Substitusjon (kap. 15.5)

Generell metode:

$u = u(x)$ velges

$du = u' \cdot dx$ regnes ut

} alt settes inn i integralet

① Lette eksempler

(u' = konstant)

Eks.

$$\int \frac{1}{1-x}\,dx = \int \frac{1}{u}\,\frac{du}{-1}$$

$u = 1 - x$

$du = -1 \cdot dx$

$dx = \frac{du}{-1}$

$$= -1 \int \frac{1}{u}\,du$$

$$= -1 \cdot \ln|u| + C$$

$$= -\ln|1-x| + C$$

$$\int \cos(\pi x - \pi)\,dx = \int \cos u \cdot \frac{du}{\pi}$$

$u = \pi x - \pi$

$du = \pi \cdot dx$

$dx = \frac{du}{\pi}$

$$= \frac{1}{\pi} \int \cos u\,du = \frac{1}{\pi} \sin u + C$$

$$= \frac{1}{\pi} \sin(\pi x - \pi) + C$$

(2) Litt vanskeligere eksempler

$x = \pm\sqrt{u-1}$

$x^2 = u - 1$

Eks: $\int \frac{x}{x^2+1}\,dx$

$u = x^2+1$

$du = 2x \cdot dx$

$dx = \frac{du}{2x}$

$= \int \frac{x}{u} \cdot \frac{du}{2x}$

$= \int \frac{1}{2u}\,du = \frac{1}{2}\ln|u| + C = \underline{\frac{1}{2}\ln(x^2+1) + C}$

$x = \sqrt[3]{u-1}$

Eks: $\int \frac{x}{x^3+1}\,dx =$

$u = x^3+1$

$du = 3x^2 \cdot dx$

$= \int \frac{x}{u} \cdot \frac{du}{3x^2}$

$= \int \frac{1}{3u \cdot x}\,du = \int \frac{1}{3u \cdot \sqrt[3]{u-1}}\,du$

går ikke med denne metoden

Eks: $\int \frac{1}{x^2+1}\,dx = \underline{\arctan(x) + C}$

(vi kommer tilbake til dette)

$$\int \tan x \, dx = \int \frac{\sin x}{\cos x} \, dx$$

$$= \int \frac{\sin x}{u} \cdot \frac{du}{-\sin x} = -1 \int \frac{1}{u} \, du$$

$$= -\ln|u| + C = -\ln|\cos x| + C$$

$u = \cos x$
$u' = -\sin x$
$du = u' \cdot dx$
$= -\sin x \cdot dx$

$dx = \frac{du}{-\sin x}$

$$\int \tan x \, dx = \int \frac{\sin x}{\cos x} \, dx$$

$$= \int \frac{u}{\cos x} \cdot \frac{du}{\cos x} = \int \frac{u}{\cos^2 x} \, dx$$

$$= \int \frac{u}{1-u^2} \, du$$

$$= \int \frac{u}{v} \cdot \frac{dv}{-2u} = -\frac{1}{2} \int \frac{1}{v} \, dv$$

$$= -\frac{1}{2} \ln|v| + C$$

$$= -\frac{1}{2} \ln|1-u^2| + C$$

$$= -\frac{1}{2} \ln|1-\sin^2 x| + C$$

$$= -\frac{1}{2} \ln|\cos^2 x| + C = -\ln|\cos x| + C$$

$u = \sin x$
$u' = \cos x$
$du = \cos x \, dx$

$\cos^2 x = 1 - \sin^2 x$
$= 1 - u^2$

$v = 1 - u^2$
$dv = -2u \cdot du$

(3) Vanskelige eksempler

Eks: $\int \frac{x}{\sqrt{x-1}}\,dx$

$x = u^2+1$

$u = \sqrt{x-1}$

$u' = \frac{1}{2\sqrt{x-1}}$

$du = \frac{1}{2\sqrt{x-1}}\,dx$

$dx = 2\sqrt{x-1}\,du$

$= \int \frac{x}{u} \cdot 2\sqrt{x-1}\,du$ ($\sqrt{x-1} = u$)

$= \int 2x\,du = \int 2(u^2+1)\,du$

$= 2\left(\frac{1}{3}u^3 + u\right) + C$

$= \frac{2}{3}\left(\sqrt{x-1}\right)^3 + 2\sqrt{x-1} + C$

$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C$

Eks: $\int \frac{x}{\sqrt{x-1}}\,dx = \int \frac{x}{\sqrt{u}} \cdot du$

$x = u+1$

$u = x-1$

$u' = 1$

$du = dx$

$= \int \frac{u+1}{\sqrt{u}}\,du = \int \left(\frac{u}{\sqrt{u}} + \frac{1}{\sqrt{u}}\right) du$

$= \int u^{1/2} + u^{-1/2}\,du = \frac{1}{3/2}u^{3/2} + \frac{1}{1/2}u^{1/2} + C$

$= \frac{2}{3}(x-1)^{3/2} + 2(x-1)^{1/2} + C$