04/05/09:

Oblig 8: frist fredag kl. 14.

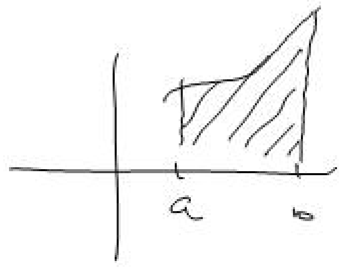
5d) — kan hoppe over
(ikke pensum)

De neste ukene:

04 - 08. mai: resten av kapittel 16
11 - 15. mai
18 - 22. mai } repetisjon
vektorregning, trigonometri
25 - 29. mai — studieuke,
prøve-eksamen (25. mai)

2. juni — eksamen.

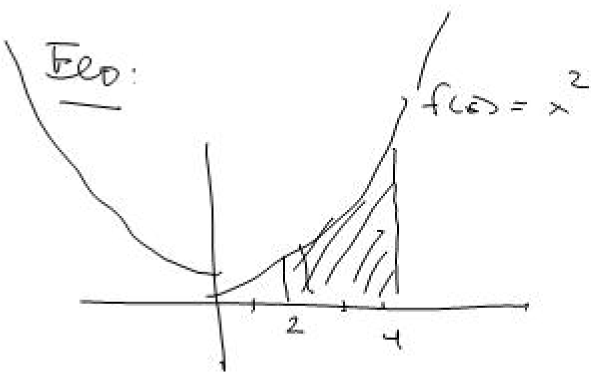
Arealberegning:



Hvis f er en kontinuerlig funksjon på $[a,b]$ og $f(x) \geq 0$ for alle $x \in [a,b]$, så er arealet begrenset av grafen til f, x-aksen, $x=a$ og $x=b$ gitt ved

$$A = \int_a^b f(x)\,dx = F(b) - F(a)$$

Eks:



Areal:

$$A = \int_2^4 x^2\,dx$$

$$= \left[\frac{1}{3}x^3\right]_2^4$$

$$= \frac{1}{3}\cdot 4^3 - \frac{1}{3}\cdot 2^3$$

$$= \frac{64}{3} - \frac{8}{3} = \frac{56}{3} \approx \underline{\underline{18,67}}$$

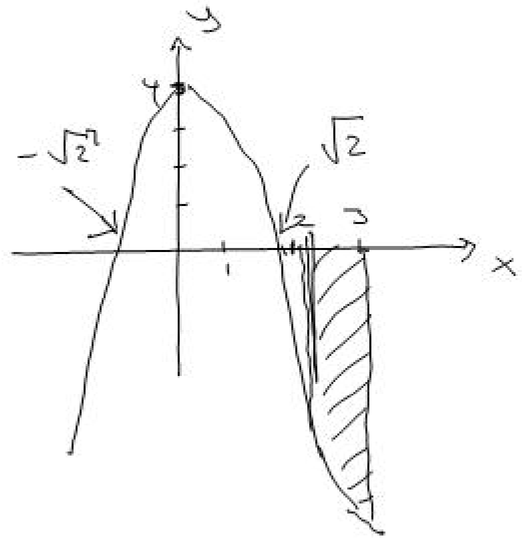
Eks: $f(x) = 4 - 2x^2$

Nullpkt. $4 - 2x^2 = 0$

$$\frac{4}{2} = \frac{2x^2}{2}$$

$$x^2 = 2$$

$$x = \pm\sqrt{2}$$



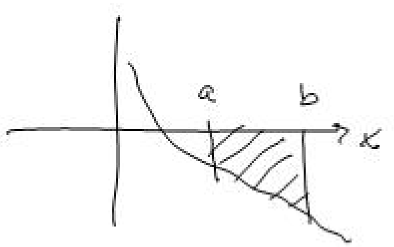
$$\text{Areal} = -\int_2^3 (4 - 2x^2)\,dx = -\left[4x - \frac{2}{3}x^3\right]_2^3$$

$$= -\left(\left(4\cdot 3 - \frac{2}{3}\cdot 3^3\right) - \left(8 - \frac{2}{3}\cdot 2^3\right)\right)$$

$$= -\left((12 - 18) - \left(8 - \frac{16}{3}\right)\right) = 6 + 8 - \frac{16}{3}$$

$$= \frac{14\cdot 3 - 16}{3} = \frac{26}{3} \approx \underline{\underline{8.67}}$$

