16/04/09:

Repetisjon : Ubestemte integral (kap. 15.1-15.5)

Regneregler:

(1) $\int (u \pm v)\,dx = \int u\,dx \pm \int v\,dx$

(2) $\int c \cdot u\,dx = c \cdot \int u\,dx$ (c konstant)

(3) $\int x^n\,dx = \frac{1}{n+1} x^{n+1} + C$ $(n \neq -1)$

(4) $\int 1/x\,dx = \ln|x| + C$

(5) $\int e^x\,dx = e^x + C$

(6) $\int \sin x\,dx = -\cos x + C$

(7) $\int \cos x\,dx = \sin x + C$

Metode:

(a) Substitusjon: $\boxed{du = u' \cdot dx}$

Eksempel:

$$\boxed{\begin{array}{l} u = x^2 \\ du = 2x \cdot dx \end{array}}$$

$$\int x \cdot \sin(x^2)\,dx$$

$$= \int \cancel{x} \cdot \sin(u) \cdot \frac{du}{2\cancel{x}} = \int \frac{1}{2} \cdot \sin(u)\,du$$

$$= \frac{1}{2} \cdot (-\cos u) + C = \underline{\underline{-\frac{1}{2} \cdot \cos(x^2) + C}}$$

(b) Delvis integrasjon (kap. 15.6)

Eks: $\int x \cdot \sin x \, dx = ?$

Produktregel for derivasjon:

$$(u \cdot v)' = u' \cdot v + u \cdot v'$$

Integrasjon:

$$\int (u' \cdot v + u \cdot v') \, dx = u \cdot v + C$$

$$\int u' \cdot v \, dx + \int u \cdot v' \, dx = u \cdot v + C$$

Delvis integrasjon: $$\boxed{\int u' \cdot v \, dx = u \cdot v - \int u \cdot v' \, dx}$$

Eks: $$\int \underbrace{x}_{u'} \cdot \underbrace{\sin x}_{v} \, dx = \frac{1}{2}x^2 \cdot \sin x - \int \frac{1}{2}x^2 \cdot \cos x \, dx$$

$u = \frac{1}{2}x^2$	$v = \sin x$
$u' = x$	$v' = \cos x$

$$\int x \cdot \sin x \, dx = \int \underbrace{\sin x}_{u'} \cdot \underbrace{x}_{v} \, dx = u \cdot v - \int u \cdot v' dx$$

$u = -\cos x$	$v = x$
$u' = \sin x$	$v' = 1$

$$= (-\cos x) \cdot x - \int (-\cos x) \cdot 1 \, dx$$

$$= -x \cos x + \int \cos x \, dx$$

$$= \underline{\underline{-x \cdot \cos x + \sin x + C}}$$

Eks: $\int x \cdot e^x \, dx = \int u' \cdot v = u \cdot v - \int u \cdot v' \, dx$

$$\boxed{\begin{array}{ll} u = e^x & v = x \\ u' = e^x & v' = 1 \end{array}}$$

$$= e^x \cdot x - \int e^x \cdot 1 \, dx$$

$$= x \cdot e^x - \int e^x \, dx$$

$$= \underline{\underline{x \cdot e^x - e^x + C}}$$

$$(x \cdot e^x - e^x + C)' = (x \cdot e^x)' - e^x + 0$$

$$= \cancel{1 \cdot e^x} + x \cdot e^x \cancel{- e^x} = \underline{x \cdot e^x}$$

Eks: $\int x \cdot \ln x \, dx = \boxed{\int u' \cdot v \, dx = u \cdot v - \int u \cdot v' \, dx}$

$$\boxed{\begin{array}{ll} u = \frac{1}{2}x^2 & v = \ln x \\ u' = x & v' = 1/x \end{array}}$$

$$= \frac{1}{2}x^2 \cdot \ln x - \int \frac{1}{2}x^2 \cdot \frac{1}{x} \, dx$$

$$= \frac{1}{2}x^2 \ln x - \frac{1}{2} \cdot \left(\frac{1}{2}x^2\right) + C$$

$$= \underline{\underline{\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C}}$$

Eks: $\int \ln x \, dx = \int \underset{u'}{1} \cdot \underset{v}{\ln x} \, dx = \boxed{\int u' \cdot v \, dx = u \cdot v - \int u \cdot v' \, dx}$

$$\boxed{\begin{array}{ll} u = x & v = \ln x \\ u' = 1 & v' = 1/x \end{array}}$$

$$= x \cdot \ln x - \int \cancel{x} \cdot \frac{1}{\cancel{x}} \, dx$$

$$= \underline{\underline{x \cdot \ln x - x + C}}$$

Eks: (litt vanskelig)

$$\int \overset{v}{x^2}\cdot \overset{u'}{\cos x}\,dx = \int u'\cdot v = u\cdot v - \int u\cdot v'\,dx$$

$$\boxed{\begin{array}{ll} u = \sin x & v = x^2 \\ u' = \cos x & v' = 2x \end{array}}$$

$$= \sin x \cdot x^2 - \int \sin x \cdot 2x\,dx$$

$$= x^2\cdot \sin x - 2\cdot \int x\cdot \sin x\,dx$$

$$= x^2\cdot \sin x - 2\left(-x\cos x + \sin x\right) + C$$

$$= \underline{\underline{x^2\cdot \sin x + 2x\cos x - 2\sin x + C}}$$

$$\int x\cdot \sin x\,dx = -\cos x\cdot x - \int (-\cos x)\cdot 1\,dx$$

$$\boxed{\begin{array}{ll} u = -\cos x & v = x \\ u' = \sin x & v' = 1 \end{array}}$$

$$= -x\cos x + \int \cos x\,dx$$

$$= \underline{-x\cos x + \sin x + C}$$