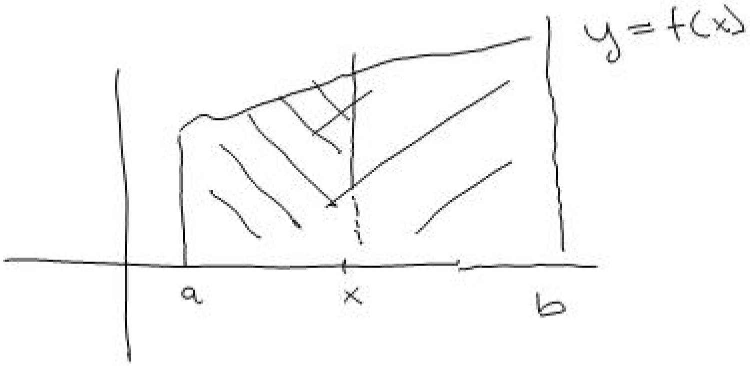
06/05/09:

Integral og areal beregning



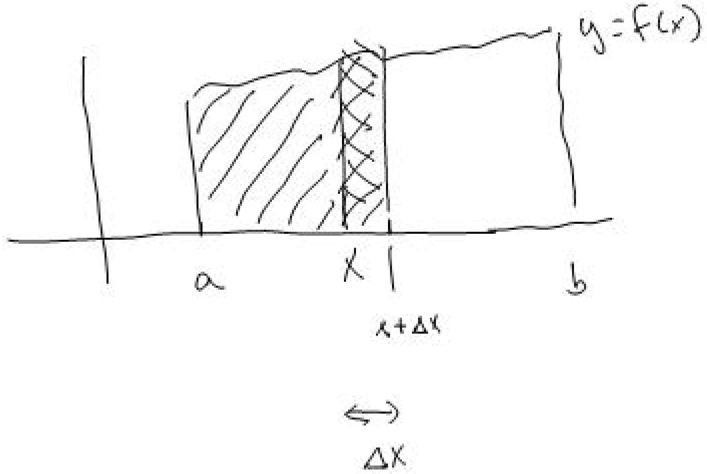
Defn: $A(x)$ = arealet mellom grafen til $y = f(x)$ fra a til x

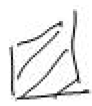
Påstand: $A(a) = 0$

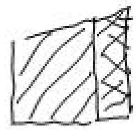
$A(b) = ?$

$$\boxed{A'(x) = f(x)}$$

Hvorfor: $A'(x) = \lim\limits_{\Delta x \to 0} \dfrac{A(x+\Delta x) - A(x)}{\Delta x}$ $(\Delta x = h)$