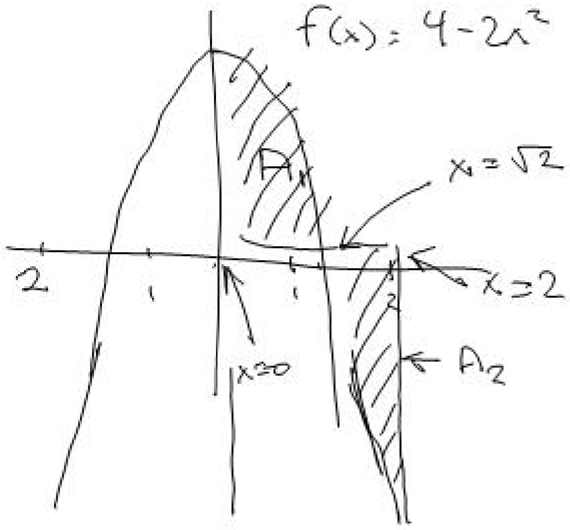
Hvis $f(x)$ er kontinuerlig på $[a,b]$ og $f(x) \leq 0$ for alle $x \in [a,b]$, så er



$$\int_a^b f(x)\,dx = -A \quad \Rightarrow$$

$$\boxed{A = -\int_a^b f(x)\,dx}$$

Eks:



Finn arealet begrenset av grafen til f, x-aksen, $x = 0$ og $x = 2$.

$$A = A_1 + A_2$$
$$= \frac{8}{3}\sqrt{2} + \left(\frac{8}{3}\sqrt{2} - \frac{8}{3}\right)$$
$$= \underline{\underline{\frac{16}{3}\sqrt{2} - \frac{8}{3}}} \approx \underline{\underline{4.88}}$$

(1) $$\int_0^{\sqrt{2}} f(x)\,dx = \int_0^{\sqrt{2}} (4 - 2x^2)\,dx = \left[4x - \frac{2}{3}x^3\right]_0^{\sqrt{2}}$$

$$= \left(4\sqrt{2} - \frac{2}{3}(\sqrt{2})^3\right) - \left(4 \cdot 0 - \frac{2}{3} \cdot 0^3\right)$$

$$= 4\sqrt{2} - \frac{2}{3} \cdot 2 \cdot \sqrt{2} = 4\sqrt{2} - \frac{4}{3}\sqrt{2} = \underline{\frac{8}{3}\sqrt{2}}$$

$$A_1 = \underline{\frac{8}{3}\sqrt{2}}$$

(2) $$\int_{\sqrt{2}}^{2} f(x)\,dx = \int_{\sqrt{2}}^{2} (4 - 2x^2)\,dx = \left[4x - \frac{2}{3}x^3\right]_{\sqrt{2}}^{2}$$

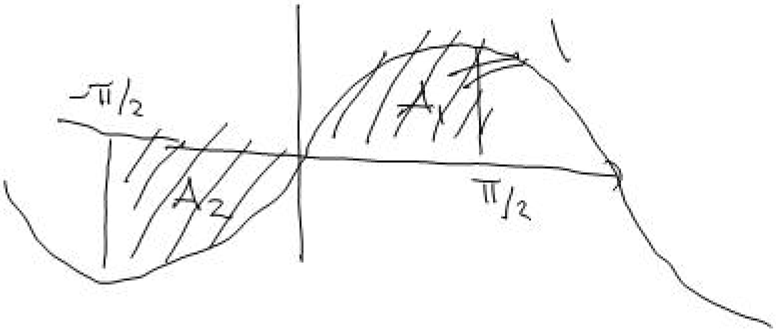
$$= \left(4 \cdot 2 - \frac{2}{3} \cdot 2^3\right) - \left(\frac{8}{3}\sqrt{2}\right)$$

$$= 8 - \frac{16}{3} - \frac{8}{3}\sqrt{2} = \frac{8}{3} - \frac{8}{3}\sqrt{2} = \frac{8}{3}(1 - \sqrt{2})$$

$$A_2 = \frac{8}{3}(\sqrt{2} - 1) = \frac{8}{3}\sqrt{2} - \frac{8}{3}$$

Eks: Finn arealet begrenset av $f(x) = \sin x$, x-aksen og linjen $x = -\pi/2$ og $x = \pi/2$.

$$\left.\begin{aligned} &\int_{-\pi/2}^{\pi/2} \sin x \, dx = \Big[-\cos x\Big]_{-\pi/2}^{\pi/2} \\ &= \left(-\cos \pi/2\right) - \left(-\cos(-\pi/2)\right) \\ &= -0 + 0 = 0 \end{aligned}\right\} \quad \underline{A_1 - A_2 = 0}$$



$$A = A_1 + A_2$$
$$= 2A_1$$
pga $\underline{\text{symmetri}}$.

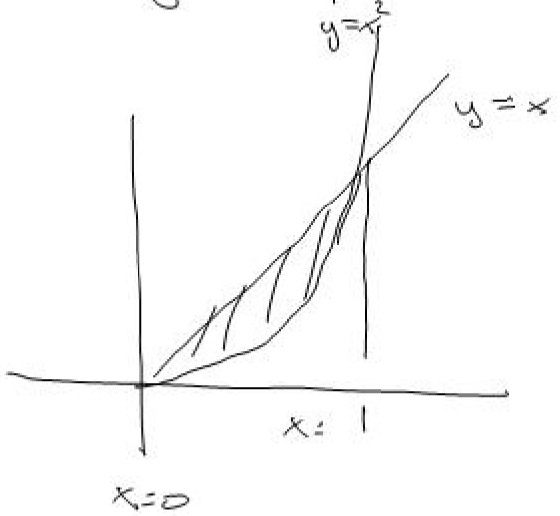
$$A_1 = \int_0^{\pi/2} \sin x \, dx = \Big[-\cos x\Big]_0^{\pi/2}$$

