

21/04/09: Integrasjonsmetoder

- (a) Substitusjon (15.5) \leftrightarrow kjerneregul
- (b) Delvis integrasjon (15.6) \leftrightarrow produktregel
- (c) Delbrøloppspalting (15.7) \leftrightarrow brøtregel

Eks: $\int \left(\frac{1}{x} - \frac{1}{x+1} \right) dx = \ln|x| - \ln|x+1| + C$

$$\frac{1}{x} - \frac{1}{x+1} = \frac{(x+1) - x}{x \cdot (x+1)} = \frac{1}{x^2+x} \leftarrow x:(x+1)$$

$$\int \frac{1}{x^2+x} dx = \ln|x| - \ln|x+1| + C$$

Husk

$$\int \frac{c}{ax+b} = c \cdot \int \frac{1}{ax+b} dx = c \cdot \frac{1}{a} \ln|ax+b| + C$$

c konstant
 a, b konstant

$$= \frac{c}{a} \cdot \ln|ax+b| + C$$

$$\int \frac{3}{2x+1} dx = \frac{3}{2} \cdot \ln|2x+1| + C$$

Eks: $\int \frac{3}{x^2-3x+2} dx = \int \left(\frac{A}{x-1} + \frac{B}{x-2} \right) dx$

Faktorisering: $x^2-3x+2 = (x-1)(x-2)$

$$\frac{3}{(x-1)(x-2)} = \frac{A}{x-1} + \frac{B}{x-2} \quad | \cdot (x-1)(x-2)$$

$$3 = A \cdot (x-2) + B \cdot (x-1)$$

Metode 1:

$$3 = Ax - 2A + Bx - B$$

$$0x + 3 = (A+B)x + (-2A-B)$$

$$\begin{cases} 0 = A+B \\ 3 = -2A-B \end{cases} \Rightarrow B = -A$$

$$3 = -2A - (-A)$$

$$3 = -2A + A = -A$$

$$A = -3, B = 3$$

$$VS = \textcircled{3}$$

$$MS = -3(x-2) + 3(x-1)$$

$$= -\cancel{3x} + 6 + \cancel{3x} - 3$$

$$= \textcircled{3}$$

Metode 2:

Setter ein neues verdier an x:

$$\underline{x=2}: \quad 3 = A \cdot 0 + B \cdot 1$$

$$3 = B \Rightarrow \underline{B=3}$$

$$\underline{x=1}: \quad 3 = A \cdot (-1) + B \cdot 0$$

$$3 = -A \Rightarrow \underline{A=-3}$$

$$\int \frac{3}{x^2 - 3x + 2} dx = \int \left(\frac{-3}{x-1} + \frac{3}{x-2} \right) dx$$

$$= -3 \ln|x-1| + 3 \ln|x-2| + C = \ln|x-1|^{-3} + \ln|x-2|^3 + C$$

$$\underline{\underline{= \ln \frac{|x-2|^3}{|x-1|^3} + C}}$$

Evo: $\int \frac{3x}{x^2-3x+2} dx = \int \left(\frac{A}{x-1} + \frac{B}{x-2} \right) dx$

$$\frac{3x}{x^2-3x+2} = \frac{A}{x-1} + \frac{B}{x-2} \quad | \cdot (x-1)(x-2)$$

$$\boxed{3x = A \cdot (x-2) + B \cdot (x-1)}$$

Methode 2:

$$\begin{array}{l} \underline{x=2}: \quad 6 = A \cdot 0 + B \cdot 1 \\ \quad \quad \quad 6 = B \quad \quad \quad \Rightarrow B = 6 \\ \underline{x=1}: \quad 3 = A \cdot (-1) + B \cdot 0 \\ \quad \quad \quad 3 = -A \quad \quad \quad \Rightarrow A = -3 \end{array}$$

Stück:

$$\begin{aligned} VS &= (3x) \\ HS &= -3(x-2) + 6 \cdot (x-1) \\ &= -3x + 6 + 6x - 6 \\ &= (3x) \end{aligned}$$

$$\begin{aligned} \int \frac{3x}{x^2-3x+2} dx &= \int \left(\frac{-3}{x-1} + \frac{6}{x-2} \right) dx \\ &= \underline{-3 \ln|x-1| + 6 \ln|x-2|} + C = \ln \frac{|x-2|^6}{|x-1|^3} + C \end{aligned}$$

Ans: $\int \frac{3x^2}{x^2-3x+2} dx = \int \left(3 + \frac{9x-6}{(x-1)(x-2)} \right) dx$

Polynomdivision: $3x^2 : (x^2-3x+2) = 3$
 $-(3x^2-9x+6)$
 $9x-6$

$\Rightarrow \frac{3x^2}{x^2-3x+2} = 3 + \frac{9x-6}{x^2-3x+2}$

$= 3x + \int \frac{9x-6}{(x-1)(x-2)} dx = 3x + \int \left(\frac{-3}{x-1} + \frac{12}{x-2} \right) dx$

$\frac{9x-6}{(x-1)(x-2)} = \frac{A}{x-1} + \frac{B}{x-2} \quad | \cdot (x-1)(x-2)$

$9x-6 = A \cdot (x-2) + B \cdot (x-1)$

Metode 2: $x=2: \quad 18-6 = A \cdot 0 + B \cdot 1$
 $12 = B \quad \Rightarrow B=12$

$x=1: \quad 9-6 = A \cdot (-1) + B \cdot 0$
 $3 = -A \quad \Rightarrow A=-3$

$= 3x - 3 \cdot \ln|x-1| + 12 \cdot \ln|x-2| + C$

Evo: $\int \frac{x^2}{x(x-1)(x+1)} dx = \int \left(\frac{0}{x} + \frac{1/2}{x-1} + \frac{1/2}{x+1} \right) dx$

Partialbruchspaltung:

$$\frac{x^2}{x(x-1)(x+1)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1} \quad | \cdot x(x-1)(x+1)$$

$$x^2 = A(x-1)(x+1) + B \cdot x(x+1) + C \cdot x(x-1)$$

Methode 2: $x=0: \quad 0 = -A \quad \Rightarrow \quad \underline{A=0}$

$x=1: \quad 1 = B \cdot 2 \quad \Rightarrow \quad \underline{B=1/2}$

$x=-1: \quad 1 = C \cdot 2 \quad \Rightarrow \quad \underline{C=1/2}$

$$= \underline{\underline{\frac{1}{2} \ln|x-1| + \frac{1}{2} \ln|x+1| + C}}$$

Varshelij: $\text{Hins neuener er} \begin{cases} (x-1)^2 \\ x^2+1 \end{cases}$

Oppsummering: Delbrøks oppspaltning.

- ① Hvis graden til teller \geq graden til nevner bruker vi først polynomdivisjon
- ② Hvis $\frac{P(x)}{Q(x)}$ med grad til $Q >$ graden til P :
 - (a) Faktorerer nevneren $Q(x)$
 - (b) Setter på en oppspaltning av $\frac{P(x)}{Q(x)}$
 - (c) Regner ut konstantene via metode (1) eller (2).
 - (d) Integrerer.

Denne metoden fungerer alltid hvis $Q(x)$ kan faktorerises i forskjellige lineære faktorer.