$A(x) =$ 

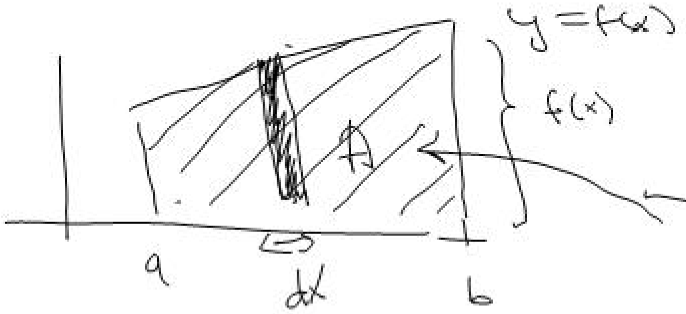
$A(x+\Delta x) =$ 

$A(x+\Delta x) - A(x) =$ 

$\cong \Delta x \cdot f(x)$

$$\frac{A(x+\Delta x) - A(x)}{\Delta x} \simeq f(x)$$

$A'(x) = f(x)$ $\quad \dfrac{dA}{dx} = f(x)$ $\quad \boxed{dA = f(x)\,dx}$



$$A'(x) = f(x)$$

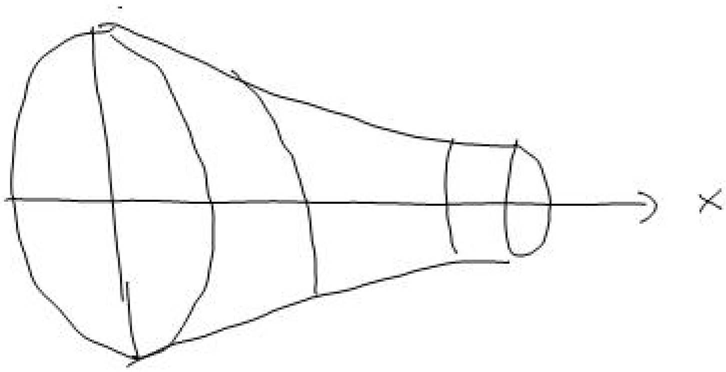
$$\int f(x)\,dx = A(x) + C$$

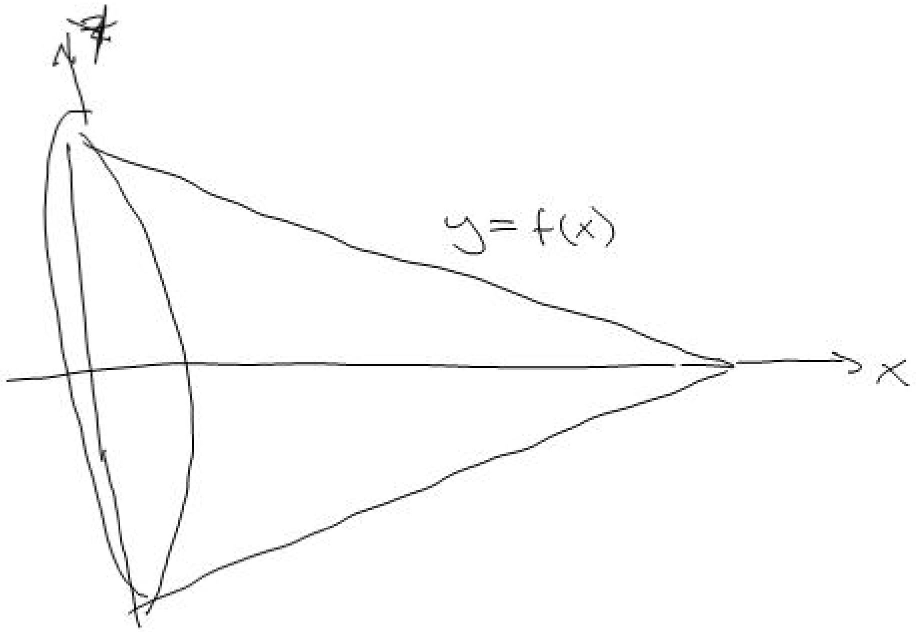
$$\int_a^b f(x)\,dx = A(b) - \underbrace{A(a)}_{0} = A(b) = A$$

<u>Konklusjon:</u>

$$\boxed{A = \int_a^b f(x)\,dx}$$

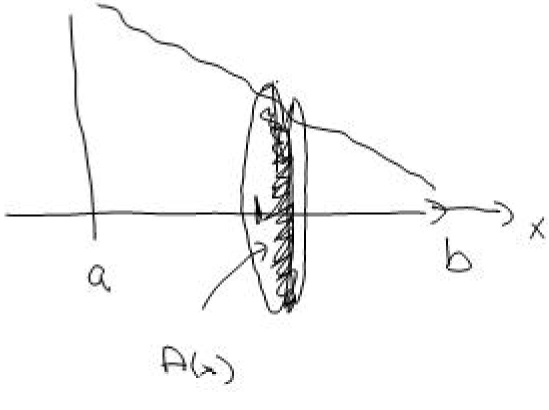
Volumberegning:



