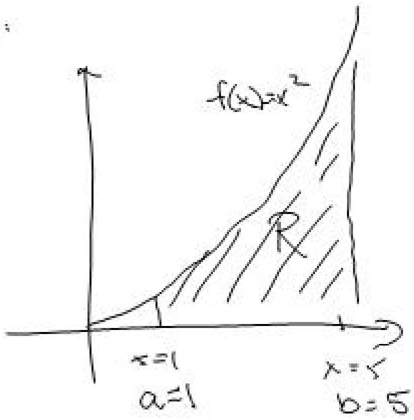
30/04/09:

Areal beregning
Bestemte integral } kap. 1.6

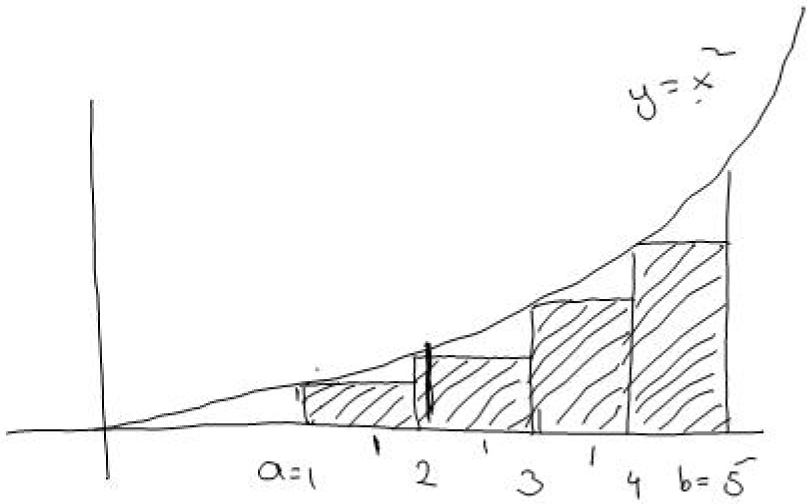
Areal beregning:

Eks:



R: området begrenset av grafen til f, x-aksen, linjene $x=1$ og $x=5$.

Arealet til R = ?

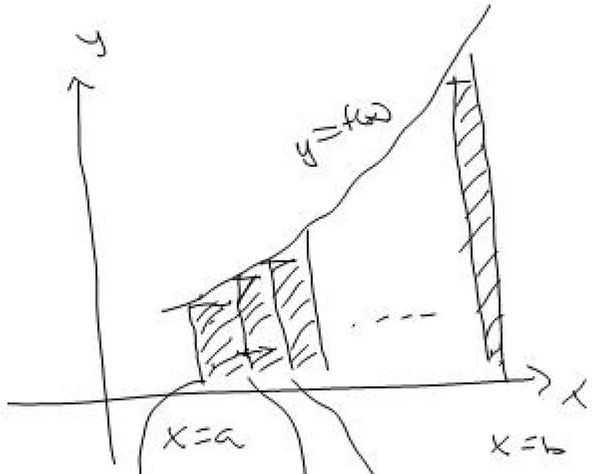


Skravert areal
$= 1\cdot1 + 1\cdot4 + 1\cdot9 + 1\cdot16$

$= 1+4+9+16 = \underline{30}$

Riemann-summer:

$$\Delta x = \frac{b-a}{n}$$



$x_0 = a$

$x_1 = a + \Delta x$

$x_2 = a + 2\Delta x$

$\vdots$

Riemann-sum:

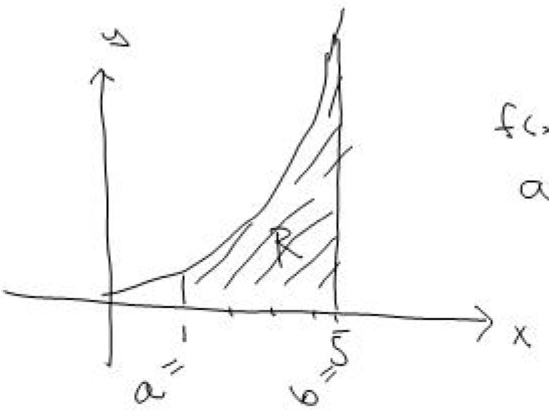
$$\Delta x \cdot f(x_0) + \Delta x \cdot f(x_1) + \Delta x \cdot f(x_2) + \dots + \Delta x \cdot f(x_{n-1})$$

Definisjon:

Areal = grenseverdien til Riemann-summen når $n \to \infty$.

Beregning av areal vha integral.

Eks:



$f(x) = x^2$

$a = 1 \quad b = 5$

Arealet av R

$$= \underbrace{\int_1^5 x^2 \, dx}_{} = \left[\frac{1}{3}x^3 + C \right]_1^5$$

bestemt integral med grenser a=1 og b=5.

$$= \left(\frac{1}{3} \cdot 5^3 + C\right) - \left(\frac{1}{3} \cdot 1^3 + C\right)$$

$$= \frac{1}{3} \cdot 125 + \cancel{C} - \frac{1}{3} \cdot 1 - \cancel{C}$$

$$= \frac{125}{3} - \frac{1}{3} = \frac{124}{3} = 41 + 1/3$$

$$\cong \underline{\underline{41,333}}$$

Bestemte integral:

$$\int_a^b f(x)\,dx = \left[F(x)\right]_a^b = F(b) - F(a)$$

der $F(x)$ er en antiderivert til $f(x)$.