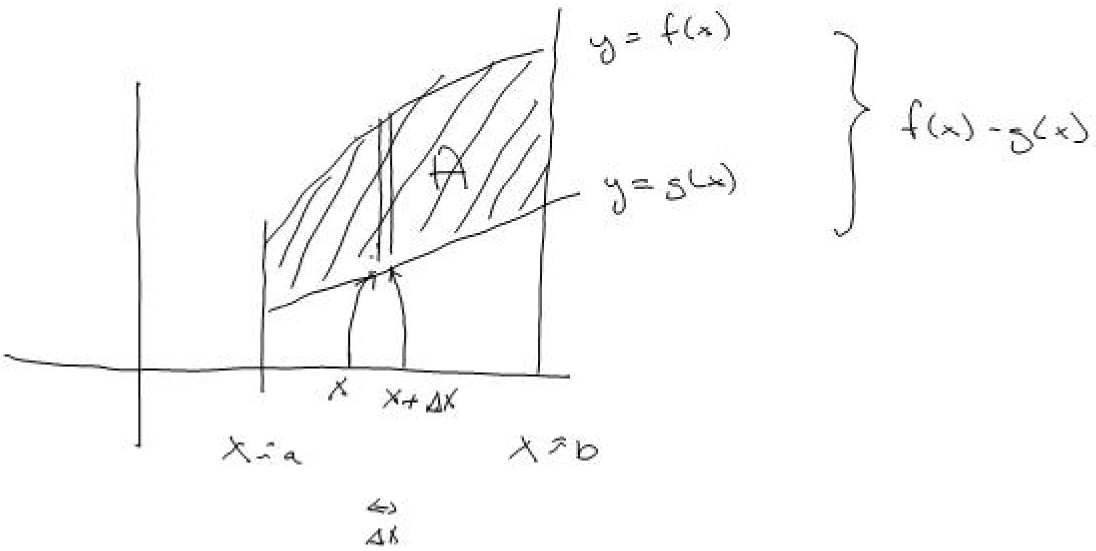
$$= \left(-\cos \frac{\pi}{2}\right) - \left(-\cos 0\right)$$

$$= -0 + 1 = \underline{1}$$

$$A = A_1 + A_2 = 2A_1 = \underline{\underline{2}}$$

Eks: Finn arealet begrenset av $y=x$, $y=x^2$, $x=0$ og $x=1$.





$$\Delta A = \left(f(x)-g(x)\right)\cdot \Delta x$$

$A = \Sigma \Delta A =$ summen av alle stripene fra $x=a$ til $x=b$.

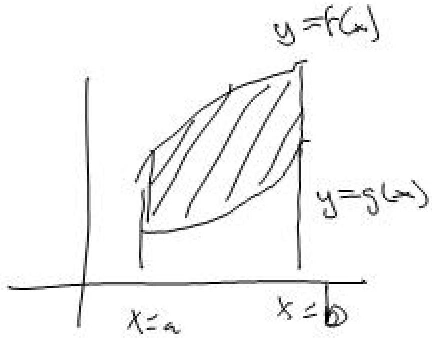
$$\Downarrow$$

$$A = \int_a^b \left(f(x)-g(x)\right)dx$$

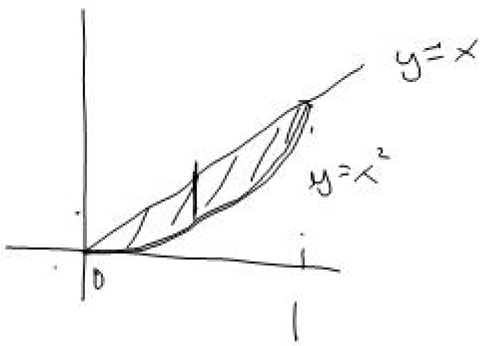
Konklusjon:

Hvis $f(x) \geq g(x)$ for alle $x \in [a,b]$
og f, g er kontinuerlige, så er
arealet begrenset av grafen til f,
grafen til g, $x=a$ og $x=b$
gitt ved

$$\boxed{A = \int_a^b \left(f(x) - g(x) \right) dx}$$



Eks:



$f(x) = x$
$g(x) = x^2$
$a=0 \quad b=1$

$$A = \int_0^1 \left(f(x) - g(x) \right) dx = \int_0^1 \left(x - x^2 \right) dx$$

$$= \left[\frac{1}{2}x^2 - \frac{1}{3}x^3 \right]_0^1 = \left(\frac{1}{2} \cdot 1^2 - \frac{1}{3} \cdot 1^3 \right) - 0$$

$$= \frac{1}{2} - \frac{1}{3} = \frac{3}{6} - \frac{2}{6} = \underline{\underline{\frac{1}{6}}}$$