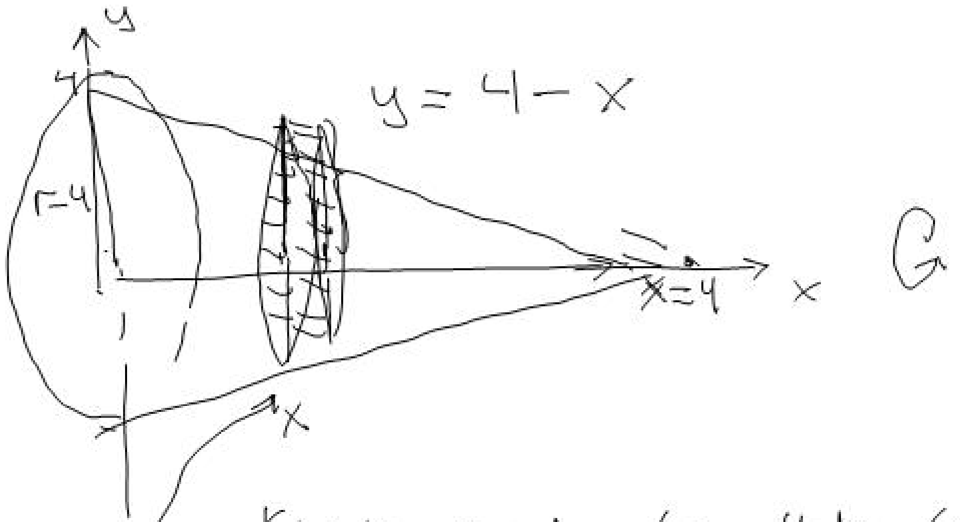


Tverrsnitt ved x: Areal $A(x)$

$$\boxed{V = \int_a^b A(x)\,dx}$$



Eks:



sirkelskive med volum $\simeq A(x)\cdot dx$

Kjegle med Grunnflate $G = \pi \cdot 4^2 = \underline{16\pi}$

$h = 4$

<u>Volum:</u> $V = \frac{1}{3}\cdot G \cdot h$

$= \frac{1}{3}\cdot 16\pi \cdot 4 = \underline{\underline{\frac{64}{3}\pi}}$

ok.

<u>Vha integral:</u>

<u>$x \in [0,4]$:</u> tverrsnitt ved x er en sirkel med radius $\underline{r(x) = 4 - x}$.

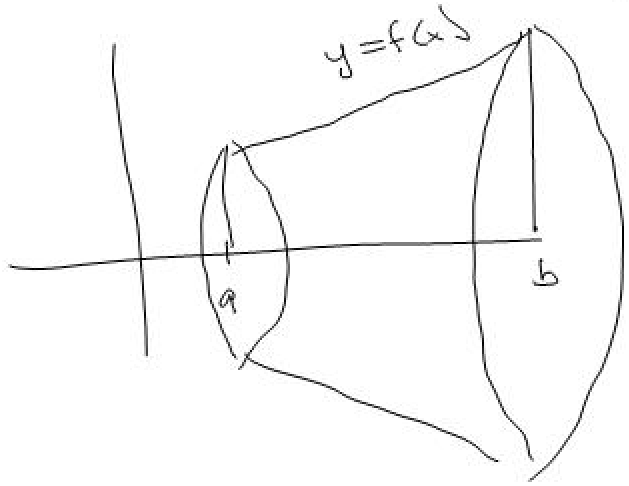
$A(x) = \pi \cdot r(x)^2 = \underline{\pi\cdot(4-x)^2}$

$A(x+\Delta x) = \pi \cdot r(x+\Delta x)^2 = \dots$

$$V = \int_0^4 A(x)\,dx = \int_0^4 \pi\cdot(4-x)^2\,dx = \pi\int_0^4 (16 - 8x + x^2)\,dx$$

