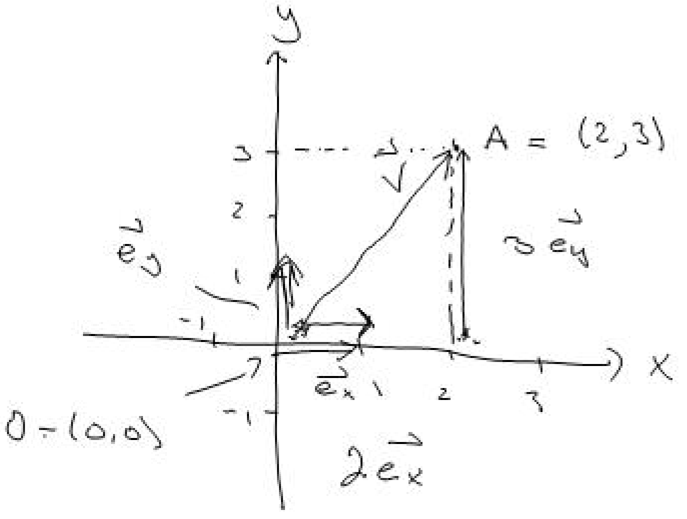
* Skalar multiplikasjon:

$2\vec{v} = 2\cdot\vec{v}$ $\quad -\vec{v} = -1\cdot\vec{v}$



skalar = tall
vektor = størrelse og retning

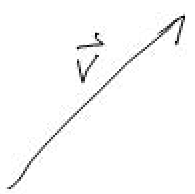
Vektorer i planet – med koordinater



Koordinatsystem

* aksen står normalt på hverandre

(kartesisk koordinatsystem)



$\vec{v} = (2,3)$

$\vec{v} = \overrightarrow{OA}$

- parallell forskyver vektoren slik at den starter i Origo,
- leser av koordinatene til sluttpunktet

Enhetsvektorer:

$\vec{e_x}$ → lengde 1 langs pos. x-aksen

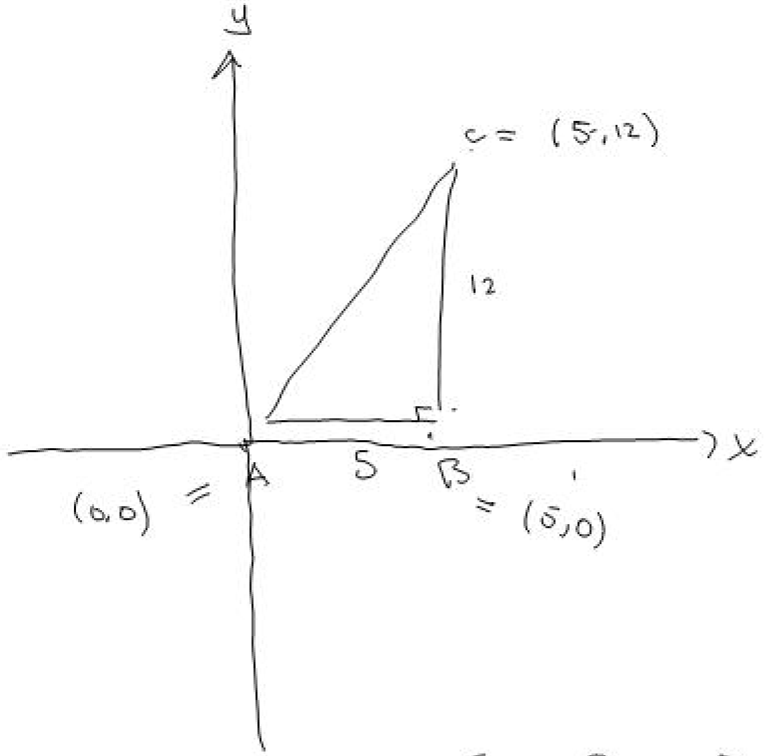
$\vec{e_y}$ ↑ lengde 1 langs pos. y-aksen

$$\vec{v} = a \cdot \vec{e_x} + b \cdot \vec{e_y}$$

Eks: $\vec{v} = 2\vec{e_x} + 3\vec{e_y} = (2,3)$

$\vec{w} = (a,b)$

Eks:



$$\vec{AB} = (5,0) = (\overset{5}{x_B} - \overset{0}{x_A},\ \overset{0}{y_B} - \overset{0}{y_A})$$

$$\vec{AC} = (5,12)$$

$$\vec{BC} = (0,12) = (\overset{5}{x_C} - \overset{5}{x_B},\ \overset{12}{y_C} - \overset{0}{y_B})$$

