

22/04/09: Delbrøksopp spaltning
Integrasjonen av brøkfunksjoner

Eks: Oppgave (a) $\int \frac{12}{x^2-4} dx$

(b) $\int \frac{12}{(x-2)^2} dx$

(c) $\int \frac{12}{x^2+4} dx$

Løsning (a): $x^2-4 = (x-2) \cdot (x+2)$

$$\frac{12}{x^2-4} = \frac{A}{x-2} + \frac{B}{x+2} \quad | \cdot (x-2)(x+2)$$

$$12 = A \cdot (x+2) + B \cdot (x-2)$$

$x = -2$: $12 = A \cdot 0 + B \cdot (-4)$
 $12 = -4B \Rightarrow B = \underline{-3}$

$x = 2$: $12 = A \cdot 4 + B \cdot 0$
 $12 = 4A \Rightarrow A = \underline{3}$

$$\int \frac{12}{x^2-4} dx = \int \left(\frac{3}{x-2} + \frac{-3}{x+2} \right) dx$$

$$= 3 \ln|x-2| - 3 \ln|x+2| + C = \underline{\underline{3 \ln \left| \frac{x-2}{x+2} \right| + C}}$$

$$(b) \int \frac{12}{(x-2)^2} dx = \int \frac{12}{u^2} du$$

$$\boxed{\begin{array}{l} u = x-2 \\ du = 1 \cdot dx \end{array}}$$

$$\begin{array}{l} du = u' \cdot dx \\ dx = \frac{du}{u'} \end{array}$$

$$= 12 \int u^{-2} du = 12 \cdot (-1) u^{-1} + C$$

$$= -12 \cdot \frac{1}{u} + C = \underline{\underline{\frac{-12}{x-2} + C}}$$

$$\boxed{\int x^n dx = \frac{1}{n+1} x^{n+1} + C}$$

$$\begin{array}{l} x = u \\ n = -2 \end{array}$$

Eks: $\int \frac{12x}{(x-2)^2} dx = ?$ Partialbruchspaltung.

$$\frac{12x}{(x-2)^2} = \frac{A}{x-2} + \frac{B}{(x-2)^2} \quad | \cdot (x-2)^2$$

$$12x = A \cdot (x-2) + B \quad (= 12(x-2) + 24 = 12x - 24 + 24 = 12x)$$

Metode 2: $x=2$: $12 \cdot 2 = A \cdot 0 + B$

$$24 = B \Rightarrow \underline{\underline{B=24}}$$

$x=0$: $12 \cdot 0 = A \cdot (-2) + B$

$$0 = -2A + 24 \Rightarrow \underline{\underline{A=12}}$$

$$\begin{aligned}
 1 \int \frac{12x}{(x-2)^2} dx &= \int \left(\frac{12}{x-2} + \frac{24}{(x-2)^2} \right) dx \\
 &= \underline{\underline{12 \ln|x-2| - \frac{24}{x-2} + C}}
 \end{aligned}$$

Husk:

$$\int \frac{C}{ax+b} dx = \frac{C}{a} \cdot \ln|ax+b| + C$$

$$\int \frac{1}{u} du = \ln|u| + C$$

$$\int \frac{1}{u^2} du = -\frac{1}{u} + C$$

Husk:

$$\left\{ \begin{aligned}
 \frac{2x+3}{(x+1)^2} &= \frac{A}{x+1} + \frac{B}{(x+1)^2} \\
 \frac{x^2}{x(x-1)^2} &= \frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2}
 \end{aligned} \right.$$

Eks:

$$\begin{aligned}
 \frac{12x}{(x-2)^2} &= \frac{(12x-24)+24}{(x-2)^2} = \frac{12x-24}{(x-2)^2} + \frac{24}{(x-2)^2} \\
 &= \frac{12\cancel{(x-2)}}{(x-2)^2} + \frac{24}{(x-2)^2} = \underline{\underline{\frac{12}{x-2} + \frac{24}{(x-2)^2}}}
 \end{aligned}$$

$$\int \frac{12x}{(x-2)^2} dx = \int \left(\frac{12}{x-2} + \frac{24}{(x-2)^2} \right) dx = \dots$$

Eks: (1) $\int \frac{12}{x^2+4} dx$

(2) $\int \frac{12x}{x^2+4} dx$

} x^2+4 kan ikke
faktoriseres

(2): $\boxed{u = x^2+4}$
 $du = 2x dx$

$$\int \frac{12x}{x^2+4} dx = \int \frac{12x^6}{u} \cdot \frac{du}{2x^1}$$

$$= \int \frac{6}{u} du = 6 \cdot \ln|u| + C$$
$$= \underline{\underline{6 \cdot \ln(x^2+4) + C}}$$

$du = u' \cdot dx$

Sjekk: $6 \cdot \frac{1}{x^2+4} \cdot (2x) = \frac{12x}{x^2+4}$

(1) $\int \frac{12}{x^2+4} dx =$ noe med arctan

$$= 12 \cdot \int \frac{1}{x^2+4} dx$$

$$(\arctan x)' = \frac{1}{x^2+1}$$