Eks:

$$\int_0^{\pi/2} \cos x\,dx = \left[\sin x + C\right]_0^{\pi/2} = \left(\sin \pi/2 \cancel{+C}\right) - \left(\sin 0 \cancel{+C}\right)$$

$$= 1 - 0 = \underline{\underline{1}}$$

$$\int_0^{\ln 3} x e^x\,dx = ?$$

AtI:

$$\int x e^x\,dx = x e^x - \int e^x\,dx = \underline{x e^x - e^x + C}$$

$$\boxed{\begin{array}{ll} u = e^x & v = x \\ u' = e^x & v' = 1 \end{array}}$$

$$\int_0^{\ln 3} x e^x\,dx = \left[x e^x - e^x\right]_0^{\ln 3}$$

$$= \left(\ln 3 \cdot e^{\ln 3} - e^{\ln 3}\right) - \left(0 \cdot e^0 - e^0\right)$$

$$= \ln 3 \cdot 3 - 3) - (-1)$$

$$= \underline{\underline{3\ln 3 - 2}}$$

Alt. 2:

$$\int_0^{\ln 3} x e^x \, dx = \left[x \cdot e^x\right]_0^{\ln 3} - \int_0^{\ln 3} e^x \, dx$$

$$\boxed{\begin{array}{ll} u = e^x & v = x \\ u' = e^x & v' = 1 \end{array}}$$

$$= \left(\ln 3 \cdot e^{\ln 3} - 0 \cdot e^0\right) - \left[e^x\right]_0^{\ln 3}$$

$$= \ln 3 \cdot 3 - \left(e^{\ln 3} - e^0\right)$$

$$= 3\ln 3 - 3 + 1 = \underline{\underline{3\ln 3 - 2}}$$

Eks:

$$\int_0^4 2x \ln(x^2+1) \, dx = \int_1^{17} \cancel{2x} \cdot \ln(u) \cdot \frac{du}{\cancel{2x}}$$

$$\boxed{\begin{array}{l} u = x^2+1 \\ du = 2x \cdot dx \end{array}} \qquad = \int_1^{17} \ln(u) \, du$$

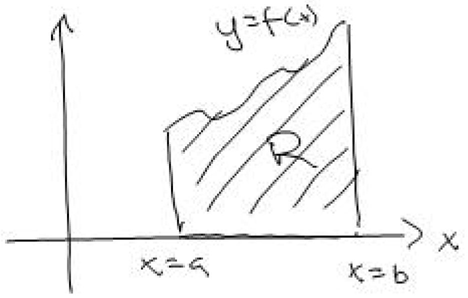
$x = 0 \qquad u = u(0) = 1$

$x = 4 \qquad u = u(4) = 17$

$$= \left[u \cdot \ln u - u\right]_1^{17} = \left(17 \ln(17) - 17\right) - \left(1 \cdot \ln 1 - 1\right)$$

$$= 17 \ln 17 - 17 + 1 = \underline{\underline{17 \ln 17 - 16}}$$

Arealberegning:

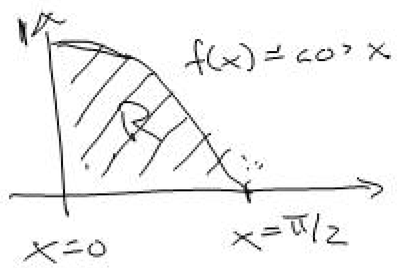


Anta $f(x)$ er en kontinuerlig funksjon på $[a,b]$ og $f(x) \geq 0$ for $x \in [a,b]$

Da er arealet av R (området begrenset av grafen til f, x-aksen og linjene $x=a$ og $x=b$) gitt ved

$$\boxed{A = \int_a^b f(x)\,dx}$$

Eks:

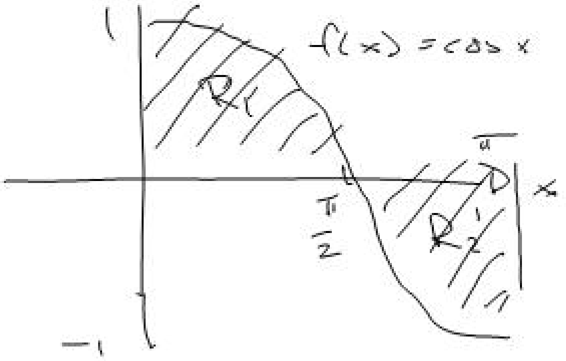


$$A = \int_0^{\pi/2} \cos x \, dx = \left[\sin x\right]_0^{\pi/2}$$

$$= \sin(\pi/2) - \sin(0) = 1 - 0 = \underline{\underline{1}}$$

Arealet av R er $\underline{\underline{1}}$.

Eks:



$$\int_0^{\pi} f(x)\,dx = \int_0^{\pi} \cos x\,dx = \left[\sin x\right]_0^{\pi}$$

$$= \sin \pi - \sin 0 = 0 - 0 = \underline{0}$$

$$\left.\begin{array}{l} \text{Areal } (R_1) = 1 \\ \text{Areal } (R_2) = 1 \end{array}\right\} \quad \text{Areal} = 2$$