$$\vec{BA} = (-5,0)$$

$$\vec{CA} = (-5,-12)$$

$$\vec{CB} = (0,-12)$$

$B = (5,0)$	$\vec{AB} = (5,0)$ $\left(\vec{AB} = [5,0]\right)$
punkt	vektor

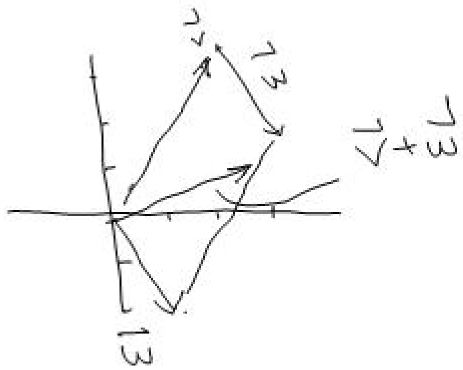
Regneoperasjoner for vektorer i planet

(a) Vektoraddisjon / subtraksjon

$\vec{v} = (v_1, v_2)$

$\vec{w} = (w_1, w_2)$

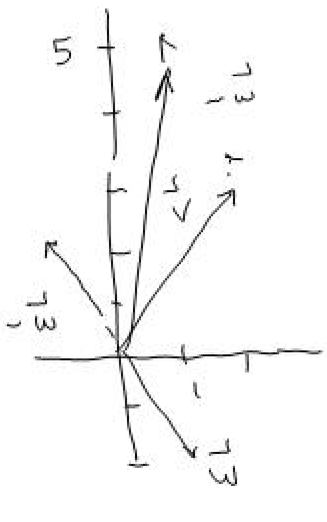
$$\boxed{\begin{aligned} \vec{v} + \vec{w} &= (v_1 + w_1,\ v_2 + w_2) \\ \vec{v} - \vec{w} &= (v_1 - w_1,\ v_2 - w_2) \end{aligned}}$$



Eks: $\vec{v} = (2, 3)$

$\vec{w} = (1, -2)$

$$\vec{v} + \vec{w} = (2,3) + (1,-2) = (2+1,\ 3+(-2)) = \underline{\underline{(3,\ 1)}}$$



$$\vec{v} - \vec{w} = (2,3) - (1,-2) = (2-1,\ 3-(-2)) = \underline{\underline{(1,5)}}$$

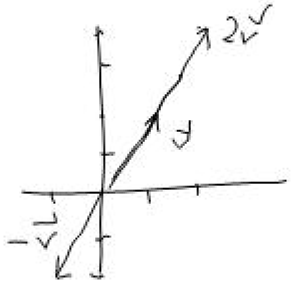
(6) <u>Skalarmultiplikasjon</u>

$\vec{v} = (v_1, v_2)$

c – et tall (skalar)

$$\boxed{c \cdot \vec{v} = c \cdot (v_1, v_2) = (cv_1, cv_2)}$$

<u>Eks:</u> $\vec{v} = (1,2)$



$2 \cdot \vec{v} = 2 \cdot (1,2) = (2 \cdot 1, 2 \cdot 2)$

$= \underline{(2,4)}$

$-1 \cdot \vec{v} = -\vec{v} = (-1 \cdot 1, -1 \cdot 2)$

$= \underline{(-1, -2)}$

$-3\vec{v} = -3 \cdot (1,2) = \underline{(-3, -6)}$

<u>Viktig:</u> $\vec{0} = (0,0)$ ← <u>nullvektoren</u>

$0 \cdot \vec{v} = 0 \cdot (1,2) = (0,0) = \vec{0}$

$\vec{v} = (v_1, v_2)$ $\qquad$ $\vec{v} = \overrightarrow{AB} = (1,2)$

$-\vec{v} = (-v_1, -v_2)$ $\qquad$ $-\vec{v} = \overrightarrow{BA} = (-1,-2)$

Regneregler for vektorregning:

a) $\vec{u}+\vec{v} = \vec{v}+\vec{u}$

b) $\vec{u}+(\vec{v}+\vec{w}) = (\vec{u}+\vec{v})+\vec{w}$

c) $\vec{u}+\vec{0} = \vec{0}+\vec{u} = \vec{u}$

d) $\vec{u}+(-\vec{u}) = \vec{0}$

e) $c\cdot(\vec{u}+\vec{v}) = c\cdot\vec{u}+c\cdot\vec{v}$

f) $(c+d)\cdot\vec{u} = c\cdot\vec{u}+d\cdot\vec{u}$

g) $c\cdot(d\vec{u}) = (cd)\cdot\vec{u}$

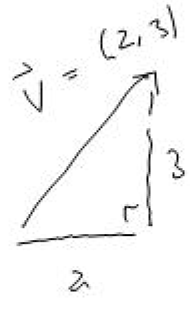
h) $1\cdot\vec{u} = \vec{u}$

$\vec{u}, \vec{v}, \vec{w}$ vektorer

c, d skalarer

(C) Lengden til en vektor

$\vec{v} = (v_1, v_2)$



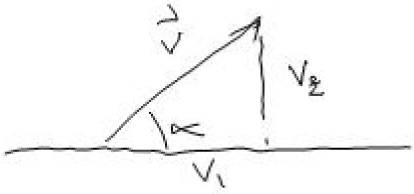
$$\boxed{|\vec{v}| = \sqrt{v_1^2+v_2^2}}$$

$|\vec{v}|$ betyr lengden til vektoren $\vec{v}$.

Eks: $|\vec{v}| = |(2,3)| = \sqrt{2^2+3^2} = \underline{\sqrt{13} \simeq 3.6}$

Vinkler mellom vektorer

$\vec{v} = (v_1, v_2)$



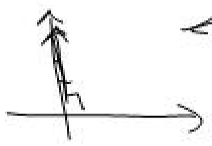
α : vinkelen til $\vec{v}$
i.f.t. horisontalplanet.

$$\tan \alpha = \frac{v_2}{v_1} \implies \boxed{\alpha = \tan^{-1}(v_2/v_1)}$$

Eks: $\vec{v} = (2,3)$

$\tan \alpha = 3/2 \Rightarrow \alpha = \tan^{-1}(3/2) \simeq \underline{56^\circ}$

$\vec{v} = (0,3) \Rightarrow \alpha = 90^\circ$



$\vec{v} = (1,-2)$

$\alpha = \tan^{-1}(-2/1) = \tan^{-1}(-2) \simeq \underline{\underline{-63^\circ}}$