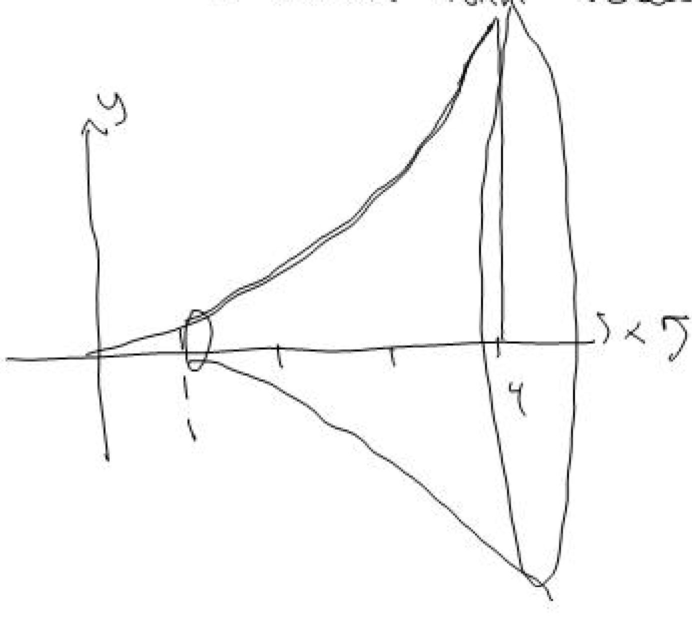
$$= \pi\left[16x - 4x^2 + \tfrac{1}{3}x^3\right]_0^4 = \pi\left(\cancel{64} - \cancel{64} + \tfrac{1}{3}4^3 - \cancel{0}\right)$$

Sirkelskivemetoden: hvert tverrsnitt en sirkelskive



$$\boxed{V = \int_a^b \pi \cdot f(x)^2 dx}$$

Eks: Grafen til $f(x) = x^2$, $x \in [1, 4]$ roteres rundt x-aksen. Finn volumet av rotasjonslegemet.



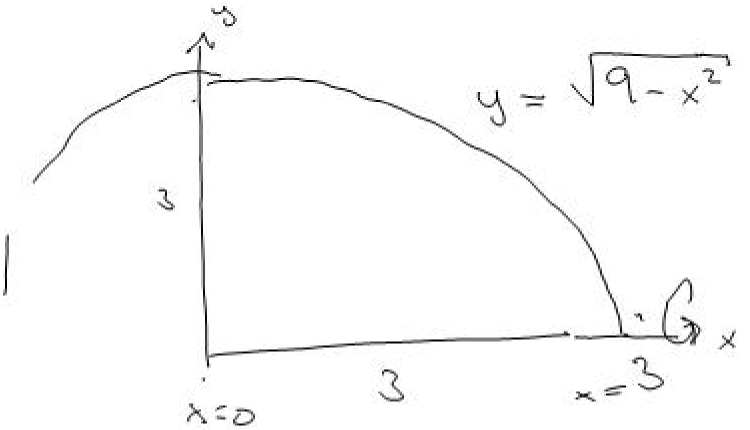
$$V = \int_1^4 \pi \cdot (x^2)^2 \cdot dx = \int_1^4 \pi \cdot x^4 dx$$

$$= \pi \left[\frac{1}{5} x^5 \right]_1^4 = \pi \left(\frac{1}{5} \cdot 4^5 - \frac{1}{5} \cdot 1^5 \right)$$

$$= \frac{\pi}{5} \cdot (4^5 - 1) = \frac{1023 \cdot \pi}{5} \approx \underline{\underline{643}}$$

Eksempel: Volum av en kule med radius 3.

Formel: $\boxed{V = \frac{4}{3}\pi R^3} = \frac{4}{3}\cdot\pi\cdot 3^3 = \underline{36\pi}$



$$x^2 + y^2 = 3^2$$
$$y^2 = 9 - x^2$$
$$y = \sqrt{9-x^2}$$

Volumet av en halvkule med radius 3:

$$V_{1/2} = \int_0^3 \pi \cdot f(x)^2\,dx$$
$$= \pi \cdot \int_0^3 \left(\sqrt{9-x^2}\right)^2 dx = \pi \int_0^3 (9-x^2)\,dx$$
$$= \pi \left[9x - \tfrac{1}{3}x^3\right]_0^3$$
$$= \pi\left((27-9)-0\right) = 18\pi$$

Volumet av hele kulen:

$V = V_{1/2}\cdot 2 = \underline{\underline{36\pi}}$ stemmer med formelen $V = \frac{4}{3}\pi R^3$.